TECHNISCHE UNIVERSITÄT DRESDEN

Almost Symmetric Linear Arc Monadic Datalog and Transitive Tournaments

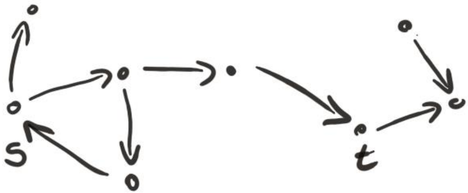
Florian Starke

joint work with Sebastian Meyer

European Research Council
Established by the European Commission

Directed Reachability and Datalog

Directed Reachability
and Datalog



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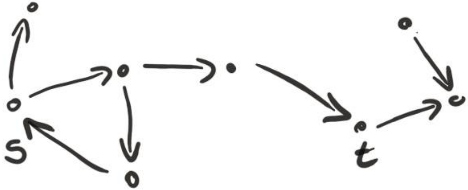
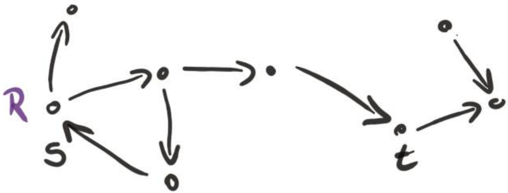


Π_{DR}

$$R(x) :- S(x) \mid \exists y\, R(y) \wedge (y \to x)$$

$$\text{goal} :- R(x) \wedge t(x)$$

Directed Reachability and Datalog



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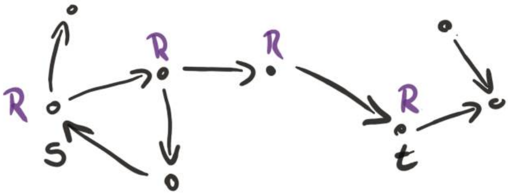

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Directed Reachability and Constraint Satisfaction Problems

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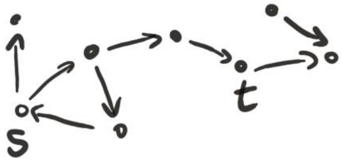

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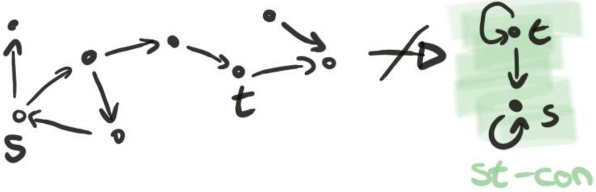

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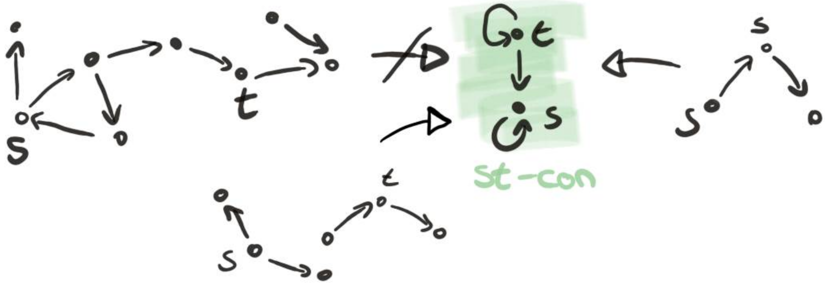

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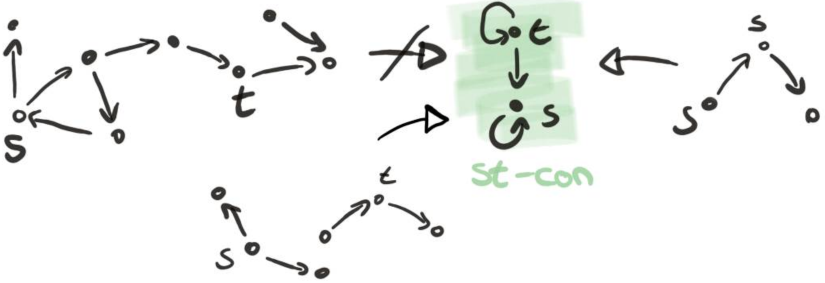

