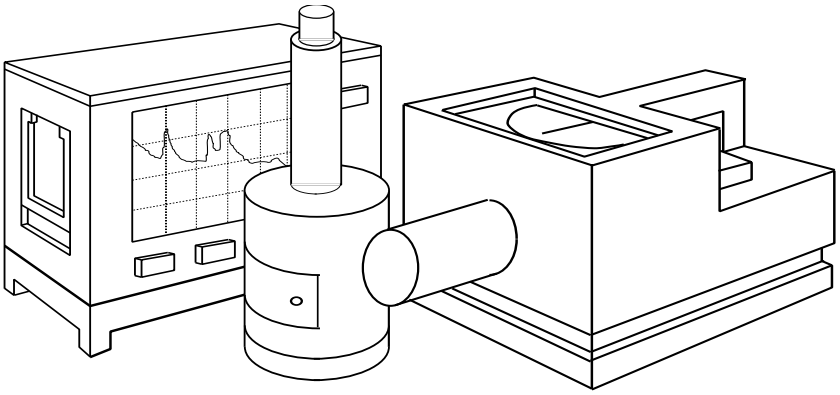
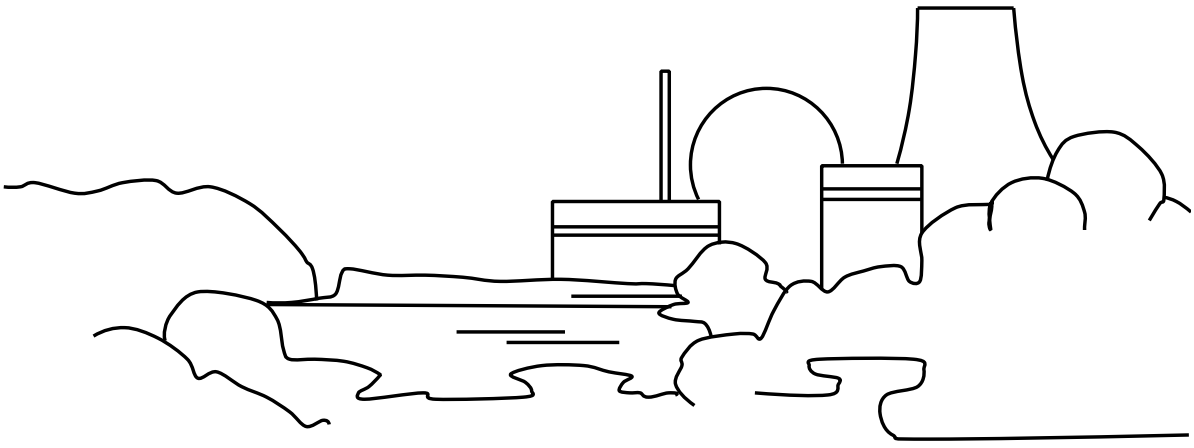
TECHNICAL UNIVERSITY DRESDEN
Institute of Power Engineering
Training Reactor



Reactor Training Course

Experiment

"Control Rod Calibration"



Instruction for
Experiment "Control Rod Calibration"

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(issued: March 2015)

1. Motivation

For the safe operation of a nuclear reactor it is of high importance to know:
- the reactivity characteristics of all control units (mainly the control rods),
- the maximum available positive reactivity reserve (excess reactivity), and
- the shut-down reactivity.

By calibrating a control rod, the influence on the reactivity by inserting a neutron absorber into the reactor core will be investigated quantitatively and the theoretical relation with other quantities of reactor physics will be illustrated.

2. Tasks

1. A control rod of the reactor AKR-2 has to be calibrated in dependence on the rod position by the **method of stable positive reactor period** measurement. Based on measured doubling times of the reactor power the corresponding stable reactor periods and, furthermore, the reactivity equivalents according to the INHOUR equation have to be calculated.
   If the obtained positive reactivity change is compensated by movement of another control rod such that the reactor returns to criticality, it is possible to calibrate a second control rod by the **compensation method** at the same time.

2. Graphical representations of
   - the **differential** control rod characteristics, as well as
   - the **integral** control rod characteristics
   both with respect to the rod position have to be prepared for two control rods of the AKR-2.

3. Including the reactivity characteristic of the third control rod of the AKR-2, which is available at the facility:
   - the **excess reactivity**, and
   - the **shut-down reactivity** (for partial shut-down)
   have to be calculated and discussed.

3. Introduction

The characteristic reactivity values of the control rods (and of all other control units) are essential reactor-physical parameters of a nuclear reactor. They need to be known on account of nuclear safety. Consequently, the reactivity efficiencies of the control rods need to be quantified in the cases of first criticality of a reactor or first criticality after substantial changes in the reactor core (materials and/or geometry).

