

REMARKS TO A NOVEL NONLINEAR BWR STABILITY ANALYSIS APPROACH (RAM-ROM METHODOLOGY)

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ABSTRACT

A new approach for nonlinear stability analysis is presented. In the framework of this approach, integrated BWR (system) codes and reduced order models (ROM's) are used as **complementary tools** to examine the stability characteristic of fixed points and periodic solutions of the nonlinear differential equations describing the stability behaviour of a BWR loop. Hence the methodology demonstrated in this paper is a novel one in a specific sense: We analyse the highly nonlinear processes of the BWR dynamics by application of system codes (in which numerical diffusion properties are examined in advance so far as possible) and by application of some sophisticated methods of the nonlinear dynamics (bifurcation analysis). We claim, the complementary application of independent methodologies to examine the nonlinear stability behaviour can increase the reliability of BWR stability analysis. This work is a continuation of the previous work at the Paul Scherrer Institute (PSI, Switzerland) and at the University of Illinois (USA) on this field. The current ROM was extended by adding the recirculation loop model. The necessity of consideration of the effect of subcooled boiling in an approximated manner was discussed. Furthermore, a new calculation methodology for the feedback reactivity was implemented. The modified ROM is coupled with the code BIFDD which performs semi-analytical bifurcation analysis. In addition to the ROM extensions, a new approach for calculation of the ROM input data was developed. The new approach for nonlinear BWR stability analysis is presented for NPP Leibstadt. This investigation is carried out for an operational point for which an out-of-phase power oscillation has been observed during a stability test at the beginning of cycle 7 (KKL cycle 7 record #4). The modified ROM and the new approach for the calculation of the ROM input data are qualified for BWR stability analysis in the framework of the approach (RAM-ROM methodology) demonstrated in this paper.

1. OBJECTIVE

In general, the dynamics of a BWR can be described by a system of coupled nonlinear partial differential equations. From the nonlinear dynamics point of view, it is well known that such systems show, under specific conditions, a very complex temporal behaviour which is reflected in the solution manifold of the corresponding equation system. Consequently, to understand the nonlinear stability behaviour of a BWR and to get an overview over types of instabilities, the solution manifold of the differential

equation systems must be examined. In particular, with regard to the existence of operational points where stable and unstable power oscillations are observed, stable or unstable fixed points and stable or unstable oscillatory solutions are of paramount interest [1-4]. Notice, stable or unstable oscillatory (periodical) solutions correspond to stable or unstable limit cycles. Saddle-node bifurcation of cycles (turning points or fold bifurcations), period doubling and other nonlinear phenomena which are important from the reactor safety point of view are also of interest in this work[5-8].

It is worthy to mention that a linear stability analysis is not able to examine the existence of stable and unstable limit cycles, e.g. if the unstable limit cycle is "born" in a subcritical Hopf bifurcation point [7], stable fixed points and unstable limit cycles will coexist in the linear stable region (close to the bifurcation point) in which the asymptotic decay ratio is less than one ($DR < 1$). This example show that conceivably unstable conditions (from the nonlinear point of view) are not recognized and the operational safety limits could be violated. Hence, in order to reveal this kind of phenomena, nonlinear BWR stability analysis like bifurcation analysis is necessary.

2. METHODOLOGY

In the framework of the approach applied in this work, integrated BWR (system) codes (RAMONA5, Studsvik/Scandpower) and simplified BWR models (**reduced order model, ROM**) are used as **complementary tools** to examine the stability characteristic of fixed points and periodic solutions [1,4]. This new approach is called **RAM-ROM** method, where RAM is a synonym for system code. The intention is, firstly, to identify the stability properties of certain operational points by performing ROM analysis and, secondly, to apply the system code for a detailed stability investigation in the neighbourhood of these operational points. Some (constitutive) essential characteristics of system codes and ROMs are summarized in Fig. 1.

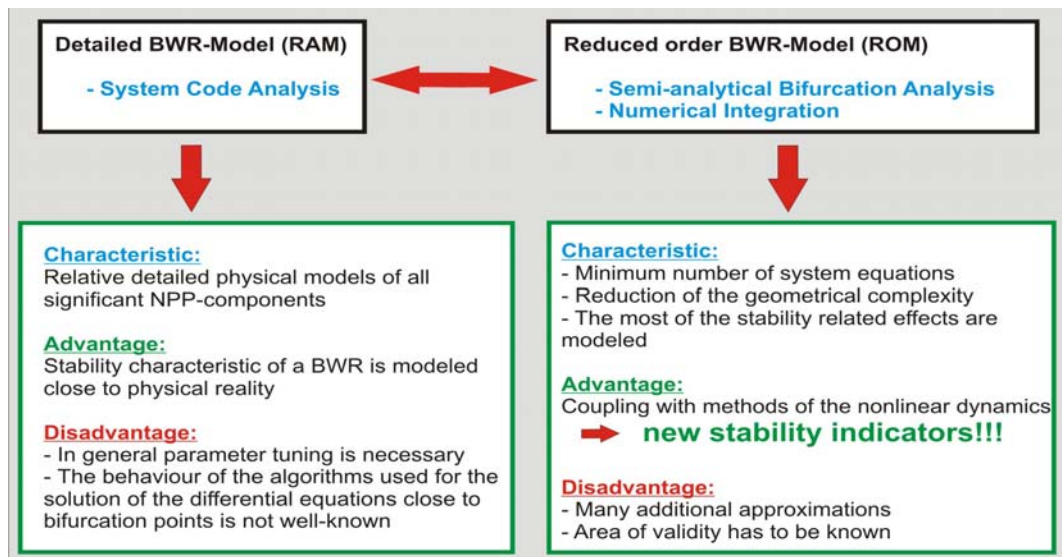


Fig. 1 Overview over the methodology applied for the nonlinear BWR stability analyses where RAMONA5 and ROM are used as complementary tools.

System codes are computer programs which include sufficiently detailed physical models of all nuclear power plant components which are significant for a particular transient analysis (including 3D core model). Therefore, such detailed BWR models should be able to represent the stability characteristics of a BWR close to the physical reality. Nonlinear BWR stability analysis using large system codes is currently common practice in many laboratories [1]. A particular requirement is the integration of a 3D neutron kinetic model, which permits the analysis of regional or higher mode stability behaviour [3]. A detailed investigation of the complete solution manifold of the nonlinear equations describing the BWR stability behaviour by applying system codes needs comprehensive parameter variation studies which require large computational effort. Hence system codes are inappropriate to reveal the complete nonlinear stability characteristics (with other words: the whole solution manifold of the DE system) of a BWR. Furthermore, user of system codes must pay attention to the stability behaviour of the algorithms employed. In particular, physical and numerical effects regarding power oscillations and the behaviour of numerical damping of the algorithms should be known in detail. Numerical diffusion, for example, can corrupt the results of system codes significantly. Therefore, reduced order analytical models could be helpful to get a first overview over the stability landscape to be expected.

The objective of the ROM development is to develop a model as simple as possible from the mathematical and numerical point of view while preserving the physics of the BWR stability behaviour [4]. The resulting equation system is characterized by a minimum number of system equations which is realized by the reduction of the geometrical complexity. One demand on our ROM is, because the ROM sub-models should be as close as possible to the sub-models used in RAMONA, that the solution manifold of the RAMONA model should be as close as possible to the solution manifold of the ROM. E.g., both neutron kinetic models (ROM and RAMONA) are based on the two neutron energy group diffusion equations. Both thermal-hydraulic two phase flow models are represented by models which consider the mechanical non-equilibrium (different velocities of the phases of the fluid). The main advantage of employing ROM's is the possible coupling with codes including methods of nonlinear dynamics like bifurcation analysis. Bifurcation analysis of a BWR system, for example, leads to an overview over types of instabilities in selected parameter spaces. The existence of stable and unstable periodical solutions (corresponds to limit cycles) can be examined reliably. Thus, the stability behaviour of global and regional power oscillation states can be investigated in detail and can be interpreted in physical terms.

In the scope of the present ROM analyses two independent techniques are employed. These are the semi-analytical bifurcation analysis with the bifurcation code BIFDD [7] and the numerical integration of the ROM differential equation system [1]. Bifurcation analysis with BIFDD determines the stability properties of fixed points and periodical solutions (correspond to limit cycle) [7,8]. For independent confirmation of these results, the ROM system will be solved directly by numerical integration for selected parameters [1,4,8].

