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Abstracts

Abutalipova, Shynar
L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
The subgroups of tame automorphisms of the automorphism groups of free metabelian Lie algebras of rank greater than or equal to 4

We study the automorphism groups of free metabelian Lie algebras of rank greater than or equal to 4. The description of subgroups of tame automorphisms of these groups are obtained.

Aichinger, Erhard
Johannes Kepler University Linz, Linz, Austria
Polynomial completeness properties of algebras with Mal'cev terms

For each finite algebra with a Mal'cev term, the clone of polynomial operations is determined by finitely many finitary relations. Polynomial completeness results provide us with concrete sets of such relations for certain classes of algebras. We will give a survey on completeness results for expansions of groups.
One of the most thoroughly studied completeness property is the property of affine completeness. An algebra is called *affine complete* if and only if every congruence preserving function is a polynomial. In establishing affine completeness, we usually face two challenges: one is, for proving that a given algebra is affine complete, to construct an interpolating polynomial function for each congruence preserving function; the other one, for disproving affine completeness, to construct a congruence preserving function that, for some reason, cannot be polynomial.
In the present talk, we will present construction methods for polynomial and congruence preserving functions involving universal algebra, near-ring theory, computer algebra, and lattice theory.

Ali, Majid
Sultan Qaboos University, Muscat, Oman
Multiplication modules and Flat modules

The purpose of this work is to study some properties of multiplication and flat modules. We give some properties of multiplication modules that characterize arithmetical rings. We investigate Ohm type properties for multiplication and flat modules. We also characterize F-modules and FGF-modules.

Ameri, Reza
Tehran, Tehran, Iran
$\mathbb{P}$-fuzzy Congruences of Multialgebras

Let L be a complete lattices. First the notion of L-multialgebras (also called L- hyperalgebras) is introduced and the notion of $\mathbb{P}$-fuzzy relation on multialgebras is given, where $\mathbb{P}$ is any down directed ordered set. These notions will be applied to introduce the notions of $\mathbb{P}$-fuzzy congruences and $\mathbb{P}$-fuzzy strong congruences of multialgebras. Finally, the lattices of $\mathbb{P}$-fuzzy and strong $\mathbb{P}$-fuzzy congruence of multialgebras are constructed and some basic properties of these lattices are investigated.

Anantpinitwatna, Amporn
Mahasarakham, Maha Sarakham, Thailand
The classes of triregular leftmost without loop and reverse arc graph varieties of type (2,0)

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2,0). We say that a graph G satisfies a term equation $s \approx t$ if the corresponding graph algebra $\underline{A(G)}$ satisfies $s \approx t$. A class of graphs $\mathcal{V}$ is called a graph variety if $\mathcal{V} = Mod_g\Sigma$ where Σ is a subset of $T(X) \times T(X)$. A graph variety $\mathcal{V}' = Mod_g\Sigma'$ is called a triregular leftmost without loop and reverse arc graph variety if Σ' is a set of triregular leftmost without loop and reverse arc term equations. A term equation $s \approx t$ is called an identity in a graph variety $\mathcal{V}$ if $\underline{A(G)}$

satisfies $s \approx t$ for all $G \in \mathcal{V}$. An identity $s \approx t$ of a graph variety $\mathcal{V}$ is belong to the class $\mathcal{V}$ whenever $Mod_g\{s \approx t\} = \mathcal{V}$.
In this paper we characterize the class of each triregular leftmost without loop and reverse arc graph variety. For identities, varieties and other basic concept of universal algebra see e.g.[K. Denecke and S.l. Wismath, Universal Algebra and Applications in Theoretical Computer Science, Chapman and Hall/CRC, 2002].

