Relation Algebras and CSPs

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DFG Research Training Group 1763

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  - Labeled homogeneous graphs (Cherlin).
  - Translation of RBCP into a classification question about CSPs.

Proper Relation Algebras

Definition

Let D be a set and $E \subseteq D^2$ an equivalence relation. Then $(\mathcal{P}(E); \cup, \bar{\ }, 0, 1, 1', \breve{\ }, \circ)$ is a relation algebra for the following interpretation of function symbols:

1. $A \cup B := A \cup B$,
2. $\bar{A} := E \setminus A$,
3. $0 := \emptyset$,
4. $1 := E$,
5. $1' := \{(x, x) \mid x \in D\}$,
6. $A^{\smile} := \{(x, y) \mid (y, x) \in A\}$,
7. $A \circ B := \{(x, z) \mid \exists y \in D : (x, y) \in A \text{ and } (y, z) \in B\}$.

A subalgebra of $(\mathcal{P}(E); \cup, \bar{\ }, 0, 1, 1', \breve{\ }, \circ)$ is called proper relation algebra.

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For model theorists:

For a proper relation algebra $\mathcal{R}$ we view $\mathbb{R} = (D; \mathcal{R})$ as a relational structure.

Relation Algebras

Definition

A relation algebra $\mathcal{A}$ is an algebra $(A; \cup, ^{-}, 0, 1, 1', ^{\smile}, \circ)$ of type $(2, 1, 0, 0, 0, 1, 2)$ satisfying the following laws:

1. $(A; \cup, ^{-}, 0, 1)$ is a boolean algebra,
2. $(x \circ y) \circ z = x \circ (y \circ z)$,
3. $(x \cup y) \circ z = x \circ z \cup y \circ z$,
4. $x \circ 1' = x$,
5. $(x^{\smile})^{\smile} = x$,
6. $(x \cup y)^{\smile} = x^{\smile} \cup y^{\smile}$,
7. $(x \circ y)^{\smile} = y^{\smile} \circ x^{\smile}$
8. $(x^{\smile} \circ \overline{(x \circ y)}) \cup \bar{y} = \bar{y}$.

Examples

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The minimal non-trivial relations with respect to inclusion are called atoms.

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Point Algebra:

The set $\{=, <, >, \leq, \geq, \varnothing, \neq, \mathbb{Q}^2\}$ together with the "natural" relation algebra operations and the table.

$\circ$	$=$	$<$	$>$
$=$	$=$	$<$	$>$
$<$	$<$	$<$	$\mathbb{Q}^2$
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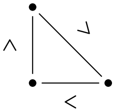

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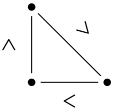

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The set $\{=, E, N, E \cup =, E \cup N, N \cup =, \varnothing, V^2\}$ together with the "natural" relation algebra operations and the table.

$\circ$	$=$	E	N
$=$	$=$	E	N
E	E	$N \cup =$	$E \cup N$
N	N	$E \cup N$	V^2

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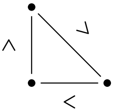

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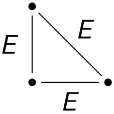

Examples II

Metric spaces:

Let $\{=, 1, 2, 3, 4\}$ be binary predicates associated with integer distances.

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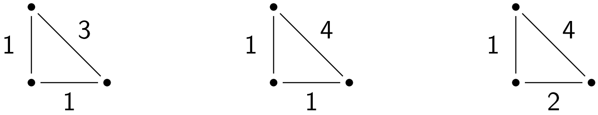

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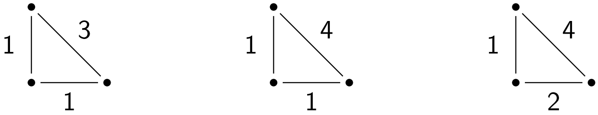

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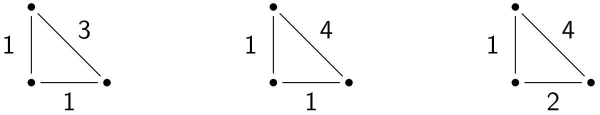

Define a relation algebra on $\mathcal{P}(\{=, 1, 2, 3, 4\})$ with the following multiplication table.