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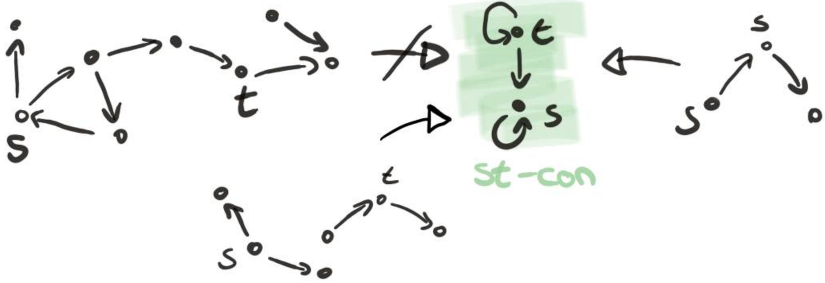

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$$= \{ G \mid G \text{ finite, there is } \underline{\text{no}} \text{ directed path from } s \text{ to } t \text{ in } G \}$$

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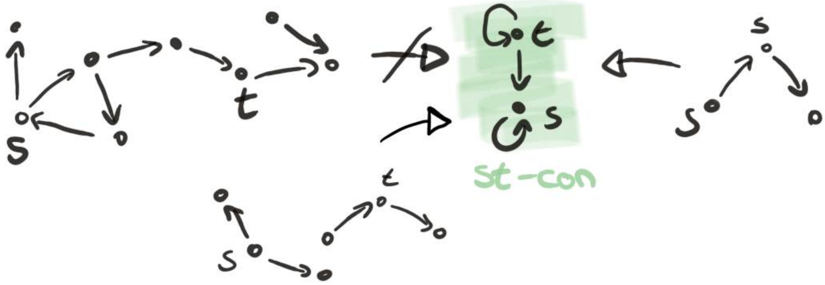

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$$= \{ G \mid G \text{ finite}, \Pi_{DR} \text{ does } \underline{\text{not}} \text{ derive Goal on } G \}$$

Datalog | vs | Constraint Satisfaction Problems

Datalog vs Constraint Satisfaction Problems

DL Program Π with EDBs from τ

Datalog VS Constraint Satisfaction Problems

DL Program Π with EDBs from τ | CSP ($\mathbb{B}$) template
(finite) τ-structure

Datalog vs Constraint Satisfaction Problems

DL Program Π with EDBs from τ

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$\mathbb{C} \to \mathbb{A}$

Datalog vs Constraint Satisfaction Problems

DL Program Π with EDBs from τ | CSP (B) template

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$C \to A$

Π derives Goal on C $\Rightarrow$ Π derives Goal on A

Datalog vs Constraint Satisfaction Problems

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Datalog vs Constraint Satisfaction Problems

DL Program Π with EDBs from τ	$\mathrm{CSP}(\mathbb{B})$ template (finite) τ-structure

$$C \to A$$

Π derives Goal on C $\Rightarrow$ Π derives Goal on A	$\begin{tikzcd} C \arrow[r] \arrow[rr, bend left] & A \arrow[r] & B \end{tikzcd}$

Datalog vs Constraint Satisfaction Problems

DL Program Π with EDBs from τ	CSP($\mathbb{B}$) template (finite) τ-structure

$$\mathbb{C} \to \mathbb{A}$$

Π derives Goal on $\mathbb{C}$ $\Rightarrow$ Π derives Goal on $\mathbb{A}$	$\begin{tikzcd} \mathbb{C} \arrow[r] \arrow[rr, bend left] & \mathbb{A} \arrow[r] & \mathbb{B} \end{tikzcd}$ $\mathbb{C} \not\to \mathbb{B} \Rightarrow \mathbb{A} \not\to \mathbb{B}$

Datalog vs Constraint Satisfaction Problems

DL Program Π with EDBs from τ	CSP(B) template; (finite) τ-structure

$C \to A$

Π derives Goal on C $\Rightarrow$ Π derives Goal on A	$C \to A \to B$; $C \not\to B \Rightarrow A \not\to B$