The procedure of the nonlinear BWR stability analysis applying the RAM-ROM method is depicted in **Fig. 2**.

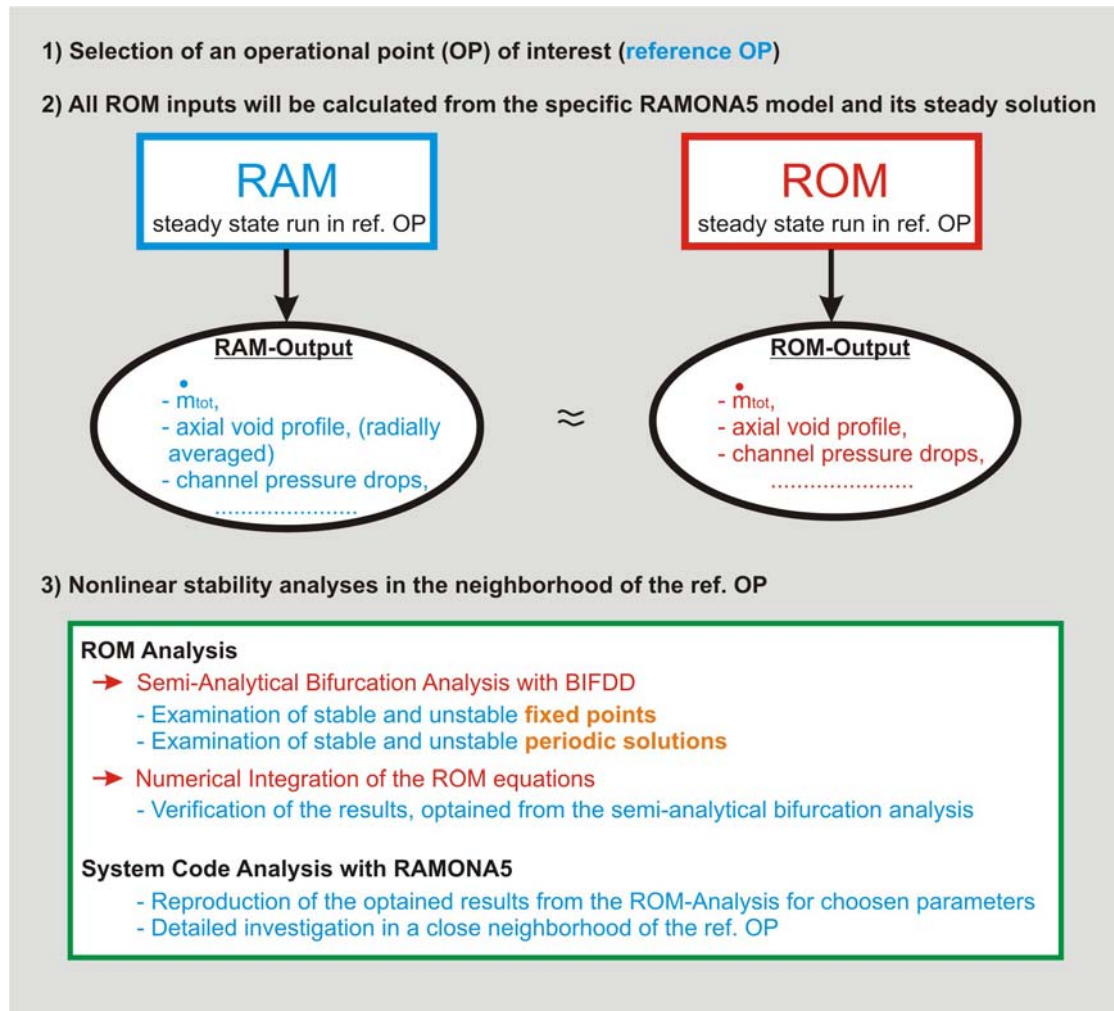


Fig. 2 This figure depicts the new approach for nonlinear BWR stability analyses using RAMONA5 and ROM as complementary tools.

The goal is to simulate the stability behavior of the power plant with the ROM as close as possible to that one calculated by RAMONA5 in the neighborhood of a selected operational point. Hence, at first, the reference OP has to be selected for which the nonlinear BWR stability analysis will be performed. Secondly, the new procedure for the ROM input calculation is applied. Thereby, all ROM input data are calculated from the specific RAMONA5 model and its steady state solution corresponding to the reference point. We demand: the ROM should provide the correct steady state values in the reference operational point. Thereby the most essential values (for the BWR stability behaviour) are the mode feedback reactivity coefficients, the core inlet mass flow, the axial void profile and the channel pressure drops over the reactor vessel components along the closed flow path. E.g. the subcooling number and the pressure loss coefficients cannot be calculated directly from the RAMONA5 model (and its steady state output)

because the models describing the axial power profile and the pressure drop along the closed flow path are different in both codes. Hence a special calculation procedure for the pressure loss coefficients and the core inlet subcooling of the ROM is developed and applied. Finally, after the calculation of the ROM input parameters, nonlinear BWR analysis is performed by using ROM and RAMONA5 as complementary tools.

To summarize and roughly speaking, the ROM analysis provides an overview about the stability characteristics of fixed points and limit cycles in selected parameter spaces. Besides, the occurrence of further types of nonlinear phenomenon such as saddle-node bifurcation of cycles can be revealed [8]. Hence, the system code analysis can be performed with more pre-information (is made more reliable).

2.1 Local bifurcation analysis using BIFDD

In the framework of the bifurcation analysis with BIFDD, the so-called Poincaré-Andronov-Hopf bifurcation (PAH-B) theorem plays a dominant role [1,4,5,7,8]. This theorem guarantees the existence of stable and unstable periodic solutions of nonlinear differential equations if certain conditions are satisfied. A mathematical description is given in [1,7-9] and thus is not repeated here. In order to get information about the stability property of the periodic solution, the (linear) Floquet theory is applied (and several techniques such as Lindstedt-Poincaré asymptotic expansion, centre manifold reduction, transformation into the Poincaré normal form presented in [7-10]) where the so-called Floquet exponent (Floquet parameter) β appears [1,7-9] which determine the stability of the periodic solution. If $\beta < 0$, the periodic solution is stable (supercritical bifurcation) while if $\beta > 0$, the periodic solution is unstable (subcritical bifurcation). Roughly speaking, the Floquet parameter can be interpreted to be a stability indicator for limit cycles and is a result of a special technique from nonlinear dynamics. **Notice, the existence of a Hopf bifurcation is the (mathematical) reason for the sudden appearance of periodic oscillations (limit cycles).** These periodic solutions will be observed in the BWR reactor dynamics as global (in-phase) or regional (e.g. out-of-phase or azimuthal mode) power oscillations.

The bifurcation analysis is carried out with the bifurcation code BIFDD developed by Hassard [7]. The user of BIFDD has to provide the input parameter vector, a set of nonlinear ODEs, the corresponding Jacobian matrix and the initial guess for the phase space variables. The bifurcation analysis starts with selection of the so called iteration and bifurcation parameter. Thereby the iteration parameter will be varied in the interval defined by the user. For each iteration step BIFDD computes the critical value $\gamma_{k,c}$ of the bifurcation parameter, the amplitude ε of the oscillation, the Floquet parameter $\beta \approx \beta_2$ and further certain expansion parameters defined in [1,7-10]. As a result of the bifurcation analysis using BIFDD, a set of fixed points where the Hopf conditions are fulfilled will be obtained in the two dimensional parameter space, which is spanned by the iteration and bifurcation parameter (left diagram of Fig. 3 b)) [4]. This set of fixed points is called linear stability boundary. In each of these fixed points a periodical solution is born whose stability property is determined by the Floquet

exponent [1,4]. The right diagram of Fig. 3 b) shows the Floquet exponent $\beta \approx \beta_2$ for each iteration step.

