

Erweiterungskörper $GF(2^3)/M(x) = x^3 + x^2 + 1$

Elemente des $GF(2^3)$	Polynomreste $\alpha^i \bmod M(x = \alpha)$	Koeffizienten der Polynomreste	Minimalpolynome $m_i(x)$
Nullelement	0	000	
α^0	1	001	$m_0(x) = x + 1$
α^1	α	010	$m_1(x) = x^3 + x^2 + 1$
α^2	α^2	100	$m_2(x) = x^3 + x^2 + 1$
α^3	$\alpha^2 + 1$	101	$m_3(x) = x^3 + x + 1$
α^4	$\alpha^2 + \alpha + 1$	111	$m_4(x) = x^3 + x^2 + 1$
α^5	$\alpha + 1$	011	$m_5(x) = x^3 + x + 1$
α^6	$\alpha^2 + \alpha$	110	$m_6(x) = x^3 + x + 1$
α^7	1	001	

Zyklen der Elemente eines Erweiterungskörpers:

$$\alpha^{2^{j-1}i \bmod p} \quad (j = 1, 2, \dots, k_1 (= \text{grad } M(x)))$$

$$= \alpha^i, \alpha^{2i}, \alpha^{4i \bmod p}, \dots$$

$GF(2^3)$:

$$\alpha^0$$

$$\alpha^1, \alpha^2, \alpha^4$$

$$\alpha^3, \alpha^6, \alpha^{12 \bmod 7} = \alpha^5$$

Entsprechend gilt:

$$m_0(x)$$

$$m_1(x) = m_2(x) = m_4(x) = M_1(x)$$

$$m_3(x) = m_6(x) = m_5(x)$$

Erweiterungskörper $GF(2^4)/M(x) = x^4 + x + 1$

Elemente des $GF(2^4)$		Minimalpolynome $m_i(x)$
α^i	Polynomreste	
0	0	0 0 0 0
1	1	0 0 0 1
α^1	α	0 0 1 0
α^2	α^2	0 1 0 0
α^3	α^3	1 0 0 0
α^4	$\alpha + 1$	0 0 1 1
α^5	$\alpha^2 + \alpha$	0 1 1 0
α^6	$\alpha^3 + \alpha^2$	1 1 0 0
α^7	$\alpha^3 + \alpha + 1$	1 0 1 1
α^8	$\alpha^2 + 1$	0 1 0 1
α^9	$\alpha^3 + \alpha$	1 0 1 0
α^{10}	$\alpha^2 + \alpha + 1$	0 1 1 1
α^{11}	$\alpha^3 + \alpha^2 + \alpha$	1 1 1 0
α^{12}	$\alpha^3 + \alpha^2 + \alpha + 1$	1 1 1 1
α^{13}	$\alpha^3 + \alpha^2 + 1$	1 1 0 1
α^{14}	$\alpha^3 + 1$	1 0 0 1

$$\begin{array}{ll}
 \alpha^0 & m_0(x) \\
 \alpha^1, \alpha^2, \alpha^4, \alpha^8 & m_1(x) = m_2(x) = m_4(x) = m_8(x) \\
 \alpha^3, \alpha^6, \alpha^{12}, \alpha^{24 \bmod 15} = \alpha^9 & m_3(x) = m_6(x) = m_{12}(x) = m_9(x) \\
 \alpha^5, \alpha^{10} & m_5(x) = m_{10}(x) \\
 \alpha^7, \alpha^{14}, \alpha^{13}, \alpha^{11} & m_7(x) = m_{14}(x) = m_{13}(x) = m_{11}(x)
 \end{array}$$

$$g(x) = x^3 + x^2 + 1 :$$

Multiplikationsverfahren

i	a_i^*	a_i
0	0000	00000000
1	0001	0001101
2	0010	0011010
3	0011	0010111
4	0100	0110100
5	0101	0111001
6	0110	0101110
7	0111	0100011

i	a_i^*	a_i
8	1000	1101000
9	1001	1100101
10	1010	1110010
11	1011	1111111
12	1100	1011100
13	1101	1010001
14	1110	1000110
15	1111	1001011