Π solves CSP(B) iff

CSP(B) $= \{ A \mid A$ finite, Π does not derive Goal on $A \}$

Not all CSPs are solved by Datalog

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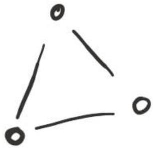

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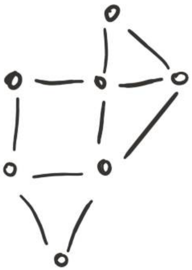

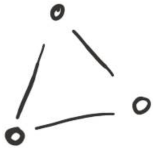

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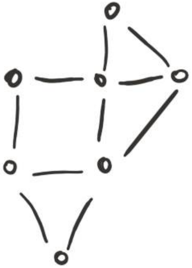

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NP-complete

Not all CSPs are solved by Datalog

CSP(△) = { G | G is 3-colourable }

NP-complete, so not solved by Datalog

Polimorphismes

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$$\mathrm{Pol}(\text{st-con}) \ni \min$$

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$$\begin{tikzcd} 0 \arrow[loop below] \arrow[r] & 1 \arrow[loop below] \end{tikzcd}$$

$t = \{0\}$

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$$\begin{array}{l} \min(1 \quad 1 \quad \dots \quad 0) = 0 \\ \qquad\ \ \uparrow \quad \uparrow \qquad\quad \uparrow \qquad \uparrow \\ \min(0 \quad 1 \quad \dots \quad 0) = 0 \end{array}$$

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Theorem

CSP($\mathbb{B}$) is solved by an AM Datalog Program

iff

Pol($\mathbb{B}$) contains totally symmetric Polimorphisms of all arities

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IDBs are unary

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Theorem

$\boxed{\text{CSP}(\mathbb{B}) \text{ is solved by an AM Datalog Program iff Pol}(\mathbb{B}) \text{ contains totally symmetric polimorphisms of all arities}}$

at most one EDB in body

IDBs are uninary

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A minor condition is a set of identities of the form
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$\mathbb{A} \leq \mathbb{B}$ if all minor conditions satisfied by $\mathrm{Pol}(\mathbb{A})$ are satisfied by $\mathrm{Pol}(\mathbb{B})$

Minor Conditions & Datalog Fragments

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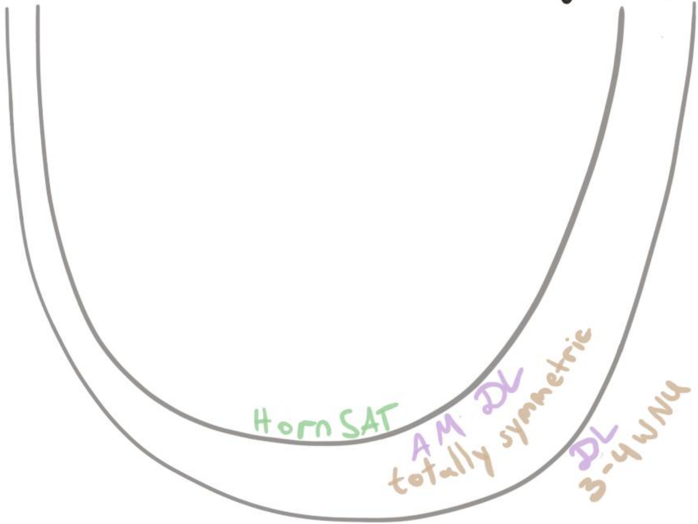

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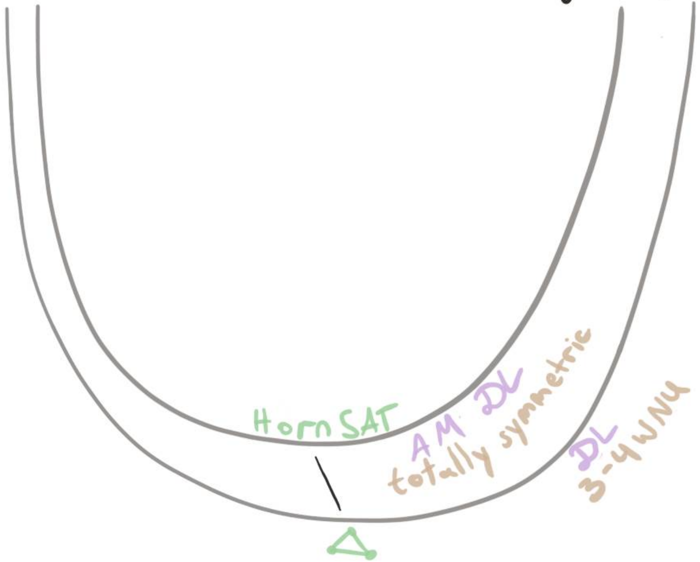