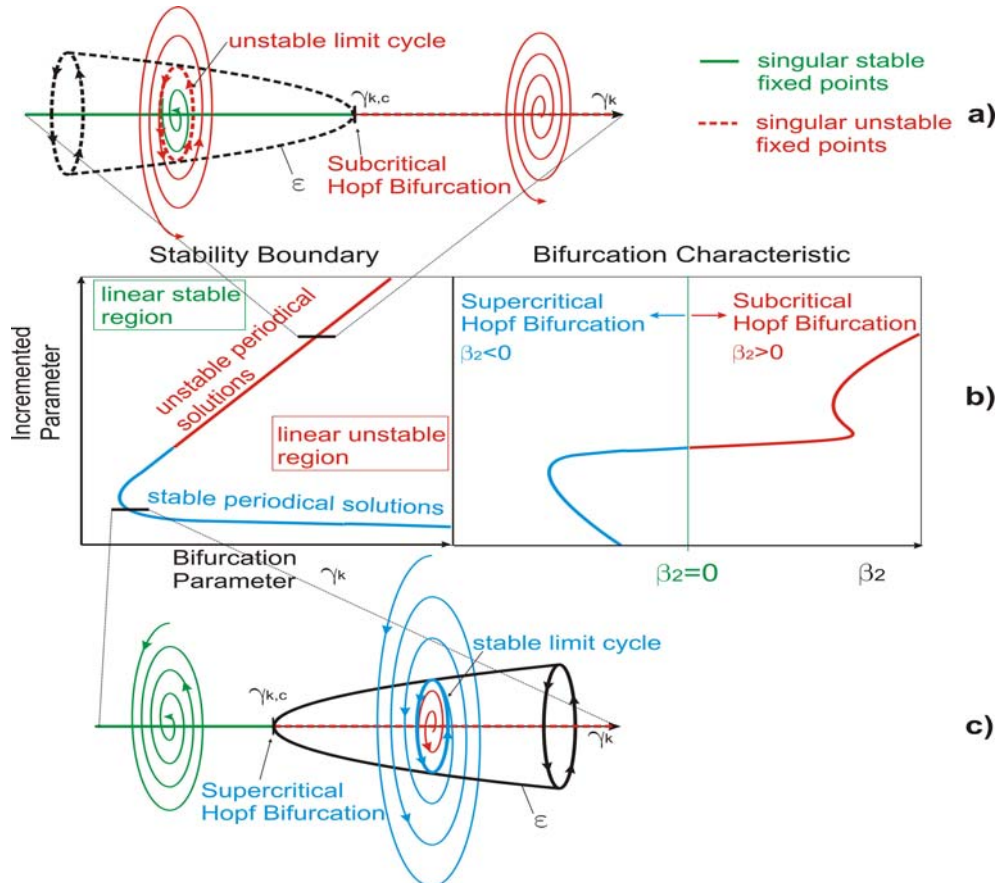


Fig. 3 Stability boundary in the two dimensional parameter space which is spanned by the iteration and bifurcation parameter and the corresponding bifurcation characteristic.

In Fig. 3b is shown a selected stability map (incremental parameter, e.g. N_{sub} , vs. control or bifurcation parameter, γ_k , left hand side) and the corresponding bifurcation characteristic (β_2 vs. incremental parameter, right hand side). The bifurcation characteristic predicts unstable limit cycles coexisting with stable fixed points (see Fig. 3a) and stable limit cycles coexisting with unstable fixed points (see Fig. 3c) in the environment of the stability line. Notice, an unstable limit cycle “born” in a subcritical Hopf bifurcation separates a set of trajectories (in phase space) which spiral into the steady state solution (singular fixed point) from a set of trajectories which spiral away ad infinitum of the phase space (the state variables diverges in an oscillatory manner; the system behaves unstable).

2.2 Numerical integration

Semi-analytical bifurcation analysis is only valid in the vicinity of the critical bifurcation parameter (SB) in a small neighbourhood of the singular fixed point. In order

to get information of the stability behaviour beyond the local bifurcation findings numerical integration (in the time domain) of the set of the ODEs is necessary. Besides, the predictions of the semi-analytical bifurcation analysis can be confirmed independently [1,4,8].

3. THE ROM

The current BWR reduced order model consists of three coupled sub-models. These are a neutron kinetic model, a fuel heat conduction model and a two-channel thermal-hydraulic model (presented in [1, 2]). Fig. 4 depicts a schematic sketch of the ROM. In this paper, only the most important ROM assumptions are presented.

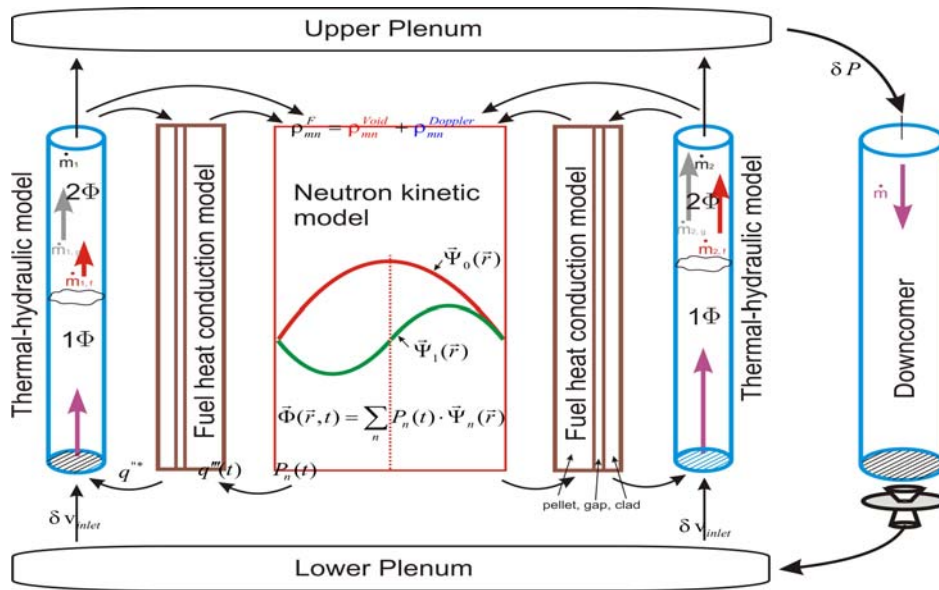


Fig. 4 Schematic sketch of the ROM

3.1 Neutron kinetic model

The neutron kinetics model is based on (an effective) two energy groups (thermal and fast neutrons). The spatial mode expansion approach of the neutron flux in terms of lambda modes (λ -modes) [12] is used (thereby it is assumed that the amplitude function is independent on the energy). Only the first two modes (fundamental and the first mode) and one effective one group of delayed neutron precursors are considered. The contribution of the delayed neutron precursors to the feedback reactivity is neglected. **A new calculation methodology for the mode feedback reactivities is implemented (close coupling to the system code RAMONA: the mode feedback reactivities are calculated with the aid of the 3D power distributions calculated by the LAMBDA code [12]; in turn, RAMONA provides the macroscopic cross sections).** Taking into account these assumptions, four mode kinetic equations could be developed, coupled with the equations of the heat conduction and the thermal-hydraulic via the feedback reactivity terms (void and Doppler feedback reactivities).

3.2 Fuel rod heat conductions

The heat conduction model assumptions (this sub-model is completely adopted from Karve (1998) [2]): (1) Two axial regions, corresponding to the single and two-phase regions, are considered; (2) three distinct radial regions, the fuel pellet, the gap and the clad are modeled in each of the two axial regions; (3) azimuthal symmetry for heat conduction in the radial direction is assumed; (4) heat conduction in the z-direction is neglected; (5) time-dependent, spatially uniform volumetric heat generation is assumed. These assumptions result in a one-dimensional (radial) time dependent partial differential equation (PDE). By assuming a two-piecewise quadratic spatial approximation for the fuel rod temperature, the PDE can be reduced to a system of ODEs by applying the variation principle approach. A detailed derivation is presented in [1,2].

