Parabolic Cataland

A Type-*A* Story

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Combinatoire Énumérative et Analytique, IRIF

Outline

1. Parabolic Cataland
2. Bijections in Parabolic Cataland
3. Posets in Parabolic Cataland
4. Chapoton Triangles in Parabolic Cataland

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1 Parabolic Cataland

2 Bijections in Parabolic Cataland

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4 Chapoton Triangles in Parabolic Cataland

Parabolic 231-Avoiding Permutations

Parabolic quotients

- $w \in \mathfrak{S}_n$

12 3 11 13 1 2 6 4 9 10 15 7 8 14 5

Parabolic 231-Avoiding Permutations

- $w \in \mathfrak{S}_\alpha$

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$\alpha = (1,3,1,2,4,3,1)$

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- **$(\alpha, 231)$-pattern**: a triple (i,j,k) with $i < j < k$ in different α-regions such that $w(i) < w(j)$ and (i,k) is a descent

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- **$(\alpha, 231)$-avoiding**: does not have an $(\alpha, 231)$-pattern $\rightsquigarrow \mathfrak{S}_\alpha(231)$

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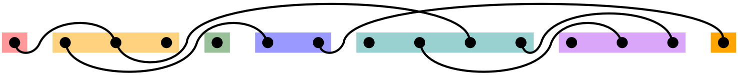

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- **noncrossing**: no bumps cross in the diagram $\rightsquigarrow NC_\alpha$

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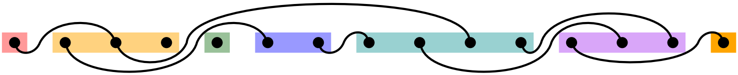

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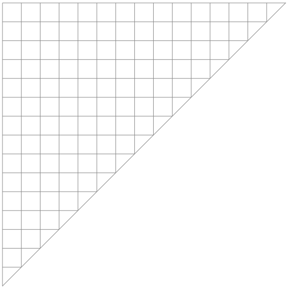

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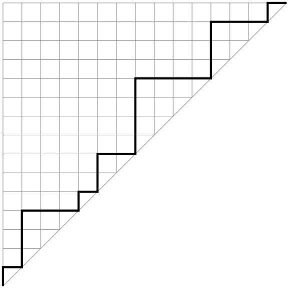

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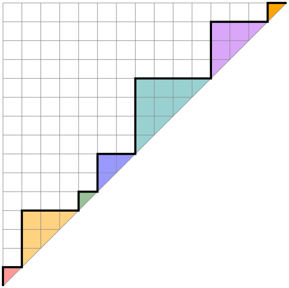

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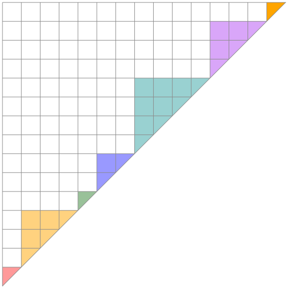

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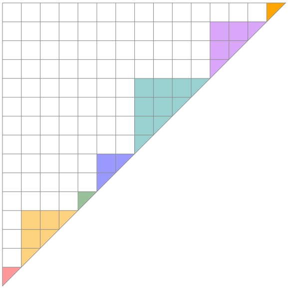

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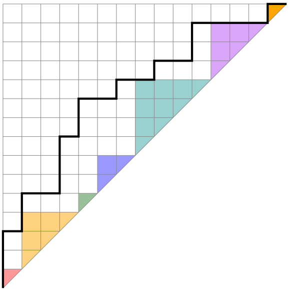

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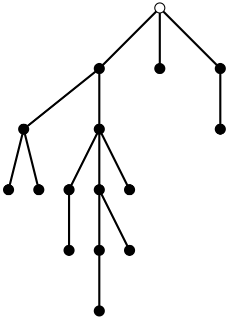

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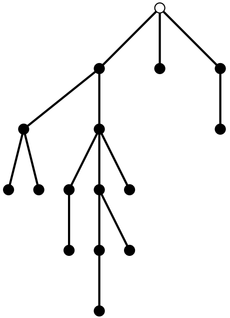

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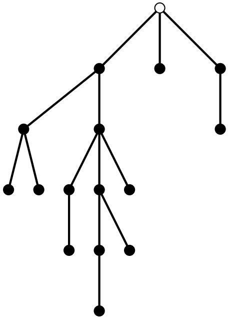

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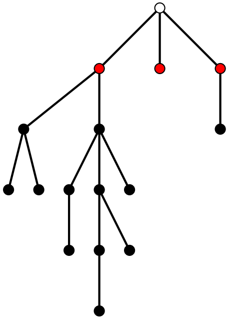

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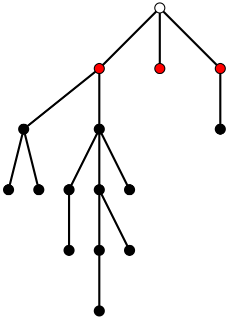
$3 < 4 \rightsquigarrow$ Failure!



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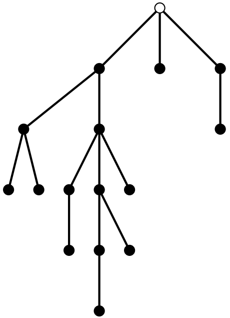

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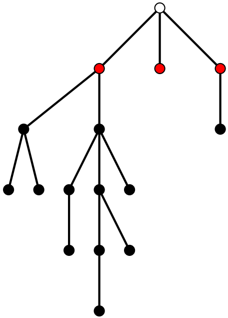

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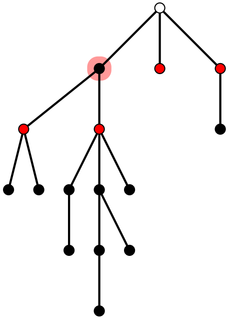

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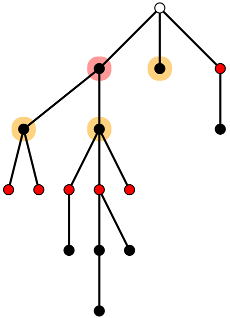

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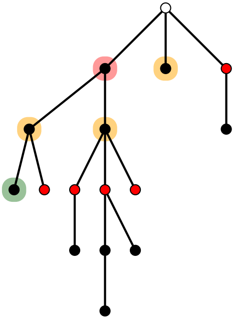

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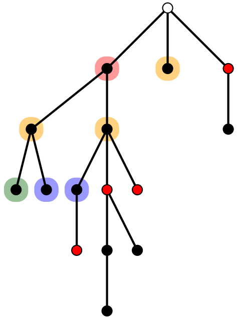

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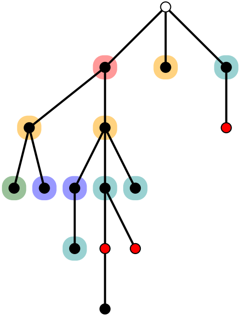

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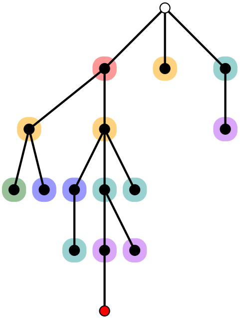

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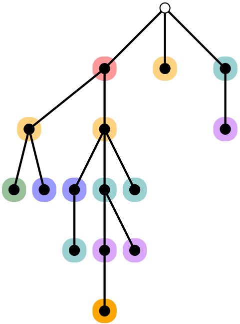

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Theorem (, N. Williams; 2015)

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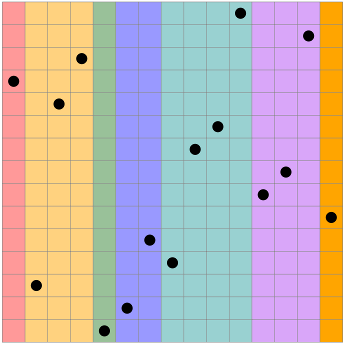

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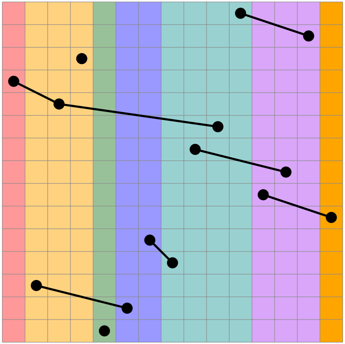

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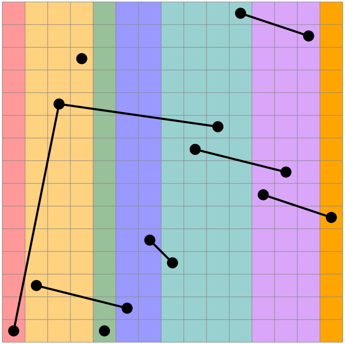

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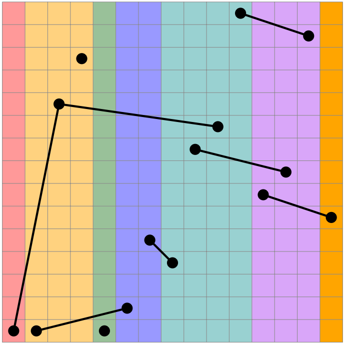

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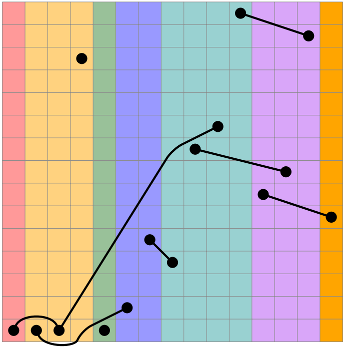

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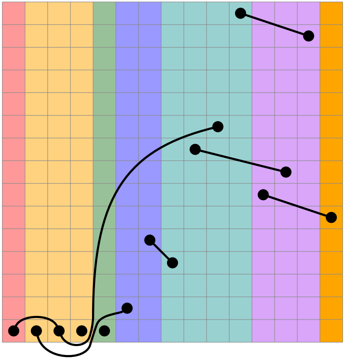

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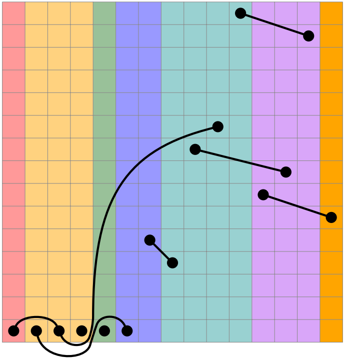

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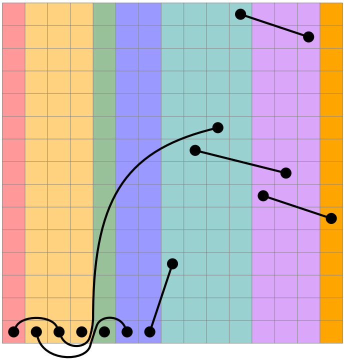

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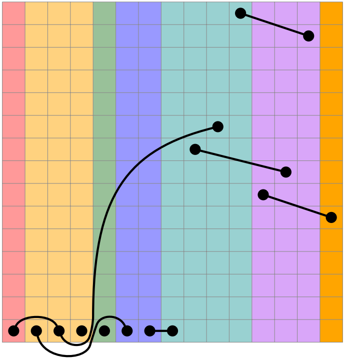

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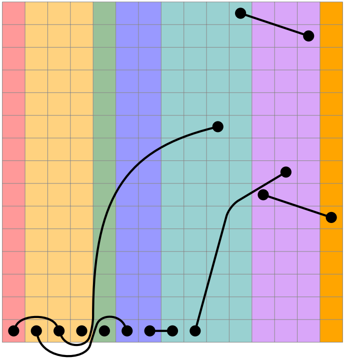

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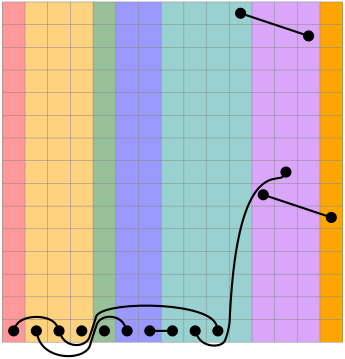

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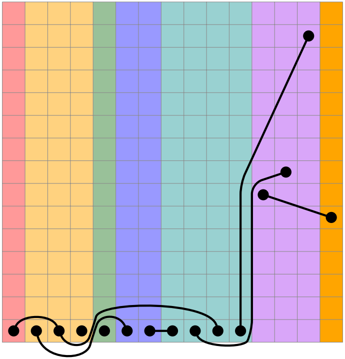

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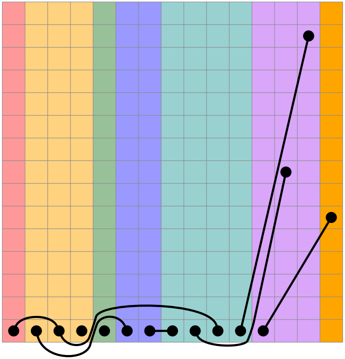

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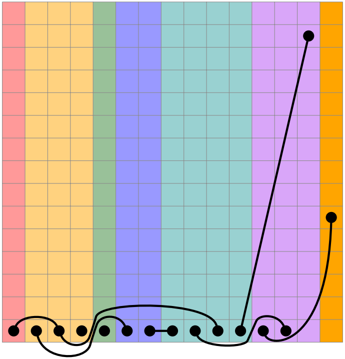

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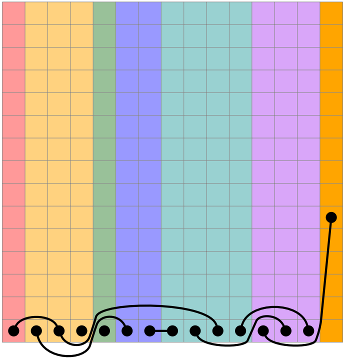

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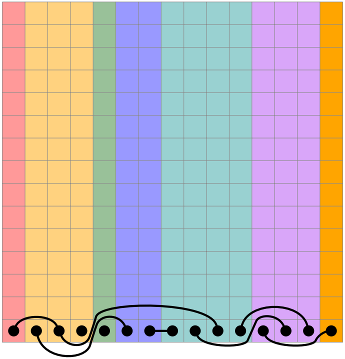

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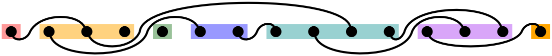

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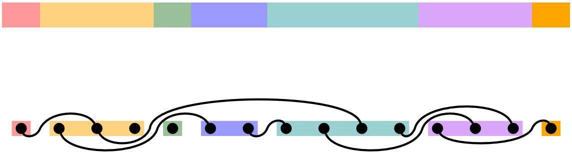

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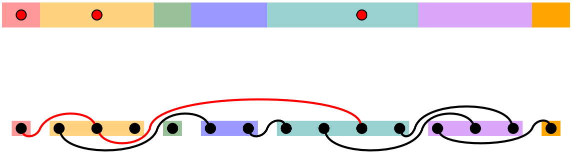

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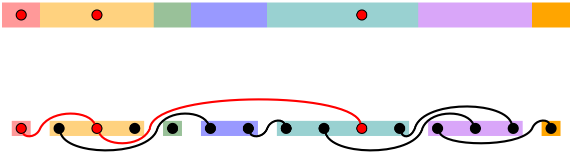

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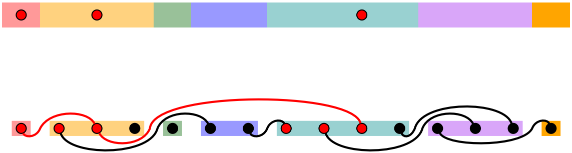

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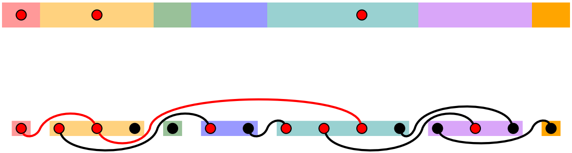

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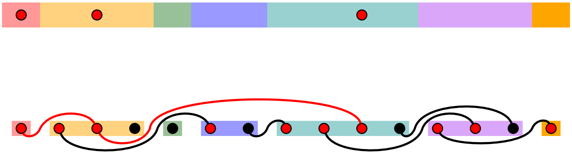

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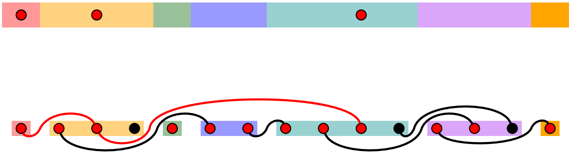

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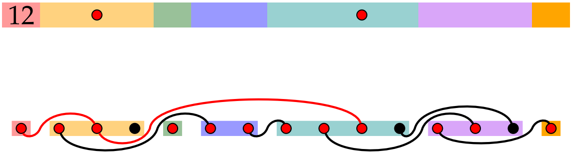

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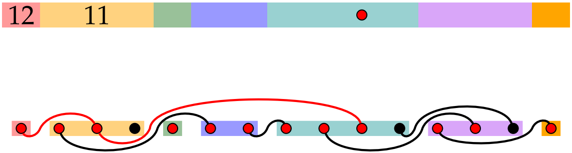

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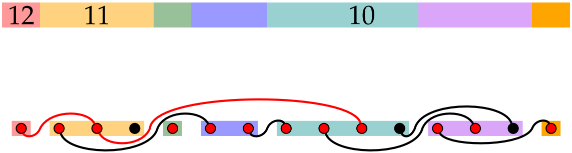

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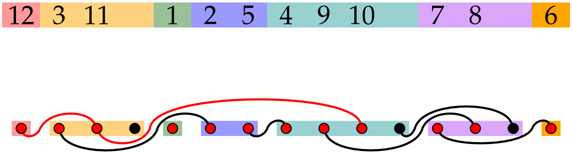

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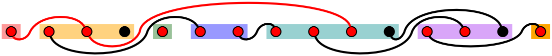

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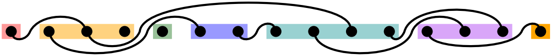

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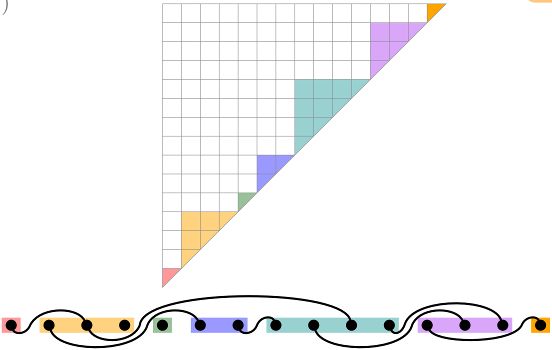

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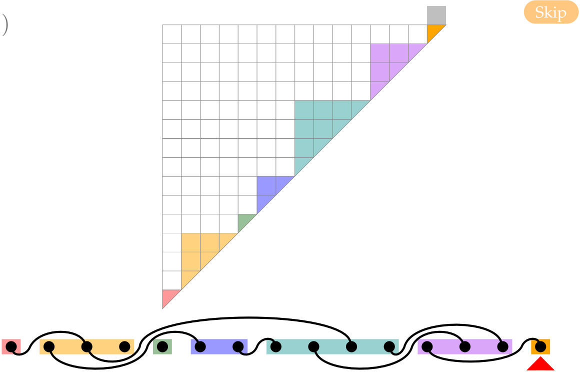

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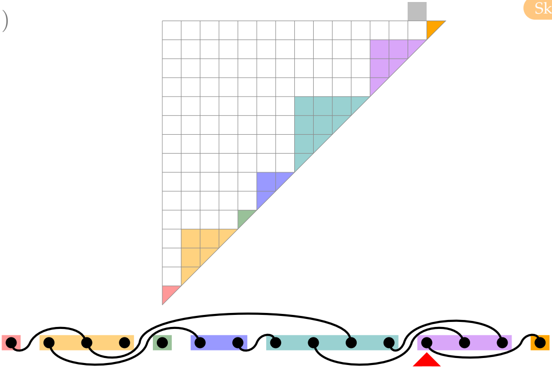

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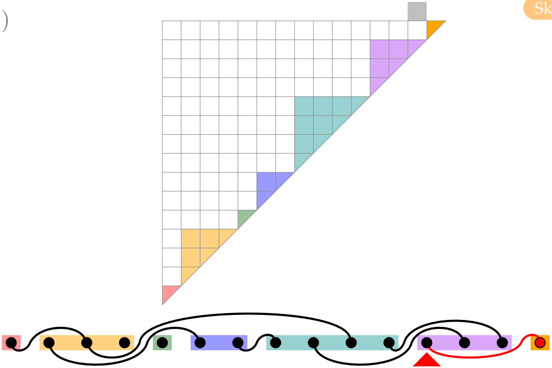

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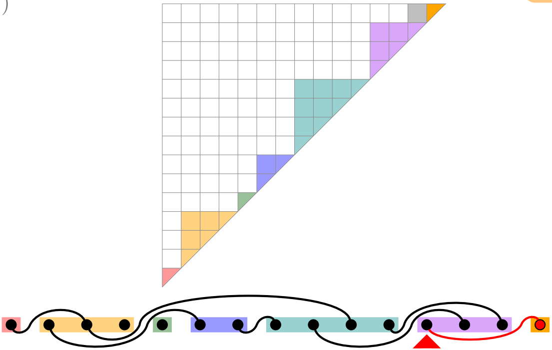

