# Valued Constraint Satisfaction in Structures with an Oligomorphic Automorphism Group

Doctoral defense


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# Outline

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1. Introduction to VCSPs
2. Contributions

# Optimization problems

- **least correlation clustering** (LCC)

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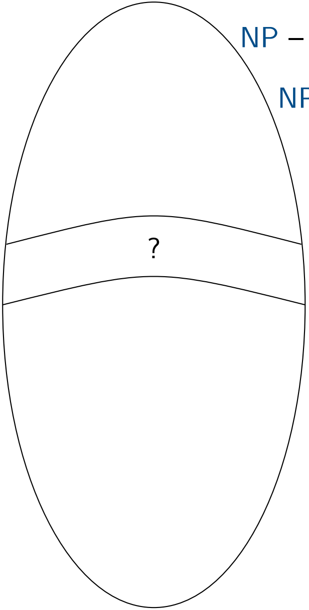
# Computational problems

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NP-complete – hardest in NP

NP-intermediate

P – efficiently solvable

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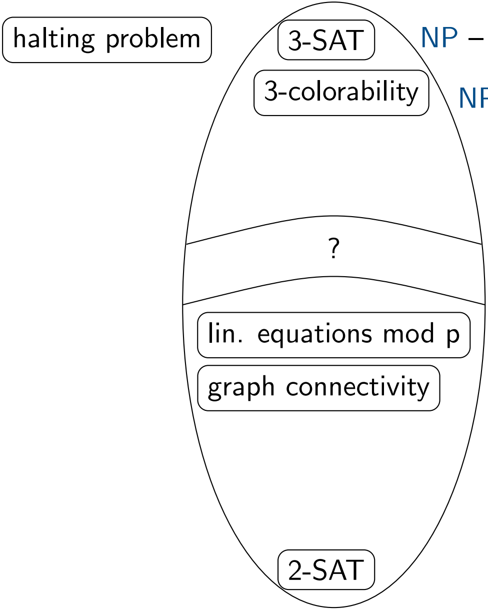


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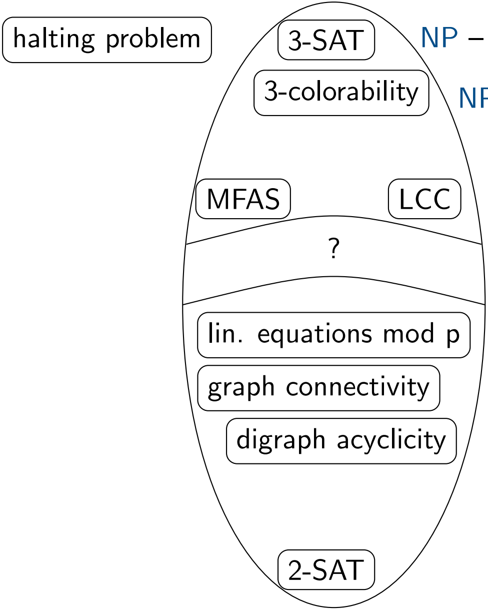


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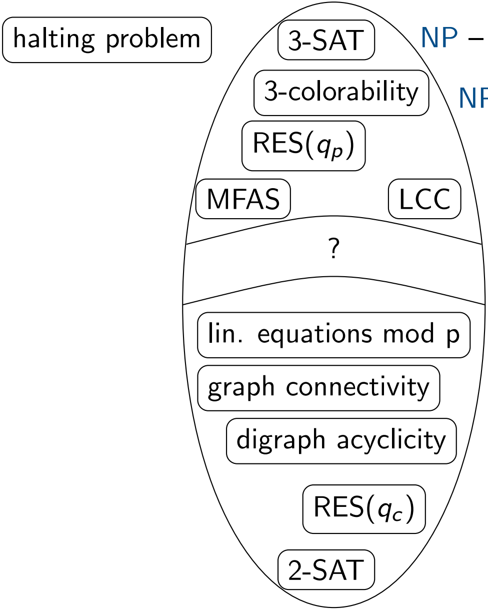


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$q_p$ – 2-path query
$q_c$ – 2-cycle query