3.3 Thermal-hydraulic model

The thermal-hydraulic behavior of the BWR is represented by two heated channels coupled by the neutron kinetics and by the recirculation loop. This sub-model is based on the following assumptions: (1) the heated channel, which has a constant flow cross section, is divided into two axial regions, the single and the two-phase region; (2) all thermal-hydraulic values are averaged over the flow cross section; (3) the dynamical behavior of the two-phase region is presented by a drift flux model (DFM, Rizwan-uddin, 1981) where mechanical non equilibrium (difference between the two phase velocities, and a radial non-uniform void distribution is considered) is assumed (the DFM represents the stability behavior of the two-phase more accurately than a homogeneous equilibrium model, in particular for high void content); (4) the two phases are assumed to be in thermodynamic equilibrium; (5) the system pressure is considered to be constant; (6) the fluid in both axial regions and the downcomer is assumed to be incompressible; (7) the following terms are neglected in the energy balance: the kinetic energy, potential energy, pressure gradient, friction dissipation; (8) the PDEs (three-dimensional mass, momentum and energy balance equation) are converted into the final ODEs by applying the weighted residual method in which spatial approximations (spatially quadratic but time-dependent profiles) for the single phase enthalpy [2] and the two-phase quality are used (is equivalent to a coarse grained axial discretization). **The thermal-hydraulic model is extended by a recirculation loop model based on the following assumptions:** (9) the downcomer (constant flow cross section) region is considered to be a single phase region; (10) all physical processes which lead to energy increase and energy decrease are neglected in the downcomer; (11) the pump head due to the recirculation pumps is considered to be constant ($\Delta P_{head} = const$); (12) the effect of the subcooled boiling phenomenon on the BWR stability behaviour is assessed. (Profile-Fit model [Levy, 1967])

To summarize, the dynamical system of the reduced BWR model consists of 22 ODEs, four from the neutron kinetic model, eight to describe the fuel rod heat conduction (two equations for each phase, in each channel) and ten that describe the thermal-hydraulic model (five for each channel) [1].

4. NONLINEAR STABILITY ANALYSIS FOR NPP LEIBSTADT (KLLc7_rec4)

In order to gather stability data for cycle 7 of the NPP Leibstadt (KKL) a stability test was performed on September 6th 1990 [11]. Comprehensive post-calculation stability analyses using system codes were conducted for KKL measurement record #4 and #5 (KLLc7_rec4-OP and KLLc7_rec5-OP). The paper contains a short presentation of results of the nonlinear stability analyses for KLLc7_rec4-OP [12] where increasing regional power oscillations occurred at approximately 60% power and 37% core mass flow [12]. Fig. 5 and Fig. 6 show the time evolutions of LPRM signals of the measurement and the RAMONA5 calculation.

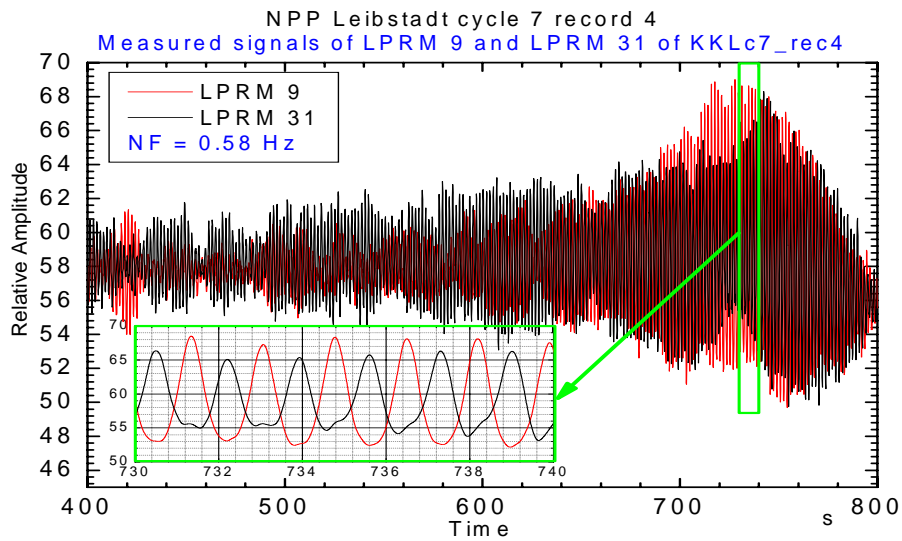


Fig. 5 Time evolution of the LPRM signals, measured in KLLc7_rec4-OP.

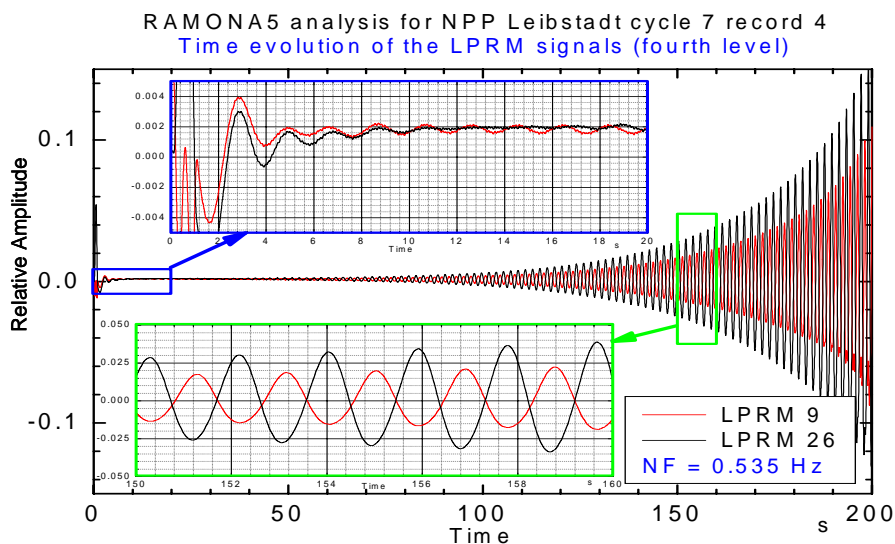


Fig. 6 RAMONA5 result for the reference OP. The relative amplitudes of signals are shown for LPRM 9 and LPRM 26. Both LPRM signals have a phase shift of π .

The transient behaviour presented in Fig. 6 was initiated by introducing a 2 node sinusoidal control rod movement resulting in a perturbation of the state variables of the BWR system. The signals of the LPRM 9 and 26 of the fourth level are located in different core half's (RAMONA predicts a fixed symmetry line for the present case []). As can be seen, an increasing out of phase power oscillation is occurring in the reference OP. RAMONA5 predicts a natural frequency of $NF^* = 0.537s^{-1}$ while the measurement gives $NF^* = 0.58s^{-1}$. All RAMONA5 investigations for the reference OP and its close neighbourhood have shown that the out of phase power oscillation will not discharge into a stable limit cycle. The existence of a stable limit cycle cannot be verified by the RAMONA5 and measurement results but it must not be excluded.

For the ROM analyses, KKLc7_rec4-OP is defined to be the reference OP. All design parameters of the ROM are calculated from the specific KKL-RAMONA5 model. The operating parameters are estimated from the steady state solution provided by RAMONA5 for KKLc7_rec4-OP where the new calculation procedure for the ROM-input was applied. The values of the heated channel pressure drops simulated by the ROM are close to the reference values provided by RAMONA5. In addition to that the axial void profiles calculated by RAMONA5 and ROM are in good agreement. Thereby the deviation of the total volumetric void fraction is less than 1%. These steady state results are presented in [].

The results of the numerical integration of the ROM equation system in the reference OP confirm the transient behavior predicted by RAMONA5. The time evolution of the fundamental mode $n_0(t)$, first azimuthal mode $n_1(t)$ and the channel inlet velocities $v_{1,inlet}(t)$ and $v_{2,inlet}(t)$ are shown in Fig. 7.

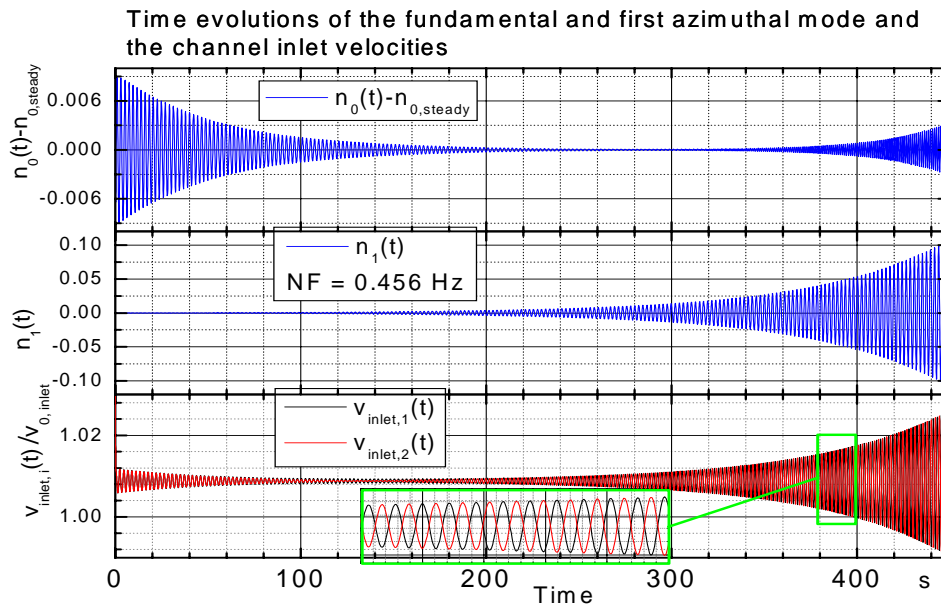


Fig. 7 Time evolutions of the fundamental $n_0(t)$ and first azimuthal mode $n_1(t)$ and the channel inlet velocities $v_{1,inlet}(t)$ and $v_{2,inlet}(t)$.