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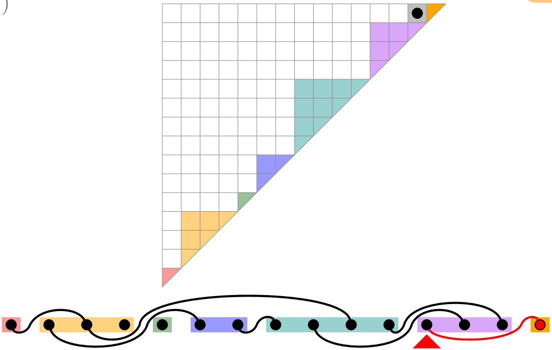

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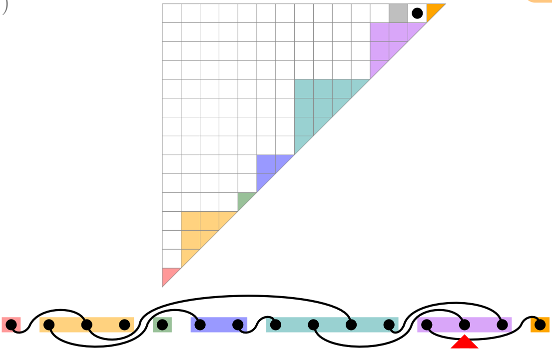

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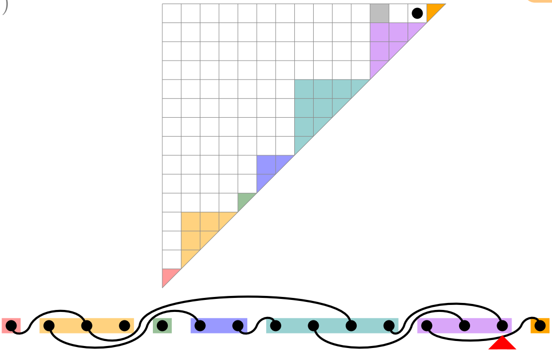

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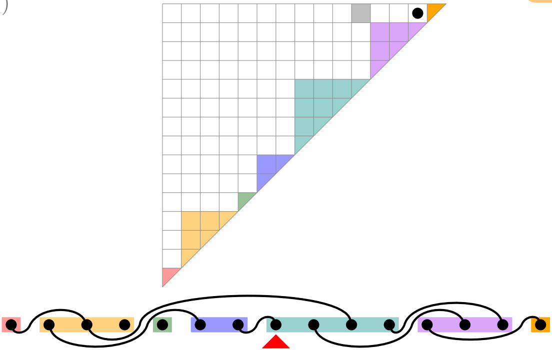

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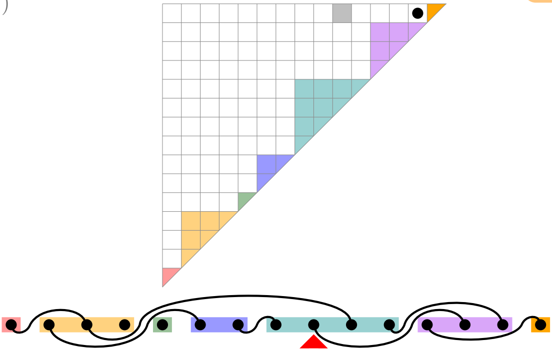

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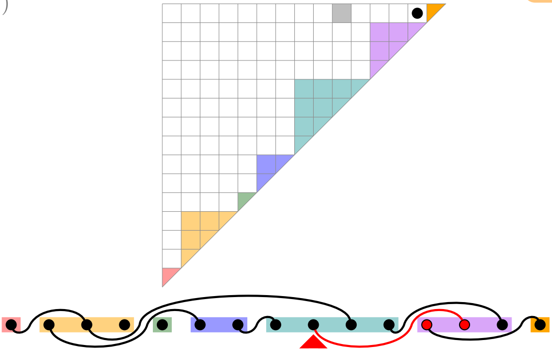

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Parabolic Cataland

Henri Mühle

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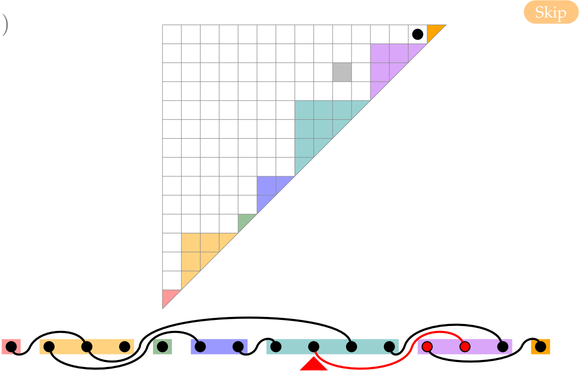

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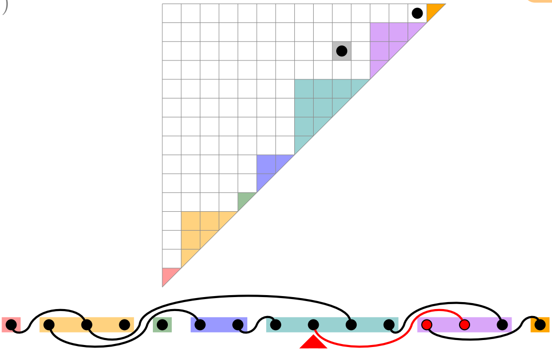

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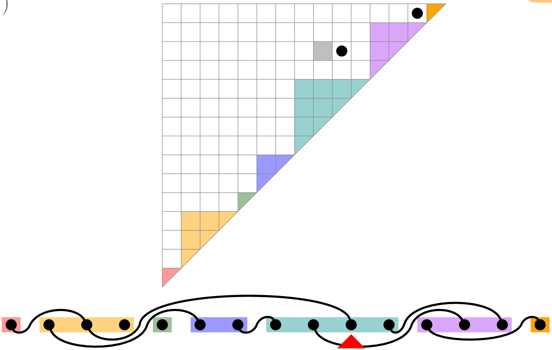

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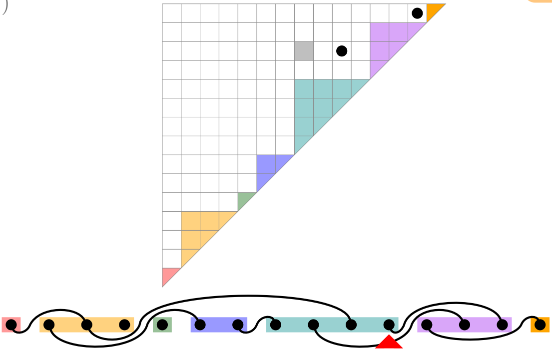

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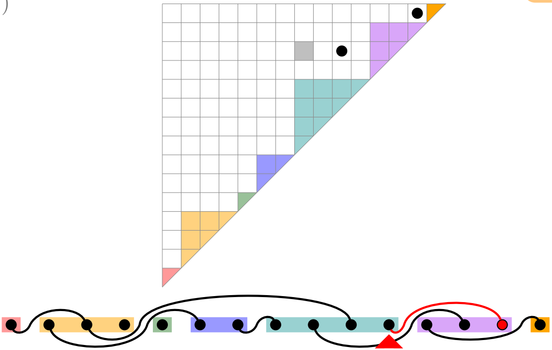

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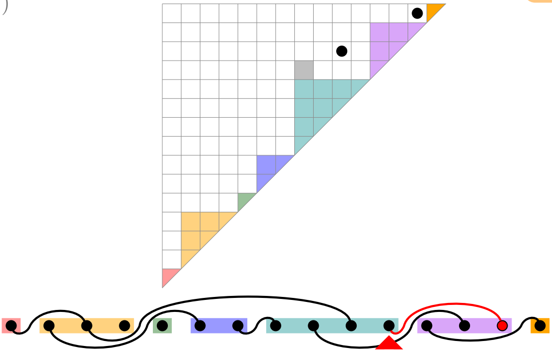

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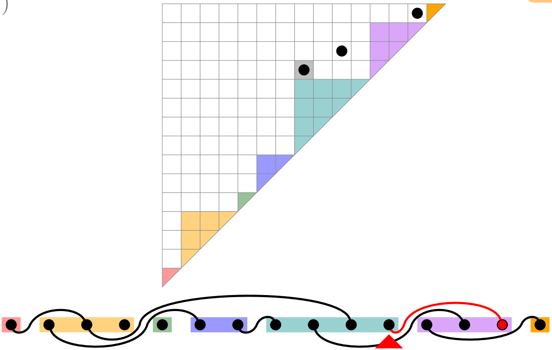

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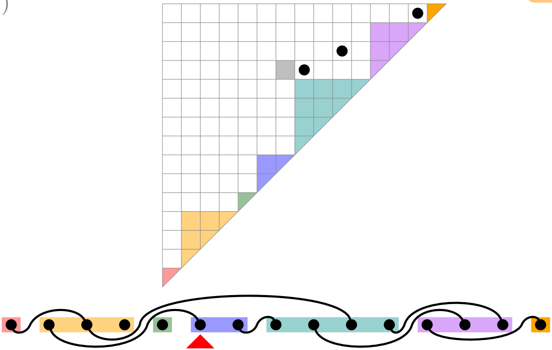

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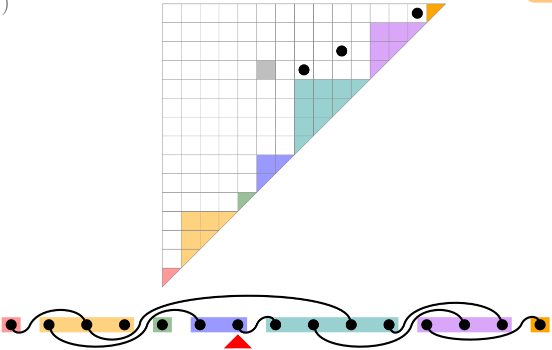

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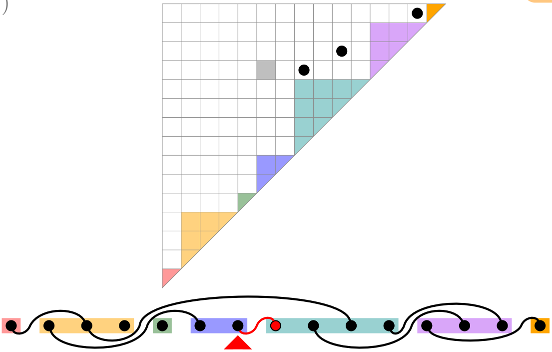

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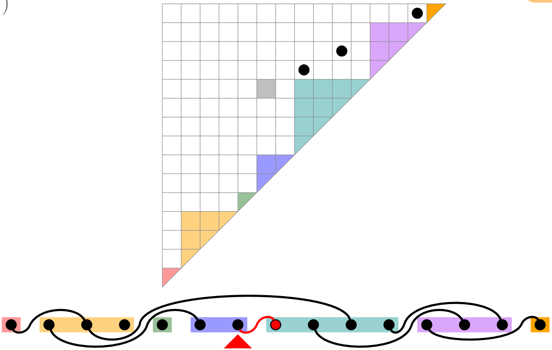

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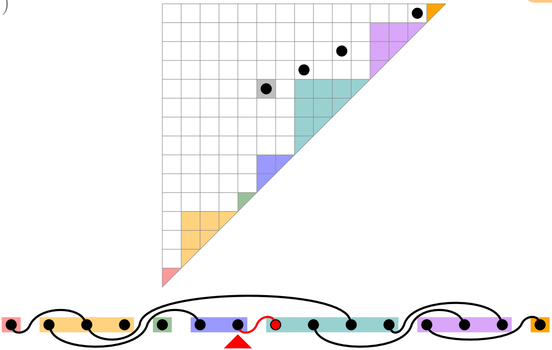

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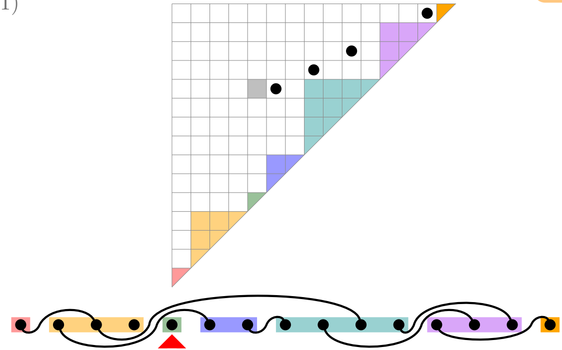

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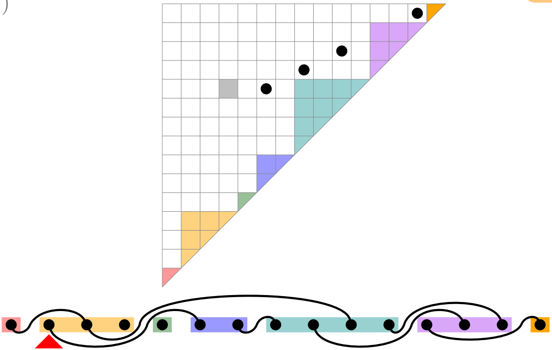

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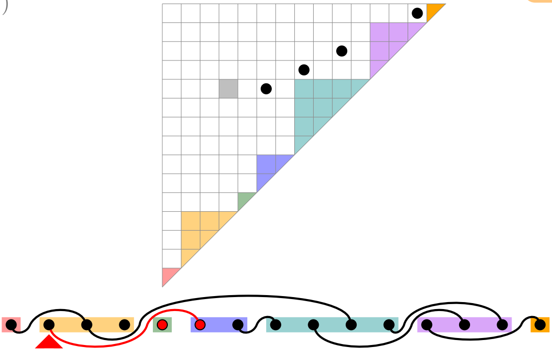

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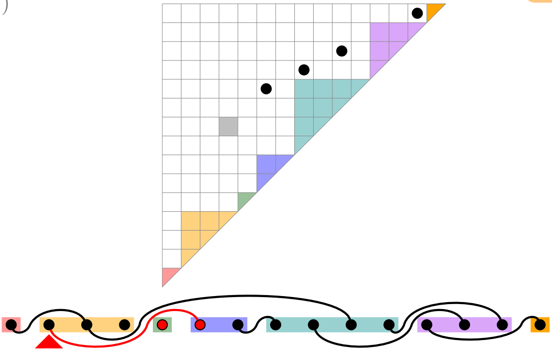

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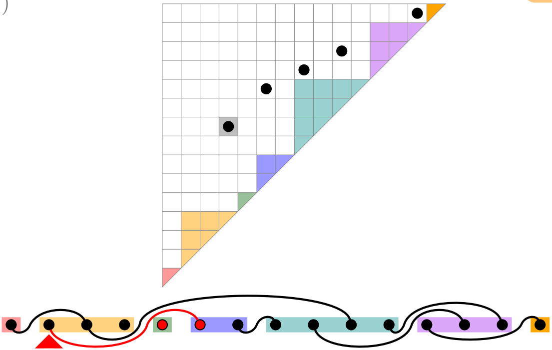

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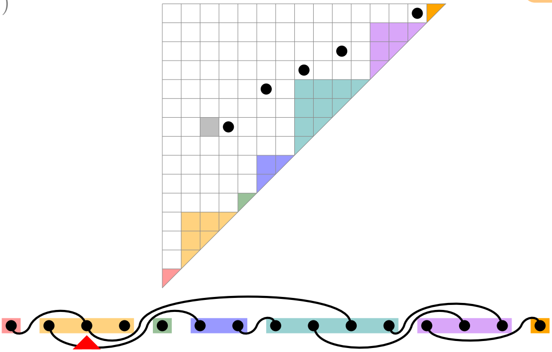

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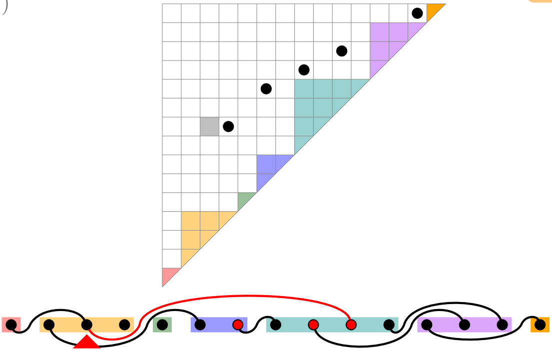

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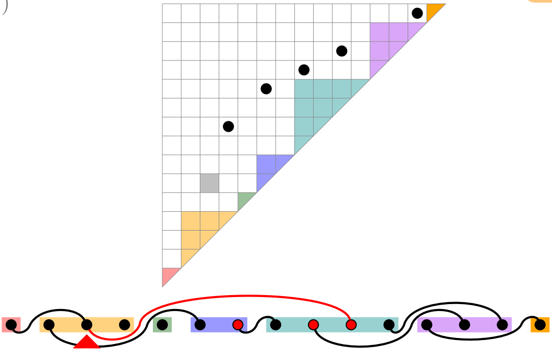

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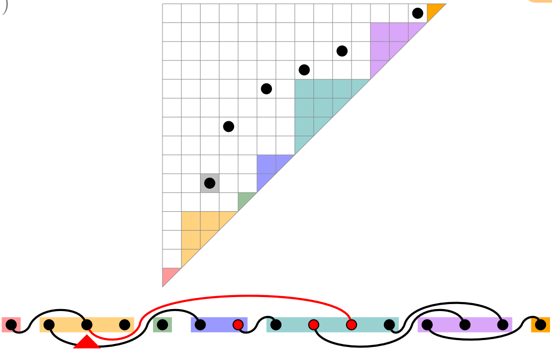

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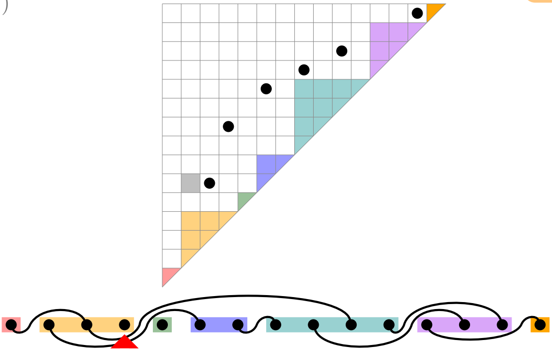

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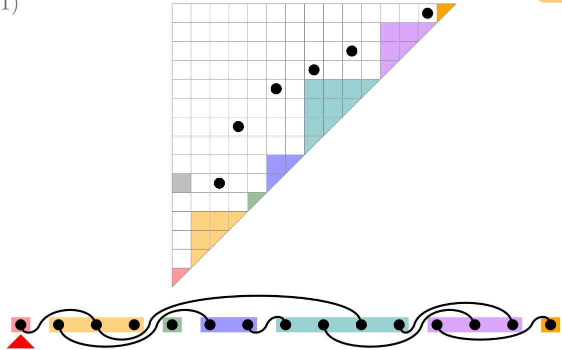

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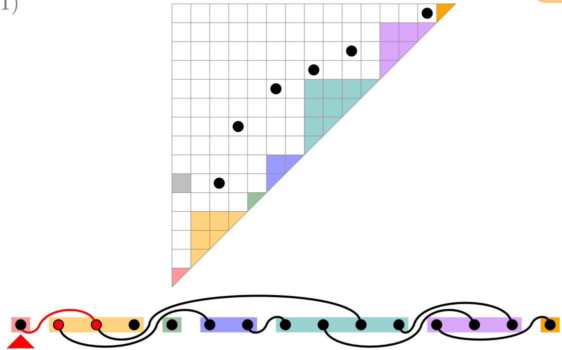

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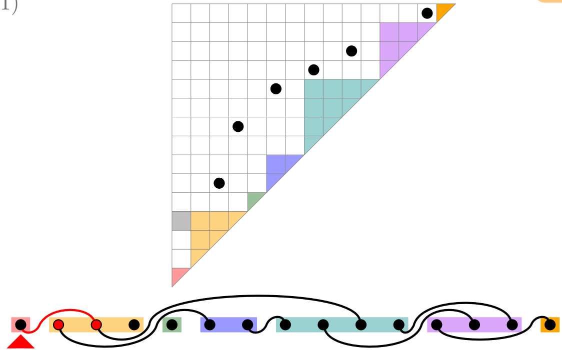

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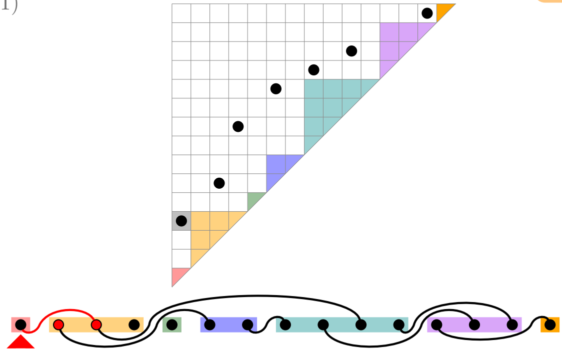

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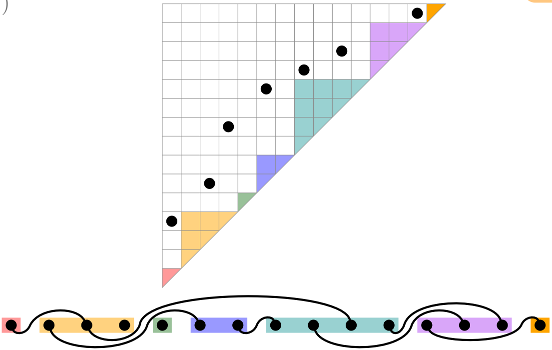

