Potential and limitations of the soap film analogy

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Abstract During the pre-computer era the soap film analogy was used to analyse potential flow in complex geometries. Such devices were developed around 1960 in Dresden to investigate the flow in high-pressure turbine cascades. The present paper briefly describes the technique and reports on the comparison between historical results, achieved with the soap film analogy, and numerical simulations using a small computer code. The juxtaposition of both illustrates the capabilities of the methods and the historical progress made since then.

1 Introduction

During the 1950ies and 1960ies, computers were not yet widely available, so that other simple and versatile methods were invented to investigate flows in complex geometries. Analogy methods are based on the following reasoning. (1) Certain physical phenomena can be described by partial differential equations. (2) Various systems might, under appropriate simplifications, be described by the same differential equations. (3) If this occurs, observations in system A can provide information about system B, if an appropriate rule for conversion of quantities is established. Werner Albring and his co-workers hence conceived such devices and improved the corresponding methods. Three different analogies were addressed: the shallow-water analogy for compressible flow, the analogy of electric potential for potential flow, and the soap film analogy (SFA), also for potential flow [1]. The present contribution addressed the third of them and provides a few details about this technology. It is a substantially enlarged version of a conference paper on this topic [2].

Dresden at that time was a centre of turbine construction, and the SFA was used to investigate the flow in high-pressure turbine cascades [3]. The method works as follows. A thin soap film is stretched over a model of the cross section of the turbine cascade. It minimizes its surface energy, revealing a solution of the Young-Laplace law [3]

$$4\sigma H = \Delta p, \qquad H = \left(\frac{1}{r_1} + \frac{1}{r_2}\right), \tag{1}$$

with H the mean curvature, and r_1 and r_2 the principal curvatures at any given point of the soap film. For small mean curvatures, equation (1) can be transformed into a Laplacian differential equation for the elongation z of the soap film. With zero pressure difference Δp between both sides of the soap film, this leads to a Laplace differential equation for z. On the other hand, the stream function Ψ of an irrotational, inviscid, solenoidal flow also obeys a Laplace equation. This leads to the analogy

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$$4\sigma\left(\frac{d^2z}{dx^2}+\frac{d^2z}{dy^2}\right)=0 \qquad \Leftrightarrow \qquad \frac{d^2\Psi}{dx^2}+\frac{d^2\Psi}{dy^2}=0 \tag{2}$$

From the stream function one can derive the fluid velocity of a potential flow field $\boldsymbol{u}(x,y)$ in two dimensions by differentiation according to

$$\boldsymbol{u}=\left(\frac{d\Psi}{dy},-\frac{d\Psi}{dx}\right). \tag{3}$$

Using the analogy between Ψ and z, one can derive isoclines and isotachs of the potential flow by analysing the tilt of the soap film lamella based on the following correspondences:

$$\begin{aligned} &isotachs: |\nabla z| = const. &\Leftrightarrow\quad &|\boldsymbol{u}| = const. \\ &isoclines: \nabla z/|\nabla z| = const. &\Leftrightarrow\quad &\boldsymbol{u}/|\boldsymbol{u}| = const. \end{aligned} \tag{4}$$

2 The soap film apparatus

The historical example was concerned with the flow through a linear cascade of high-pressure profiles. Plane blades of the same shape as in the industrial application were constructed and mounted in a frame, as indentified by the label A in Figure 1. The soap film was created in the vertical plane containing the space between the blades and an exterior frame. The latter can be tilted and shifted perpendicularly to the vertical plane to realize different flow conditions. To determine the shape and tilt of the soap film, a picture of the reflection of a bright spherical lattice was taken (Figure 1 and Figure 2). If the orientation of the soap film fulfills certain geometrical conditions, it reflects a certain element of the white lattice and appears bright in the picture taken by the camera. All pieces of the soap film with the same magnitude of tilt reflect the same circle with a given latitude of the lattice. This mechanism is independent of the direction of the tilt. Due to the analogy, the reflections draw the shape of an isotach in the picture. Similarly, all pieces of the soap film with the same direction of tilt reflect the same meridian of the spherical lattice. This mechanism is independent of the magnitude of the tilt. Due to the analogy, the reflections draw an isocline in the picture. In that way, isoclines and isotachs of a potential flow through the model turbine cascade can be derived [3,4].