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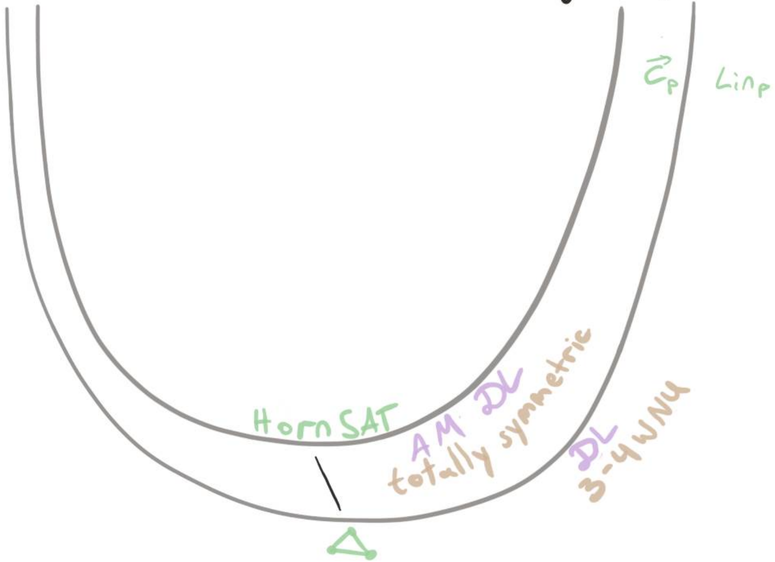

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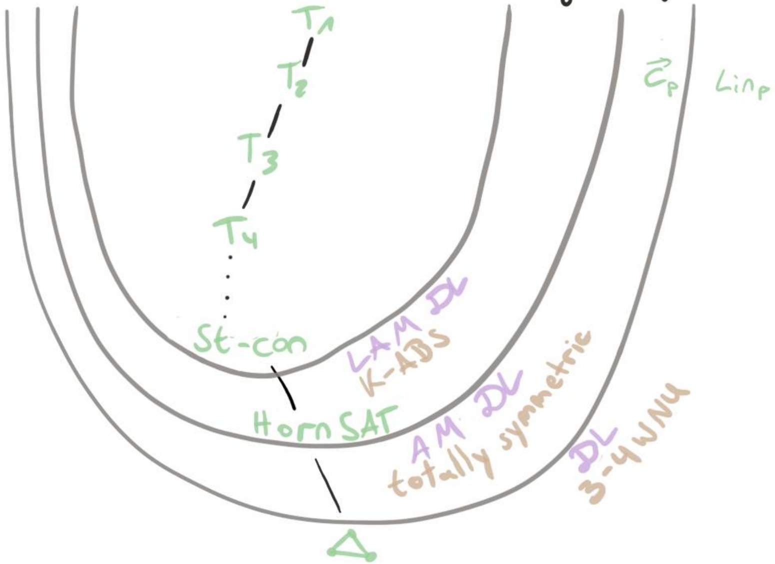