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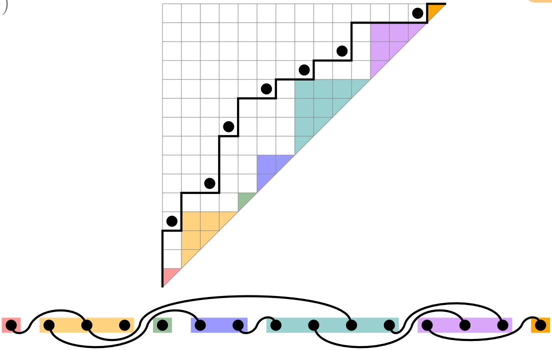

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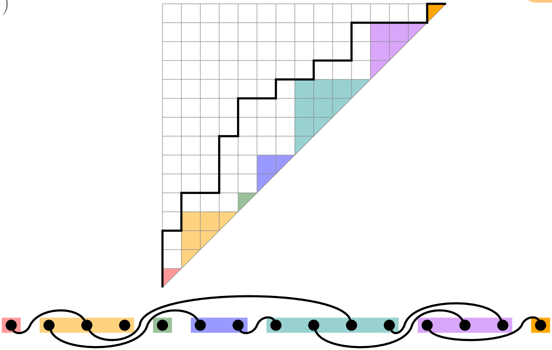

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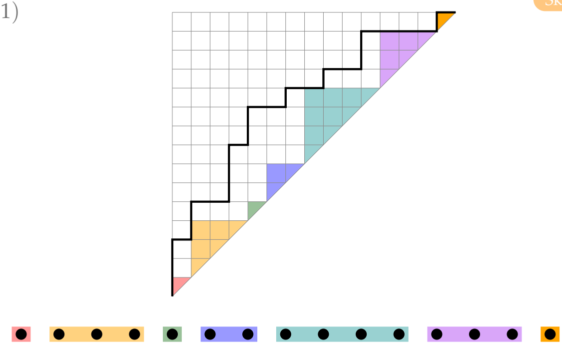

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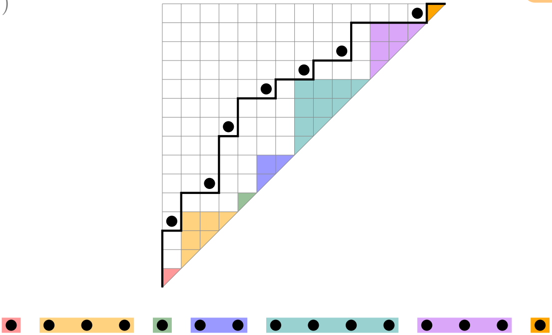

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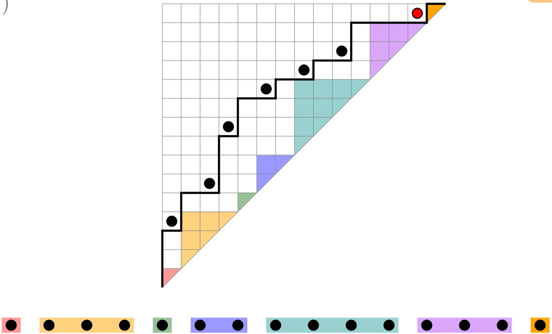

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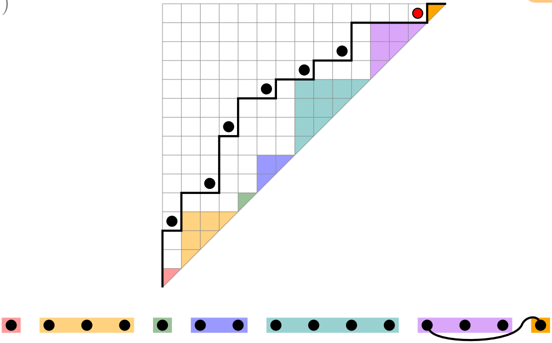

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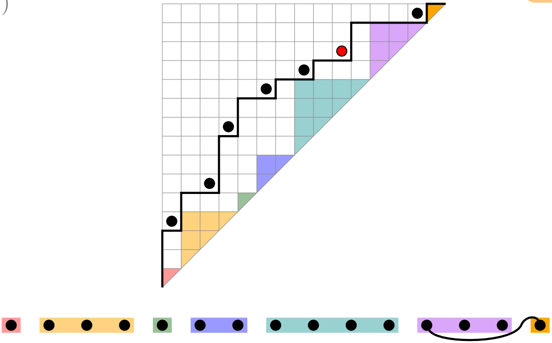

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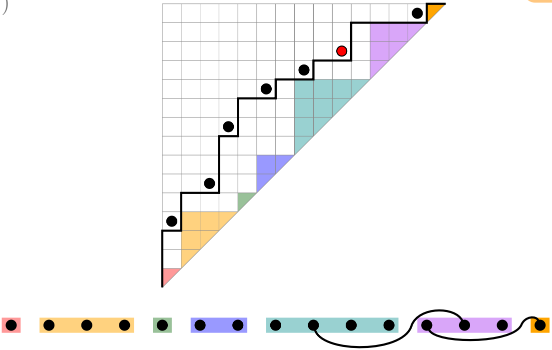

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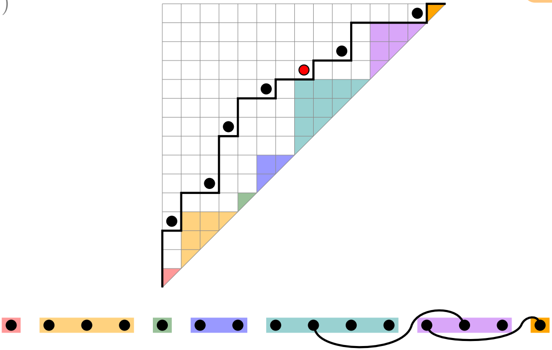

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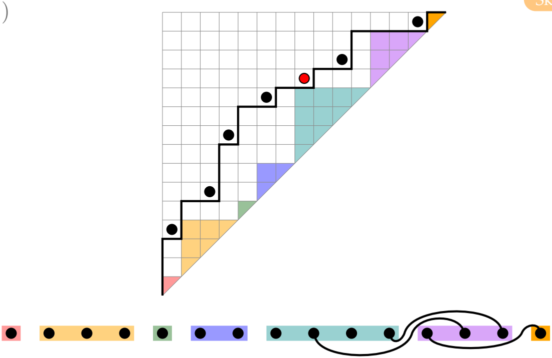

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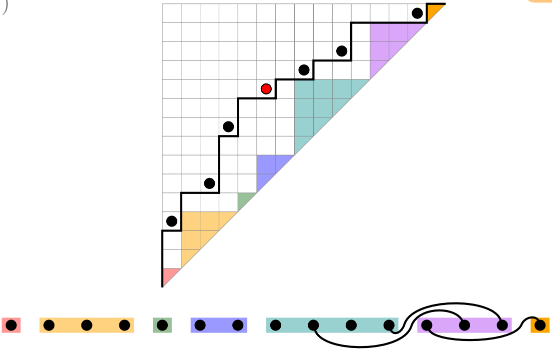

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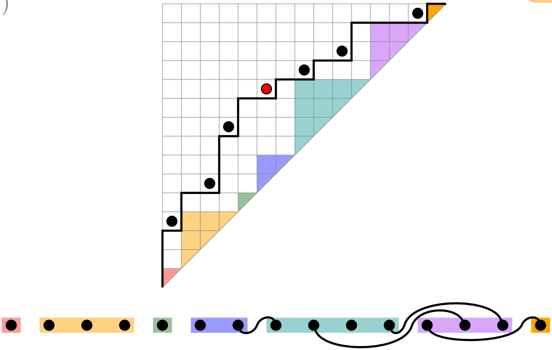

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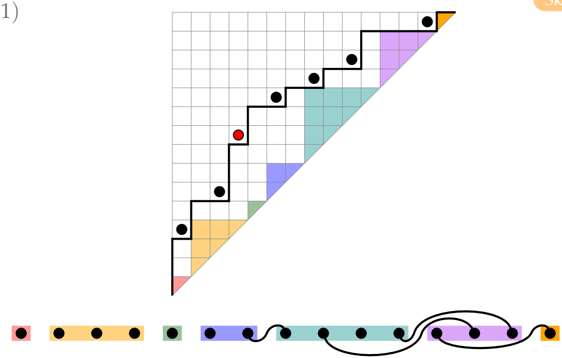

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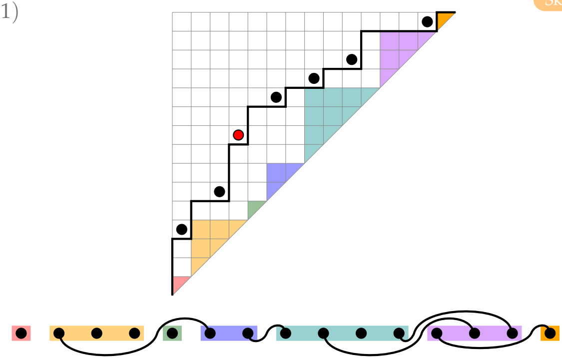

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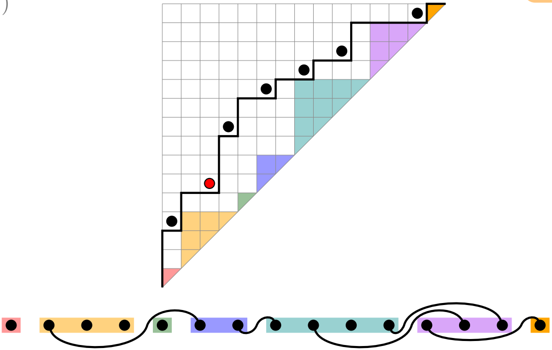

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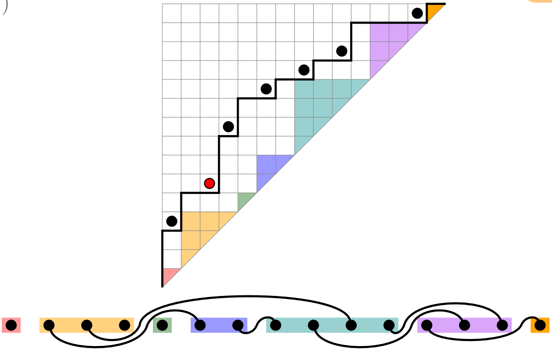

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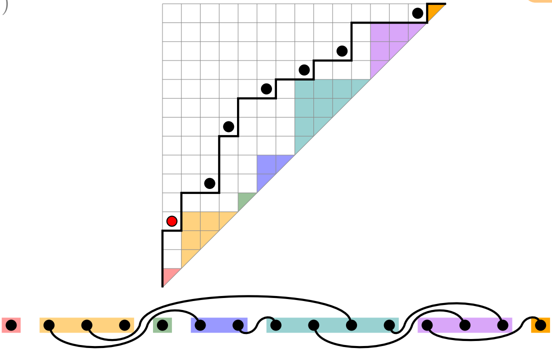

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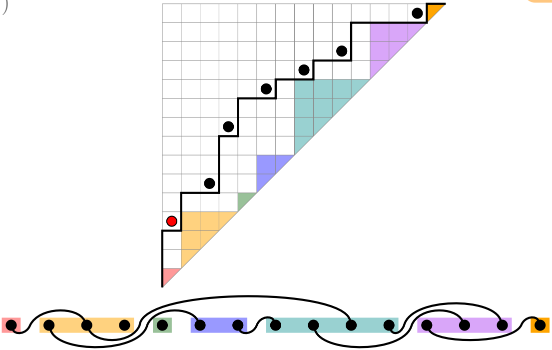

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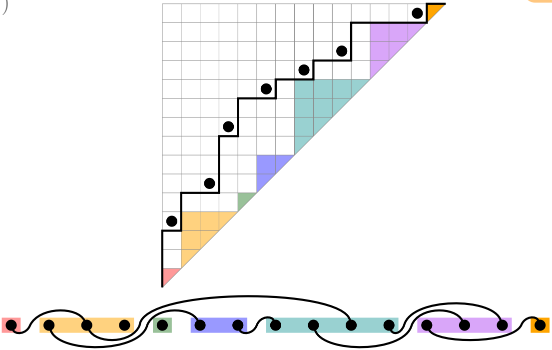

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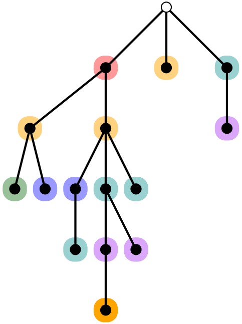

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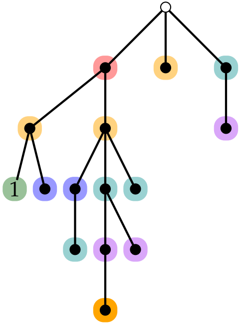

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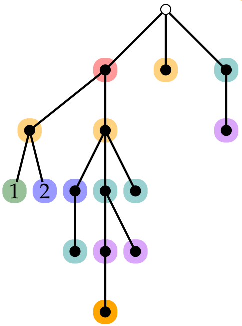

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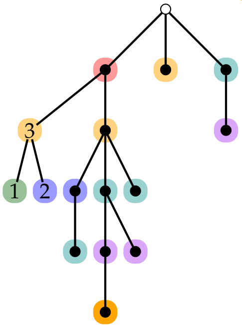

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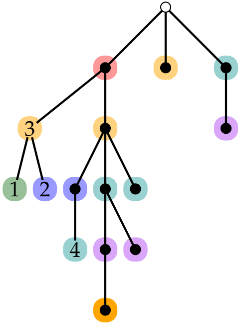

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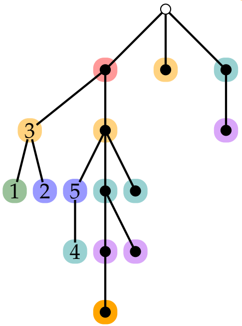

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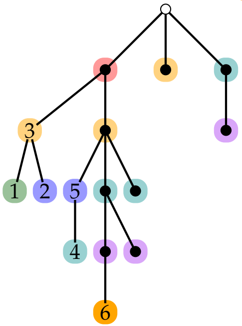

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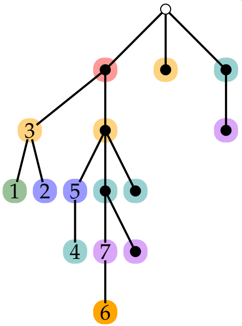

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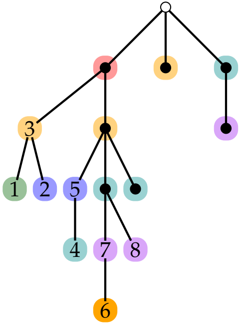

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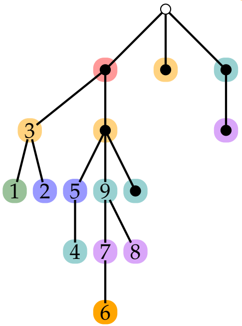

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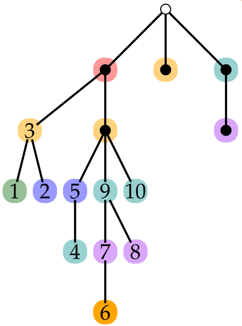

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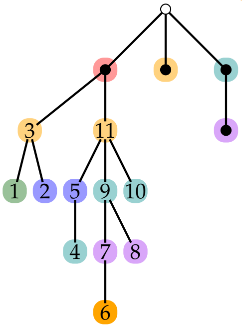

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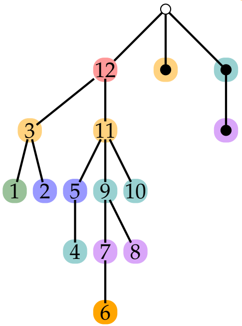

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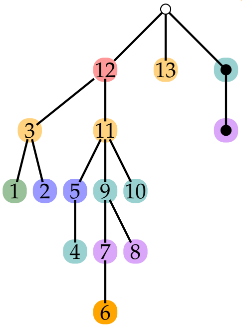

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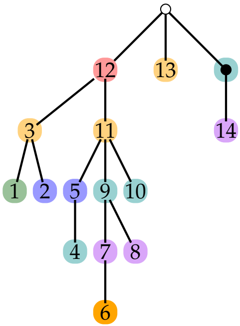

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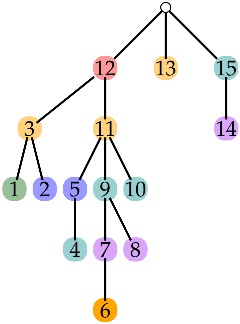

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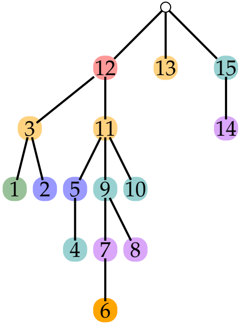

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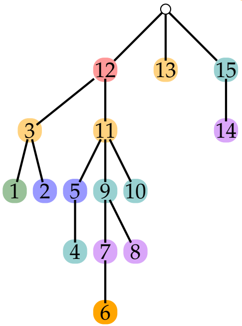

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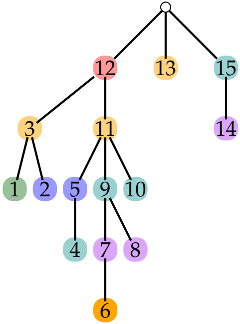

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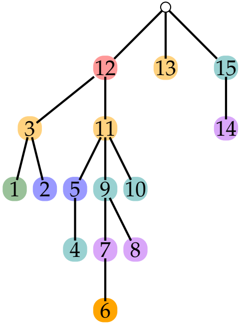

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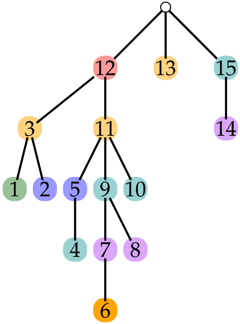

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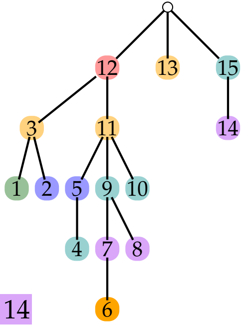

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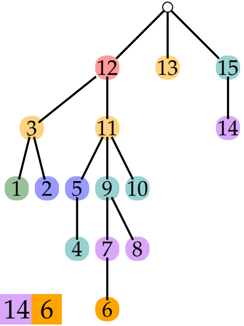

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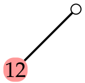

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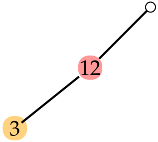

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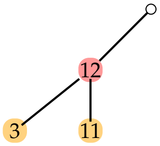

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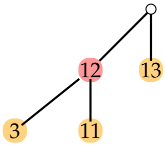

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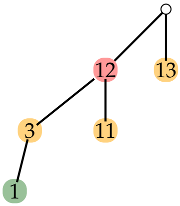

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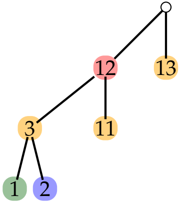

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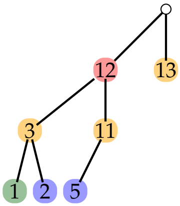

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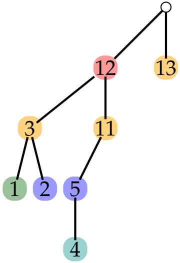

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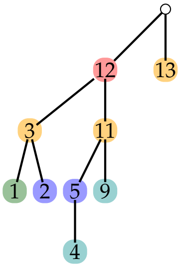

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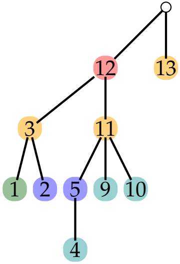

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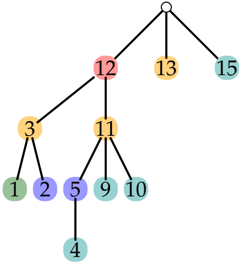

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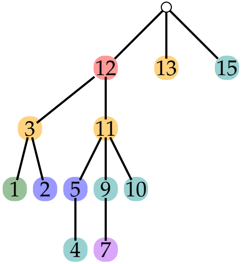

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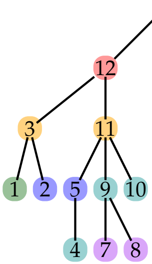

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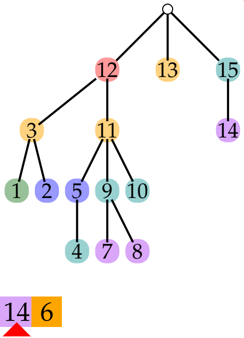

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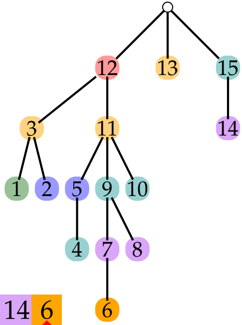

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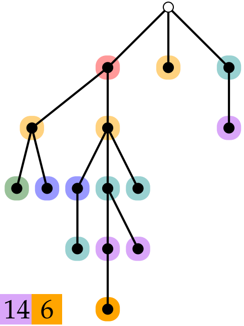

