Steven Dale Cutkosky (University of Missouri), On the construction of valuations and generating sequences on hypersurface singularities

Suppose that (K, ν) is a valued field, $f(z) \in K[z]$ is a unitary and irreducible polynomial and (L, ω) is an extension of valued fields, where $L = K[z]/(f(z))$. The description of these extensions is a classical subject. We deal here with the more delicate situation where A is a local domain with quotient field K dominated by the valuation ring of ν and $f(z)$ is in $A[z]$, and we want to describe the extensions ω of ν to $A[z]/(f(z))$. A motivation is the problem of local uniformization in positive characteristic: assuming that the valuation ν on A can be uniformized, when can ω on $A[z]/(f(z))$ also be uniformized?

In joint work with Hussein Mourtada and Bernard Teissier, we give an algorithm which in many cases produces a finite set of elements of $A[z]/(f(z))$ whose images in $\mathrm{gr}_\omega A[z]/(f(z))$ generate it as a $\mathrm{gr}_\nu A$-algebra as well as the relations between these images. We also work out the interactions of our method with phenomena which complicate the study of ramification and local uniformization in positive characteristic, such as the non tameness and the defect of an extension. For a valuation ν of rank one and a separable extension of valued fields $(K, \nu) \subset (L, \omega)$ as above our algorithm produces a generating sequence in a local birational extension A_1 of A dominated by ν if and only if there is no defect. In this case, $\mathrm{gr}_\omega A_1[z]/(f(z))$ is a finitely presented $\mathrm{gr}_\nu A_1$-module.