If the reactivity characteristics of the control rods are known, it is also possible to determine the excess reactivity of a reactor using the positions of its calibrated control rods in the critical condition. Furthermore, the control rod characteristics can be used for easily quantifying the changes in reactivity, which may be induced by various reasons.

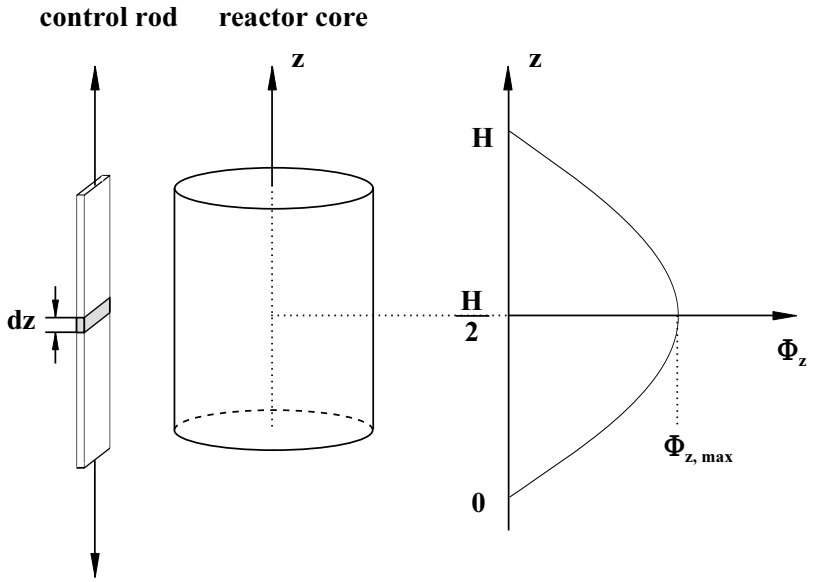

Fig. 1, Cylindrical reactor core with control rod and axial neutron flux density distribution Φ_z

The reactivity characteristic of any control rod can also be determined by theoretical considerations. But the determination of the rod reactivities by calculation is a rather difficult problem, which can be solved with reasonable effort only by applying various approximations. Anyways, calculated reactivity values need to be proven by experiments.

For illustration, consider the arrangement shown in Fig. 1. A neutron absorber moves vertically parallel to the z-axis of the cylindrical core of a thermal reactor. Considering the point of origin to be in the middle plane of the reactor core, the axial component of the neutron flux density can be written as

$$\Phi_z = \Phi_{z,\,max} \cdot \sin\left(\frac{\pi \cdot z}{H}\right) \qquad \textbf{(1)}$$

The parameter H represents the height of the core. By moving a neutron absorber into the core, the number of free neutrons decreases, i.e. the reactivity is lowered. The decrease in reactivity which is caused by a differential rod slice dz being located at the position z is the larger,
- the larger the macroscopic absorption cross section Σ_a,
- the larger the neutron flux density Φ_z at the position z, and
- the longer the slice dz is.

Moreover, the influence of losing free neutrons across the surface of the reactor (leakage) needs to be considered. Those neutrons originating in the center of the reactor core have the maximum probability to remain in the core and to cause fission reactions. On the other hand, the probability for neutron leakage has its maximum in the region of low neutron density near the surface of the core. Consequently, the various neutrons contribute unequally to the neutron balance and hence, they have an unequal influence on the reactivity. Thus, this function of influence, also called **adjoint flux function**, varies in space approximately in same way as the neutron flux density, i.e. both functions are proportional to each other, and the magnitude of reactivity loss

is determined by two effects
- absorption ($\sim \Sigma_a \cdot \Phi_z \cdot dz$)
- space-dependent adjoint flux ($\sim \Phi_z$)

According to probability theory the total probability is the product of the particular contributions. Therefore, the total effect can be written as

$$d\rho \sim \Sigma_a \cdot \Phi_z^2 \cdot dz \qquad \textbf{(2)}$$

By introducing a constant of proportionality C, the equation for the **differential control rod characteristic** results in

$$\frac{d\rho}{dz} = C \cdot \Sigma_a \cdot \phi_z^2 \qquad \textbf{(3)}$$

A control rod which is moved into the reactor core by a finite length $\Delta z = z2 - z1$ changes the reactivity by

$$\Delta\rho = \int_{z_1}^{z_2} d\rho = \int_{z_1}^{z_2} C \cdot \Sigma_a \cdot \phi_z^2 \cdot dz. \qquad \textbf{(4)}$$