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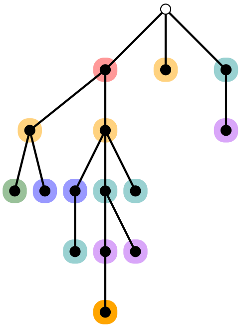

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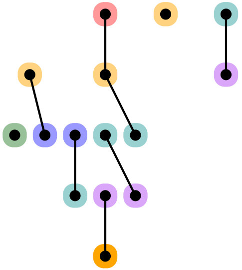

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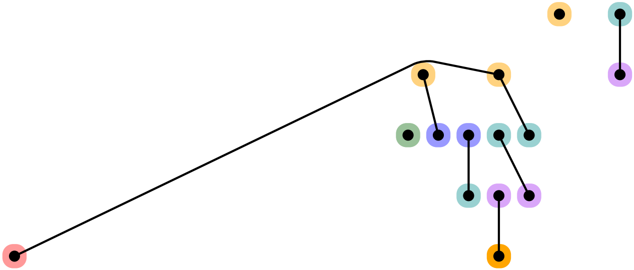

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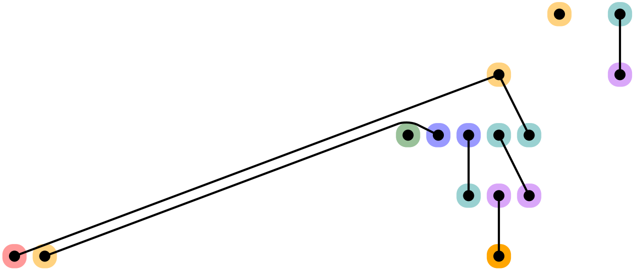

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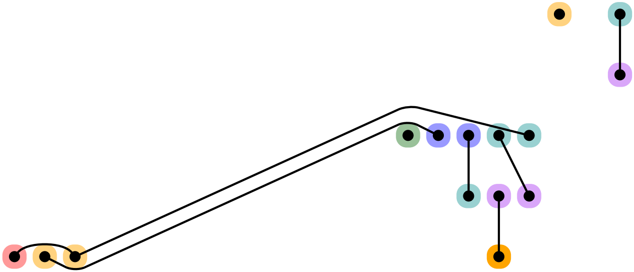

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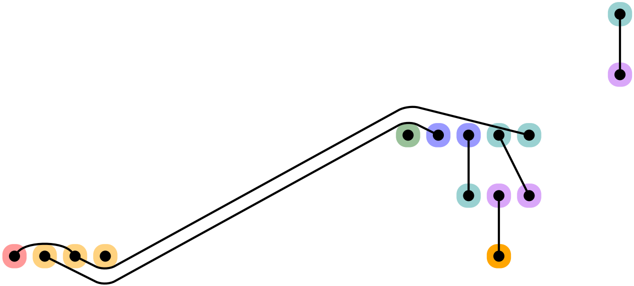

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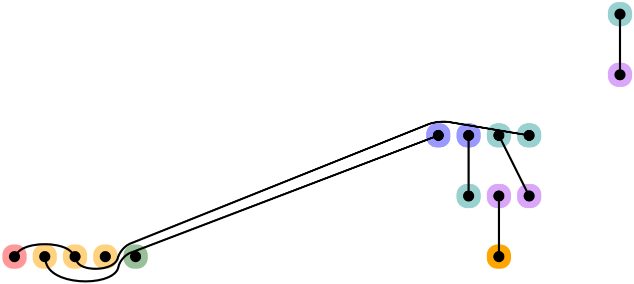

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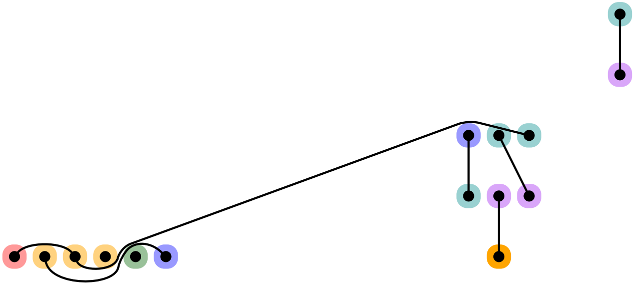

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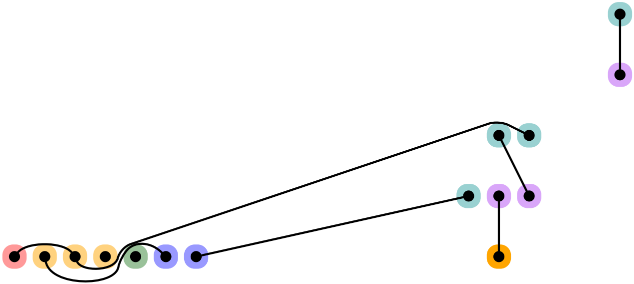

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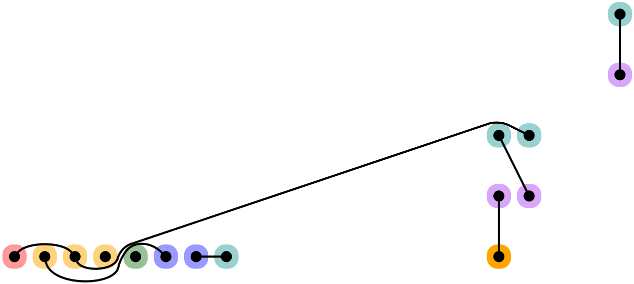

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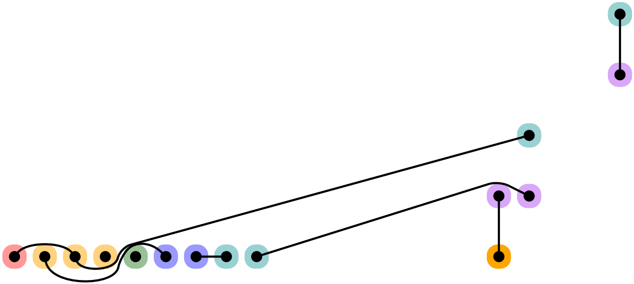

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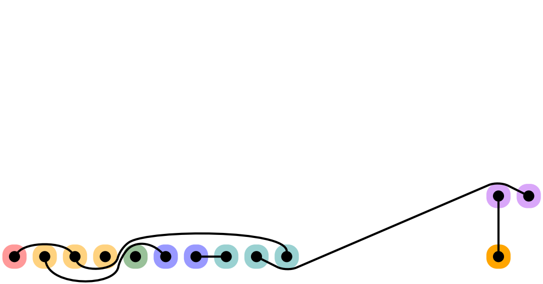

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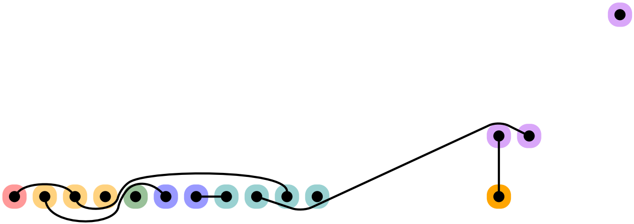

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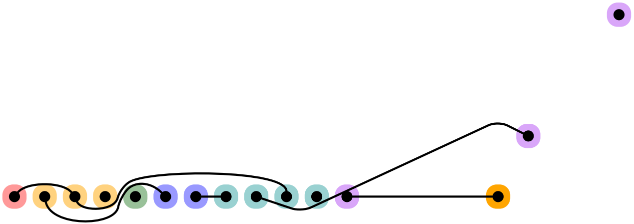

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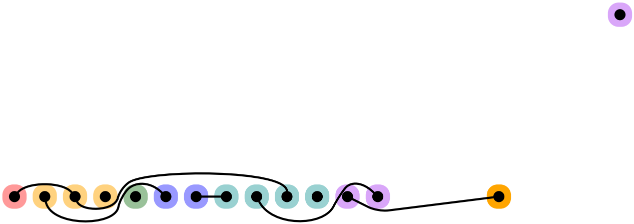

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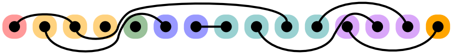

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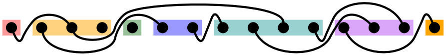

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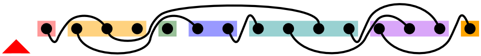

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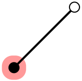

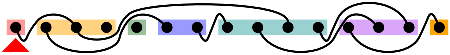

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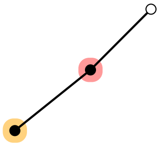

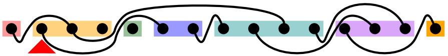

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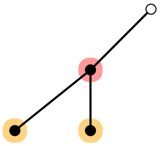

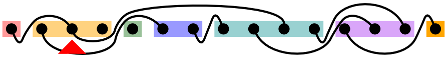

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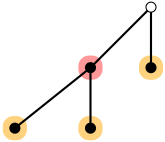

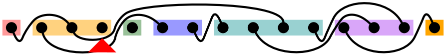

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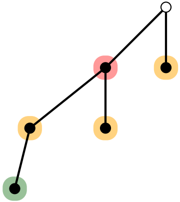

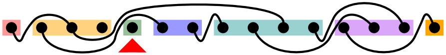

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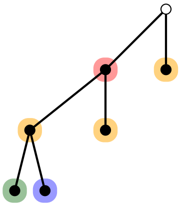

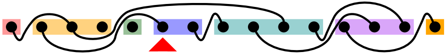

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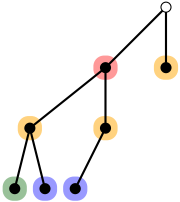

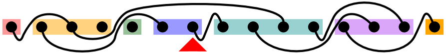

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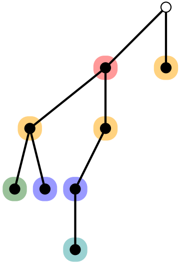

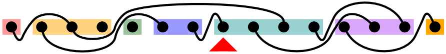

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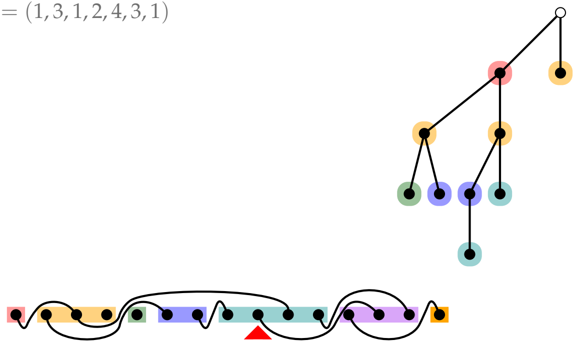

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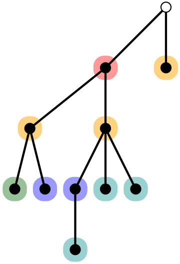

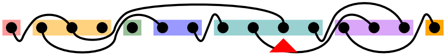

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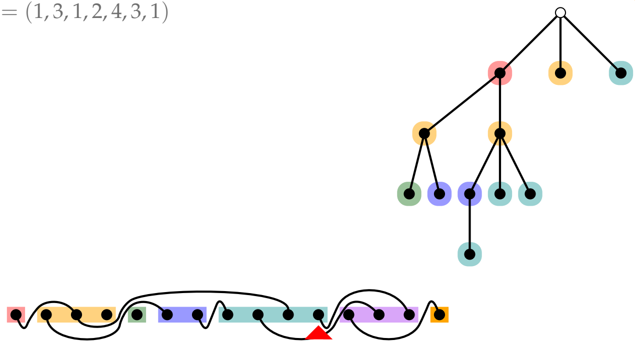

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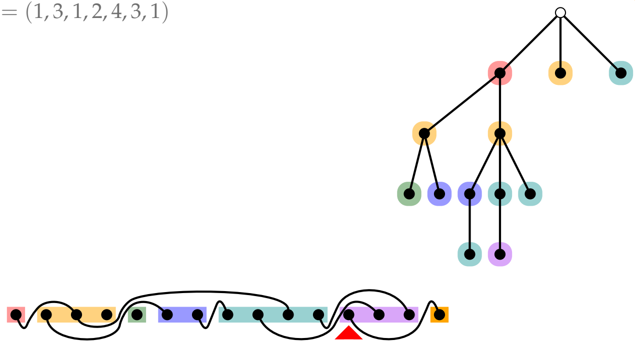

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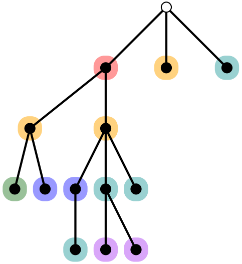

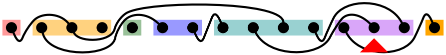

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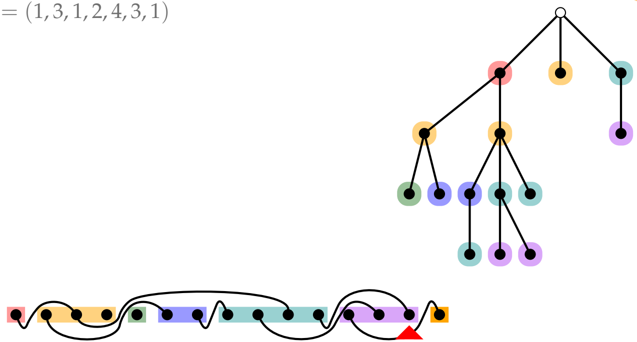

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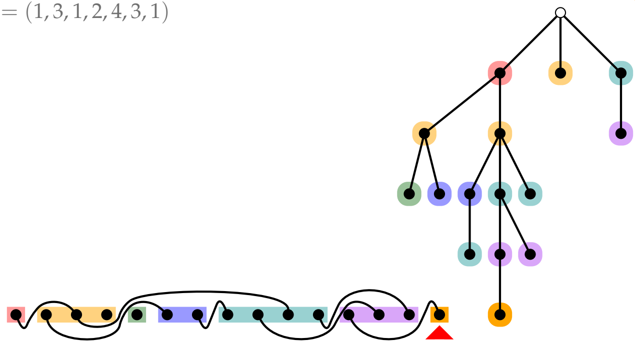

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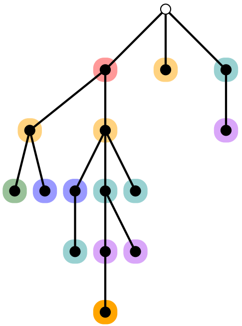

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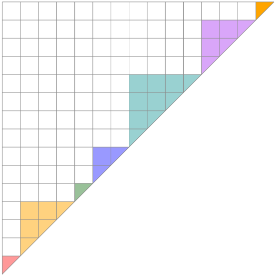

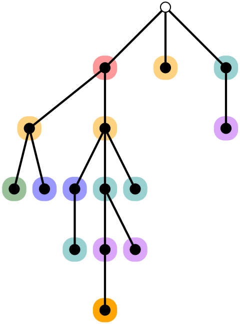

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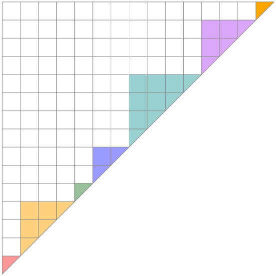

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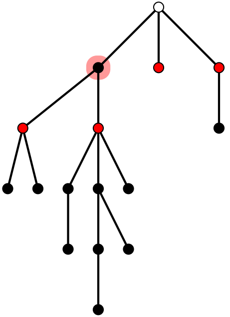

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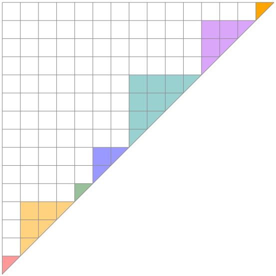

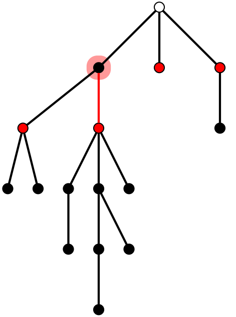

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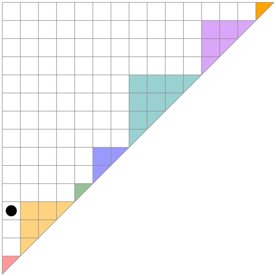

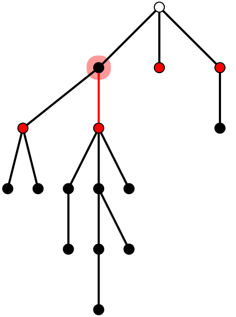

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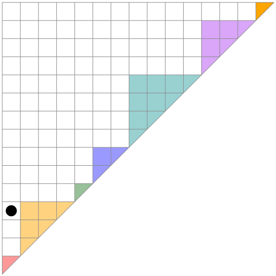

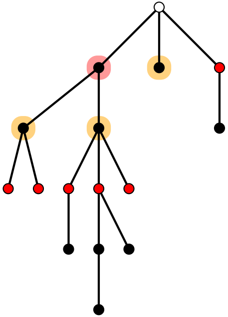

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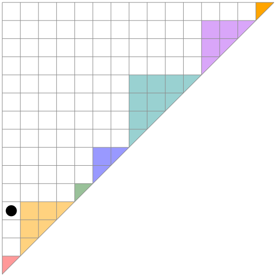

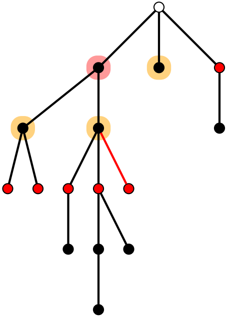

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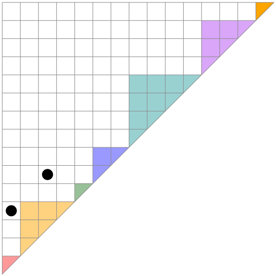

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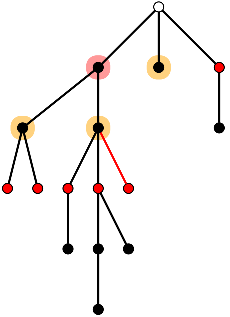

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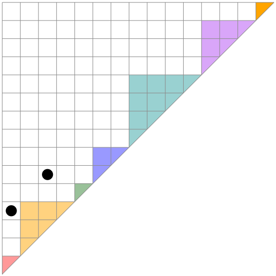

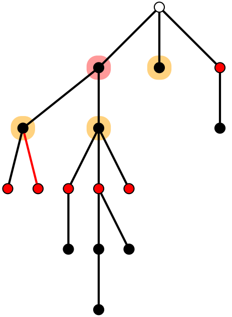

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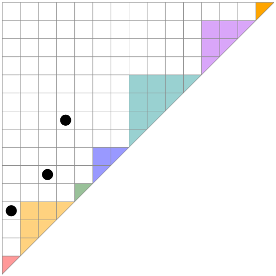

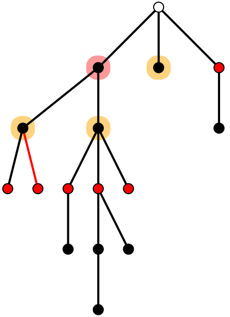

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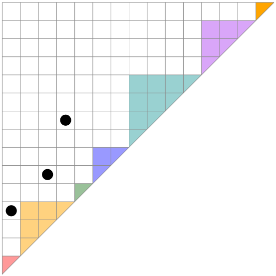

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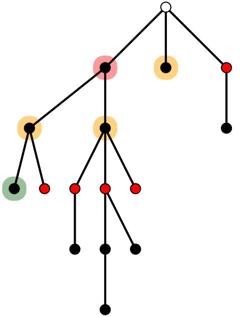

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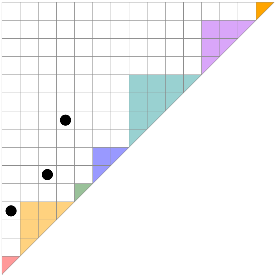

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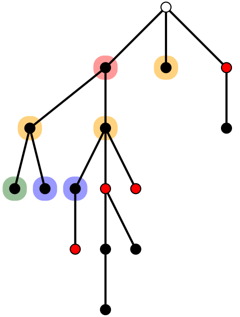

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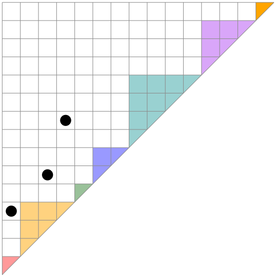

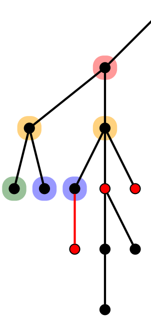

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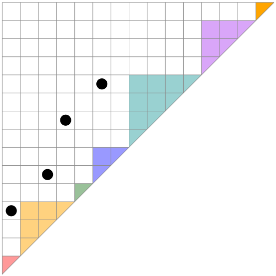

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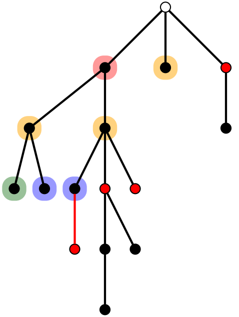

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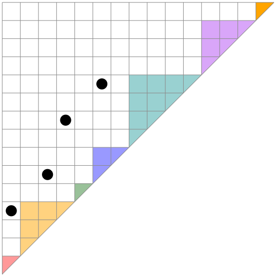