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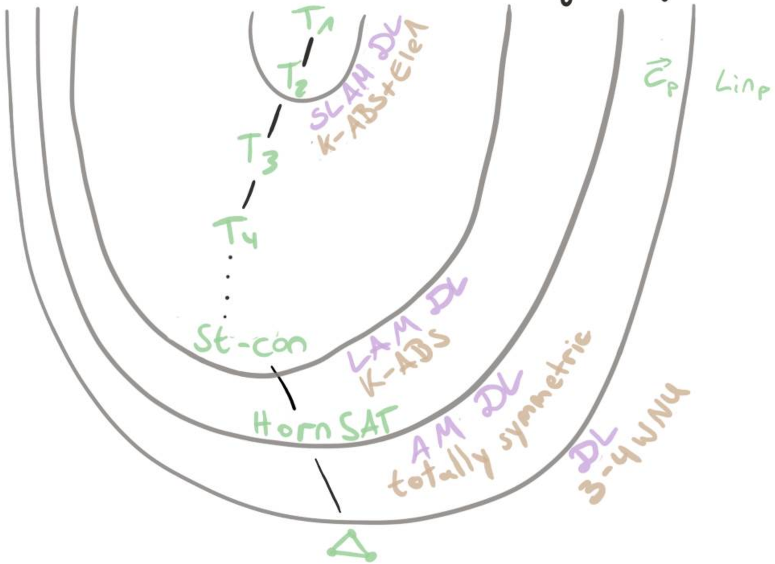

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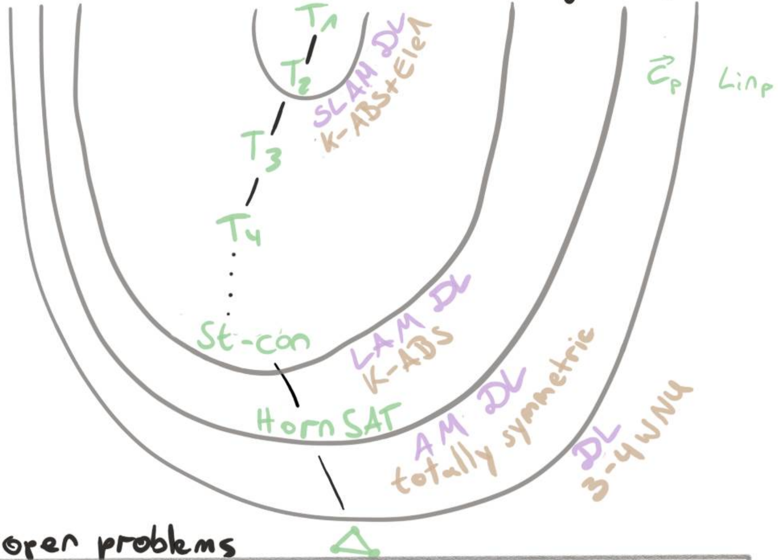

Big open problems

$\boxed{\text{find minor conditions for } l\text{ DL and } sl \text{ DL}}$

CSPs of Transitive Tournaments

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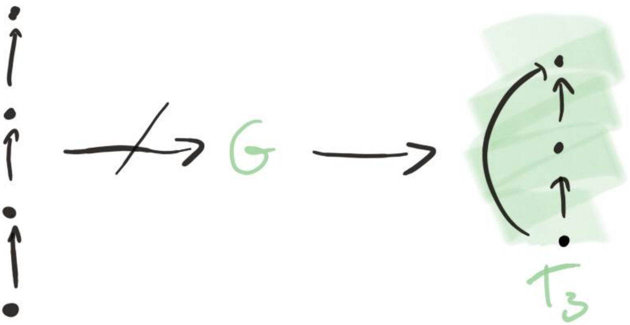

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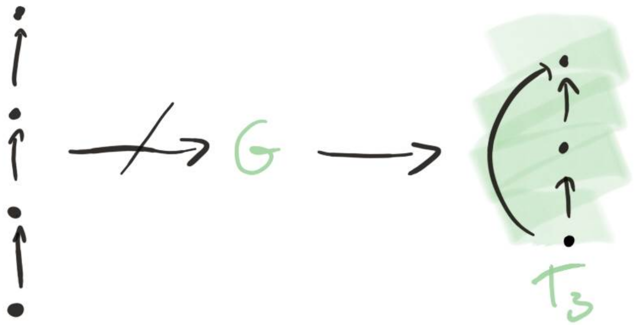

