Exercise 8.1 Clearly, the $\mathcal{ELH}$ concept $(\exists r. A) \sqcap (\exists r. B)$ subsumes $\exists r. (A \sqcap B)$. Using the similarity measure $\sigma_d$ from the lecture, we can generalize this fact for the concepts

- $C_1 = (\exists r_1. A_1) \sqcap (\exists r_1. B_1)$
- $C_2 = \exists r_2. (A_2 \sqcap B_2)$

which are already expanded w.r.t. their respective ontologies $O_1$ and $O_2$. Let the weighting factor $\beta$ be 0.6 and the primitive measures $\sigma_a'$ and $\sigma_b'$ be defined as follows:

<table>
<thead>
<tr>
<th></th>
<th>$(A_1, A_2)$</th>
<th>$(B_1, B_2)$</th>
<th>$(A_1, B_2)$</th>
<th>$(B_1, A_2)$</th>
<th>$(r_1, r_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_a'$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_b'$</td>
<td>0.9</td>
<td>0.6</td>
<td>0.4</td>
<td>0.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Compute the directed similarities $\sigma_d(C_1, C_2)$ and $\sigma_d(C_2, C_1)$ using…

(a) …the primitive measure $\sigma_a'$. What does this measure essentially encode?

(b) …the primitive measure $\sigma_b'$.

Exercise 8.2 Given the ontology $O = (\emptyset, T, R)$ with abbreviations for the axioms as follows:

$T = \{ \begin{align*} C \sqcap \exists s. E & \sqsubseteq F \sqcap D_1, \quad & T_1 \\ C & \sqsubseteq A \sqcap B, \quad & T_2 \\ D & \equiv D_1 \sqcap \exists r. A_1 \sqcap \exists r. A_2, \quad & T_3 \\ A & \sqsubseteq D_1 \sqcap \exists s.(A_1 \sqcap A_2), \quad & T_4 \\ B & \sqsubseteq D, \quad & T_5 \\ F & \sqsubseteq \exists r.(A_1 \sqcap A_2) \end{align*} \right\}$ $R = \{ s \sqsubseteq r, \quad & R_1 \\ \text{ran}(s) = E \}$ $R_2$

(a) Compute all justifications for the consequence

$$C \sqcap \exists s. T \sqsubseteq D \text{ w.r.t. } O.$$

(b) Use the Hitting Set Tree (HST) algorithm to compute all diagnoses.

Exercise 8.3 Prove that $\text{Just}_O(\alpha) = \text{MHS}_O(\text{Diag}_O(\alpha))$. 