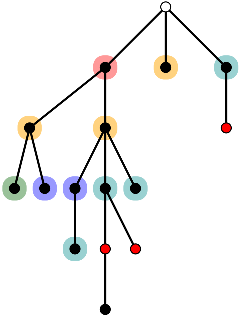

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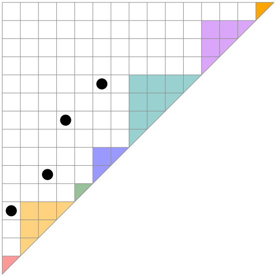

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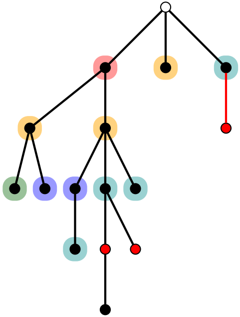

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Parabolic Cataland

Henri Mühle

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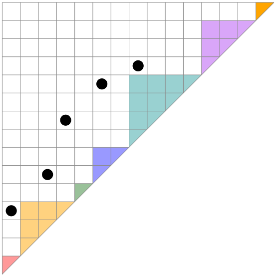

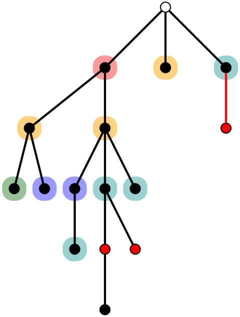

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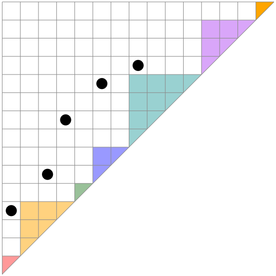

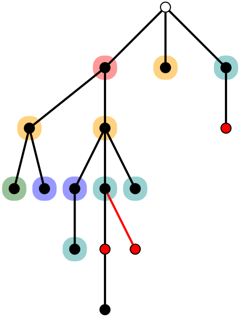

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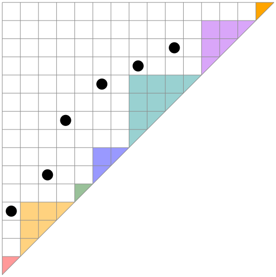

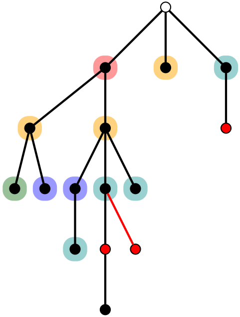

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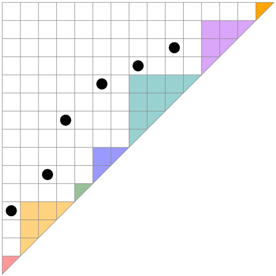

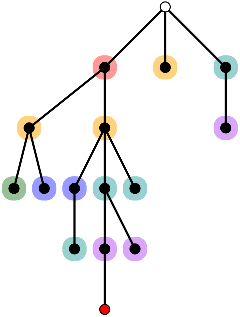

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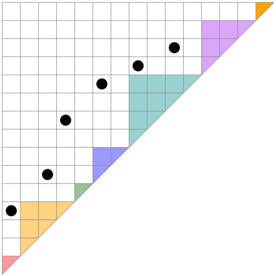

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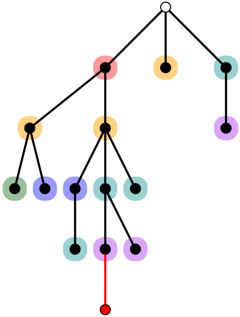

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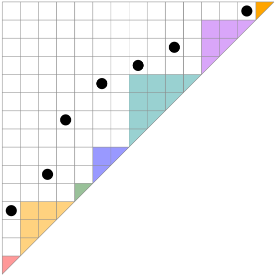

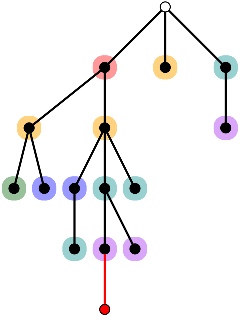

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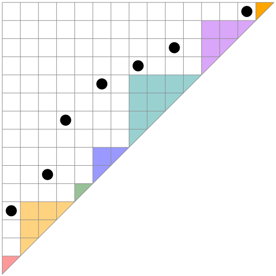

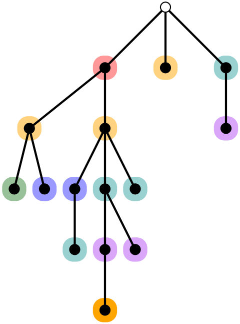

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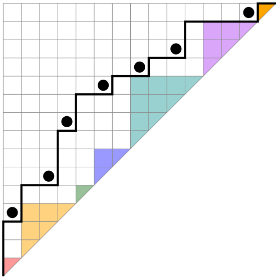

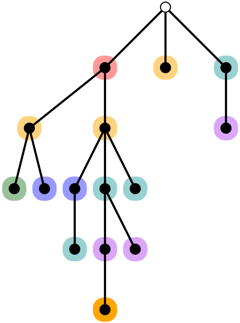

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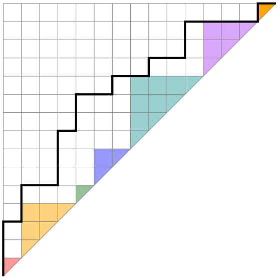

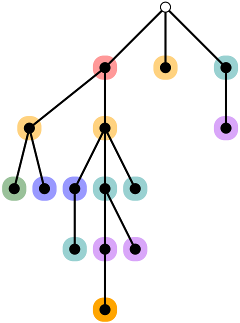

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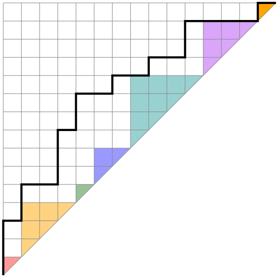

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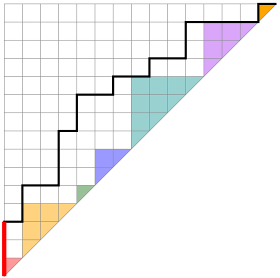

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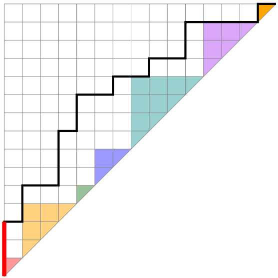

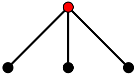

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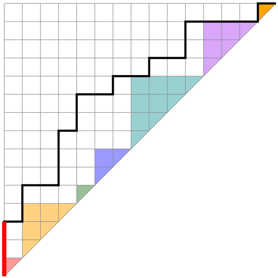

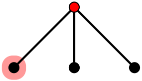

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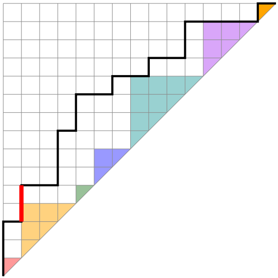

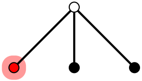

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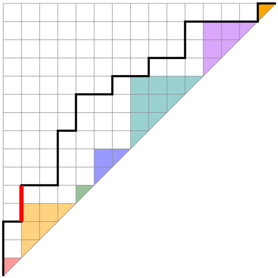

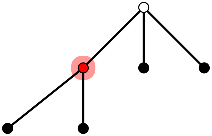

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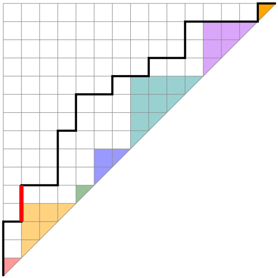

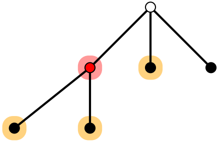

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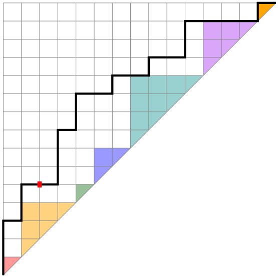

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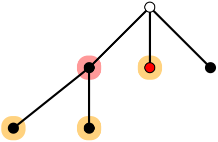

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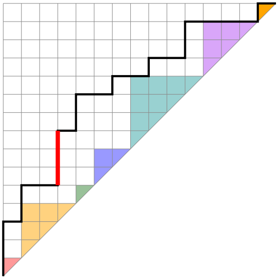

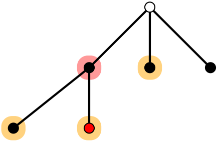

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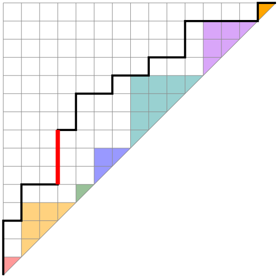

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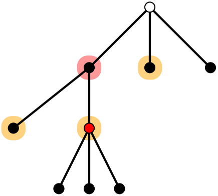

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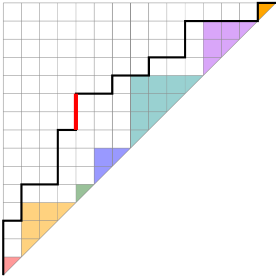

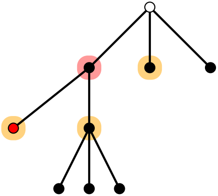

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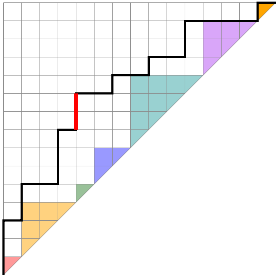

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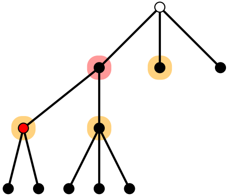

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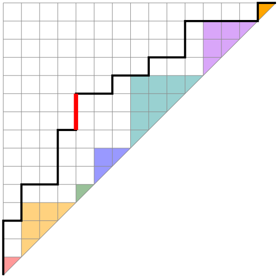

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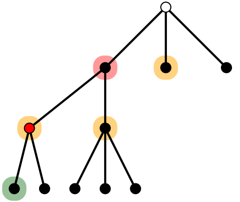

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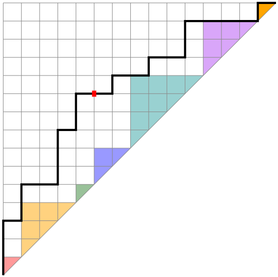

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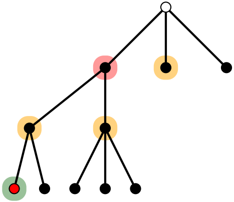

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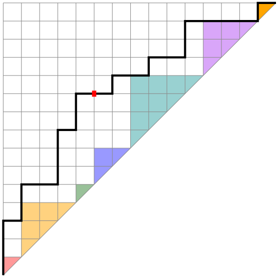

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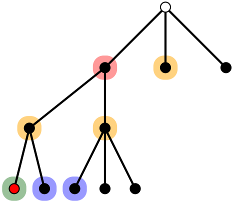

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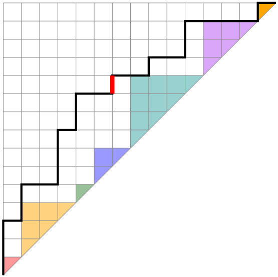

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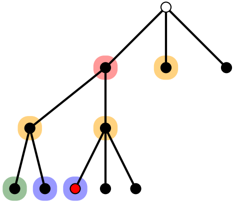

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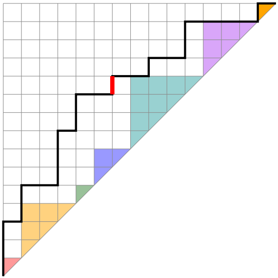

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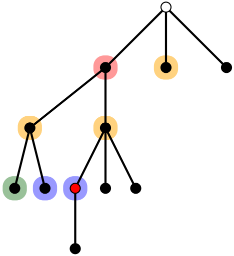

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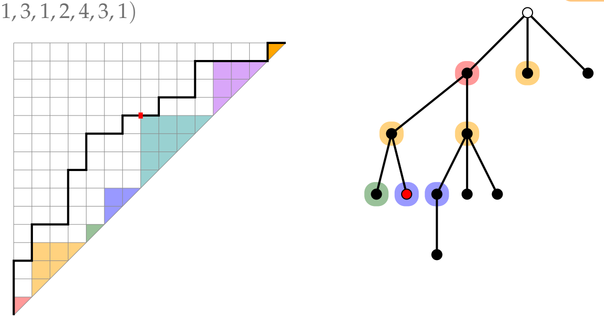

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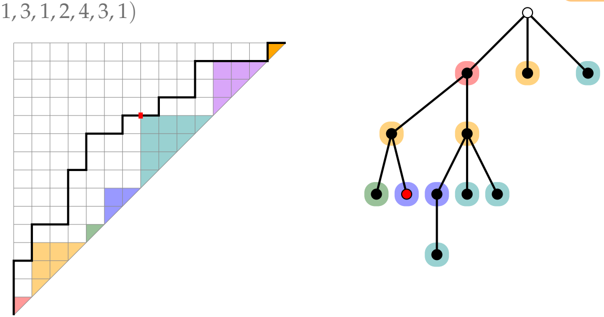

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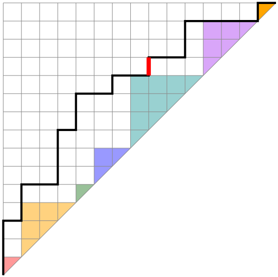

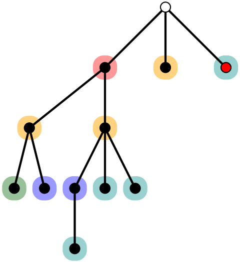

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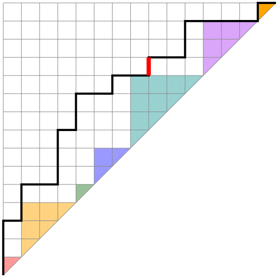

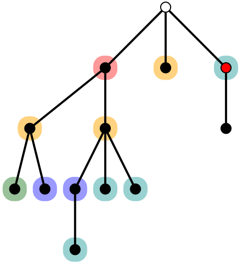

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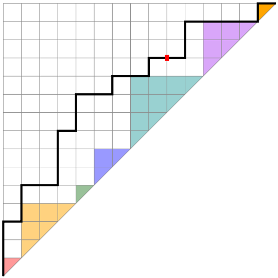

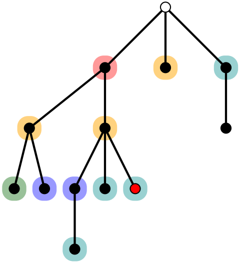

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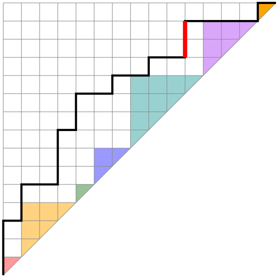

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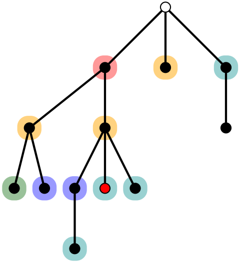

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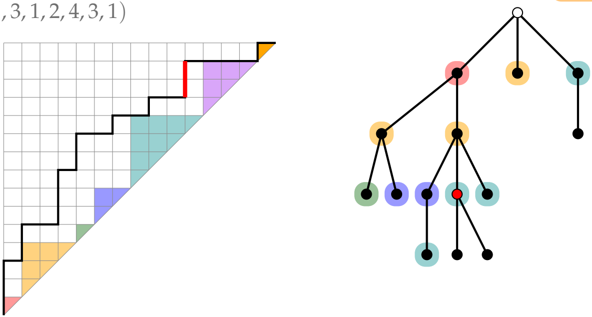

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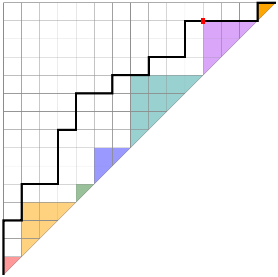

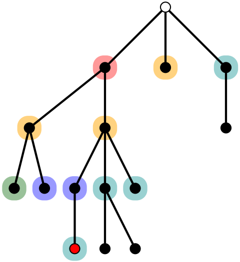

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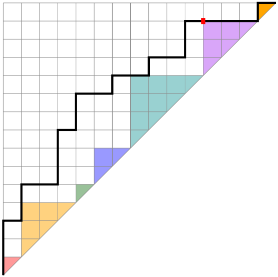

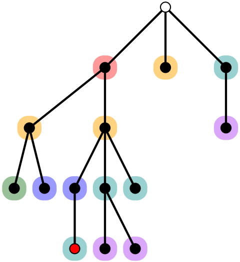

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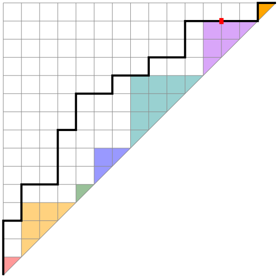

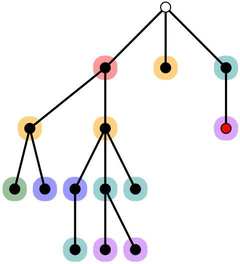

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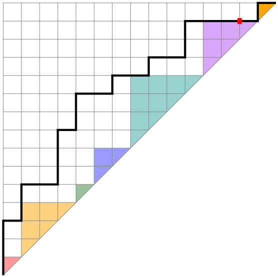

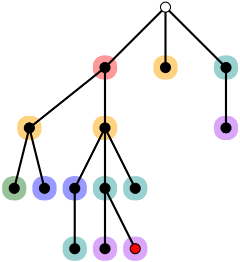

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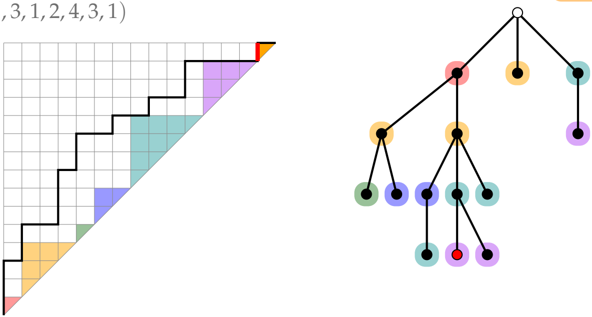

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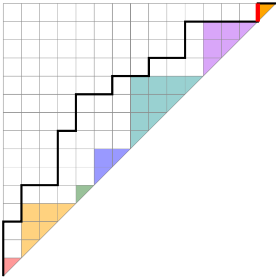

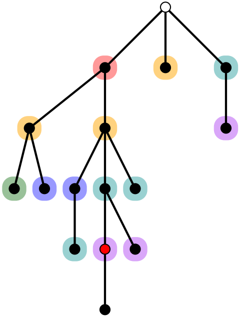

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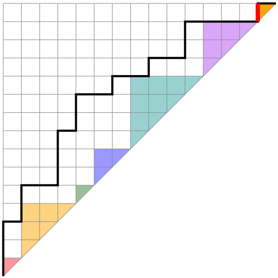

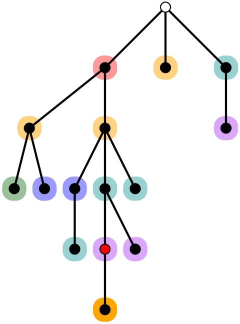

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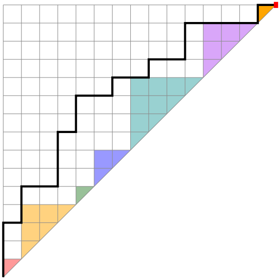

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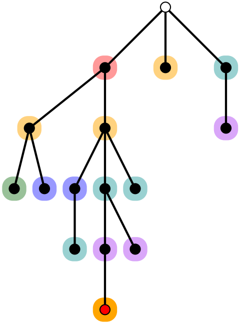

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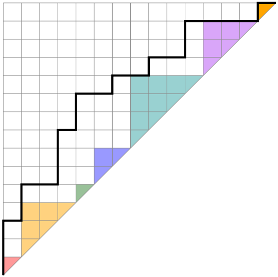

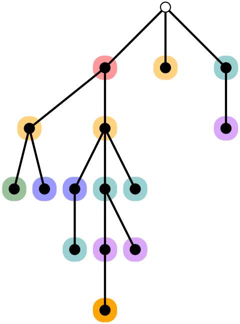

Outline

1. Parabolic Cataland
2. Bijections in Parabolic Cataland
3. Posets in Parabolic Cataland
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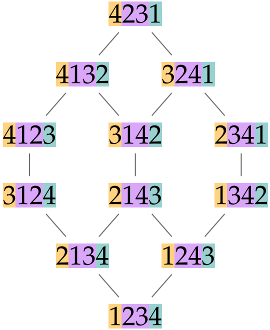

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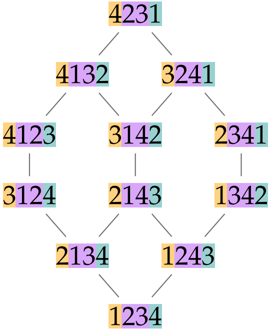

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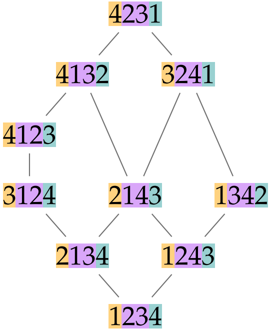

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Theorem (, N. Williams; 2015)

For every integer composition α, the poset $\mathcal{T}_\alpha$ is a quotient lattice of $(\mathfrak{S}_\alpha, \leq_L)$.