Anantpinitwatna, Apinant
Mahasarakham, Maha Sarakham, Thailand
Special m-hyperidentities in (x(yz))z with loop graph varieties of type (2,0)

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2,0). We say that a graph G satisfies a term equation $s \approx t$ if the corresponding graph algebra $\underline{A(G)}$ satisfies $s \approx t$. A class of graph algebras V is called a graph variety if $V = Mod_g\Sigma$ where Σ is a subset of $T(X) \times T(X)$. A graph variety $V' = Mod_g\Sigma'$ is called a (x(yz))z with loop graph variety if Σ' is a set of (x(yz))z with loop term equations. A term equation $s \approx t$ is called an identity in a variety V if $\underline{A(G)}$ satisfies $s \approx t$ for all $G \in V$. An identity $s \approx t$ of a variety V is called a hyperidentity of a graph algebra $\underline{A(G)}$, $G \in V$ whenever the operation symbols occuring in s and t are replaced by any term operations of $\underline{A(G)}$ of the appropriate arity, the resulting identities hold in $\underline{A(G)}$. An identity $s \approx t$ of a variety V is called an M-hyperidentity of a graph algebra $\underline{A(G)}$, $G \in V$ whenever the operation symbols occuring in s and t are replaced by any term operations in a subgroupoid M of term operations of $\underline{A(G)}$ of the appropriate arity, the resulting identities hold in $\underline{A(G)}$.
In this paper we characterize special M-hyperidentities in each (x(yz))z with loop graph variety. For identities, varieties and other basic concepts of universal algebra see e.g. in [K. Denecke and S.l. Wismath, Universal Algebra and Applications in Theoretical Computer Science, Chapman and Hall/CRC, 2002].

Auinger, Karl
University of Vienna, Wien, Austria
The finite basis problem for matrix semigroups with involution

It is nowadays a classical result that the full matrix semigroup $\langle M_n(K), \cdot \rangle$ over a field K admits a finite basis for its equational theory if and only if the field K is infinite. There are basically two different methods to prove the non-existence of a finite basis in case K is finite: (i) the use of "critical subsemigroups" (Volkov) and (ii) the use of "inherently non-finitely based semigroups" (Sapir). The situation for the involutory semigroups $\langle M_n(K), \cdot, ^T \rangle$ where ^{T} denotes transposition of matrices is somehow different. One can use both methods to clarify the situation completely but each of them on its own is not sufficient. One of the main results presented in the talk is the following Theorem:

Theorem 1 *Let $n \geq 2$ and K be a finite field. Then*

1. *$\langle M_n(K), \cdot, ^T \rangle$ is not finitely based;*
2. *$\langle M_n(K), \cdot, ^T \rangle$ is inherently non-finitely based if and only if either $n \geq 3$ or $n = 2$ and $|K| \not\equiv 3 \pmod 4$.*

(The results are joint work with I. Dolinka and M. V. Volkov.)

Behrisch, Mike
TU Dresden, Dresden, Germany
On a characterisation of categorical equivalence of finite semigroups

Algebras **A** and **B** are categorically equivalent if there is a categorical equivalence F between the varieties generated by **A** and **B**, viewed as categories with morphisms being homomorphisms, such that $F(\mathbf{A}) = \mathbf{B}$.
Algebras **A** and **B** are called weakly isomorphic, if **A** is isomorphic to some algebra **A′** that shares its type with **A** and its carrier set and clone of term operations with **B**. Putting this condition differently, **A** is isomorphic to some algebra that is term equivalent to **B**. Such algebras are known to be categorically equivalent.
For finite semigroups the converse also holds. It is shown that finite semigroups are categorically equivalent if and only if they are weakly isomorphic.
This presents joint work with Tamás Waldhauser, University of Szeged, and extends earlier results about finite groups by László Zádori.

Botur, Michal
Palacký University Olomouc, Olomouc, Czech Republic
State Morphism Algebra - General Approach

We present a complete description of subdirectly irreducible state BL-algebras as well as of subdirectly irreducible state-morphism BL-algebras. In addition, we present a general theory of state-morphism algebras, that is, algebras of general type with state-morphism which is an idempotent endomorphism. We define a diagonal state-morphism algebra and we show that every subdirectly irreducible state-morphism algebra can be embedded into a diagonal one. We describe generators of varieties of state-morphism algebras, in particular ones of state-morphism BL-algebras, state-morphism MTL-algebras, state-morphism non-associative BL-algebras, and state-morphism pseudo MV- algebras.

Chajda, Ivan
Palacký University Olomouc, Olomouc, Czech Republic
Groupoids assigned to relational systems

By a relational system is meant a couple (A, R) where A is a set and R is a binary relation on A, i. e. $R \subseteq A \times A$. To every directed relational system $\mathcal{A} = (A, R)$ is assigned a groupoid $\mathcal{G}(\mathcal{A}) = (A, \cdot)$ on the same base set where $xy = y$ if and only if $(x, y) \in R$. We characterize basic properties of R by means of identities satisfied by $\mathcal{G}(\mathcal{A})$ and show how homomorphisms between those groupoids are related to certain homomorphisms of relational systems.