Integration over the whole rod length up to the position z gives the **integral control rod characteristic** $\rho(z)$

$$\rho(z) = \int_0^z C \cdot \Sigma_a \cdot \Phi_z^2 \cdot dz. \qquad \textbf{(5)}$$

Applying the neutron flux distribution shown in Fig. 1 into equation (5) gives

$$\rho(z) = C \cdot \Sigma_a \cdot \Phi_{z,\,\max}^2 \cdot \frac{H}{\pi} \left[\frac{\pi \cdot z}{2 \cdot H} - \frac{1}{4} \sin\left(\frac{2 \cdot \pi \cdot z}{H} \right) \right] \qquad \textbf{(6)}$$

and integration up to the full length 0 ... H gives the integral reactivity value of the complete rod

$$\rho_{max} = C \cdot \Sigma_a \cdot \Phi_{z,\,\max}^2 \cdot \frac{H}{2} \qquad \textbf{(7)}$$

After normalising the reactivity to the maximum value ρ_{max}, the integral control rod characteristic gets the shape which is shown in Fig. 2.

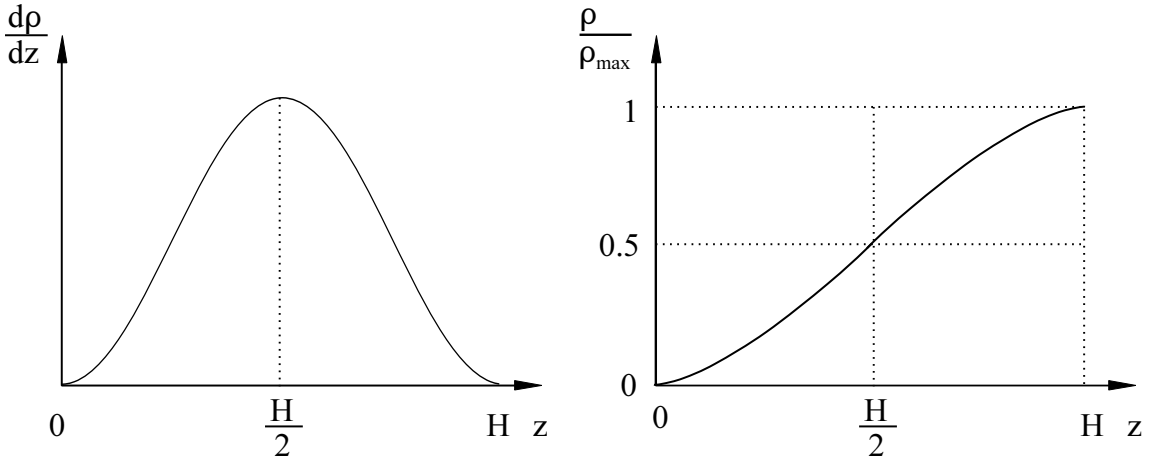

Fig. 2, Control rod characteristics, differential (left) and integral (right, normalised)

The control rods are made of materials having a large macroscopic absorption cross section Σ_a. For practical applications in nuclear reactors, also the mechanical and the chemical properties of these materials have to be considered. In addition, the sensitivity with respect to radiation exposure and economical aspects decide about applicability of a certain material. Most common materials for the control rods of thermal reactors are boron and cadmium.

The control rod calibration is the experimental determination of the function $\frac{d\rho}{dz}(z)$ and, subsequently, to calculate the function $\rho(z)$.

4. Methods for Reactivity Determination

The reactivity is always determined via the time dependence of the reactor power, which follows from the emission rates of prompt and delayed neutrons. The time dependent behaviour of the reactor power after a sudden (positive) change in the reactivity is shown in Fig. 3.

The production rate of prompt neutrons immediately reacts on the reactivity change (prompt jump). In contrast, the production rate of the delayed neutrons initially remains constant. Since by the prompt neutrons alone the reactor is subcritical, the system remains in a quasi-stationary condition until the production rate of the delayed neutrons starts to increase. This is about 100 ms after the reactivity change.

Then, the production rate of delayed neutrons noticeably starts to increase and after about 100 s, an equilibrium condition of the production of prompt and delayed neutrons has been established. The change in reactor power is then determined by the so-called stable reactor period T_s.

Negative reactivity jumps give analogous system answers but a decrease in power.