$\circ$	$=$	1	2	3	4
$=$	$=$	1	2	3	4
1	1	$1 \cup 2 \cup =$	$1 \cup 2 \cup 3$	$3 \cup 4$	$3 \cup 4$
2	2	$1 \cup 2 \cup 3$	$1 \cup 2 \cup 3 \cup 4 \cup =$	$1 \cup 2 \cup 3 \cup 4$	$2 \cup 3 \cup 4$
3	3	$2 \cup 3 \cup 4$	$1 \cup 2 \cup 3 \cup 4$	$1 \cup 2 \cup 3 \cup 4 \cup =$	$1 \cup 2 \cup 3 \cup 4$
4	4	$3 \cup 4$	$2 \cup 3 \cup 4$	$1 \cup 2 \cup 3 \cup 4$	$1 \cup 2 \cup 3 \cup 4 \cup =$

Representations

Definition

Let $\mathcal{A}$ be a relation algebra. A relational structure $\mathbb{B}$ is called a representation of $\mathcal{A}$ if

- $\mathbb{B}$ is an A-structure,
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- $(\mathbb{Q}; =, <, >, \leq, \geq, \varnothing, \neq, \mathbb{Q}^2)$ is a representation of the Point Algebra.
- The countable, universal, homogeneous, triangle-free graph

$$\mathbb{H} = (V; =, E, N, E \cup =, E \cup N, N \cup =, \varnothing, V^2)$$

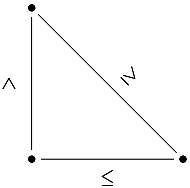
is a representation of the Henson Algebra.

Networks

Definitions

Let $\mathcal{A}$ be a relation algebra. An $\mathcal{A}$-network $(V; f)$ is a finite set of nodes V together with a function $f\colon V \times V \to A$.

Point Algebra Network:



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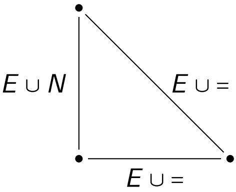

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Let $\mathbb{B}$ be a representation of $\mathcal{A}$. An $\mathcal{A}$-network $(V; f)$ is satisfiable in $\mathbb{B}$ if there exists an assignment $s\colon V \to B$ such that for all $x, y \in V$:

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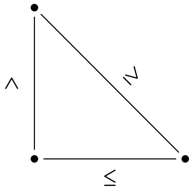

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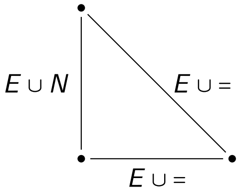

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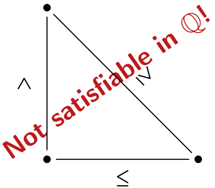

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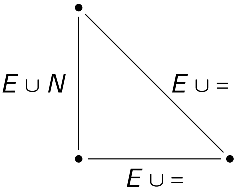

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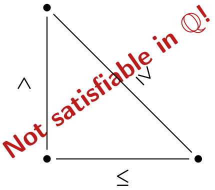

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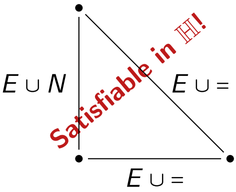

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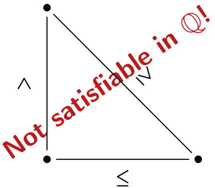

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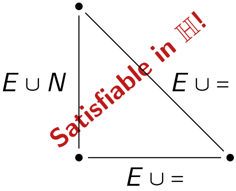

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Definition

Let $\mathcal{A}$ be a finite relation algebra. An atom $S \in A$ is flexible if for all $B, C \in A \setminus \{1'\}$ it holds that $S \leq B \circ C$.
$\rightarrow$ "All triangles that contain a S are allowed."