It can be seen that the amplitudes of the fundamental mode oscillation are decaying for the first 250 s and then increasing while the first azimuthal mode oscillation is increasing continuously, after the perturbation (an in-phase oscillation was triggered) was imposed on the system. This means, an increasing out of phase power oscillation is occurring in the reference OP. Thus, the prediction of the RAMONA5 investigation in the reference OP could be verified by the ROM. The oscillation frequency predicted by the ROM is $NF^* = 0.457s^{-1}$. The behaviour of the fundamental mode oscillation (decaying for the first 250 s and then increasing) can be explained by the solution of a linearized system where each component of the solution depends on each eigenvalue of the Jacobian matrix. This means, if there is at least one pair of complex conjugated eigenvalues with a real part larger than zero, all components of the solution will diverge asymptotically in an oscillatory manner. This was shown in detail in [1].

4.1 Local bifurcation analysis with BIFDD

The results of the semi-analytical bifurcation analysis of the ROM equation system are presented in the $N_{sub}-DP_{ext}$ -operating plane (N_{sub} and DP_{ext} are defined in **Nomenclature** at the end of this paper). Notice, a variation of DP_{ext} (steady state external pressure drop) corresponds to a movement on the rod-line which crosses the reference OP while the 3D-distributions will not be affected. Thus the stability properties of operational points along a fixed rod-line (fixed control rod configuration) and its close neighborhood are analyzed. The stability boundary (SB) and the bifurcation characteristic are shown in Fig. 8. The SB is defined as the set of fixed points for which the Hopf conditions are fulfilled. Roughly speaking, this means, in each of these fixed points a limit cycle is "born" and exists either in the (linear) stable or (linear) unstable region. The stability characteristic of the limit cycle is determined by the Floquet parameter β_2 [7-9]. In Fig. 9 is shown the SB transformed into the power flow map.

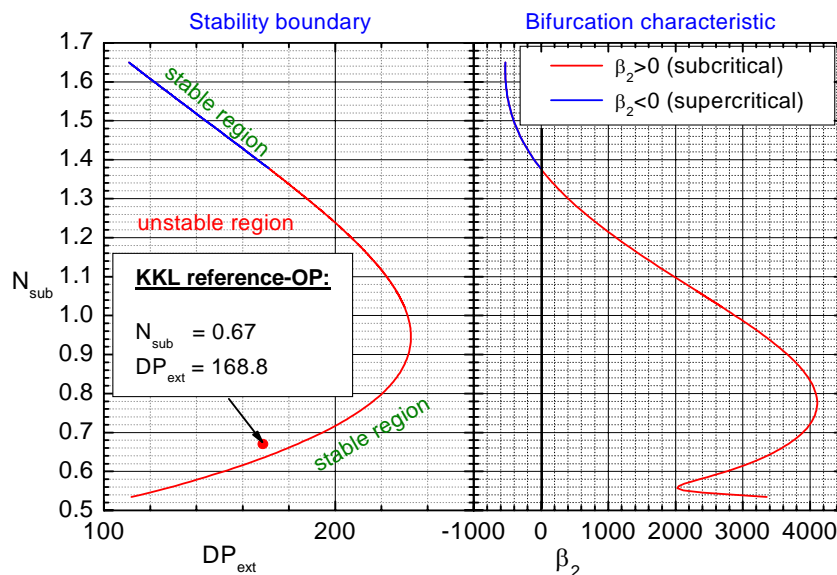


Fig. 8 Stability boundary and the bifurcation characteristic (nature of the Poincarè-Andronov-Hopf bifurcation) for the reference OP.

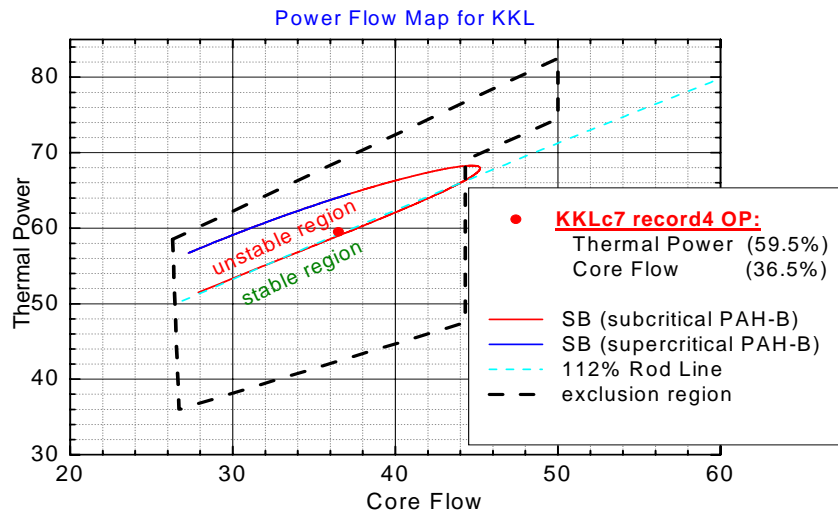


Fig. 9 Stability boundary transformed into the power flow map.

Unstable periodical solutions (unstable limit cycle) close to the KKLc7_rec4-OP are predicted by the semi-analytical bifurcation analysis (for $0.53 < N_{sub} < 1.38$, see Fig. 8). These solutions are located in the linear stable region close to the stability boundary. This means, in this region coexist stable fixed points and unstable limit cycles (see Fig. 3a). Notice, the asymptotic decay ratio (linear stability indicator) is less than 1 ($DR < 1$) in this region. A linear stability analysis is not capable to examine the stability properties of limit cycles.

4.2 Numerical Integration: Local Consideration

For independent confirmation of the results which are predicted by the bifurcation analyses, numerical integration of the ROM equation system (in the time domain) has been carried out for selected parameters. The ROM equations are solved in an operational point that is located in the (linear) stable region (see Fig. 10) close to the SB. In this region unstable limit cycles are predicted by the bifurcation analysis.

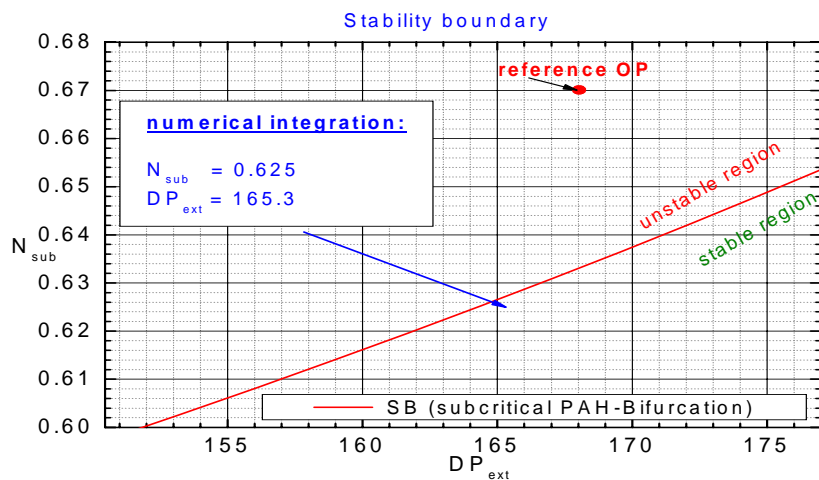


Fig. 10 SB and the point for which the unstable limit cycle will be verified by numerical integration.