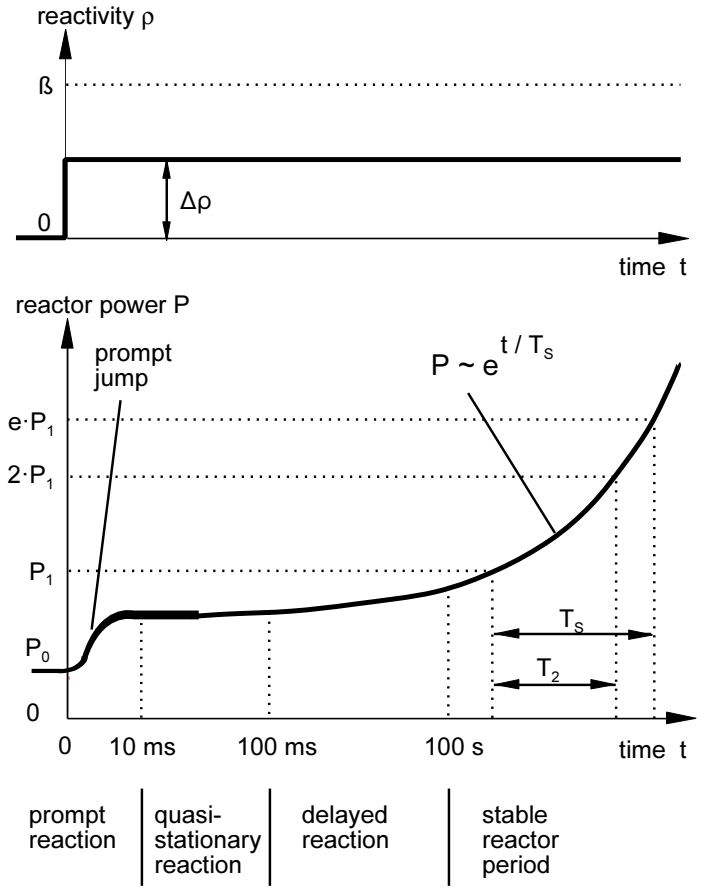

Fig. 3, Time behaviour of the reactor power after a positive reactivity jump ($0 < \rho << \beta$ at time t = 0)

The calibration of the control rods can be done applying several methods, which all use various specific effects:

Rod-Drop Method
Before and after dropping a control rod, the time behaviour of the reactor power has to be registered. The resulting change in reactivity due to the rod drop can be determined from the magnitude of the so-called prompt jump, which marks the decrease in power.

Periodic Power Modulation
Periodic changes in the position of the control rods induce periodic power changes. The corresponding reactivity change can be determined from the amplitudes of these power oscillations.

Method of Inverse Kinetics
This method bases on mathematically modelling the time-dependent processes in the reactor. In the model, the reactivity is continuously regulated such that the actual reactor power meets the calculated reactor power in every single instant. This method is used by so-called reactivity meters that display the reactivity permanently. The technical effort for such reactivity meters is rather high.

Positive period method (measurement of stable reactor period T_s)
This is a conventional method for the determination of the reactivity. It will be applied also in the given experiment.
The relation between the measured stable reactor period and the unknown value of reactivity is given by the so-called INHOUR equation (derived from "inverse hour"). This equation follows from the reactor-kinetic equations (for a detailed derivation see the appendix)

$$\frac{dn}{dt} = \frac{\rho - \beta}{l^*} \cdot n + \sum_{i=1}^{6} \lambda_i \cdot C_i + S$$
$$\frac{dC_i}{dt} = \frac{\beta_i}{l^*} \cdot n - \lambda_i \cdot C_i \qquad (i = 1, \ldots, 6) \qquad \textbf{(8)}$$

with:
l^* = average effective life-time of prompt neutrons,
ρ = reactivity,
β_i = absolute fraction of the i-th group of delayed neutrons,
C_i = concentration of the delayed-neutron precursors of the i-th group,
λ_i = decay constant of the delayed-neutron precursors of the i-th group, and
S = source strength of the external neutron source in the reactor.

This system of ordinary differential equations has 7 solutions corresponding to the 7 eigenvalues ω_i. The system can be solved using the ansatz

$$n_i(t) = n_{i,0} \cdot e^{\omega_i \cdot t} \qquad resp. \qquad C_i(t) = C_{i,0} \cdot e^{\omega_i \cdot t} \qquad \textbf{(9)}$$

As a consequence of the linearity of the system the neutron density n(t) is a linear combination of the 7 solutions, which have the form given in equation (9):

$$n(t) = n_0 \cdot e^{\omega_0 \cdot t} + n_1 \cdot e^{\omega_1 \cdot t} + \ldots + n_6 \cdot e^{\omega_6 \cdot t} \qquad \textbf{(10)}$$

Its solutions ω_i are given schematically in Fig. 4.