Examples of Flexible Atoms

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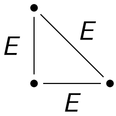

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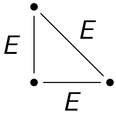

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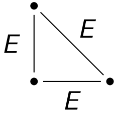

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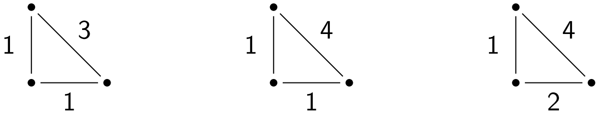
Metric space $+F$:

The Boolean algebra on $\mathcal{P}(\{=, 1, 2, 3, 4, F\})$ with the multiplication specified by the forbidden triangles:



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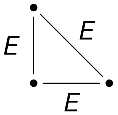

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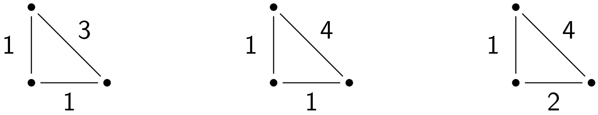

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Cherlin: Classification is open.

Normal Representations

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- homogeneous if every isomorphism of finite substructures of $\mathbb{B}$ can be extended to an automorphism;
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Proposition

Let $\mathcal{A}$ be a finite relation algebra with normal representation $\mathbb{A}$.
Then $\mathbb{A}$ is finitely bounded and NSP($\mathcal{A}$) equals CSP($\mathbb{A}$) (up to some cosmetic differences in the formalisation) and is therefore in NP.

NSP as CSP

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Remark: There exists a finite relation algebra with undecidable NSP (Hirsch 1999)!

Result restated

Theorem

Let $\mathcal{A}$ be a finite relation algebra with a flexible atom. Then $\mathcal{A}$ has a normal representation Γ and $\mathrm{CSP}(\Gamma)$ is in P or NP-complete.
Moreover, it is decidable which of the two cases holds.

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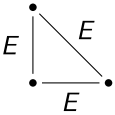
- Important result by Hubička and Nešetřil: Γ with a generic linear order is a Ramsey structure.
- Use of the Bulatov-Zhuk Dichotomy Theorem for finite-domain CSPs.

Examples Classified

Henson Algebra:
The Boolean algebra on $\{=, E, N, E \cup =, E \cup N, N \cup =, \varnothing, V^2\}$ with the multiplication specified by the forbidden triangle:



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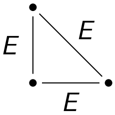

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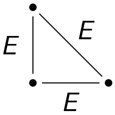

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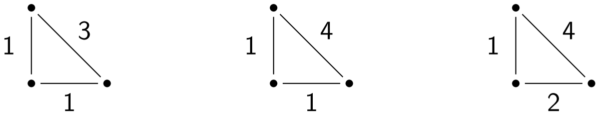
Metric space $+F$:

The Boolean algebra on $\mathcal{P}(\{=, 1, 2, 3, 4, F\})$ with the multiplication specified by the forbidden triangles:



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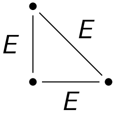

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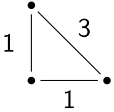

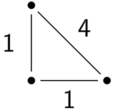

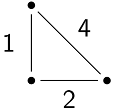

NSP of the "Metric+F Algebra" is polynomial-time solvable!

Thank you for your attention!

Result

Theorem

Let Γ be a normal representation of a finite integral relation algebra with a flexible atom. One of the following holds:

1. There exists for every two atoms A and B of the algebra a polymorphism $f_{A,B}$ of Γ that is canonical and the induced function on $\{A, B\}$ is of Schaefer-type, then Γ has a canonical pseudo-Siggers polymorphism. Then CSP(Γ) is in P.
2. CSP(Γ) is NP-complete.