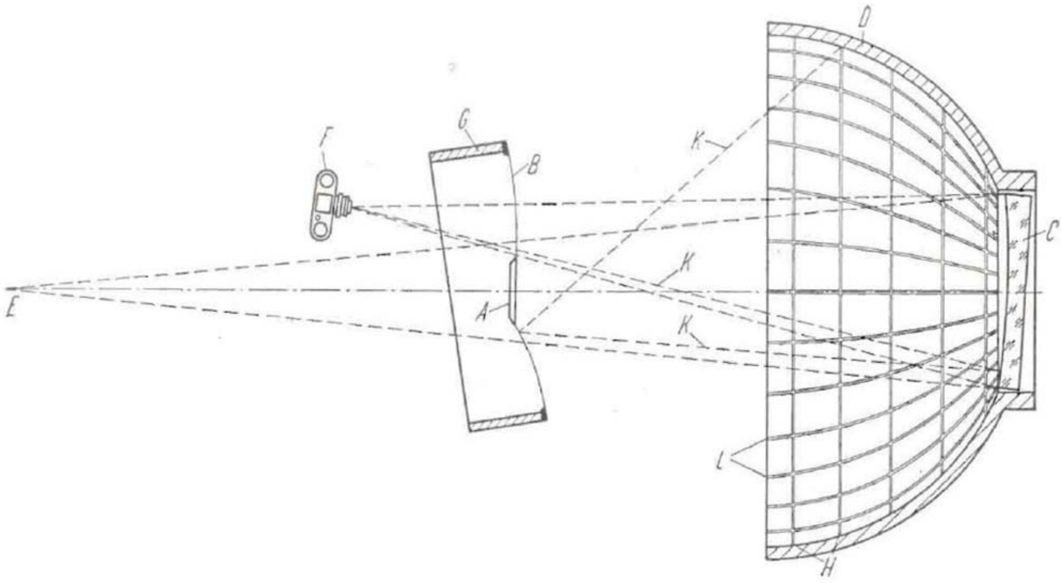

Figure 1. Sketch of the light path between the illuminated lattice and the camera [4]. Light from a meridian L (iscolines) or a circle of latitude H (isotachs) of the illuminated lattice D is reflected by the curved soap film B, then reflected by the concave mirror C, passes through the soap film and is recorded by the camera F.

The boundary conditions for the Laplacian differential equation (1) are imposed by fixing the soap film on a purpose-shaped frame (Figure 2a), featuring the desired slopes corresponding to the desired in- and outflow conditions.