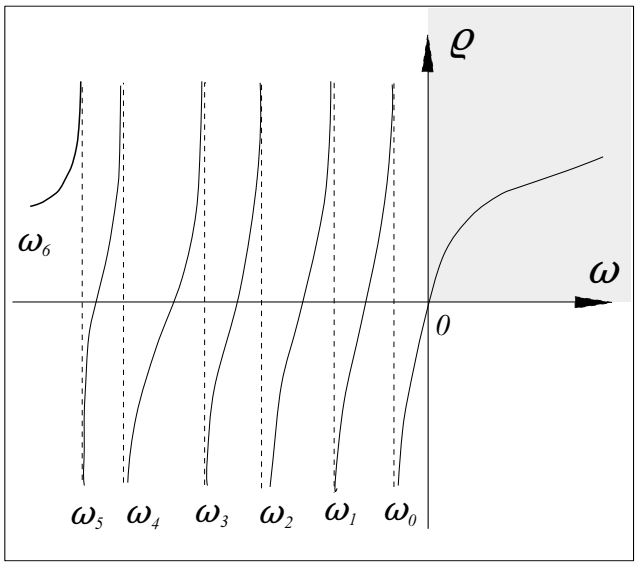

Fig. 4, Schematic presentation of the solutions ω_i of the INHOUR equation

Having a positive amount of reactivity ($\rho > 0$) only one of the 7 values ω_i is positive. Consequently, 6 of the 7 contributions in equation (10) decay within a short period of time and vanish.

If finally the remaining terms

$$n(t) = n_0 \cdot e^{\omega \cdot t} \qquad resp. \qquad C_i(t) = C_{i,\,0} \cdot e^{\omega \cdot t} \tag{11}$$

are inserted into the reactor-kinetic equations (8) and, additionally, respecting the relation $\omega = 1 / T_S$, then the **INHOUR equation** follows as:

$$\rho = \frac{l^*}{T_S} + \sum_{i=1}^{6} \frac{\beta_i}{1 + \lambda_i \cdot T_S} \tag{12}$$

Considering only small positive reactivity values, i.e. for large stable reactor periods, in equation (12), the denominator of the sum can be approximated by $\lambda_i \cdot T_S$ (since $1 << \lambda_i \cdot T_S$) and the first term l^*/T_S can be neglected against the sum. Thus, equation (12) can be approximated by

$$\rho \approx \frac{1}{T_S} \cdot \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i} \tag{13}$$

The reactivity ρ may be normalised to the fraction of delayed neutrons ß. This is reasonable, because the quotient ρ / ß occurs in any investigation concerning the kinetic reactor behaviour. Although this quantity ρ' = ρ / ß carries no physical unit, it gets attached by the artificial unit Dollar ($) with 1 $ = 100 ¢ to distinguish it from the non-normalised quantity ρ. In this representation, the INHOUR equation has the form

$$\rho' = \frac{\rho}{\beta} = \frac{l^* / \beta}{T_S} + \sum_{i=1}^{6} \frac{a_i}{1 + \lambda_i \cdot T_S} \qquad \textbf{(14)}$$

with $a_i = \beta_i / \beta$ being the fraction of the delayed neutrons of group i relative to the total fraction of all delayed neutron groups. Some group specific information is summarised in Tab. 1.

At the AKR, the parameter l^* / β has the value $l^* / \beta = 0.0051$ s.

i	λ_i [s^{-1}]	$a_i = ß_i / ß$
1	0.0124	0.033
2	0.0305	0.219
3	0.111	0.196
4	0.301	0.395
5	1.14	0.115
6	3.01	0.042

Tab. 1, Data of delayed neutron groups for the application in the INHOUR equation

5. Procedure of the Experiment

The reactor has to be set to a critical condition at low power (about 0.3 W), which has to be done with leaving one rod completely in the core (position 0), pulling out another rod completely (position 4000), and adjusting the critical condition with the third rod. The neutron source has to be removed from the reactor.

Then, the rod which has to be calibrated (that rod being in position 0) is drawn out by a certain defined length (e.g. position 800). Consequently, the corresponding change in reactivity $\Delta\rho$ causes a supercritical condition with increasing power. Initially, all solutions of the INHOUR equation contribute to the increase. Only after the higher solutions have decayed, i.e. after equilibrium of delayed and prompt neutrons has established (about 60 … 100 s after moving the rod), the stable reactor period can be measured. It is that time interval which the reactor takes for a power increase by a factor of e = 2.71. In practical cases, it is more convenient to measure the doubling time T_2, instead. This quantity is related to the stable reactor period by

$$T_S = \frac{T_2}{\ln 2} \qquad \textbf{(15)}$$

Inserting the resulting value of the stable reactor period T_S into the INHOUR equation (12) or (14), gives the corresponding value of reactivity.