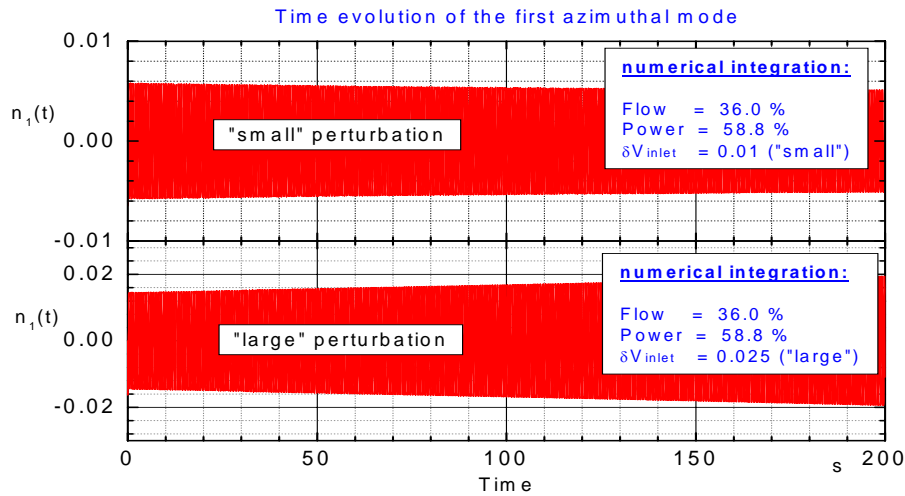


Fig. 11 Numerical integration is carried out in an operational point in which an unstable limit cycle is predicted by the bifurcation analysis. The transient was initiated by imposing perturbations of the core inlet mass flow with (small $\delta v_{inlet} = 0.01$, large $\delta v_{inlet} = 0.025$).

In order to verify the existence of the unstable limit cycle, perturbations of different amplitudes are imposed on the system. This means, according to $\vec{X}(t) = \vec{X}_0 + \delta\vec{X}(t)$, the steady state solution \vec{X}_0 is perturbed by different perturbation amplitudes $\delta\vec{X}(t)$ and the transient behavior of the system state $\vec{X}(t)$ is calculated by numerical integration of the ROM equation system. If a sufficient small perturbation is imposed on the system, the state variables will return to the steady state solution. The terminus “sufficient small perturbation” means that the trajectory starts within the basin of attraction of the fixed point. Roughly speaking, the perturbation amplitude is less than the repeller amplitude (see phase space portrait depicted in Fig. 3a). But if a sufficient large perturbation is imposed on the system, the state variables will diverge in an oscillatory manner. The terminus “sufficient large perturbation” means that the perturbation amplitude is larger than the repeller amplitude. In this case the trajectory will start out of the basin of attraction of the fixed point. As shown in Fig. 11, the results of the numerical integration method confirm locally the prediction of the bifurcation analysis.

4.3 Numerical Integration: Global Consideration

The local ROM analysis has shown that the bifurcation analyses using BIFDD and the numerical integration method provide locally in the origin of the dynamical system (phase or state space) in the vicinity of the critical parameter value $\gamma_{k,c}$ (parameter space; index k is ignored in the following discussion) consistent results. The terminus “locally in the origin of the dynamical system” means that the close neighbourhood of the steady state solution \vec{X}_0 (the singular fixed point) is taken into account in the phase space. Further analyses in the reference OP and its neighbourhood, whereby numerical integration is carried out for a time period of 800 s revealed the existence of stable limit cycles. The result of the time integration in the reference OP using the numerical integration code is shown in Fig. 12.

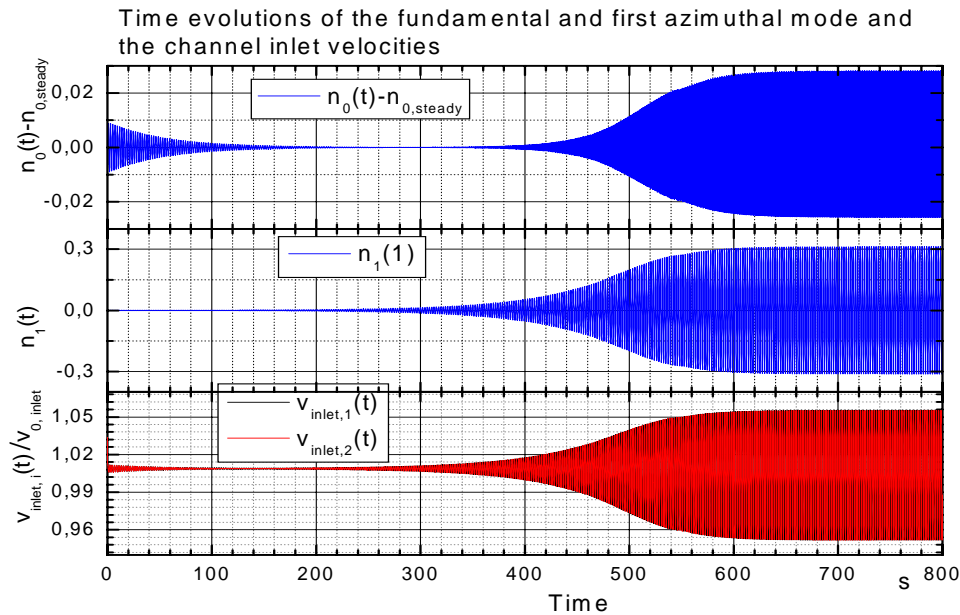


Fig. 12: Result of (a long time) numerical integration in the reference OP where a long time integration is carried out (reference OP of KKLc7rec4).

The (cursory) conclusion is: The existence of a stable limit cycle in the linear unstable region is inconsistent with the result of the bifurcation analysis which delivers subcritical Hopf bifurcations. Hence, unstable limit cycles are predicted in the linear stable region for this analysis case. In order to understand this behaviour, more in depth considerations are necessary.

The above analysis reveals that the system behaviour cannot be examined only by local considerations such as semi-analytical bifurcation analysis using BIFDD. The coexistence of a subcritical bifurcation point (where an unstable limit cycle is born) and stable limit cycles in the linear unstable region could be a unique indicator for a possible existence of global bifurcation. In contrast to the Hopf bifurcation, global bifurcations involve large regions of the phase space rather than just the neighbourhood of a singular fixed point [6]. Thus, in the scope of the present work, the post-bifurcation state can only be determined through numerical integration of the ROM equations. For this purpose, the amplitudes of the limit cycles vs. the core inlet subcooling will be determined by numerical integration. Thereby, all the other parameters are fixed. The results are plotted in Fig. 13 and Fig. 14. The diagram shown in these figures is also known as bifurcation diagram. In particular, the global behaviour in the close neighbourhood of the stability boundary will be analysed.

The subcooling number N_{sub} is varied between 0.6 and 0.9 and the stable limit cycle amplitudes of the first azimuthal mode $A(n_1)$ are determined. Fig. 13, Fig. 14 and Fig. 15 summarises results of this analysis. The results show, that limit cycle amplitudes $A(n_1)$ decreases with decreasing core inlet subcooling. Below the critical value $N_{sub,c} \approx 0.63547$ (the Hopf conditions are fulfilled at $N_{sub,c}$) stable limit cycles still exist

(see Fig. 14). This means, **stable and unstable** limit cycles **coexist** in the linear stable region. The coexistence of stable and unstable limit cycles for $N_{sub} < N_{sub,c}$ is verified by numerical integration for $N_{sub} = 0.632$ by imposing different perturbation amplitudes $\delta \vec{X}(t)$ on the system. A sufficient small perturbation leads to a stable behaviour. But when a sufficient large perturbation is imposed on the system, the state variables are attracted by the limit cycle. This is presented in Fig. 15.