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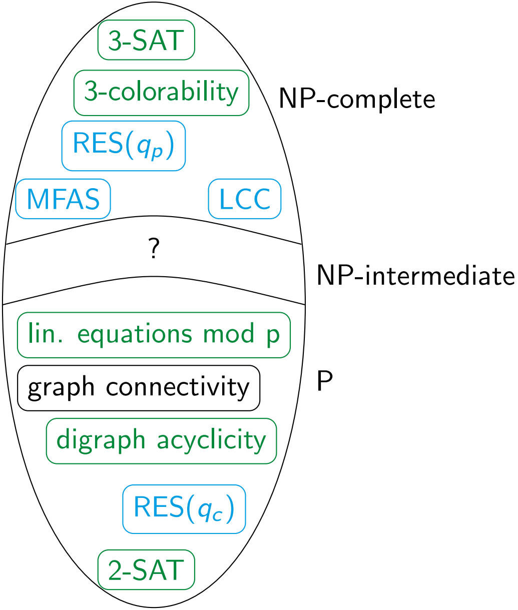
**Observation**: VCSP generalizes CSP and MinCSP.
**Proof**: Model the tuples in relations with cost 0 and outside with cost 1 (for MinCSP) or $\infty$ (for CSP).

# Complexity of CSPs and VCSPs

CSPs
VCSPs



# Valued Constraint Satisfaction Problem

A valued structure $\Gamma$ consists of:

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**Input**: $u \in \mathbb{Q}$, an expression

$$\phi(x_1, \dots, x_n) = \sum_i \psi_i,$$

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**Example**: VCSP($\{0,1\}; R$) where $R(x,y) = 0$ if $x = 0$ and $y = 1$, and $R(x,y) = 1$ otherwise is the Max-Cut problem for directed multigraphs.

# Revisiting problems from the start

- **least correlation clustering** $=$ VCSP$(\mathbb{N}; (=)_0^1, (\neq)_0^1)$

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  $\hookrightarrow$ not obvious how to model as a VCSP

# Complexity of VCSPs

**Theorem (Kozik, Ochremiak '15; Kolmogorov, Rolínek, Krokhin '15; Bulatov '17; Zhuk '17)**

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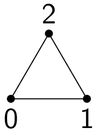
**Example**: $\mathrm{Aut}(\mathbb{Q}; (<)_0^1) = \mathrm{Aut}(\mathbb{Q}; <)$ is oligomorphic.

# Pp-constructability

$K_3$ is the valued structure on $\{0, 1, 2\}$ with single binary relation $E$ defined:

$$E(x, y) = \begin{cases} 0 \text{ if } x \neq y \\ \infty \text{ if } x = y \end{cases}$$

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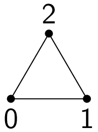


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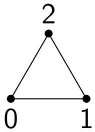


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**Corollary (Bodirsky, S., Lutz '24)**

*If* Aut(Γ) *is oligomorphic and* Γ *pp-constructs* $K_3$*, then* VCSP(Γ) *is NP-hard.*

# Fractional polymorphisms

## Definition (fractional polymorphism)

A fractional polymorphism of $\Gamma$ of arity $n$ is a probability distribution $\omega$ on the maps $f\colon C^n \to C$ such that for every $k$-ary $R \in \tau$ and $t^1, \ldots, t^n \in C^k$

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$\mathrm{Id}_n \in \mathrm{fPol}(\Gamma)$ for every Γ:

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# Classification of temporal VCSPs

A valued structure $\Gamma$ is called temporal if $\mathrm{Aut}(\mathbb{Q}; <) \subseteq \mathrm{Aut}(\Gamma)$.

**Example**:

- $(\mathbb{Q}; (=)_0^1, (\neq)_0^1)$ (models least correlation clustering)

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Building on the classification of temporal CSPs [Bodirsky, Kára '10]:

**Theorem (Bodirsky, Bonnet, S. '24)**

*Let $\Gamma$ be a temporal valued structure. Then at least one of the following:*

- $\Gamma$ *pp-constructs $K_3$ and* $\mathrm{VCSP}(\Gamma)$ *is NP-complete.*
- $\Gamma$ *is essentially crisp,* $\mathrm{fPol}(\Gamma)$ *contains* min, mx, mi, ll, *or one of their duals, and* $\mathrm{VCSP}(\Gamma)$ *is in P.*
- const $\in \mathrm{fPol}(\Gamma)$ *and* $\mathrm{VCSP}(\Gamma)$ *is in P.*
- lex $\in \mathrm{fPol}(\Gamma)$, *all crisp relations expressible in $\Gamma$ are preserved by* min, mx, mi, ll, *or one of their duals, and* $\mathrm{VCSP}(\Gamma)$ *is in P.*