By application of a trick even a second control rod can be calibrated in a single experiment at the same time (compensation method). For this purpose, after measuring the doubling time and calculating the positive reactivity, a second control rod (that is totally pulled out at the beginning in position 4000) is used to re-establish the critical condition. By this way, the same absolute amount of reactivity is fed to the reactor as before, but with negative sign. The corresponding change in the position of the second control rod can be determined from the display on the control panel.

After adjusting the critical reactor condition at about 0.3 W reactor power, again, the first rod can be drawn out of the core by a defined length for the second time. According to the procedure explained above, the doubling time has to be measured and afterwards, the positive amount of reactivity due to the movement of the first rod has to be compensated by moving in the second rod, again.

This procedure has to be repeated until the first rod has reached its upper end position (position 4000). The second rod is supposed to have the position 0 after the compensation assuming the reactivity values of both rods are identical.

The calculation of the reactivity values according to equations (12) or (14) is time consuming, since for each value of the stable reactor period, all 6 groups of delayed neutrons contribute to the sum in the equation. For this reason, preferably a programmable calculator or a computer code should be used to evaluate the measured doubling times. Such a computer program is provided at the working place.

Fig. 5 shows the relation between doubling time (or stable reactor period, respectively) and the reactivity according to the INHOUR equation using the parameter values from Tab. 1.

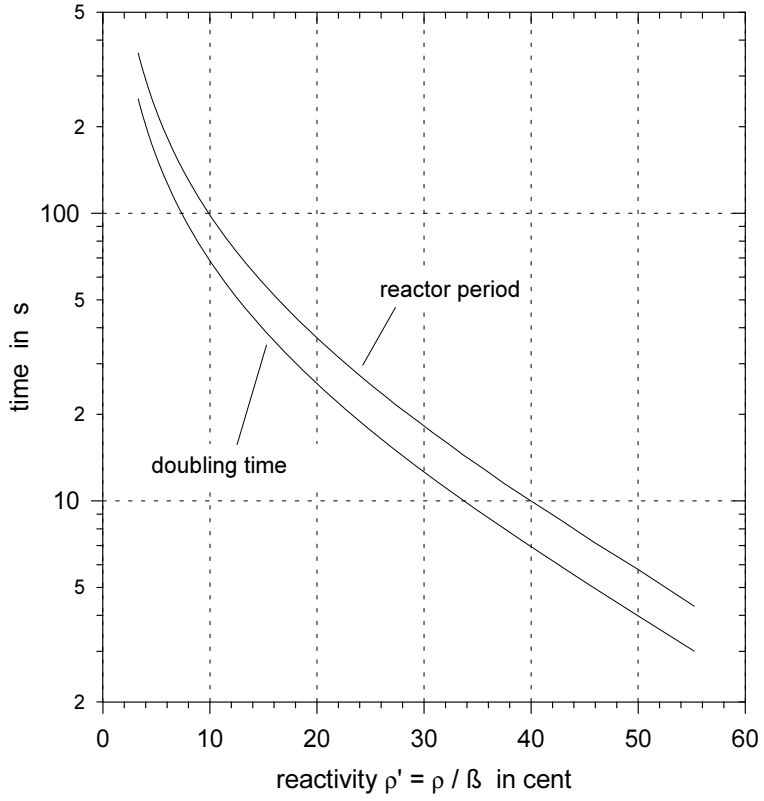

Fig. 5, Relation between doubling time or reactor period, respectively, and reactivity according to the INHOUR equation

The determination of the reactivity by measuring the stable reactor period is easy to perform at least for determination of small reactivity changes, since only a relation between the reactivity and the stable reactor period is needed. Further physical quantities describing the reactor are unnecessary to know at this point. But it must be mentioned, that the application of the IN-HOUR equation is restricted to small reactivity range (0 … 20 ¢) only, because higher reactivity values would cause too large rates of power increase and a too strong influence of drift or noise effects. Moreover, this method is rather time-consuming since for the determination of the characteristic of a complete rod, several subsequent measurements of the reactor period has to be carried out.

6. Calculation of the Total Reactivity of the Control Rods and Determination of the Excess Reactivity and of the Shut-Down Reactivity

The **total reactivity value** of a control rod is given by the the integral control rod characteristic determined over the complete rod length (Fig. 2).