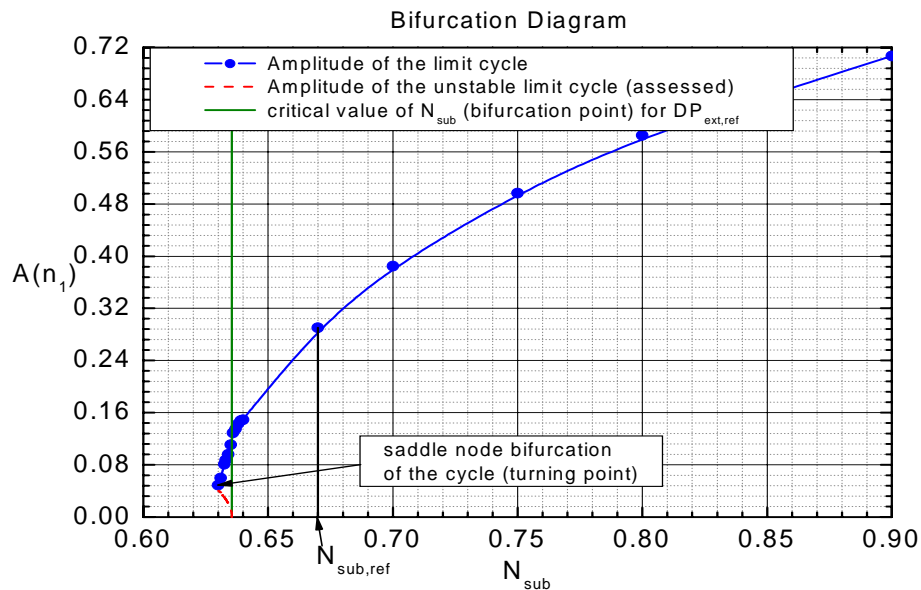


Fig. 13 The results of the numerical integration are plotted as bifurcation diagram, where N_{sub} is the bifurcation parameter and $DP_{ext} = DP_{ext,ref}$.

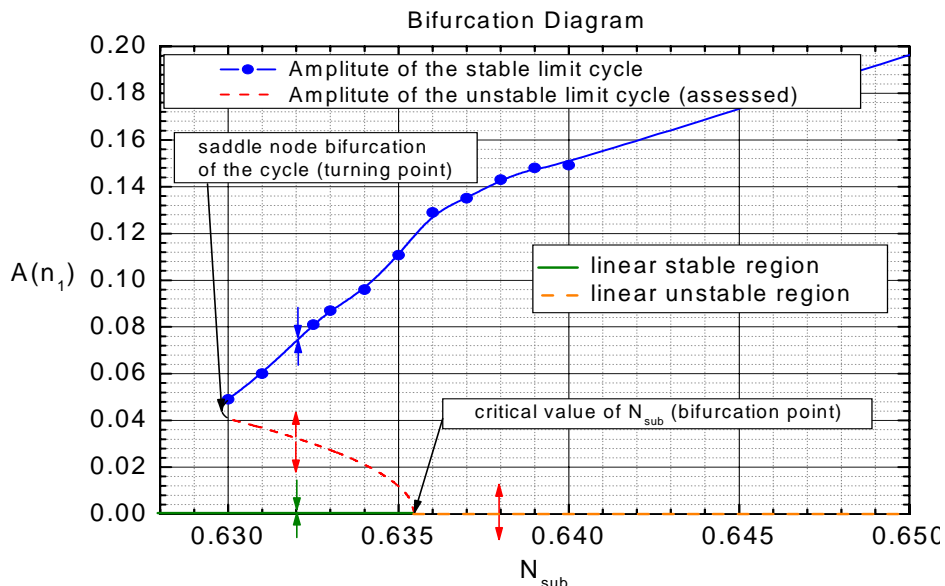


Fig. 14 ("Zoom in" of Fig. 13) Notice, the function of the amplitudes of the unstable limit cycle is an assessment only, not a calculated one.

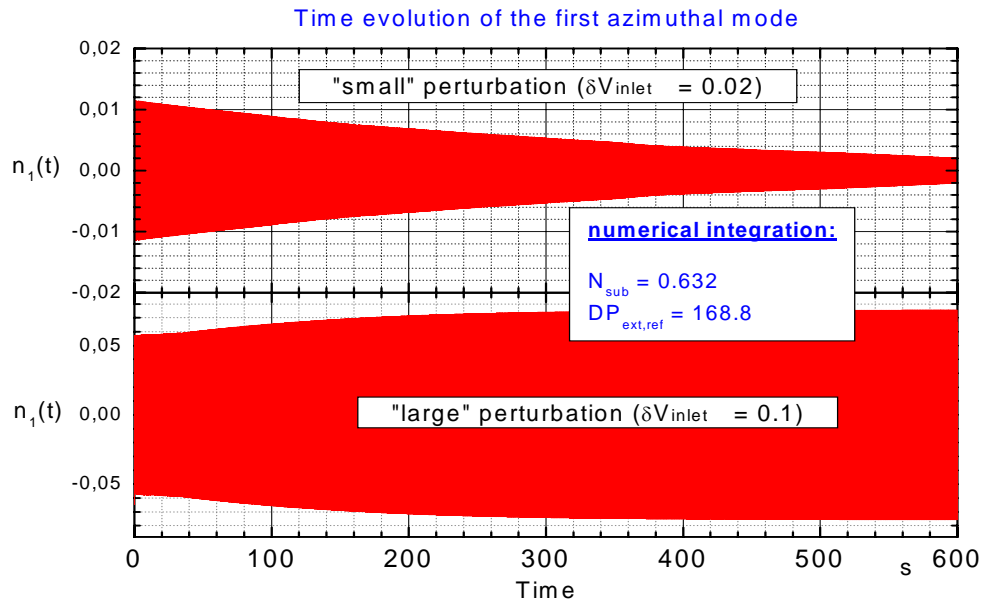


Fig. 15 This figure shows the results of numerical integration for $N_{sub} = 0.632$ and $DP_{ext} = DP_{ext,ref}$, where a sufficient small $\delta v_{inlet} = 0.02$ and sufficient large $\delta v_{inlet} = 0.1$ perturbation is imposed on the system. When a small perturbation is imposed on the system, the state variables are returning to the steady state solution (origin of the dynamical system). When a large perturbation is imposed on the system, the state variables are attracted by the limit cycle.

The amplitudes of the unstable limit cycle correspond to the boundary which separates the basin of attraction of the singular fixed point and basin of attraction of the stable limit cycle.

The analysis has shown that stable and unstable limit cycles do not exist for core inlet subcooling less than $N_{sub} < 0.63$. Hence, there is a critical core inlet subcooling $N_{sub,t}$, where the two limit cycles coalesce and annihilate. Due to (numerical) convergence problems of numerical integration, $N_{sub,t}$ cannot be calculated exactly. The estimated value of $N_{sub,t}$ is $N_{sub,t} \approx 0.63$. From this result, it can be concluded that in $N_{sub,t}$ there is a saddle-node bifurcation of a cycle. This bifurcation type belongs to the class of global bifurcations and is also referred to as turning point or fold bifurcation [6].

The behaviour of a dynamical system which undergoes a saddle-node bifurcation of a cycle is summarized in Fig. 16 (there is depicted the bifurcation diagram and the corresponding radial phase portraits). In the following Fig. 16 is discussed more in detail. To this end, the control parameter γ in Fig. 16 is assumed to be N_{sub} ($\gamma = N_{sub}$).

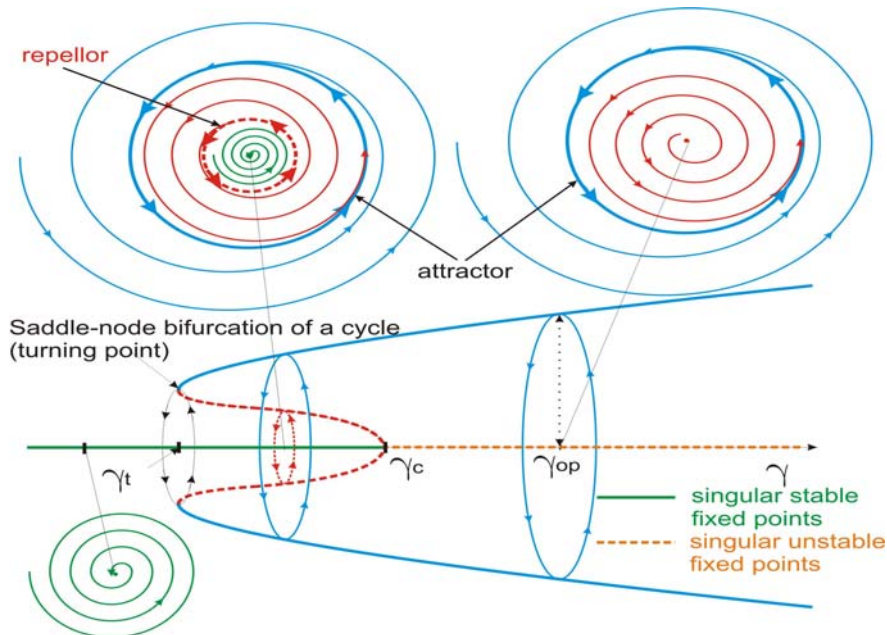


Fig. 16 This figure depicts a bifurcation diagram of a saddle-node bifurcation of a cycle and the corresponding radial phase portraits.