Cirulis, Janis
University of Latvia, Riga, Latvia
On quasi-orthomodular posets

Motivated by some recent investigations of an ordering of Hilbert space operators, we introduce a class of posets equipped with an orthogonality relation of certain type, and call them quasi-orthomodular posets (qOM-posets). Of special interest is the case when the poset has the upper bound property, i.e., is a nearsemilattice or even a nearlattice. We study the structure of qOM-posets and discover connections between qOM-posets and sectionally orthocomplemented posets as well as generalized orthomodular posets, between qOM-nearsemilattices and generalized orthoalgebras, and between qOM-nearlattices and commutative weak BCK-algebras.

Czédli, Gábor
University of Szeged, Szeged, Hungary
Semimodular lattices determined by permutations

A finite lattice L is called *slim* if no three join-irreducible elements of L form an antichain. Slim lattices are *planar*. So, they are relatively easy objects to understand. A lattice L is called (upper) *semimodular*, if $b \vee c$ covers or equals $a \vee c$ for all $a, b, c \in L$ such that b covers a. A few years ago, G. Grätzer and J. B. Nation [4] improved the classical Jordan-Hölder theorem for groups. A bit later, noticing that slim semimodular lattices arise naturally as the duals of lattices associated with two composition series of a group, we proved in [2] that the permutation occurring in [4] is uniquely determined.
Following [3] in the talk, we associate a permutation $\pi(D)$ with each planar slim semimodular lattice diagram D. We do this not only as in [2], but also in several different, equivalent ways. The main result is that $\pi(D)$ determines D (up to diagram similarity), whence it also determines the lattice represented by D.
Our description of slim semimodular lattices and their diagrams has some applications, including [1]; these will only briefly mentioned.

[1] G. Czédli, L. Ozsvárt, and B. Udvari: How many ways can two composition series intersect? Submitted.

[2] G. Czédli and E.T. Schmidt: The Jordan-Hölder theorem with uniqueness for groups and semimodular lattices. Algebra Universalis, 66 (2011), 69–79.

[3] G. Czédli and E.T. Schmidt: Intersections of composition series in groups and slim semimodular lattices by permutations. Submitted.

[4] G. Grätzer and J. B. Nation: A new look at the Jordan-Hölder theorem for semimodular lattices. Algebra Universalis 64 (2010), 309–311.

Davidov, Sergey
Yerevan State University, Yerevan, Armenia
Nucleus of Linear Invertible Algebras

We define the concepts of nucleus in invertible algebras based on the following hyperidentities:

$$X(x, Y(y, z)) = Y(X(x, y), z),$$

$$X(x, Y(y, z)) = X(Y(x, y), z),$$

$$X(x, X(y, z)) = Y(Y(x, y), z),$$

and we investigate their connections with linear invertible algebras.

[1] Movsisyan Yu.M., Introduction to the theory of algebras with hyperidentities, Yerevan State University Press, 1986.

Dimitrova, Ilinka
South-West University, Blagoevgrad, Bulgaria
On the monoid of all partial order-preserving extensive transformations

A partial transformation α on an n-element chain X_n is called order-preserving if $x \leq y$ implies $x\alpha \leq y\alpha$ for all x, y in the domain of α and it is called extensive if $x \leq x\alpha$ for all x in the domain of α. The set of all partial order-preserving extensive transformations on X_n forms a semiband POE_n. We show that

both the rank and the idempotent rank of POE_n are equal to $2n$. We characterize the maximal subsemigroups as well as the maximal subsemibands of POE_n.

Dormán, Miklós
University of Szeged, Szeged, Hungary
Finite monoidal intervals

It is a well-known that on a finite set A for every transformation monoid M the collection of all clones on A whose sets of unary operations coincide with M constitute a bounded interval in the lattice of all clones on A. Such an interval is called *monoidal interval*. These monoidal intervals yield a natural partition of the clone lattice with finitely many blocks, which may help us in understanding the structure of the clone lattice on A. A monoidal interval will be called *collapsing* if it contains only one element. In [1] we examined a class of maximal inverse transformation monoids constructed from finite lattices. We gave a necessary and sufficient condition for such an inverse transformation monoid to be collapsing. In the lecture we will study monoids that arise from such inverse monoids by adding all the unary constant operations. It turns out that the corresponding monoidal intervals are finite.