Goal: $\neg \exists x_0\, x_1\, x_2\, x_3 \quad x_0 \to x_1 \to x_2 \to x_3$

CSPs of Transitive Tournaments

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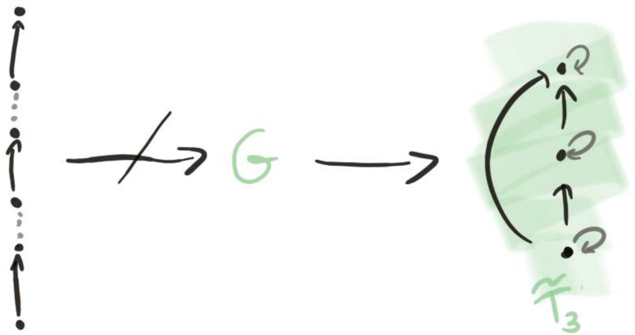

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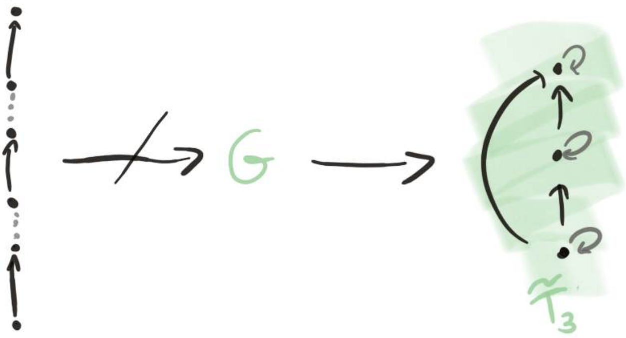

$P_1(x) :- \exists y \; y \to x$

$P_2(x) :- \exists y \; P_1(y) \wedge y \to x$

$\text{Goal} :- \exists x, y \; P_2(y) \wedge y \to x$

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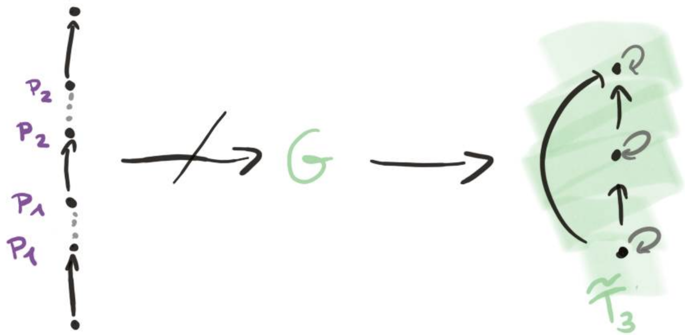

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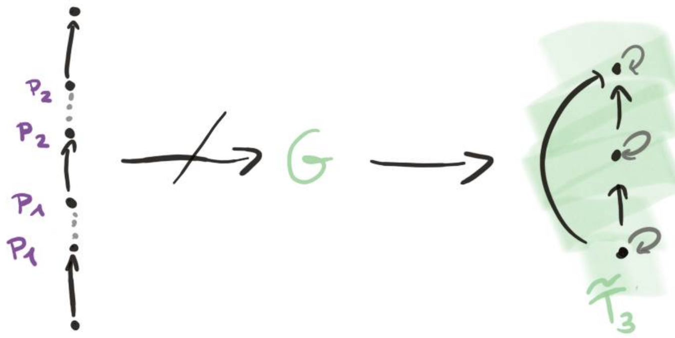

$P_1(x) :- \exists y \; y \to x \mid \exists y \; P_1(y) \land y \to x \mid \exists y \; P_1(y) \land y \leftarrow x$

$P_2(x) :- \exists y \; P_1(y) \land y \to x \mid \exists y \; P_2(y) \land y \to x \mid \exists y \; P_2(y) \land y \leftarrow x$

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n-almost symmetric linear arc monadic Datalog

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Definition

Π is a S_n LAM DL program if

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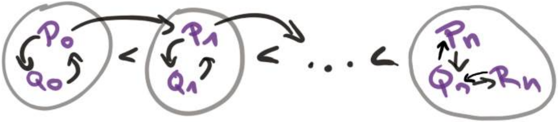

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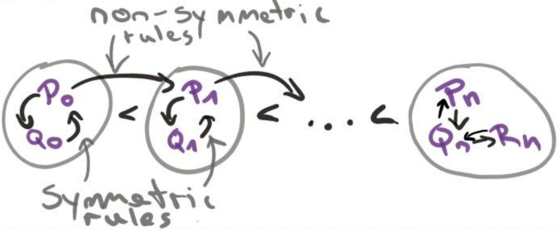