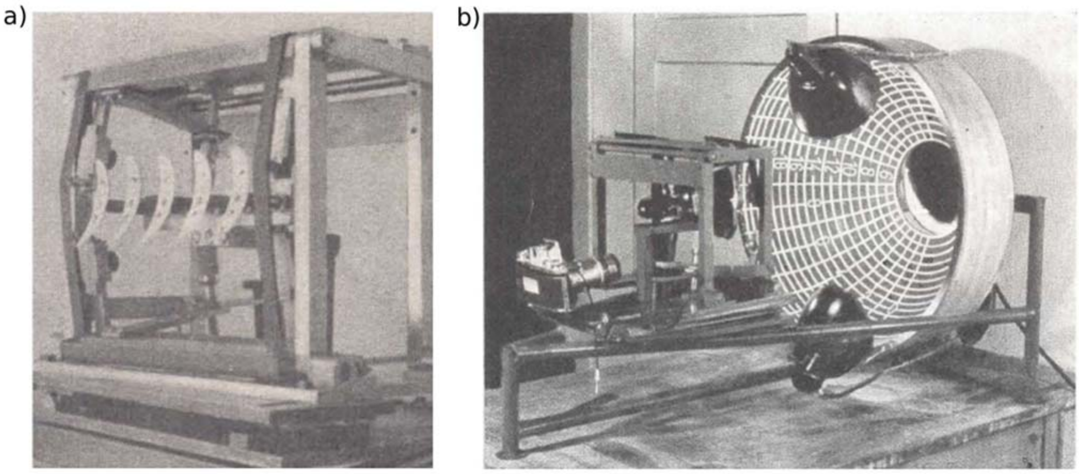

Figure 2. Pictures of the historical SFA apparatus [3]. a) Frame and model blades, to which the soap film is attached. b) Photograph of the entire SFA apparatus including the camera at the left, the blades with the film in the centre and the illuminated spherical lattice at the right.

According to equation (5), the velocity in x-direction corresponds to the slope in y-direction and vice-versa. Therefore, a tilted boundary of the frame creates what corresponds to an inflow velocity in direction $\boldsymbol{e}_n$ orthogonal to the boundary.

$$
\begin{aligned}
u_x &= \frac{d\Psi}{dy} &\Leftrightarrow\quad& \frac{dz}{dy} \\
u_y &= -\frac{d\Psi}{dx} &\Leftrightarrow\quad& -\frac{dz}{dx} \\
\boldsymbol{u} \bullet \boldsymbol{e}_n &= 0 &\Leftrightarrow\quad& z = const\,.
\end{aligned}
\tag{5}
$$

The tangential velocity cannot be imposed directly, but only in combination with other boundaries. An obstacle is implemented by putting its rim on a constant height z. In this way, the normal velocity at its rim is zero, thus implementing an impermeable boundary of the object.

3 Numerical method

An own simple Laplacian solver to determine Ψ in equation (1) was implemented and employed for comparison. It uses a central difference scheme on a cartesian grid with an immersed boundary method of first order for curved boundaries by just setting values at grid points inside a solid to a given value. Present industrial investigations, if they apply potential theory at preliminary states of the analysis, of course are conducted with different numerical methods, such as curvilinear, boundary-fitted, non-orthogonal grids. The present method is elementary on purpose for illustration of simplicity and compensates better convergence of more sophisticated schemes by a finer grid. The same Dirichlet-type boundary conditions were imposed on the boundary of the blades and at all other boundaries, as in the SFA experiment. Neumann boundary conditions at the in- and outflow and periodic conditions in the perpendicular direction would have been closer to the turbomachinery application but could not be realized with the SFA. They were hence not used for the numerical solution neither.

4 Validation using a cylinder

In order to assess the accuracy of the SFA and the present numerical method, the potential flow around a cylinder was investigated as a reference, since this flow is known analytically [5]. Figure 3a and Figure 3b show isotachs of the flow field, obtained from the SFA and the numerical solver, respectively. It was verified that the numerical result almost exactly agrees with the analytical solution (not shown here). The deviation due to the Dirichlet-type boundary condition is negligible. The SFA result shows slight differences with respect to the numerical solution and a slight break of symmetry, left-right and top-bottom. The reason for the asymmetry is most likely due to the optical projection. Also, asymmetries in the geometry of the SFA model and gravitational effects might be at the origin. Symmetric deviations might result from the finite curvature of the soap film, resulting in a small deviation from equation (2).