[1] Dormán, M., *Collapsing inverse monoids*, Algebra Universalis 56 (2007), pp. 241–261.

Goracinova-Ilieva, Lidija
FON University, Skopje, Macedonia
Mendelsohn designs arising from quasigroups of varieties with the property (2,q)

Let q be positive integer. An algebra is said to have the property $(2, q)$ if all of its subalgebras generated by two distinct elements have exactly q elements. A variety $\mathcal{V}$ of algebras is a variety with the property $(2, q)$ if every member of $\mathcal{V}$ has the property $(2, q)$. Such varieties exist only in the case of q prime power. By taking the universes of the subalgebras of any finite algebra of a variety with the property $(2, q), 2 < q$, blocks of Steiner system of type $(2, q)$ are obtained.
The stated correspondence between Steiner system of type $(2, q)$ and the finite quasigroups of the varieties with the property $(2, q)$ is a folklor. Here we discuss another connection between universal algebra and theory of combinatorial designs, and that is the relationship between such quasigroups and Mendelsohn designs. We prove that these quasigroups can be used, not only as models of Steiner systems, but for constructing Mendelsohn designs, as well.
For any two elements a and b of a groupoid, we define a sequence generated by the pair (a, b) in the following way: $w_0 = a, w_1 = b$, and $w_k = w_{k-2} \cdot w_{k-1}$ for $k \geq 2$. If there is $p > 0$ such that $w_p = a$ and $w_{p+1} = b$, then for the least number with this property we say that it is the period of the sequence generated by the pair (a, b). Then the sequence can be represented by the cycle $(w_0, w_1, \ldots, w_{p-1})$. The main result of this paper is that all of the sequences generated by pairs of distinct elements in arbitrary quasigroup of a variety with the property $(2, q)$ have the same periods (we say it is the period of the variety), and they contain unique appearance of each ordered pair of distinct elements. Thus, the cycles obtained by a finite quasigroup of a variety with period p are the blocks (all of them of order p) of a Mendelsohn design.

Grasegger, Georg
Johannes Kepler University Linz, Linz, Austria
An explicit polynomial equivalence of the rings $\mathbb{Z}_{p^2}$ and $\mathbb{Z}_p[t]/(t^2)$

In this talk we will consider the rings $\mathbb{Z}_p[t]/(t^2)$ and $\mathbb{Z}_{p^2}$. It is known that for every prime p these two rings have a three element chain as ideal lattice and they have the same number of polynomial functions.

We will show that the two rings are isomorphic to polynomially equivalent rings. We will do so by explicitly giving two binary polynomial functions f, g on $\mathbb{Z}_p[t]/(t^2)$ such that $\mathbb{Z}_{p^2}$ is isomorphic to $(\mathbb{Z}_p[t]/(t^2), f, g)$.

Grech, Mariusz
University of Wrocław, Wrocław, Poland
Definability of permutative varieties

We say that a variety of semigroups $\mathcal{V}$ is permutative if there is an identity the form $x_1 \cdots x_n = x_{\sigma(1)} \cdots x_{\sigma(n)}$, for some $n > 1$ and a non-trivial permutation $\sigma \in S_n$, which is satisfied in all subgroups belonging to $\mathcal{V}$.
We say that a variety of semigroups $\mathcal{V}$ is strongly permutative if there is $n > 1$, such that all the identity of the form $x_1 \cdots x_n = x_{\sigma(1)} \cdots x_{\sigma(n)}$ are satisfied in all subgroups belonging to $\mathcal{V}$.
Here we describe first-order definable sets and elements in the lattices of varieties of permutative and strongly permutative semigroups.