1. The bifurcation at γ_t is a saddle-node bifurcation of a cycle. In this point, a **stable and an unstable periodical solution** (limit cycle) are born (“out the clear blue sky”). As depicted in Fig. 16, the phase portrait is changing significantly when passing γ_t .
2. In the range $\gamma_t < \gamma < \gamma_c$, two qualitatively different stable states coexist, namely the origin (fixed point solutions) and the stable limit cycle. Both states are separated by an unstable limit cycle (see phase portrait in Fig. 16). In other words, due to the saddle-node bifurcation at $\gamma = \gamma_t$, a stable and an unstable limit cycle coexist with stable fixed points within the parameter range $\gamma_t < \gamma < \gamma_c$. One consequence is that the origin is stable to “small” perturbations, but not to “large” ones. In this sense the origin is locally stable, but not globally stable.
3. From the stability analysis point of view, a saddle-node bifurcation is operational safety significant, if the amplitude of the stable limit cycle is sufficient large. Supposing the system is in steady state (origin) below γ_t , and the control parameter γ is slowly increased. As long as $\gamma < \gamma_c$ the state remains at the origin. But at $\gamma = \gamma_c$ the origin loses stability and thus the slightest “nudge” will cause the state to jump to the limit cycle. In this case, the state will start to oscillate when reaching $\gamma = \gamma_c$ and the oscillations are growing as long as they will be attracted by the stable limit cycle. With further increase of γ , the state moves out along the limit cycle solution. But if γ is now reduced, the state remains on the stable limit cycle oscillation, even when γ is decreased below γ_c . The system will return to the origin when the control parameter γ is reduced below γ_t . This characteristic (called hysteresis) can be considered as a loss of reversibility as the control parameter is varied.

4. The system behaviour near the origin is similar to the behaviour of a system which undergoes a subcritical Hopf bifurcation at $\gamma = \gamma_c$.

To summarize, local ROM analyses for KKLc7_rec4 have shown that the bifurcation analyses and the numerical integration method provide consistent results only in a **small neighbourhood of the Hopf bifurcation point γ_c and in the vicinity of the origin** (local consistency; Notice, it must be differentiated between the terminus "a small neighbourhood in **parameter space**" and "a small neighbourhood in the **phase or state space**"; see Fig. 17). In order to study the global character of the nonlinear system, numerical integration is necessary. For this purpose, numerical integration of the ROM equation system have been carried out, where N_{sub} was varied in the range $[0.62, \dots, 0.9]$. The analyses have shown that in the range $N_{sub,t} < N_{sub} < 0.9$ ($N_{sub,t} = 0.63$) stable limit cycles exist, even though the bifurcation analysis predicts only unstable limit cycles for $N_{sub} < N_{sub,c}$ ($N_{sub,c}$ is the critical bifurcation parameter, for which the Hopf conditions are fulfilled). Hence, in the reference OP the state variables will also be attracted by the limit cycle. In addition to that, for $N_{sub} \in [N_{sub,t}, \dots, N_{sub,c}]$ with $N_{sub,t} < N_{sub,c}$ stable fixed point solution, unstable periodical solution and stable periodical solution coexist. Below $N_{sub,t}$, only stable fixed point solution exist.

One would think that the predictions of BIFDD and the results of the numerical integration are inconsistent. But notice: BIFDD is based on local methods. Thus the stability investigation using BIFDD is concentrated only in the origin of the system close to the critical parameter γ_c (see Fig. 17).

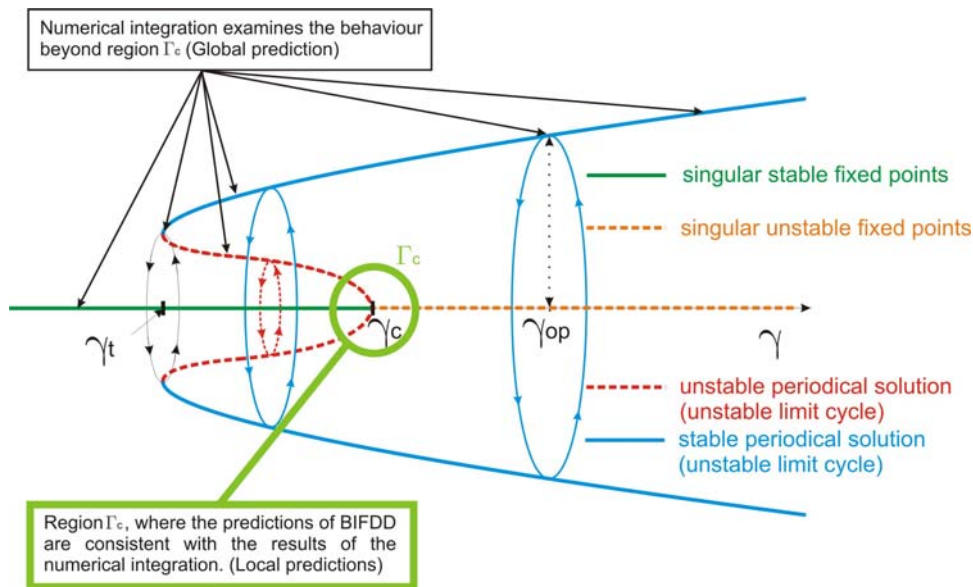


Fig. 17 Summary of the main characteristics of a system, which undergoes a saddle-node bifurcation of cycles. The results of BIFDD are only valid locally in the close neighbourhood of the origin of the system close to γ_c (region Γ_c). The global character of the nonlinear system can only be examined by numerical integration.

5. CONCLUSIONS

The nonlinear analysis has shown that the stability behaviour of the reference OP and its close neighbourhood can be simulated reliably by the new ROM. In this OP, the results of RAMONA5 and ROM are locally consistent. Under stability related parameter variations the stability behaviour calculated by both ROM and RAM are consistent. Hence, the new ROM and the new procedure for the calculation of the ROM input data are qualified for BWR stability analysis in the framework of the new approach (RAM-ROM methodology): **The ROM analysis provides an overview about types of instability which have to be expected** (in a selected BWR power-flow map region). **With the RAM (system code), these phenomena are analyzed in detail (but with pre-information received by ROM analysis).** Hence the reliability of the stability analysis is improved.

A more in depth nonlinear analysis, where global aspects are taken into account, revealed a system behavior which can be explained by the existence of a saddle-node bifurcation of cycles.

Methods and results of this work will be the basis for further nonlinear BWR stability analyses. An in-depth understanding of the BWR stability behavior should definitely contribute to an efficient design of "detection and suppression" systems.

NOMENCLATURE

The subcooling number N_{sub} represents the core inlet subcooling and appears as a boundary condition in the single phase energy equation. The subcooling number and the steady state external pressure drop are written as

$$N_{sub} = \frac{(h_{sat}^* - h_{inlet}^*)}{\Delta h_{fg}^*} \cdot \frac{\Delta \rho^*}{\rho_g^*} \quad DP_{ext} = \frac{DP_{ext}^*}{\rho_f^* v_0^{*2}}, \quad (1)$$

where h_{sat}^* is the saturation enthalpy, h_{inlet}^* inlet enthalpy, $\Delta \rho^* = \rho_f^* - \rho_g^*$ liquid-vapor density, $\Delta h_{fg}^* = h_g^* - h_f^*$ liquid-vapor enthalpy, v_0^* reference velocity and DP_{ext}^* is the steady state pressure drop over the vessel high. $\vec{X}(t)$ with $\vec{X}(t) \in \mathbb{R}^n$ is the state vector of the dynamical system. An asterisk on a variable or parameter indicates the original dimensional quantity and any quantity without an asterisk is dimensionless.

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