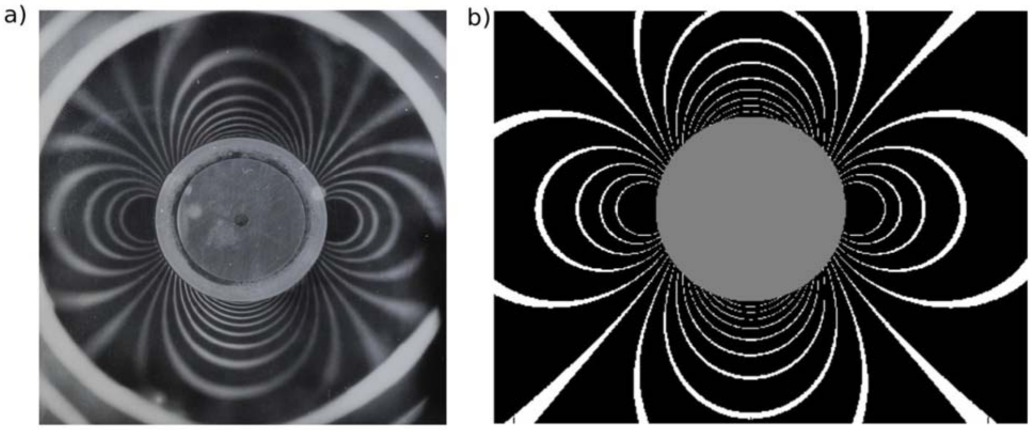

Figure 3. Isotachs of the potential flow around a circular. a) Result of the SFA [3]. b) Result of the numerical computation. Colouring of the numerical result was adjusted so as to mimic the colouring of the SFA experiment as closely as possible.

5 Results for a high-pressure turbine cascade

It has been and still is common practice to investigate the properties of stages of axil turbines under the approximation of a linear arrangement of the blades to address fundamental issues. In [3], this was done for a high-pressure cascade under various conditions. Figure 4a reproduces a characteristic result of this campaign in comparison to the numerical result for the same cascade.

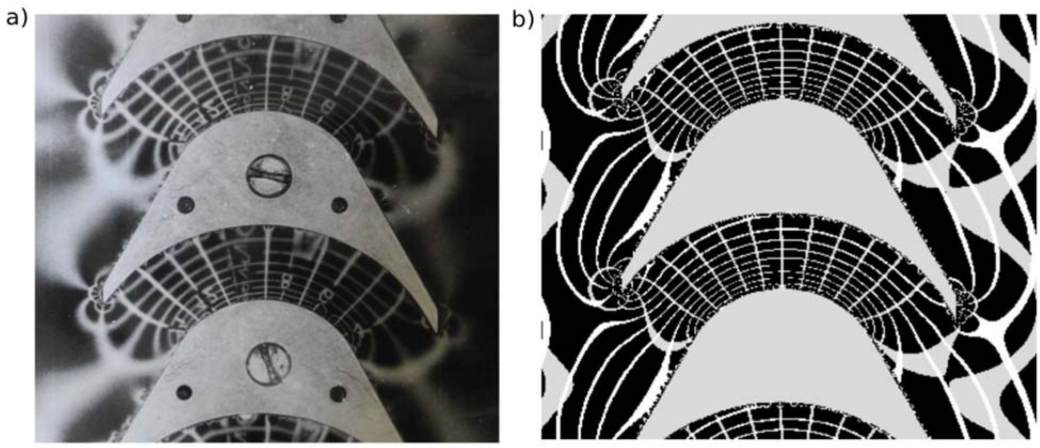

Figure 4. Isoclines and isotachs of the potential flow through a high-pressure turbine cascade. a) Results of the SFA [3]. b) Results of the numerical computation. Colouring of the numerical results was adjusted so as to mimic colouring of the SFA experiment as closely as possible.

The results displayed in Figure 4 illustrate, that with the SFA very satisfactory accuracy of this potential flow in highly complex geometry could be obtained. Comparing the two pictures provides an impression about the kind of deviation of the results obtained with the SFA from the numerical solution. Close to the blades small scale distortions appear in the SFA that could result from unevenness of the blade elements. Before and after the cascade, the lines become blurred, due to the small gradients in these regions. For the turbine cascade an additional degree of freedom is the value of Ψ on the surface of the blades, equivalent to the vertical position of the model blades with the SFA. This is linked to the angle of attack and was adjusted when conducting the SFA experiment, as well as the numerical solution, to obtain the desired value. The upstream flow is quite sensitive to this issue so that deviations of isoclines and isotachs between the two realizations can occur in this region. In the channel between the blades the isoclines and isotachs coincide well. Comparing two channels reveals that the small differences between SFA and simulation are systematic and cannot result from tilting of the blades in the model.