Guo, Lankun
Hunan University, Changsha, China
A representation of completely distributive algebraic lattices by formal contexts

Formal concept analysis plays an important role in characterizing order structures, especially complete lattices. In this work, we give an equivalent description of the notion of rough approximable concept, which provides an alternative approach to represent algebraic lattices. It also suggests a new way to generate other types of concepts by combining classical ones. particularly, we propose the notion of weak rough concept and investigate the order theoretical properties of the associated hierarchical concept structures. The results show that this notion provides a way to represent completely distributive algebraic lattices.

Halaš, Radomír
Palacký University Olomouc, Olomouc, Czech Republic
Non-associative BL-algebras and quantum structures

Recently a non-associative version of BL-algebras, the so-called NABL-algebras, being an equivalent algebraic semantics of the logic $\mathcal{NABL}$, was presented by M. Botur, see [1]:
recall that an algebra $\mathbf{A} = (A, \vee, \wedge, \cdot, \to, 0, 1)$ of type $\langle 2, 2, 2, 2, 0, 0\rangle$ is a non-associative residuated lattice if

(A1) $(A, \vee, \wedge, 0, 1)$ is a bounded lattice,

(A2) $(A, \cdot, 1)$ is a commutative groupoid with the neutral element 1,

(A3) for any $x, y, z \in A$, $x \cdot y \leq z$ if and only if $x \leq y \to z$ (adjointness property).

Moreover, if $\mathbf{A} = (A, \vee, \wedge, \cdot, \to, 0, 1)$ satisfies both

$$(x \to y) \vee \alpha_b^a(y \to x) = 1, \qquad (\alpha\text{-prelinearity})$$

$$(x \to y) \vee \beta_b^a(y \to x) = 1, \qquad (\beta\text{-prelinearity})$$

then it is called an NABL-algebra.
Botur has shown that *NABL* forms a variety generated just by non-associative t-norms. Consequently, the corresponding logic $\mathcal{NABL}$ is the logic of non-associative t-norms and their residua [1].
The aim of our talk is to show that NABL-algebras can naturally be described in the language of very basic effect-like algebras in which the Riesz decomposition property plays an essential role. Our result generalizes the result of T. Vetterlein [6] describing BL-algebras as special NAG's, naturally ordered abelian groupoids.

[1] M. BOTUR: *A non-associative generalizaton of Hájek's BL algebras*, Fuzzy Sets and Systems, **178**(1) (2011), 24–37.

[2] M. BOTUR, R. HALAŠ: *Commutative basic algebras and non-associative fuzzy logics*, Archive for Math. Logic, **48**(3-4) (2009), 243–255.

[3] I. CHAJDA, R. HALAŠ, J. KÜHR: *Many-valued quantum algebras*, Algebra Universalis **60** (2009), 63–90.

[4] D.J. FOULIS, M.K. BENNETT: *Effect algebras and unsharp quantum logics*, Found. Phys. **24** (1994), 1325–1346.

[5] P. HÁJEK, R. MESIAR: *On copulas, quasicopulas and fuzzy logic*, Soft Comp. 12 (2008), 1239–1243.

[6] T. VETTERLEIN: *BL-algebras and quantum structures*, Math. Slovaca **54**(2) (2004), 127–141.

[7] YAGER R.R.: *Modelling holistic fuzzy implication using co-copulas*, Fuzzy Optim. Decis. Making 5, 207-226 (2006).

Horváth, Gábor
University of Debrecen, Debrecen, Hungary
Polynomial equivalence of the rings $\mathbb{Z}_{p^n}$ and $\mathbb{Z}_p[t]/(t^n)$?

Georg Grasegger proved that the rings $\mathbb{Z}_{p^2}$ and $\mathbb{Z}_p[t]/(t^2)$ are polynomially equivalent. He gave a bijection $\Phi\colon \mathbb{Z}_{p^2} \to \mathbb{Z}_p[t]/(t^2)$ and binary polynomials f, g over $\mathbb{Z}_p[t]/(t^2)$ such that $\Phi(a+b) = f(\Phi(a), \Phi(b))$ and $\Phi(a \cdot b) = g(\Phi(a), \Phi(b))$ for every $a, b \in \mathbb{Z}_{p^2}$.
We investigate the polynomial equivalence of the rings $\mathbb{Z}_{p^n}$ and $\mathbb{Z}_p[t]/(t^n)$ for some p and $n > 2$.
This research has been supported by the Austrian Science Fund (FWF): P24077 at the Institute for Algebra of the Johannes Kepler University Linz.