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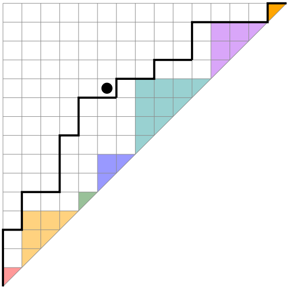

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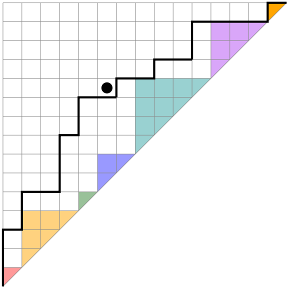

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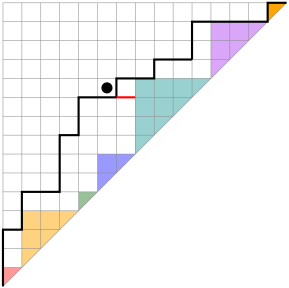

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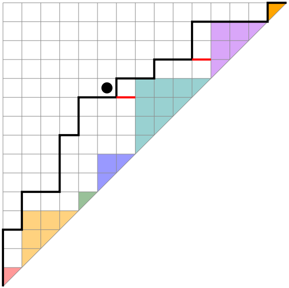

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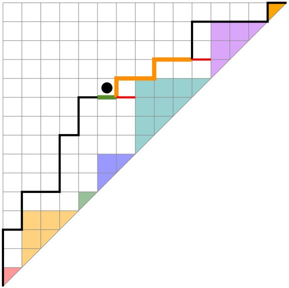

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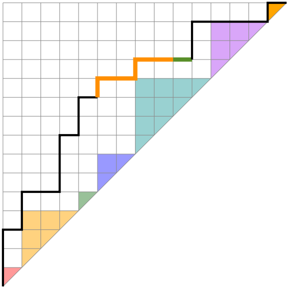

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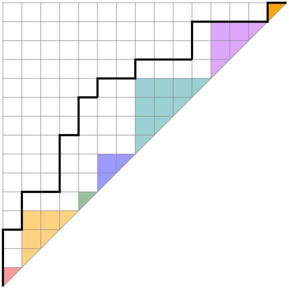

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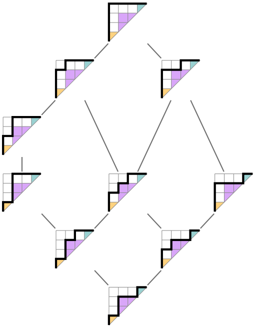

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Holds for arbitrary Dyck paths ν.

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Galois graphs

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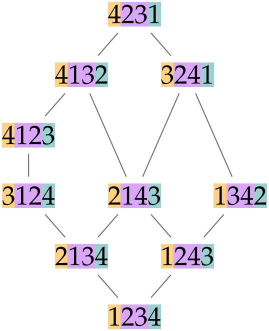

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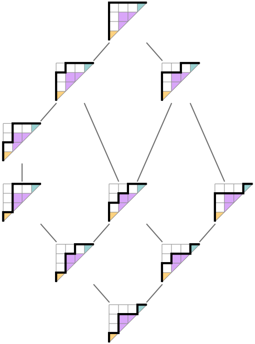

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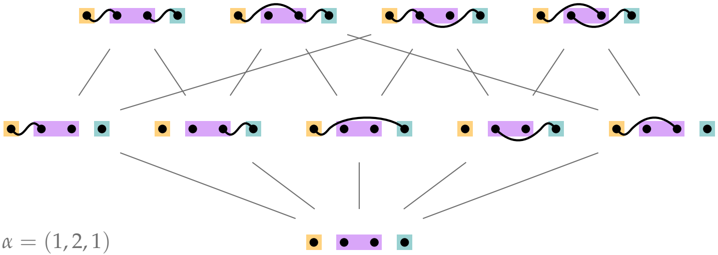

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$\mathcal{NC}_\alpha$ is a lattice if and only if $\alpha = (n)$ or $\alpha = (1,1,\ldots,1)$.

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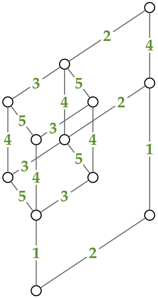

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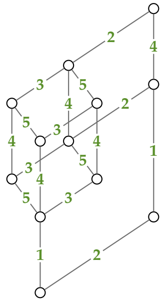

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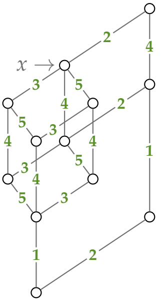

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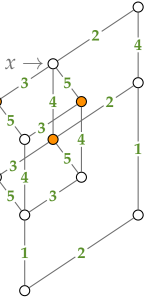

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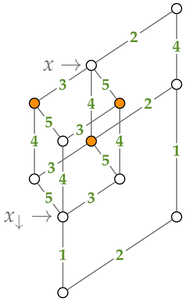

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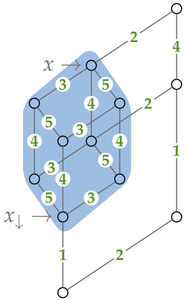

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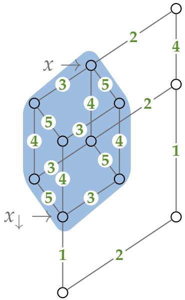

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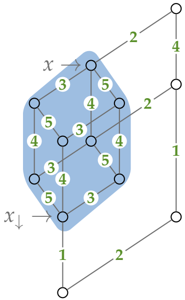

$\Psi_{\lambda}(x) = \{3, 4, 5\}$

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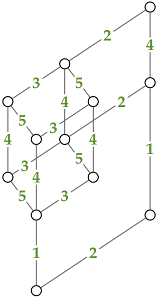

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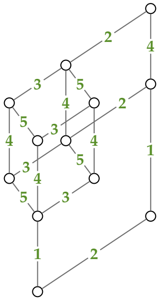

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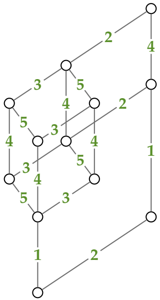

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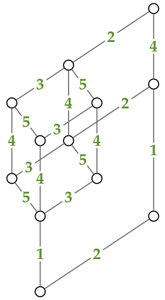

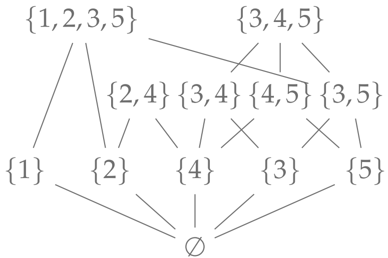

The Core Label Order of $\mathcal{T}_\alpha$

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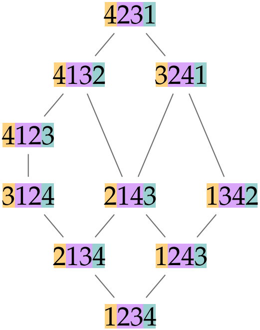

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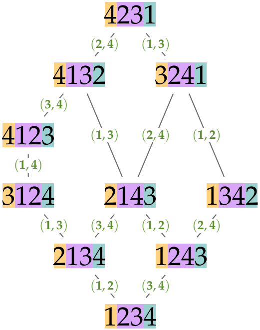

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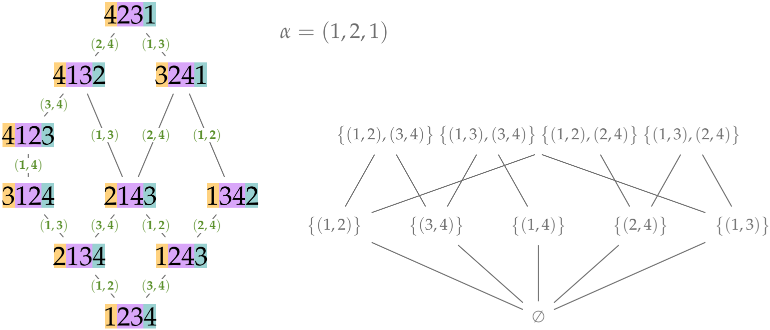

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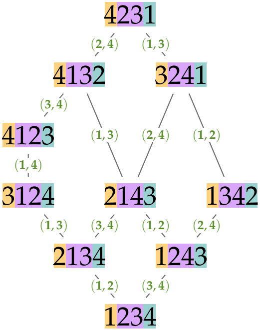

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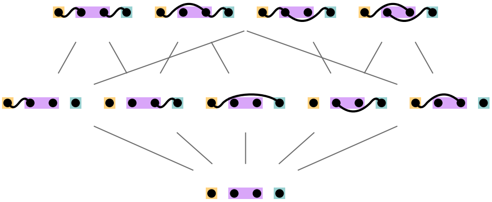

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Theorem (; 2018)

Let α be an integer composition of n. We have $\mathrm{CLO}_{\lambda_\alpha}(\mathcal{T}_\alpha) \cong \mathcal{NC}_\alpha$ if and only if $\alpha = (a,1,1,\ldots,1,b)$ for some $a,b \geq 0$.

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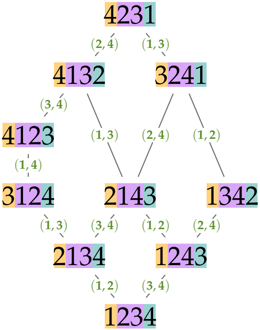

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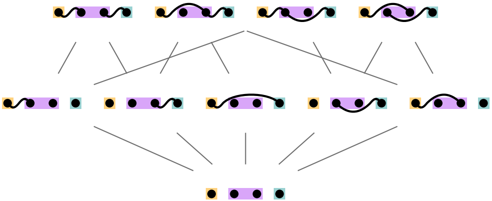

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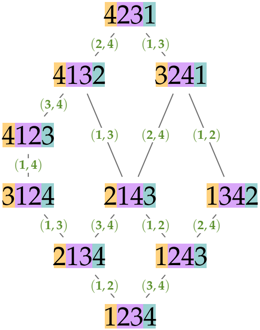

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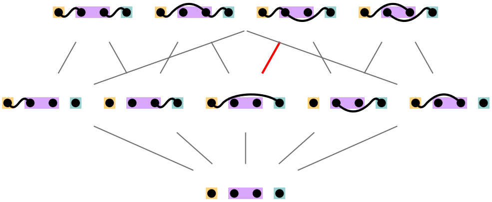

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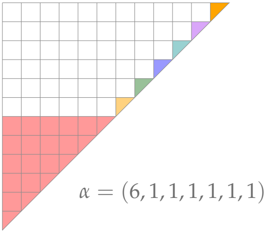

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Theorem (; 2018)

For $n > 0$ and $1 \leq t \leq n$, the common cardinality of the sets $\mathfrak{S}_{\alpha_{(n;t)}}(231)$, $NC_{\alpha_{(n;t)}}$, $\mathcal{D}_{\alpha_{(n;t)}}$, and $\mathbb{T}_{\alpha_{(n;t)}}$ is

$$\mathrm{Cat}\bigl(\alpha_{(n;t)}\bigr) \stackrel{\text{def}}{=} \frac{t+1}{n+1}\binom{2n-t}{n-t}.$$

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Theorem (🌬; 2018)

For $n > 0$ and $1 \leq t \leq n$, the number of noncrossing $\alpha_{(n;t)}$-partitions with exactly k bumps is

$$\binom{n}{k}\binom{n-t}{k} - \binom{n-1}{k-1}\binom{n-t+1}{k+1}.$$

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For $n>0$ and $1\leq t\leq n$, we have $\mathrm{CLO}\bigl(\mathcal{T}_{\alpha_{(n;t)}}\bigr)\cong\mathcal{NC}_{\alpha_{(n;t)}}$.

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Theorem (C. Krattenthaler; 2019)

For $n > 0$ and $1 \leq t \leq n$, the zeta polynomial of $\mathcal{NC}_{\alpha_{(n;t)}}$ is

$$\mathcal{Z}_{\mathcal{NC}_{\alpha_{(n;t)}}}(q) = \frac{t(q-1)+1}{n(q-1)+1}\binom{nq-t}{n-t}.$$

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Outline

1 Parabolic Cataland

2 Bijections in Parabolic Cataland

3 Posets in Parabolic Cataland

4 Chapoton Triangles in Parabolic Cataland

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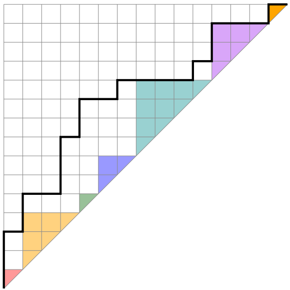
$\alpha = (1,3,1,2,4,3,1)$



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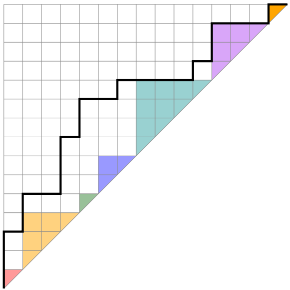

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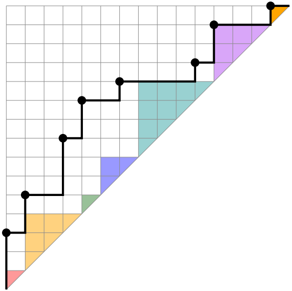

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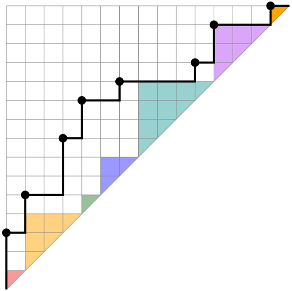

Statistics on Dyck Paths

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- **bounce peak**: common peak of μ and ν_α

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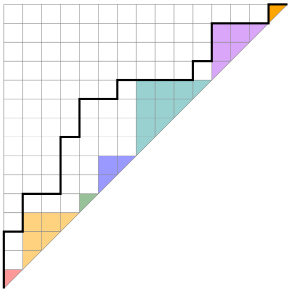
$\text{peak} = 8$



Statistics on Dyck Paths

- $\mu \in \mathcal{D}_\alpha$

- **peak**: coordinate preceded by N and followed by E
- **bounce peak**: common peak of μ and ν_α

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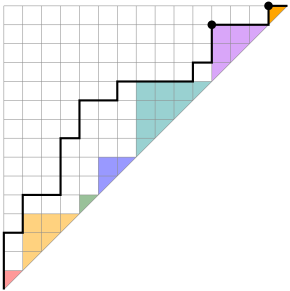

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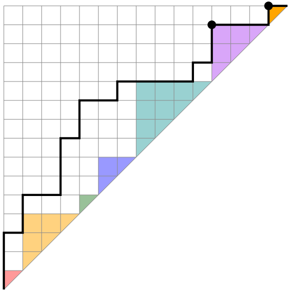

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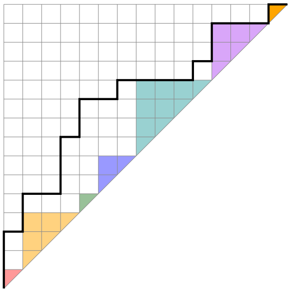

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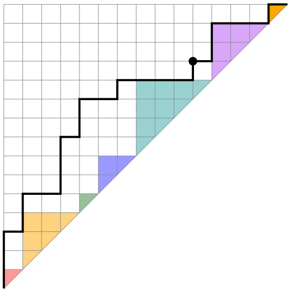

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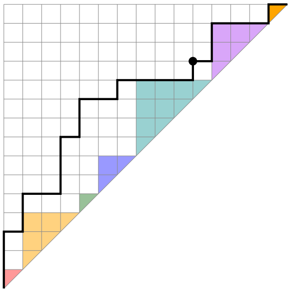

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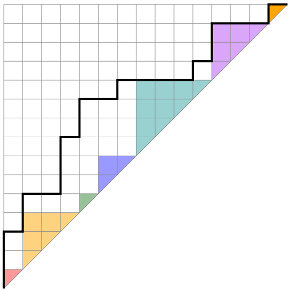
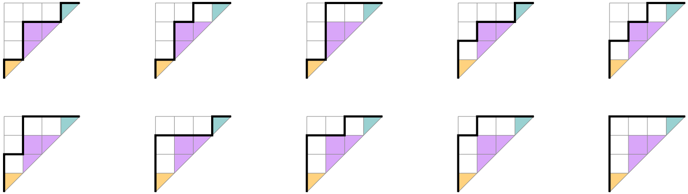


The Parabolic H-Triangle

$\alpha = (1,2,1)$



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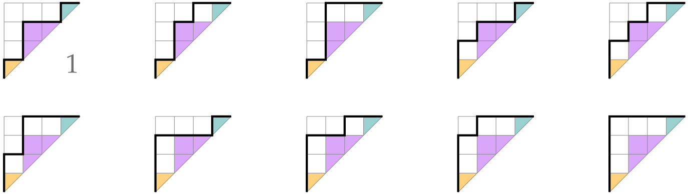

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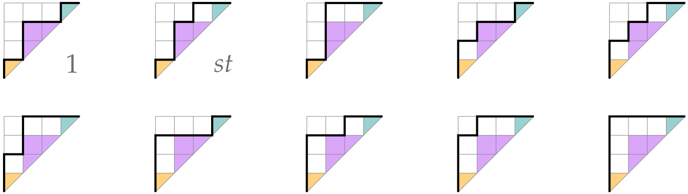

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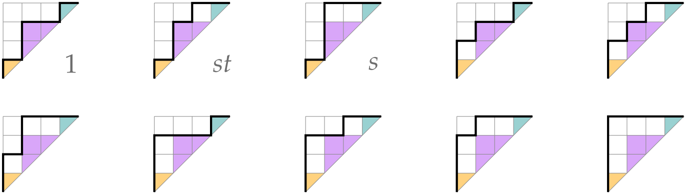

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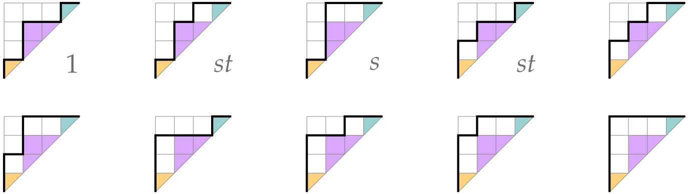

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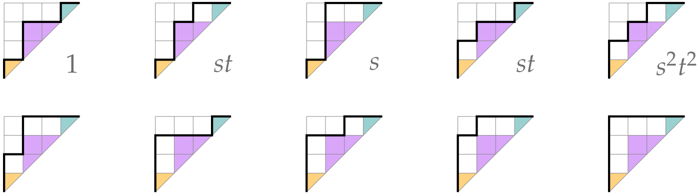

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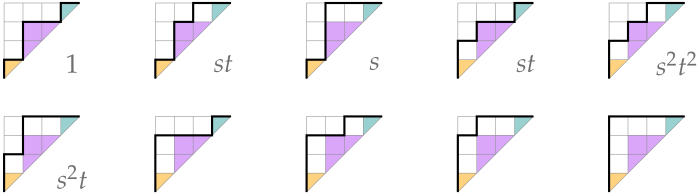

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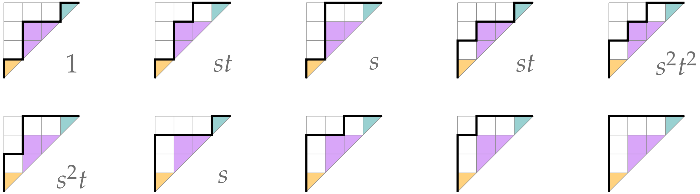

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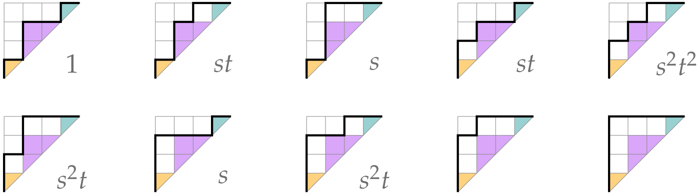

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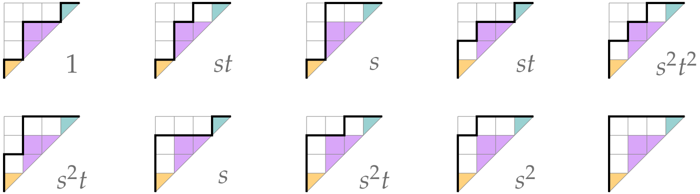

The Parabolic H-Triangle

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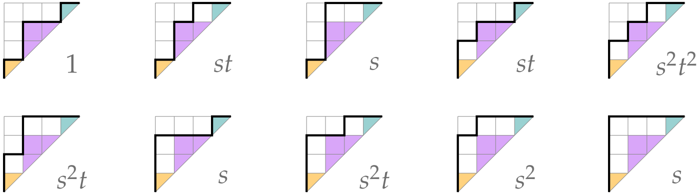
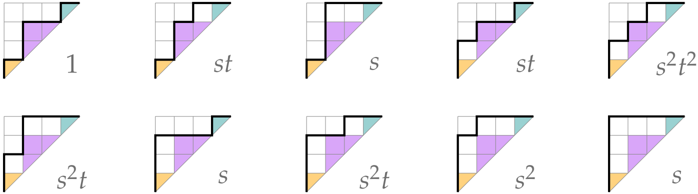


