LATTICES OF IDEMPOTENT THEORIES

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Let $\mathcal{K}$ be a class of algebraic structures of similarity type σ and $Th(\mathcal{K})$ the first-order theory of class $\mathcal{K}$. On the set of all theories $T(\sigma)$ of a countable language σ we define a binary operation $\{\cdot\}$ by the following:

$$T \cdot S = Th(\{A \times B \mid A \models T \text{ and } B \models S\}),$$

for any two theories $T, S \in T(\sigma)$. It easy that the algebra $\langle T(\sigma); \cdot \rangle$ forms a semigroup. We show that the set of all idempotent elements of this semigroup forms a complete lattice with respect to the partial order $\leq$ defined as $T \leq S$ iff $T \cdot S = S$, for any $T, S \in T(\sigma)$. We provide some examples of such idempotent theories which establish that, in general, this lattice has uncountable many elements. Also we discussed some problems related to these lattices.

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