Hrbek, Michal
Charles University in Prague, Prague, Czech Republic
Modules with a minimal generating set

The central topic of the talk is a characterization of modules over Dedekind domains having a minimal set of generators. We will provide a characterization for local Dedekind domains (i.e. discrete valuation rings). By localization we also obtain a characterization for torsion modules over general Dedekind domain. On the other hand the conditions for torsion-free modules are essentially different in the case of non-local domains.

Hyčko, Marek
Slovak Academy of Sciences, Bratislava, Slovakia
Pre pseudo-effect algebras - generalizations and finite models

Recently, I. Chajda and J. Kühr [1] have introduced a notion of an pre effect algebra, a certain generalization of an effect algebra such that its lattice ordered models characterize ortholattices. It was shown that it is sufficient to omit the uniqueness of the orthosupplement condition. Moreover the authors defined the notion of a generalized pre effect algebra as a generalization of a generalized effect algebras (no top element present).
The main aim of this contribution is to introduce a non-commutative generalization of pre effect algebras and generalized pre effect algebras, characterize the most important results and estimate the complexity of the search for finite models altogether with certain improvements that enabled to generate all models of pre effect algebras with the size up to 10 elements in reasonable time.

[1] I. Chajda, J. Kühr, *A generalization of effect algebras and ortholattices*, Math. Slovaca, submitted.
[2] A. Dvurečenskij, T. Vetterlein, *Pseudoeffect algebras. I. Basic properties*, Inter. J. Theor. Phys. **40** (2001), 83–99.

Jakubíková-Studenovská, Danica
P.J.Šafárik University, Košice, Slovakia
On formations of finite monounary algebras

A class of algebras is said to be a formation if it is closed under homomorphic images and finite subdirect products. This concept appeared first in connection with finite groups. Several authors investigated formations of concrete algebraic structures, e.g., formations of lattices, of ℓ-groups or of GMV-algebras. Shemetkov and Skiba in their monograph proved that if all finite algebras of a given type have modular congruence lattice, then the lattice of all formations is modular.
The following assertion, which is a slight modification of the well-known Jónsson's lemma, was shown for any class $\mathcal{K}$ of lattices (this result is still valid if we replace lattices by any algebras forming a congruence distributive variety): If A is a finitely subdirectly irreducible lattice and A belongs to the formation generated by $\mathcal{K}$, then A is a homomorphic image of some member of $\mathcal{K}$.
The variety of all monounary algebras fails to be congruence distributive or even congruence modular. Nevertheless, the structure of subdirectly irreducible monounary algebras is relatively simple and this allows us to provide an analogous characterization as it can be done in the case of a congruence distributive variety. In order to obtain this characterization we introduce the set of almost subdirectly irreducible monounary algebras.
The aim of this talk is to present that the collection of all formations of finite monounary algebras is isomorphic to the lattice of all hereditary subsets of a certain poset.

Kaleta, Joanna
Warsaw University of Agriculture, Warsaw, Poland
Some two-generated directly indecomposable entropic quasigroups with quasi-identity

We show that there exists infinite family of pairwise non-isomorfic quasigroups with quasi-identity which are two-generated and are not monogenic. Moreover all quasigroups belonging to this family are directly indecomposable.

Kazda, Alexandr
Charles University, Prague, Czech Republic
Polymorphisms of conservative digraphs

Let $\mathbb{A}$ be a conservative digraph with multiple kinds of edges allowed, i.e. $\mathbb{A}$ is a finite relational structure with relations of arity at most two and $\mathbb{A}$ contains all the possible unary relations. We show that the algebra of polymorphisms of $\mathbb{A}$ either has no Taylor operation (i.e. the Constraint Satisfaction Problem with right hand side fixed to $\mathbb{A}$ is NP-complete), or the algebra of polymorphisms of $\mathbb{A}$ generates a meet semidistributive variety (i.e. the Constraint Satisfaction Problem with right hand side fixed to $\mathbb{A}$ has bounded width).
An earlier combinatorial proof of this result was given by P. Hell and A. Rafiey.