6 Current state

For the STT 2014 the original apparatus was taken from the archives, cleaned and put to use again. It will be on display in working conditions during the event, so that attendants can appreciate of the method themselves.

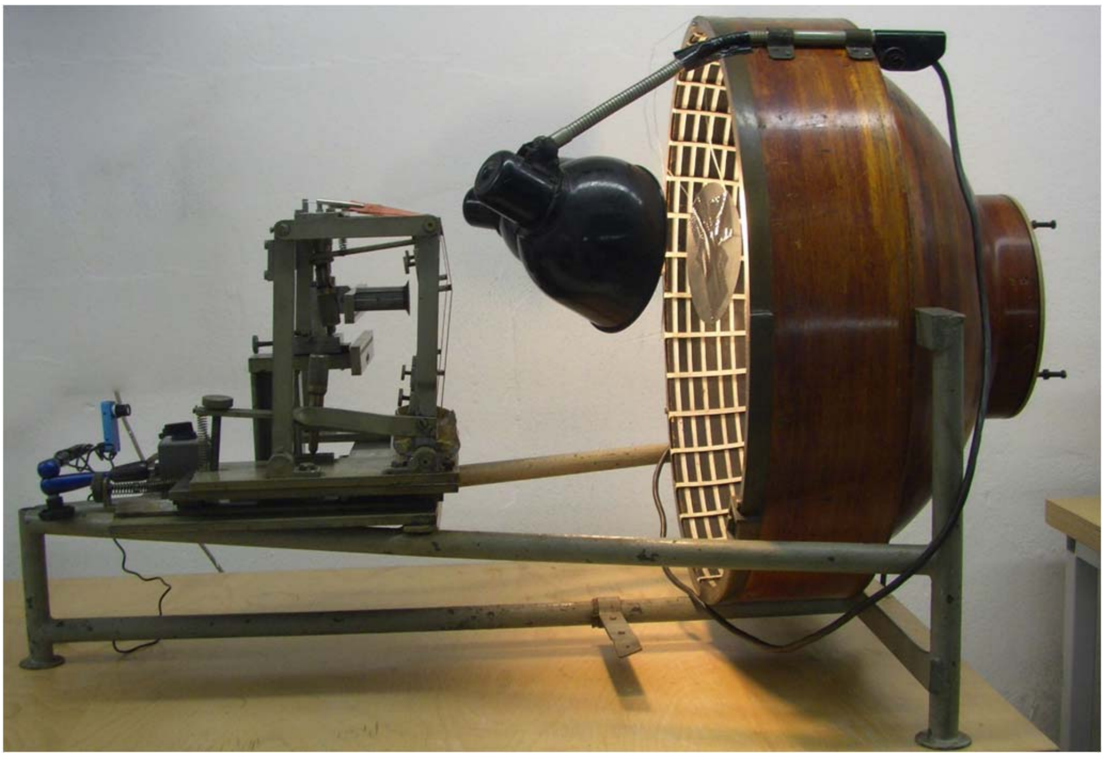

Figure 5. The historical apparatus mounted again in 2014. Progress is visible in that the analogue *Leica* camera was replaced by a small digital electronic camera.

7 Conclusions

The SFA is an adequate tool for the estimation of potential flow in complex geometries. However, one is restricted in the choice of boundary conditions. Only the value of the stream function, e.g. its tangential slope, can be prescribed and hence the normal velocity. Also the accuracy is limited due to deviations in the optical imaging and additional terms in equation (1) for higher curvatures. In terms of expenditure of time and money the SFA apparatus requires about 5000 Euro and 4 weeks to be manufactured by a workshop. Programming and validating a Laplacian solver required about two weeks and a normal Desktop PC. Furthermore, the numerical solution is obtained almost instantly, so that today the numerical method is preferred for investigating potential flow. Nevertheless, the SFA offers an alternative way of solving potential flow in complex geometries that enabled researchers before the computer era to investigate potential flows in practically relevant cases and to predict key data of turbine cascades which could not be obtained otherwise.

Acknowledgements

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