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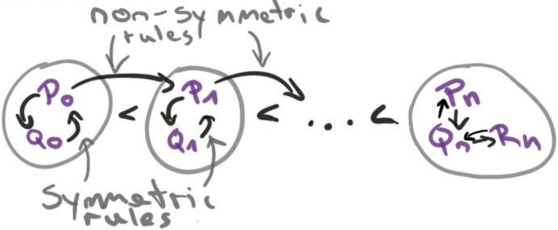

every derivation of Π applies non-symmetric rules at most n times

Results

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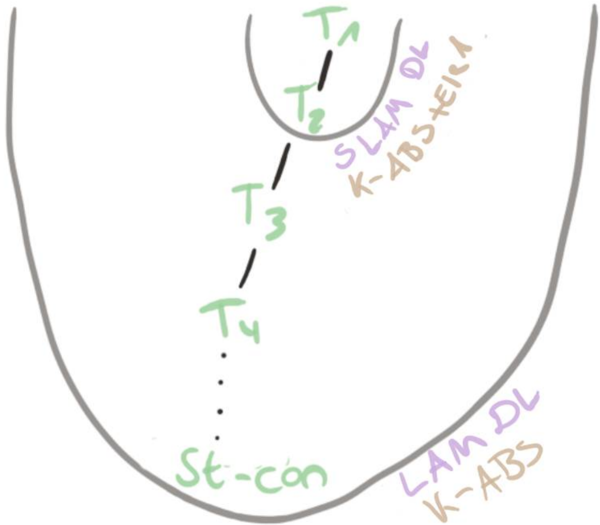

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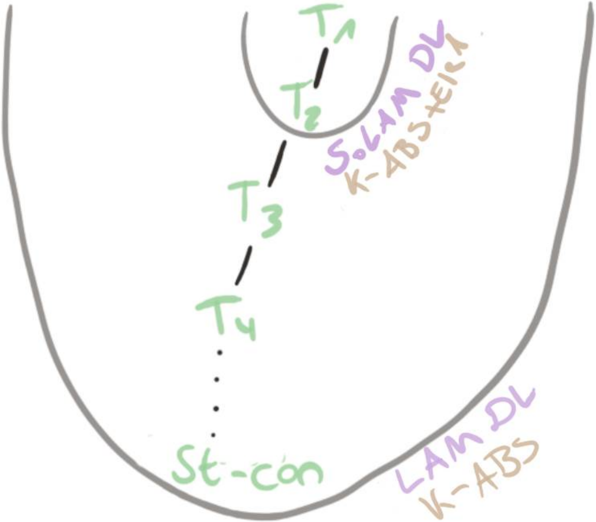

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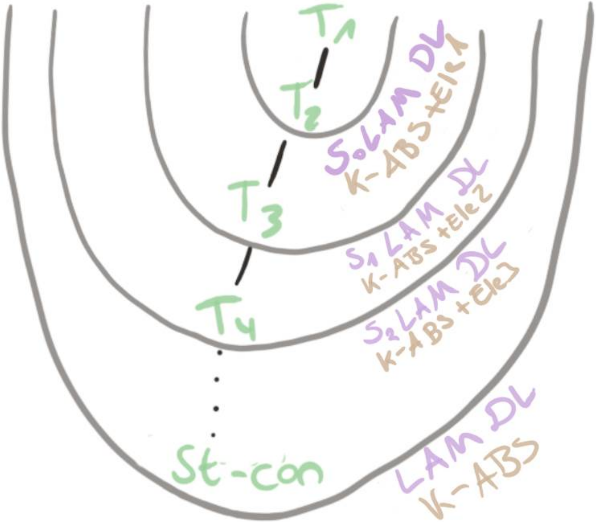

T1
T2
T3
T4
St-con
S0LAM DL
K-ABS+Ele1
S1LAM DL
K-ABS+Ele2
S2LAM DL
K-ABS+Ele3
LAM DL
K-ABS

Results

Thank you for your attention