Kearnes, Keith
University of Colorado, Boulder, USA
Dualizable algebras in congruence modular varieties

I will talk about a new condition that is sufficient to establish that a finite algebra in a congruence modular variety is the algebra of character values for a duality theory. For certain types of algebras (groups, rings, modules) the condition is both necessary and sufficient.

Kerkhoff, Sebastian
TU Dresden, Dresden, Germany
On how duality theory enables us to use elementary topology for the benefit of clone theory

We will study coclones in the category of topological spaces in a rather abstract fashion. In fact, we will investigate how elementary properties such as the connectedness of a topological space influences the coclones that are defined over it. Subsequently, we will use duality theory to see what these abstract results mean for clones in the classical sense. In other words, we try to harvest the results from the topological investigation for the benefit of (classical) clone theory. To illustrate the approach, we will take several well-known dual equivalences such as the Stone duality, the Priestley duality or the Gelfand duality to conclude some results for clones over Boolean algebras, distributive lattices and C^*-algebras.

Kisielewicz, Andrzej
University of Wroclaw, Wroclaw, Poland
Progress in the Černý Conjecture

In my talk I present a recent result by M. Grech and myself on a new class of synchronizing n-state automata having a reset word of length not exceeding $(n-1)^2$.

Klin, Mikhail
Ben-Gurion University of the Negev, Beer Sheva, Israel
Construction of infinite families of non-Schurian association schemes of order $2p^2$, p an odd prime

We report about the existence of at least four infinite series of non-Schurian association schemes of order $2p^2$ (for every odd prime $p > 3$). Our starting object $\mathcal{M}$, which is called *biaffine coherent configuration*, appears via intransitive permutation group $\mathbb{Z}_p^2 \rtimes \mathbb{Z}_p$ of order p^3, degree $2p$ and two orbits of lengths p^2. We define four colour graphs $\mathcal{M}_i$, $1 \leq i \leq 4$, which are mergings of $\mathcal{M}$ and prove that these graphs $\mathcal{M}_i$ provide non-Schurian association schemes. Diverse properties of these schemes will be briefly mentioned, as well as their links with some known graphs in extremal graph theory.
The obtained objects were initially discovered (for small values of p) with the aid of computer algebra packages linked to GAP.

Kondo, Michiro
Tokyo Denki University, Inzai, Japan
States on bounded commutative residuated lattices

We define states on bounded commutative residuated lattices and consider their property. We show that, for a bounded commutative residuated lattice X,

(1) If s is a state, then $X/\ker(s)$ is an involutive MTL-algebra.

(2) If s is a state-morphism, then $X/\ker(s)$ is a linearly ordered MV-algebra.

Koppitz, Jörg
Potsdam University, Potsdam, Germany
Coregular Semigroups of special types

In [2], R. Gray and J.D. Mitchell give a series of results that allow us to determine whether a set of mappings in the full transformation semigroup T_n generates a semigroup of a given type. Instead of a set of any mappings we take a set of coregular transformations. A transformation α is called coregular if $\alpha^3 = \alpha$ (Bijev and Todorov have established in 1980 the concept of a coregular element α of a semigroup S as $\alpha\beta\alpha = \beta\alpha\beta = \alpha$ for some $\beta \in S$ [1]). We give a series of results that allow us to determine whether a set of coregular transrormations generates a semigroup of a given type. These types are: left zero semigroups, right zero semigroups, inverse semigroups, groups, completely simple semigroups, and Clifford semigroups.
A semigroup is called coregular if it contains only coregular elements. We give results that allow us to determine whether a set of coregular transformation generates a coregular semigroup of a given type. These results bases on the description of the idempotent elements in T_n. (Note, α^2 is an idempotent for any coregular element α)

[1] G. Bijev, K. Todorov, Coregular Semigroups, Notes on Semigroups VI, Budapest (1980-1984), 1-11.

[2] R. Gray and J.D. Mitchell, Largest subsemigroup of the full transformation monoid.

Kovács, István
University of Primorska, Koper, Slovenia
Characterization of cyclic Schur groups

A finite group G is called a Schur group if all Schur rings over G are schurian, i.e., arise from suitable permutation groups. In this talk we prove that a cyclic group of order n is a Schur group if and only if n belongs to one of the following (partially overlapped) families: p^k, p^kq, $2p^kq$, pqr and $2pqr$, where p, q, r are distinct primes and $k \geq 0$ is an integer.
This is a joint work with Ilya Ponomarenko and Sergei Evdokimov.