The **excess reactivity** is that amount of positive reactivity which results from drawing out all control rods each to its upper end position, starting from a critical reactor condition. Based on the integral control rod characteristic of each control rod i (at the AKR is i = 1, 2, 3), it is determined which positive reactivity $\rho_{i,\,excess}$ can be still fed into the reactor by complete pulling out of all control rods starting from their positions in critical reactor condition. For the determination of the total excess reactivity, the individual critical positions of the rods relative to each other are irrelevant, since the total excess reactivity is the sum of the excess reactivities of all rods

$$\rho_{excess} = \rho_{1,excess} + \rho_{2,excess} + \rho_{3,excess}$$

Fig. 6 illustrates the idea of calculating the excess reactivity from the measured control rod characteristics.

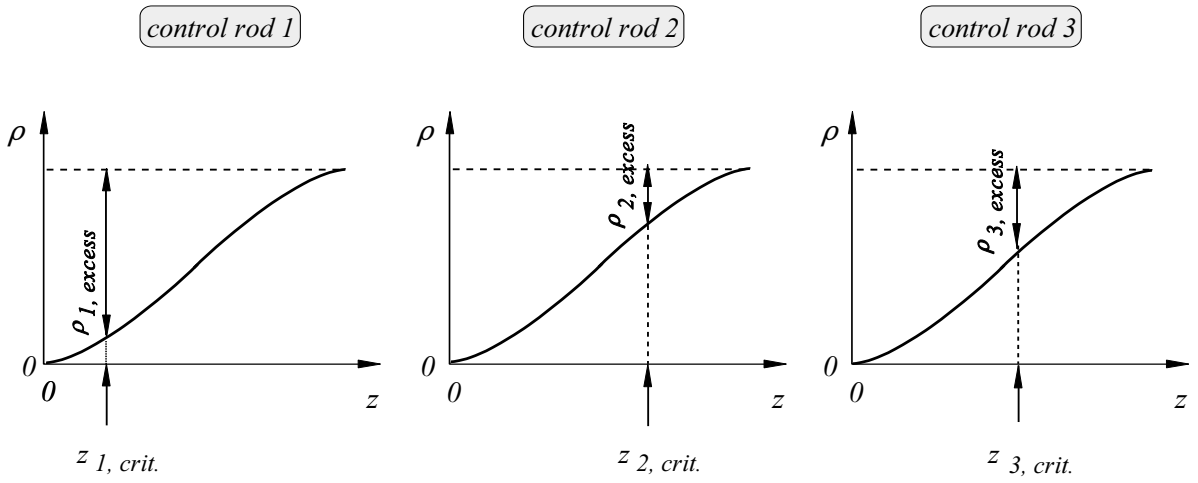

Fig. 6, Determinationof the excess reactivity based on measured control rod characteristics

In the given reactor experiment at AKR-2, usually only two of the three control rods are calibrated. Since the calculation of the total excess reactivity requires the characteristic of the third rod, too, this characteristic will be approximated by the average of the integral rod characteristics of the other two rods. Such an assumption is justified, because all three rods are designed identically and positioned symmetrically in the reactor.

The starting point for the determination of the excess reactivity is always a critical reactor condition - no matter what particular positions of the rods relative to another mark the critical condition. Conveniently, that critical rod configuration should be chosen which marks the critical condition at the start of the experiment. Doing so, the entire reactivity of the first rod contributes to the excess reactivity (full rod length is available) while for the second rod no contribution has to be taken into account (rod is already in its upper end position). Only the excess reactivity of the third rod has to be determined by interpolation of the corresponding characteristic.

The total excess reactivity is an important criterion for the nuclear safety of a nuclear reactor. If the total excess reactivity ρ_{excess} of a reactor does not exceed the fraction of delayed neutrons ß, i.e. for keeping $\rho_{excess} < ß$ (or identically $\rho'_{excess} < 1$ \$), such a reactor can never reach a prompt-supercritical condition - neither due to technical failure nor due to wrong action of the reactor operator. At the AKR, the criterion $\rho_{excess} < ß$ is fulfilled and defined in the operation licence of the facility. An important result of the given experiment is to prove this condition!

In contrast to the previously discussed quantity, the **shut-down reactivity** (of the partially shut-down reactor) is that amount of negative reactivity which results from dropping all control rods to their lower end position 0, starting again from critical condition. The determination of the shut-down reactivity goes along with that of the excess reactivity but with opposite direction of rod moving. In case the excess reactivity is already known, the shut-down reactivity can be calculated as the difference of the total reactivity and the excess reactivity

$$\rho_{shut-down} = \rho_{total} - \rho_{excesss}$$

7. Comments on the Protocol

- short description of the experiment and its procedure
- table containing all measured values including measurement uncertainties
- numerical analysis and discussion of the errors
- graphical representation of the (differential and integral) control rod characteristics
- determination of the excess reactivity and the shut-down reactivity (for partial shut-down)

8. Questions to Answer

1. Explain the derivation of the INHOUR equation and discuss its solutions for the cases:
   a) $\rho < 0$ b) $\rho = 0$ c) $\rho > 0$!