The Parabolic H-Triangle

$\alpha = (1,2,1)$

$$H_{(1,2,1)}(s,t) = s^2t^2 + 2s^2t + s^2 + 2st + 3s + 1$$



Statistics on Noncrossing Partitions

- $\mathbf{P} \in NC_\alpha$

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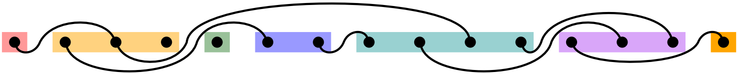

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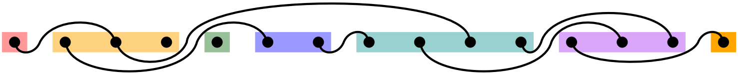

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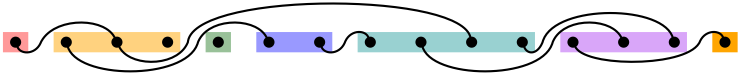
$\mathrm{bump} = 7$



Statistics on Noncrossing Partitions

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Möbius Function

$\alpha = (1,3,1,2,4,3,1)$

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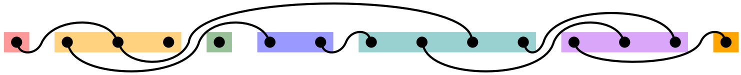

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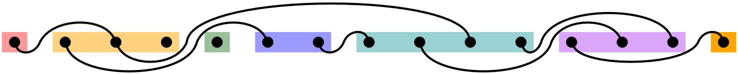

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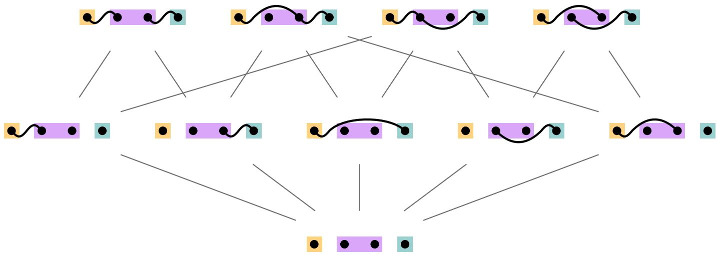
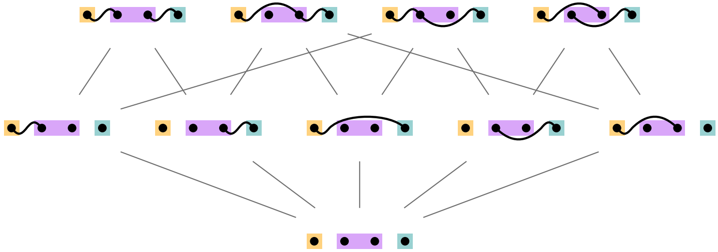


The Parabolic M-Triangle

$\alpha = (1,2,1)$

$$M_{(1,2,1)}(s,t) = 1$$



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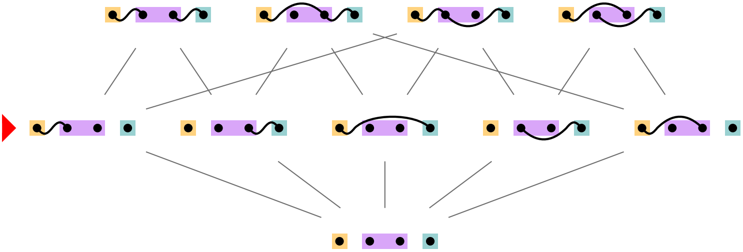
$$M_{(1,2,1)}(s,t) = 1 + 5st$$



The Parabolic M-Triangle

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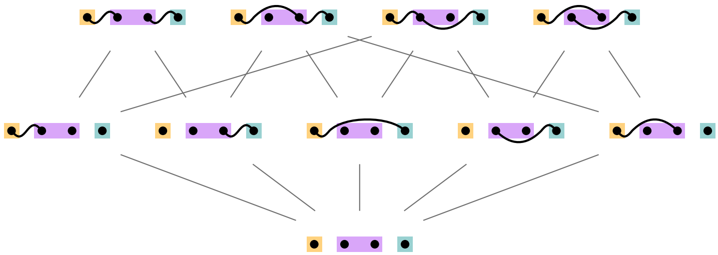
$$M_{(1,2,1)}(s,t) = 1 + 5st + 4s^2t^2$$



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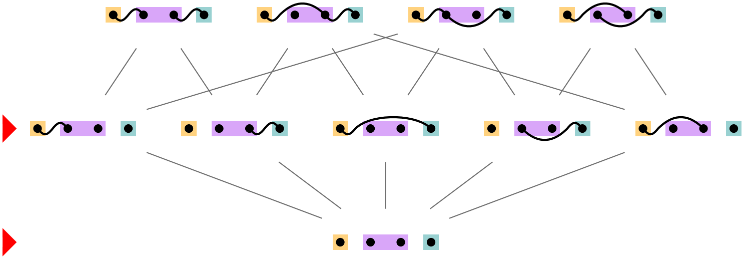
$$M_{(1,2,1)}(s,t) = 1 + 5st + 4s^2t^2 - 5s$$



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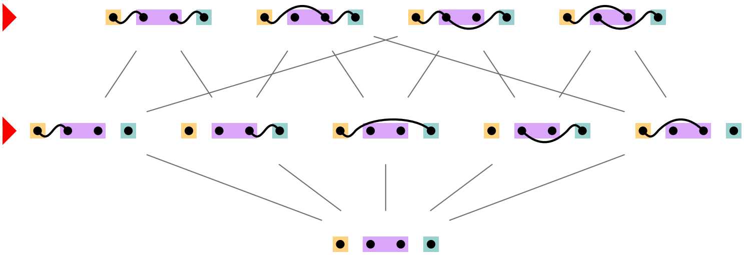

The Parabolic M-Triangle

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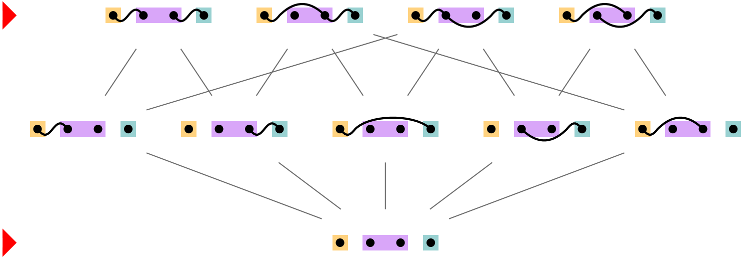
$$M_{(1,2,1)}(s,t) = 1 + 5st + 4s^2t^2 - 5s - 10s^2t + 6s^2$$



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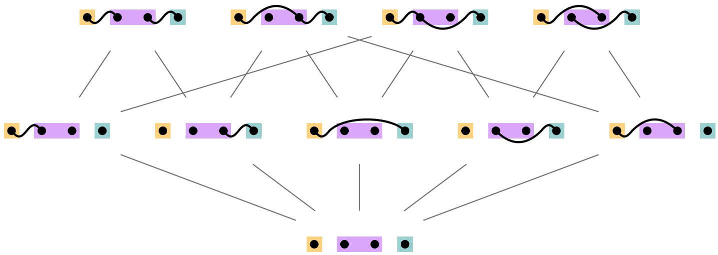

An Enumerative Connection

- ***H*-triangle**:
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Conjecture (; 2018)

The following equation holds if and only if α has r parts, where either the first or the last may exceed 1:
$$H_\alpha(s,t) = \bigl(s(t-1)+1\bigr)^{r-1} M_\alpha\left(\frac{s(t-1)}{(s(t-1)+1}, \frac{t}{t-1}\right).$$

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Conjecture (; 2018)

The following rational function is a polynomial with positive integer coefficients if and only if α has r parts, of which at most one exceeds 1:
$$F_\alpha(s,t) \overset{\text{def}}{=} s^{r-1} H_\alpha\left(\frac{s+1}{s}, \frac{t+1}{s+1}\right).$$

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Question

Which family of combinatorial objects realizes F_α? What are the statistics?

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$\alpha = (1,2,1)$

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$$F_{(1,2,1)}(s,t) = 5s^2 + 4st + t^2 + 9s + 4t + 4$$

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$\alpha = (2,2)$

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$$F_{(2,2)}(s,t) = \frac{5s^2+st+6s+1}{s}$$

Thank You.

Parabolic Quotients of the Symmetric Group

Back

- $\mathfrak{S}_n$: symmetric group of degree n
- $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_r)$: composition of n

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- **parabolic quotient**:

$$\mathfrak{S}_\alpha \overset{\text{def}}{=} \mathfrak{S}_n / (\mathfrak{S}_{\alpha_1} \times \mathfrak{S}_{\alpha_2} \times \cdots \times \mathfrak{S}_{\alpha_r})$$

Parabolic Quotients of the Symmetric Group

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- $\mathfrak{S}_n$: symmetric group of degree n
- $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_r)$: composition of n
- let $s_i \stackrel{\text{def}}{=} \alpha_1 + \alpha_2 + \cdots + \alpha_i$
- **α-region**: a set $\{s_i + 1, s_i + 2, \ldots, s_{i+1}\}$
- **parabolic quotient**:

$$\mathfrak{S}_\alpha \stackrel{\text{def}}{=} \big\{w \in \mathfrak{S}_n \mid w(k) < w(k+1) \text{ for all } k \notin \{s_1, s_2, \ldots, s_{r-1}\}\big\}$$

Parabolic Quotients of the Symmetric Group

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$n = 4$

1234	1243	1324	1342	1423	1432
2134	2143	2314	2341	2413	2431
3124	3142	3214	3241	3412	3421
4123	4132	4213	4231	4312	4321

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$\alpha = (1, 2, 1)$

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Möbius Function

Back

- $\mathcal{P} = (P, \leq)$ finite poset
- **Möbius function**: the map $\mu_{\mathcal{P}} : P \times P \to \mathbb{Z}$ given by

$$\mu_{\mathcal{P}}(x,y) = \begin{cases} 1, & \text{if } x = y, \\ -\sum\limits_{x \leq z < y} \mu_{\mathcal{P}}(x,z), & \text{if } x < y, \\ 0, & \text{otherwise} \end{cases}$$

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Theorem (G.-C. Rota; 1964)

Let $\mathcal{P} = (P, \leq)$ be a finite poset, and let $f, g \colon P \times P \to \mathbb{Z}$. It holds $f(y) = \sum_{x \leq y} g(x)$ if and only if $g(y) = \sum_{x \leq y} g(x)\mu_{\mathcal{P}}(x,y)$.

Möbius Function

- $\mathcal{P} = (P, \leq)$ finite bounded poset; $\hat{0}, \hat{1}$ least/greatest element

Back

- **Möbius function**: the map $\mu_{\mathcal{P}} : P \times P \to \mathbb{Z}$ given by

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Theorem (P. Hall; 1936)

Let $\mathcal{P} = (P, \leq)$ be a finite bounded poset. The reduced Euler characteristic of the order complex of $\bigl(P \setminus \{\hat{0}, \hat{1}\}, \leq\bigr)$ equals $\mu_{\mathcal{P}}(\hat{0}, \hat{1})$ up to sign.

Extremal Lattices

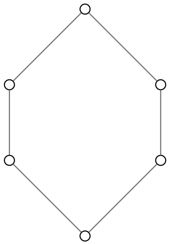
Back

- $\mathcal{L} = (L, \leq)$ finite lattice
- **join irreducible**: $j \in L$ such that $j = x \vee y$ implies $j \in \{x, y\}$ $\rightsquigarrow \mathcal{J}(\mathcal{L})$
- **meet irreducible**: $m \in L$ such that $m = x \wedge y$ implies $m \in \{x, y\}$ $\rightsquigarrow \mathcal{M}(\mathcal{L})$
- **length**: maximal length of a chain $\rightsquigarrow \ell(\mathcal{L})$

Extremal Lattices

- $\mathcal{L} = (L, \leq)$ finite lattice
- **extremal**: $\lvert\mathcal{J}(\mathcal{L})\rvert = \ell(\mathcal{L}) = \lvert\mathcal{M}(\mathcal{L})\rvert$

Back



$\lvert\mathcal{J}(\mathcal{L})\rvert = 4$

$\lvert\mathcal{M}(\mathcal{L})\rvert = 4$

$\ell(\mathcal{L}) = 3$

Extremal Lattices

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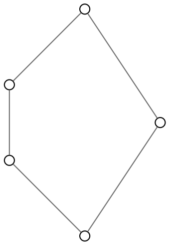

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Extremal Lattices

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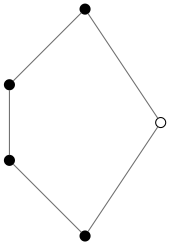
- **extremal**: $\lvert\mathcal{J}(\mathcal{L})\rvert = \ell(\mathcal{L}) = \lvert\mathcal{M}(\mathcal{L})\rvert$
- $C : x_0 \lessdot x_1 \lessdot \cdots \lessdot x_{\ell(\mathcal{L})}$

Back



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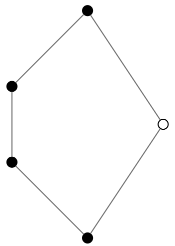
$$j_1 \vee j_2 \vee \cdots \vee j_k = x_k = m_{k+1} \wedge m_{k+2} \wedge \cdots \wedge m_{\ell(\mathcal{L})}$$



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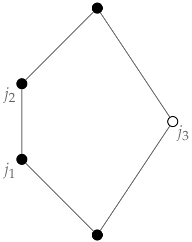

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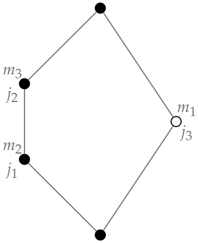

Extremal Lattices

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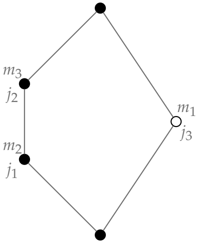

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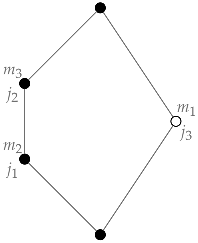

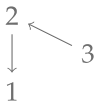

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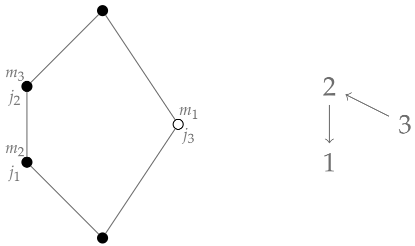

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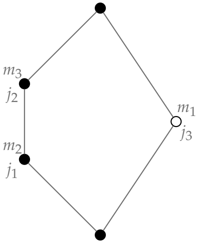

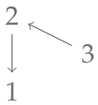

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Theorem (G. Markowsky; 1992)

Every finite extremal lattice is isomorphic to the lattice of maximal orthogonal pairs of its Galois graph.

Extremal Lattices

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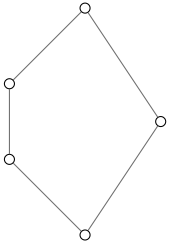

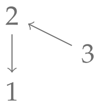

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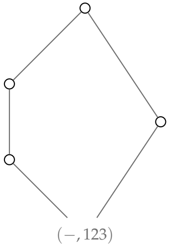

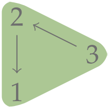

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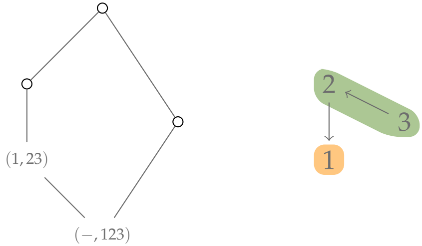

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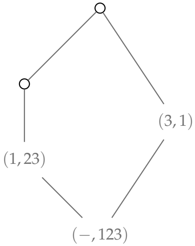

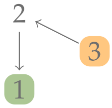

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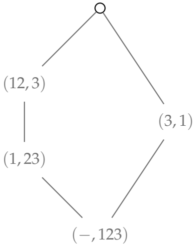

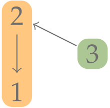

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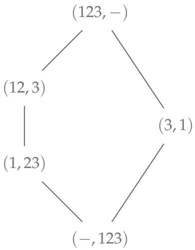
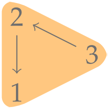
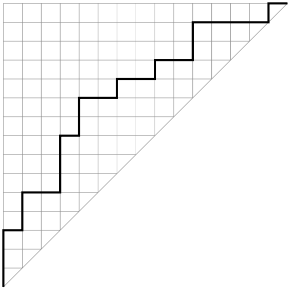




Recovering the Zeta Map

Back



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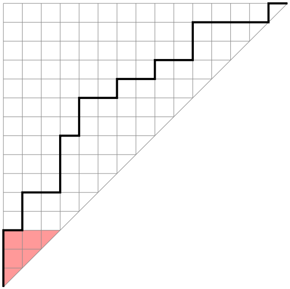

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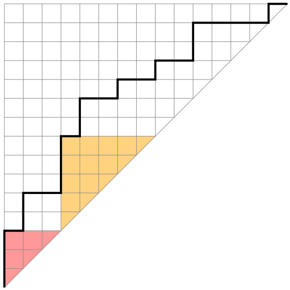

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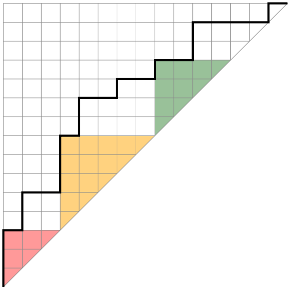

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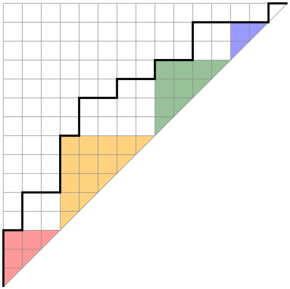

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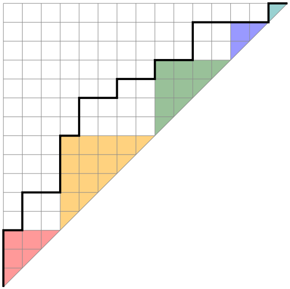

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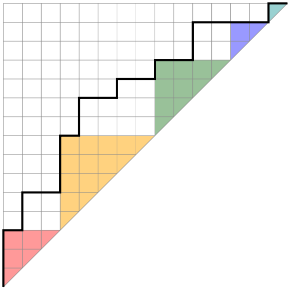

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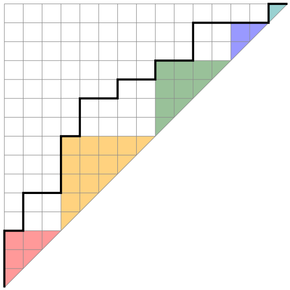

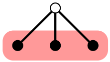

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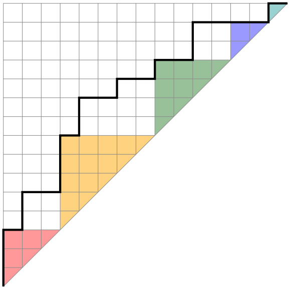

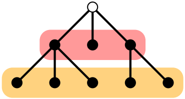

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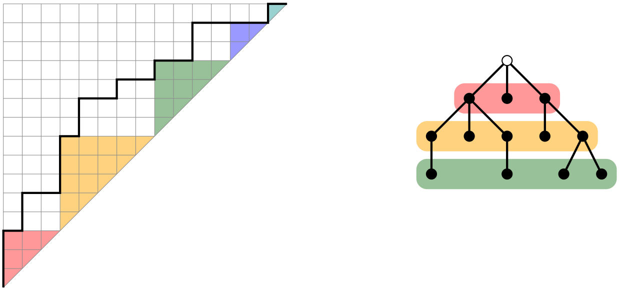

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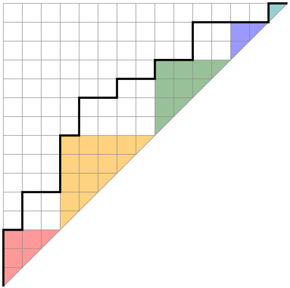

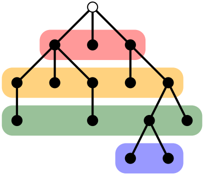

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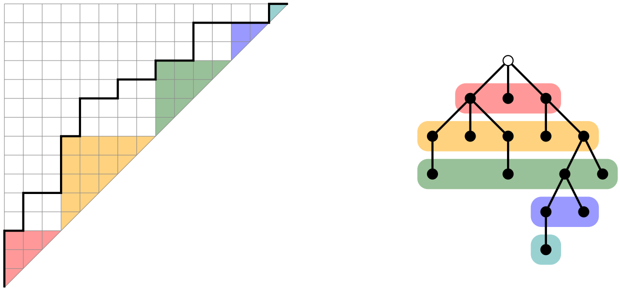

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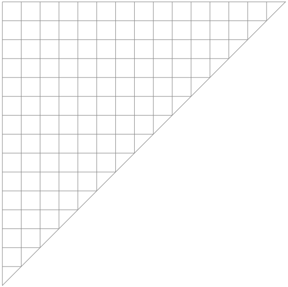