Divisionsverfahren

i	a_i^*	a_i
0	0000	00000000
1	0001	0001101
2	0010	0010111
3	0011	0011010
4	0100	0100011
5	0101	0101110
6	0110	0110100
7	0111	0111001

i	a_i^*	a_i
8	1000	1000110
9	1001	1001011
10	1010	1010001
11	1011	1011100
12	1100	1100101
13	1101	1101000
14	1110	1110010
15	1111	1111111

$$g(x) = x^3 + x + 1 :$$

Multiplikationsverfahren

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0	0000	00000000
1	0001	0001011
2	0010	0010110
3	0011	0011101
4	0100	0101100
5	0101	0100111
6	0110	0111010
7	0111	0110001

i	a_i^*	a_i
8	1000	1011000
9	1001	1010011
10	1010	1001110
11	1011	1000101
12	1100	1110100
13	1101	1111111
14	1110	1100010
15	1111	1101001

Divisionsverfahren

i	a_i^*	a_i
0	0000	00000000
1	0001	0001011
2	0010	0010110
3	0011	0011101
4	0100	0100111
5	0101	0101100
6	0110	0110001
7	0111	0111010

i	a_i^*	a_i
8	1000	1000101
9	1001	1001110
10	1010	1010011
11	1011	1011000
12	1100	1100010
13	1101	1101001
14	1110	1110100
15	1111	1111111

$$\begin{aligned} \text{Zyklen } k_1 = 3: & \alpha^0 \\ & \alpha^1, \alpha^2, \alpha^4 \\ & \alpha^3, \alpha^6, \alpha^5 \end{aligned}$$

$$\begin{aligned} \text{Zyklen } k_1 = 4: & \alpha^0 \\ & \alpha^1, \alpha^2, \alpha^4, \alpha^8 \\ & \alpha^3, \alpha^6, \alpha^{12}, \alpha^9 \\ & \alpha^5, \alpha^{10} \\ & \alpha^7, \alpha^{14}, \alpha^{13}, \alpha^{11} \end{aligned}$$

$$\begin{aligned} \text{Zyklen } k_1 = 5: & \alpha^0 \\ & \alpha^1, \alpha^2, \alpha^4, \alpha^8, \alpha^{16} \\ & \alpha^3, \alpha^6, \alpha^{12}, \alpha^{24}, \alpha^{17} \\ & \alpha^5, \alpha^{10}, \alpha^{20}, \alpha^9, \alpha^{18} \\ & \alpha^7, \alpha^{14}, \alpha^{28}, \alpha^{25}, \alpha^{19} \\ & \alpha^{11}, \alpha^{22}, \alpha^{13}, \alpha^{26}, \alpha^{21} \\ & \alpha^{15}, \alpha^{30}, \alpha^{29}, \alpha^{27}, \alpha^{23} \end{aligned}$$

$$\begin{aligned} \text{Zyklen } k_1 = 6: & \alpha^0 \\ & \alpha^1, \alpha^2, \alpha^4, \alpha^8, \alpha^{16}, \alpha^{32} \\ & \alpha^3, \alpha^6, \alpha^{12}, \alpha^{24}, \alpha^{48}, \alpha^{33} \\ & \alpha^5, \alpha^{10}, \alpha^{20}, \alpha^{40}, \alpha^{17}, \alpha^{34} \\ & \alpha^7, \alpha^{14}, \alpha^{28}, \alpha^{56}, \alpha^{49}, \alpha^{35} \\ & \alpha^9, \alpha^{18}, \alpha^{36} \end{aligned}$$

$$\begin{aligned} & \vdots \\ \text{Zyklen } k_1 = 9: & \alpha^0 \\ & \alpha^1, \alpha^2, \alpha^4, \alpha^8, \alpha^{16}, \alpha^{32}, \alpha^{64}, \alpha^{128}, \alpha^{256} \\ & \alpha^3, \alpha^6, \alpha^{12}, \alpha^{24}, \alpha^{48}, \alpha^{96}, \alpha^{192}, \alpha^{384}, \alpha^{257} \\ & \alpha^5, \alpha^{10}, \alpha^{20}, \alpha^{40}, \alpha^{80}, \alpha^{160}, \alpha^{320}, \alpha^{129}, \alpha^{258} \\ & \alpha^7, \alpha^{14}, \alpha^{28}, \alpha^{56}, \alpha^{112}, \alpha^{224}, \alpha^{448}, \alpha^{385}, \alpha^{259} \\ & \vdots \end{aligned}$$