2. What is the time behaviour of the reactor power for a negative reactivity jump?

3. What is the so-called stable reactor period in the context of the time behaviour of a reactor?

4. What is the excess reactivity and how it can be determined from the control rod characteristics? What is the maximum value of the excess reactivity such that the reactor would be safely controllable?

5. What is the idea of the calibration of a control rod simultaneously to the calibration of another control rod (compensation method)? How to determine the end value of the reactivity of the second rod, in case the total amounts of reactivity of the rods are unequal?

6. What is the benefit of the control rod calibration?

7. What are the physical and technical requirements for the material properties of a control rod?

Appendix: Derivation of the INHOUR equation

Fundamentals are the point-kinetic equations:

$$\frac{dn}{dt} = \frac{\varrho - ß}{l^*} \cdot n + \sum_{i=1}^{6} \lambda_i \cdot C_i \qquad \text{(A1)}$$

$$\frac{dC_i}{dt} = \frac{ß_i}{l^*} \cdot n - \lambda_i \cdot C_i \qquad \text{(A2)}$$

Solutions correspond to:

$$n(t) = n_0 \cdot e^{\omega \cdot t} \quad \Rightarrow \quad \frac{dn}{dt} = n_0 \cdot e^{\omega \cdot t} \cdot \omega = n \cdot \omega \qquad \text{(A3)}$$

$$C_i(t) = C_{0,i} \cdot e^{\omega \cdot t} \quad \Rightarrow \quad \frac{dC_i}{dt} = C_{0,i} \cdot e^{\omega \cdot t} \cdot \omega = C_i \cdot \omega \qquad \text{(A4)}$$

from (A2) follows by insertion of (A4):

$$C_i \cdot \omega = \frac{ß_i}{l^*} \cdot n - \lambda_i \cdot C_i$$

$$C_i \, (\omega + \lambda_i) = \frac{ß_i}{l^*} \cdot n$$

$$C_i = \frac{ß_i \cdot n}{l^* \cdot (\omega + \lambda_i)}$$

respectively

$$C_i \cdot \lambda_i = \frac{\lambda_i \cdot ß_i \cdot n}{l^* \cdot (\omega + \lambda_i)} \qquad \text{(A5)}$$

insertion of (A5) and (A3) in (A1) results in:

$$n \cdot \omega = \frac{\varrho - ß}{l^*} \cdot n + \sum_{i=1}^{6} \frac{\lambda_i \cdot ß_i \cdot n}{l^* \cdot (\omega + \lambda_i)}$$

by multiplication of both sides by l*/n follows:

$$l^* \cdot \omega = \varrho - ß + \sum_{i=1}^{6} \frac{\lambda_i \cdot ß_i}{\omega + \lambda_i}$$

and resolved to ρ:

$$\varrho = l^* \cdot \omega + ß - \sum_{i=1}^{6} \frac{\lambda_i \cdot ß_i}{\omega + \lambda_i} \qquad \text{with} \quad ß = \sum_{i=1}^{6} ß_i \quad (!)$$

$$= l^* \cdot \omega + \sum_{i=1}^{6} \left(ß_i - \frac{\lambda_i \cdot ß_i}{\omega + \lambda_i} \right)$$

$$= l^* \cdot \omega + \sum_{i=1}^{6} ß_i \cdot \left(1 - \frac{\lambda_i}{\omega + \lambda_i} \right)$$

$$= l^* \cdot \omega + \sum_{i=1}^{6} ß_i \cdot \left(\frac{\omega + \lambda_i - \lambda_i}{\omega + \lambda_i} \right)$$

$$= l^* \cdot \omega + \sum_{i=1}^{6} \frac{ß_i \cdot \omega}{\omega + \lambda_i}$$

$$= l^* \cdot \omega + \sum_{i=1}^{6} \frac{\omega}{\omega} \cdot \left(\frac{ß_i}{1 + \frac{\lambda_i}{\omega}} \right)$$

$$= l^* \cdot \omega + \sum_{i=1}^{6} \frac{ß_i}{1 + \frac{\lambda_i}{\omega}}$$

If only such positive reactivity addition is taken into account which cause a stable reactor period (i.e. $0 < \rho < ß$), it can be seen from equation (A3) $n(t) = n_0 \cdot e^{\omega \cdot t}$ that in this case ω must be $\omega = 1/T_s$.

$$\varrho = \frac{l^*}{T_s} + \sum_{i=1}^{6} \frac{ß_i}{1 + \lambda_i \cdot T_s}$$

INHOUR equation