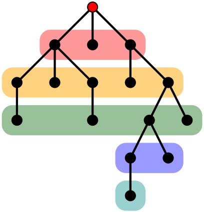

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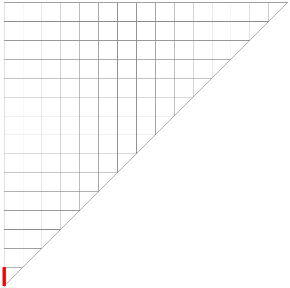

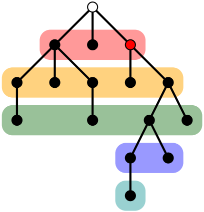

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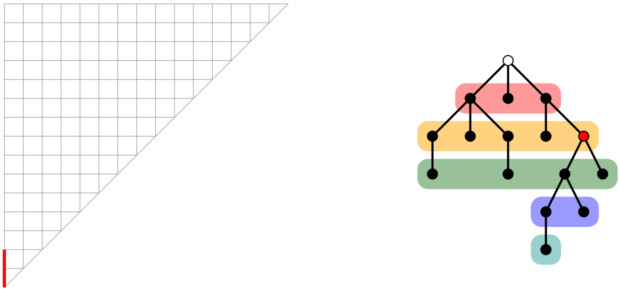

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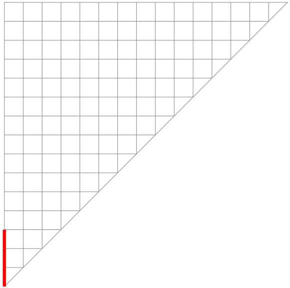

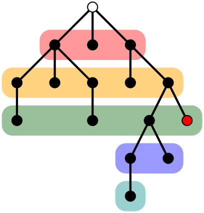

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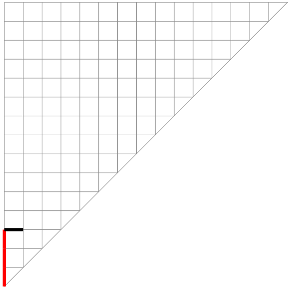

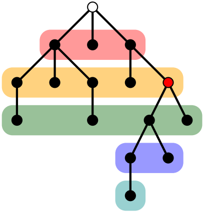

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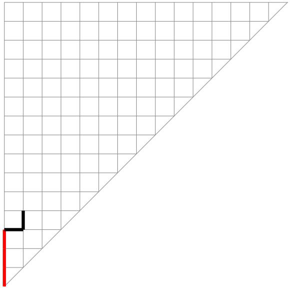

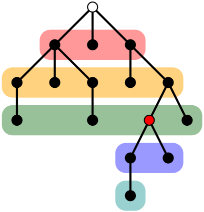

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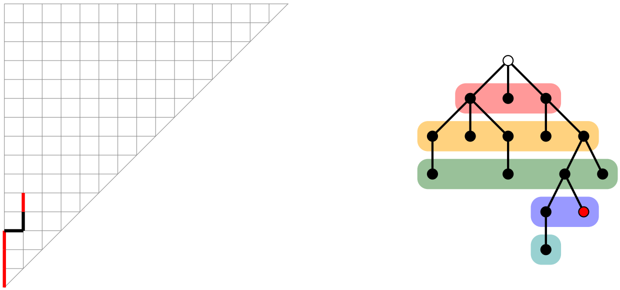

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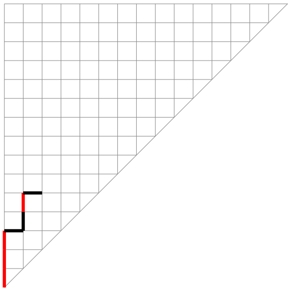

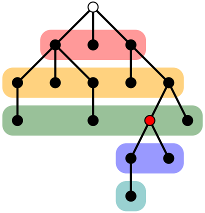

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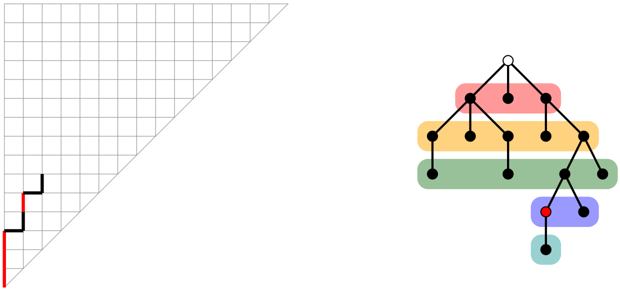

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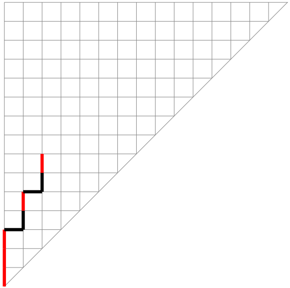

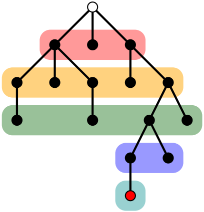

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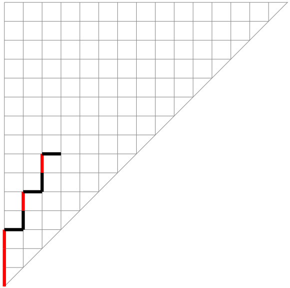

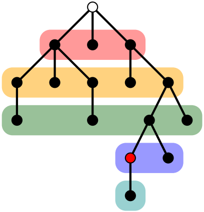

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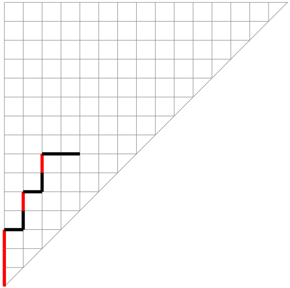

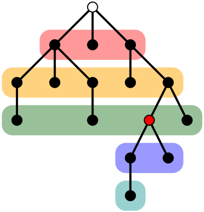

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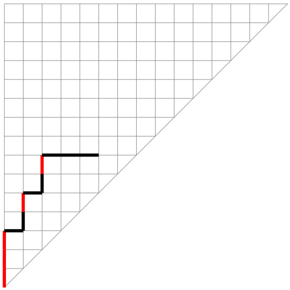

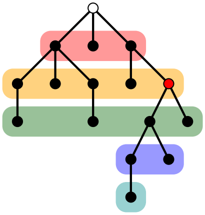

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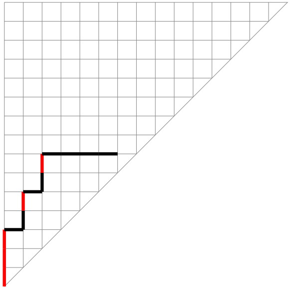

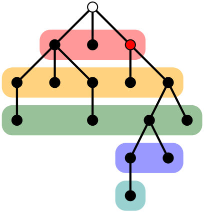

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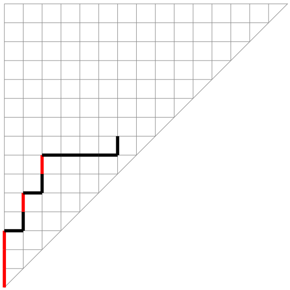

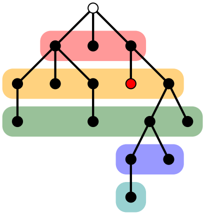

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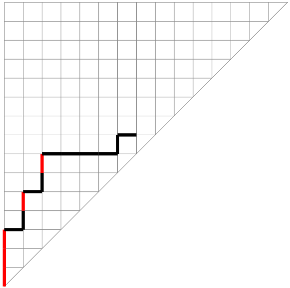

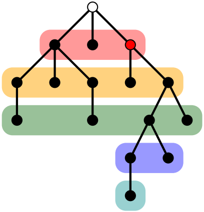

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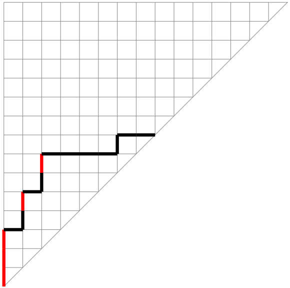

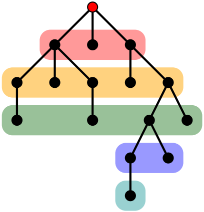

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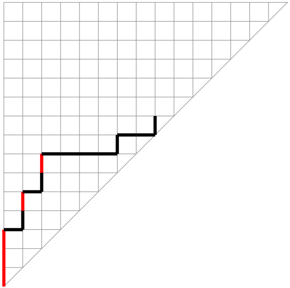

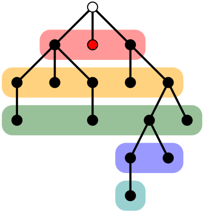

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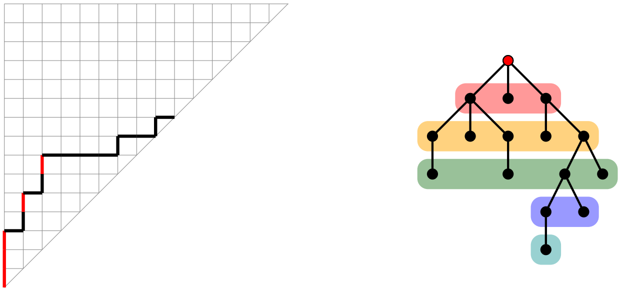

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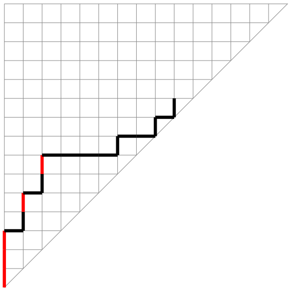

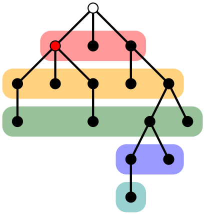

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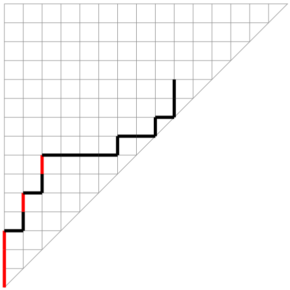

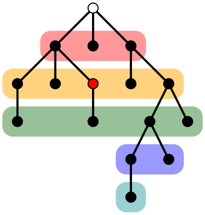

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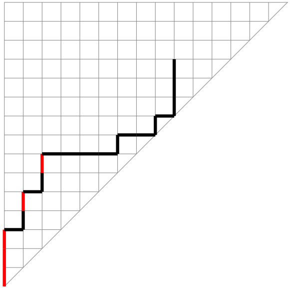

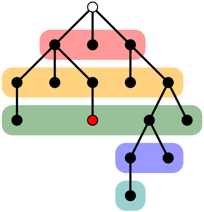

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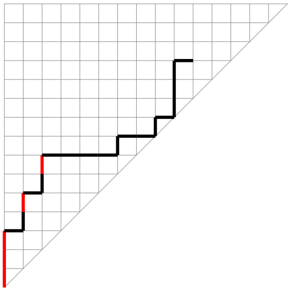

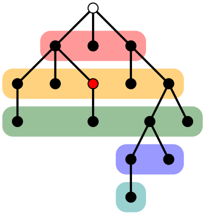

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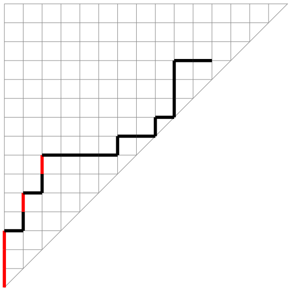

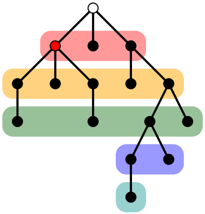

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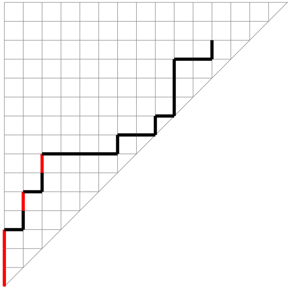

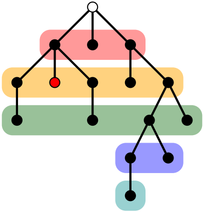

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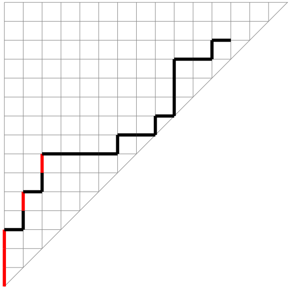

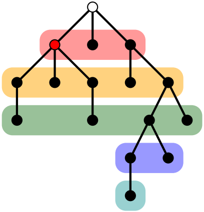

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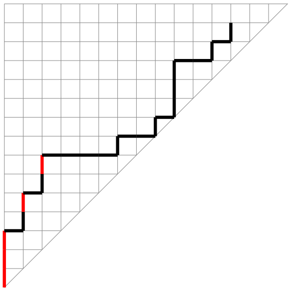

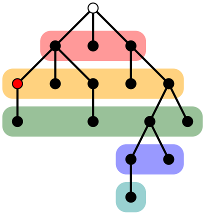

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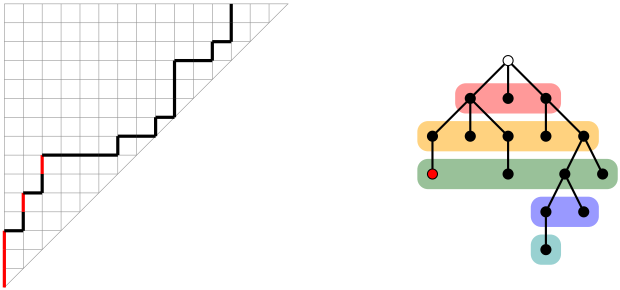

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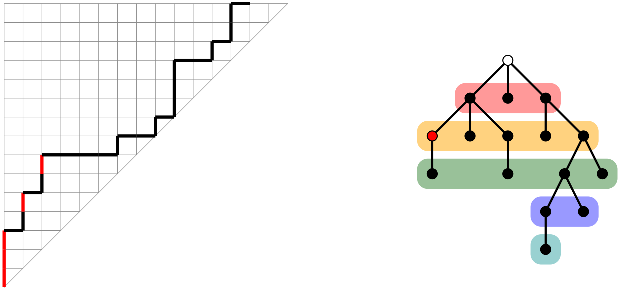

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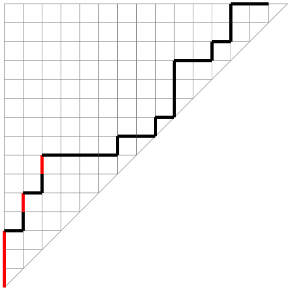

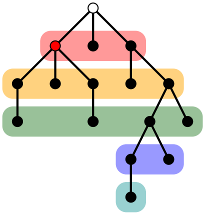

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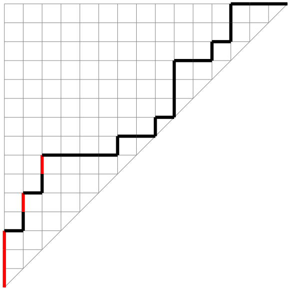

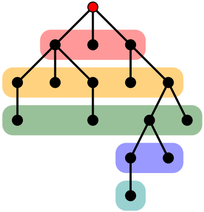

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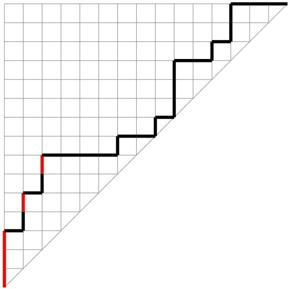

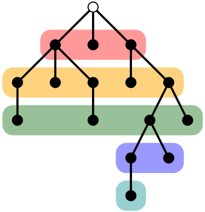

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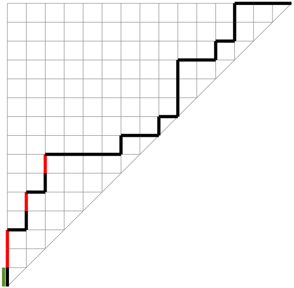

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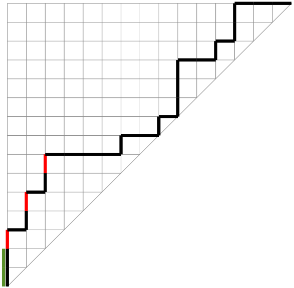

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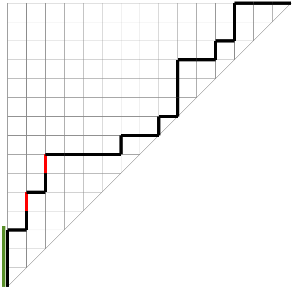

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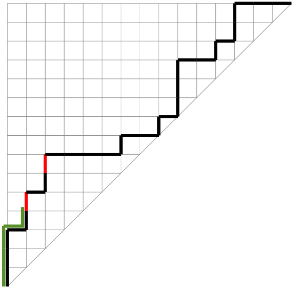

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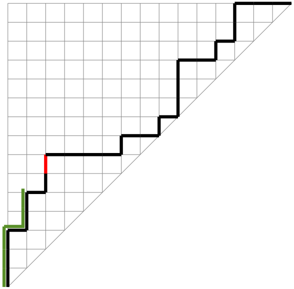

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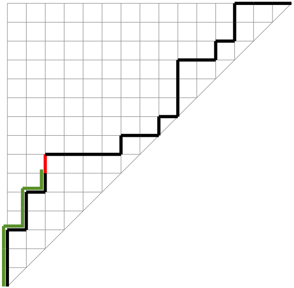

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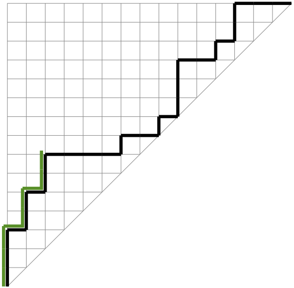

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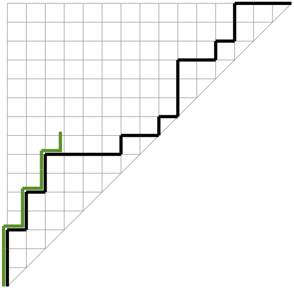

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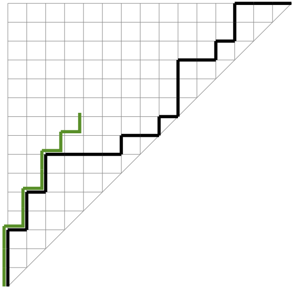

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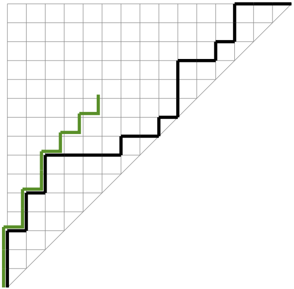

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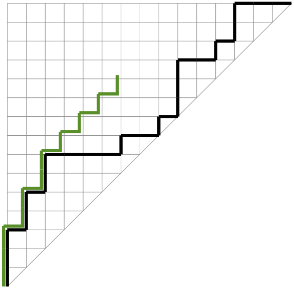

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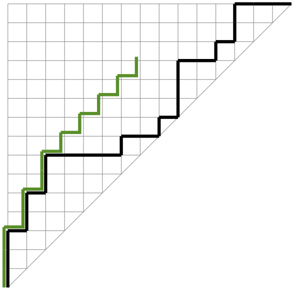

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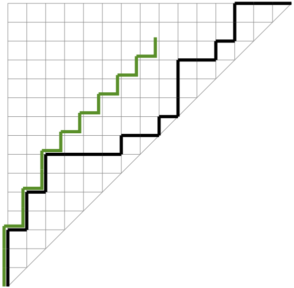

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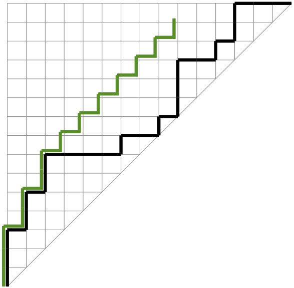
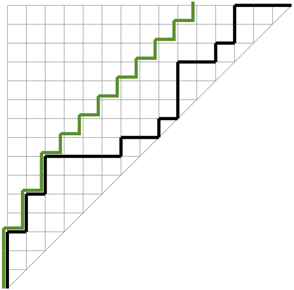


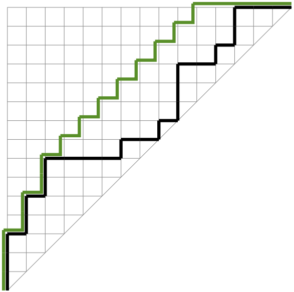
Recovering the Zeta Map

Back



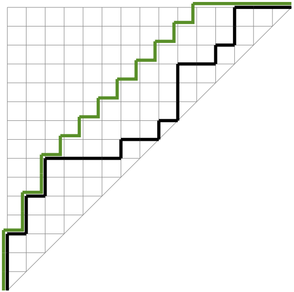
Recovering the Zeta Map

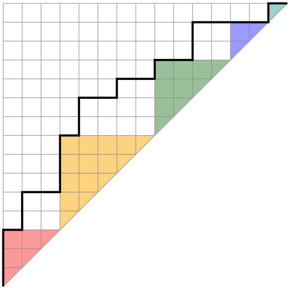
Back



Recovering the Zeta Map

Back





Recovering the Zeta Map

Back