# Resilience of queries

database – a relational structure $\mathfrak{A}$

conjunctive query – a formula $q$ of the form $\exists y_1, \ldots, y_l \, (\psi_1 \wedge \cdots \wedge \psi_m)$, where $\psi_i$ are atomic

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**Input**: a finite database $\mathfrak{A}$, $u \in \mathbb{N}$

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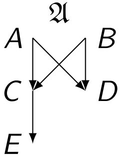
with respect to $\mathfrak{A}$ is 1 – remove $(C, E)$.



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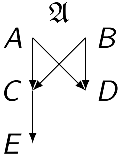


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**Goal**: Classify complexity of resilience for all $q$.

# Translation to a dual problem

**Example**:
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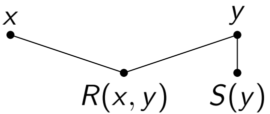

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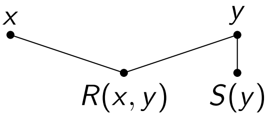


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## Theorem (Cherlin, Shelah, Shi '99)

*Let $q$ be a query. If $I(q)$ is connected, then there exists a dual structure $\mathfrak{B}_q$, such that for every finite $\mathfrak{A}$:*

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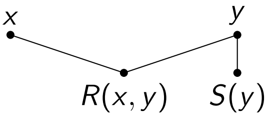


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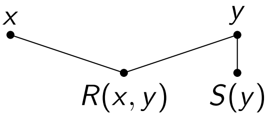

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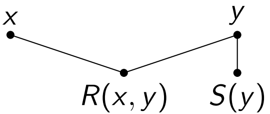


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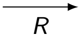
**Example:** For every finite directed graph $\mathfrak{G}$ we have:
$\mathfrak{G} \not\models \exists x, y, z(R(x, y) \wedge R(y, z)) \Leftrightarrow \mathfrak{G}$ maps homomophically to $\xrightarrow[R]{}$

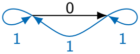
# Resilience problems as VCSPs

query $q$ (WLOG $I(q)$ connected) $\rightsquigarrow$ dual $\mathfrak{B}_q$ $\rightsquigarrow$ 0-1-valued structure $\Gamma_q$

**Example**:

$\mathfrak{B}_q$ $\Gamma_q$

$q = \exists x, y, z(R(x, y) \wedge R(y, z))$ $\rightsquigarrow$ 

$\rightsquigarrow$ 

# Resilience problems as VCSPs

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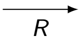


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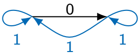


**Theorem (Bodirsky, S., Lutz '24)**

*The resilience problem for $q$ equals* $\mathrm{VCSP}(\Gamma_q)$.

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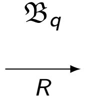


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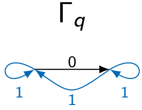


**Theorem (Bodirsky, S., Lutz '24)**

*The resilience problem for q equals* VCSP($\Gamma_q$).

**Remark**: We need to consider bag databases: tuples have multiplicity $\geq 1$.

# Complexity results for resilience

Direct consequence for queries with a finite dual:

**Corollary (Bodirsky, S., Lutz '24)**

*Let $q$ be a conjunctive query such that $I(q)$ is acyclic. Then the resilience problem for $q$ is in P or NP-complete.*

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General tractability criterion:

**Theorem (Bodirsky, S., Lutz '24)**

*If $\Gamma_q$ has a fractional polymorphism which is canonical and pseudo cyclic with respect to $\mathrm{Aut}(\Gamma_q)$, then $\mathrm{VCSP}(\Gamma_q)$ and hence resilience of $q$ is in P.*

# Future classification goals

**Resilience problems**:

**Conjecture**

For a query $q$, whenever $\Gamma_q$ does not pp-construct $K_3$, it satisfies the tractability condition from the previous slide.

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**Graph VCSPs**:

- Classify the complexity of VCSP($\Gamma$) where Aut($\Gamma$) contains the automorphism group of the countable random graph.
- Is VCSP($\Gamma$) in $P$ whenever $\Gamma$ does not pp-construct $K_3$?

# Thank you for your attention

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