Krähmann, Daniel
TU Dresden, Dresden, Germany
When does a sequence of finite groups yield an expander-sequence?

One motivation to study expander-sequences is to construct economical robust networks. A lot of other applications in theoretical computer science have resulted from this approach for example REINGOLD's proof of $SL = L$. In this talk we want to discuss mathematical questions of the theory of expander-sequences behind these practical applications.
We consider finite graphs which may contain loops and multiple edges. A sequence $(G_n)_{n\in\mathbb{N}}$ of regular graphs is called an expander-sequence if the following three conditions are satisfied:

(1) The sequence $(|G_n|)_{n\in\mathbb{N}}$ in $\mathbb{N}$ tends to infinity as n tends to infinity where $|G_n|$ is the order of G_n for all $n \in \mathbb{N}$.

(2) The sequence $(\deg(G_n))_{n\in\mathbb{N}}$ in $\mathbb{N}$ is bounded where $\deg(G_n)$ is the degree of G_n for all $n \in \mathbb{N}$.

(3) The sequence $(\mathrm{h}(G_n))_{n\in\mathbb{N}}$ in $\mathbb{R}$ is bounded away from zero where $\mathrm{h}(G_n)$ describes the CHEEGER-constant of G_n for all $n \in \mathbb{N}$.

The existence of expander-sequences isn't obvious because the conditions listed above are opposite. Having introduced and motivated all mathematical objects and concepts we show necessary conditions for a sequence of graphs to be an expander-sequence. Here we consider mainly sequences of CAYLEY-graphs which are induced by sequences of finite groups. This leads to the question which is formulated as the title. We investigate CAYLEY-graphs of ABELIAN groups and present the Subgroup-Nonexpansion-Principle. Then we give a possibility to construct an expander-sequence. These results enable a interesting access to the so-called MOORE-graphs. In particular we get some ideas to proof or disproof the existence of the missing 57-regular MOORE-graph with diameter 2 and girth 5.

Krapez, Aleksandar
Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia
Explicit formulas for solutions of quadratic quasigroup equations

A general solution of all quadratic quasigroup equations is found by S. Krstić in his PhD thesis using graph theory. We give a new method of solution using trees and dichotomies (a special equivalence relations). Our solution has two advantages:

- General solutions are given by closed formulas
- There is an easy generalization to systems of quadratic quasigroup equations not possible by Krstić's method.

Kühr, Jan
Palacký University Olomouc, Olomouc, Czech Republic
Triple construction of BCK-algebras

Let A be a BCK-algebra with least element 0. If we define $x' = x \to 0$, then the skeleton $S(A) = \{a \in A\colon a'' = a\}$ is a subuniverse of A and the set $D(A) = \{a \in A\colon a' = 0\}$ of dense elements is a filter of A. This situation is similar to Stone algebras, and hence we discuss the question whether the initial BCK-algebra A is characterized by the triple $T(A) = (S(A), D(A), \varphi(A))$ where $\varphi(A)\colon S(A) \to 2^{D(A)}$ is defined by $\varphi(A)(a) = \{x \in D(A)\colon x \geq a\}$. We also focus on the same problem in case of pseudo-BCK-algebras and integral residuated lattices.

Lehtonen, Erkko
University of Luxembourg, Luxembourg, Luxembourg
Generalized entropy in algebras with neutral element and in inverse semigroups

An algebra $\mathbf{A} = (A; F)$ is said to have the *generalized entropic property* if for every n-ary $f \in F$ and every m-ary $g \in F$, there exist m-ary term operations $t_1, \ldots, t_n$ of $\mathbf{A}$ such that $\mathbf{A}$ satisfies the identity

$$g(f(x_{11}, x_{21}, \ldots, x_{n1}), \ldots, f(x_{1m}, x_{2m}, \ldots, x_{nm})) \approx f(t_1(x_{11}, x_{12}, \ldots, x_{1m}), \ldots, t_n(x_{n1}, x_{n2}, \ldots, x_{nm})).$$

We investigate the relationships between the generalized entropic property and the commutativity of the fundamental operations of an algebra. In particular, we show that an algebra with a neutral element has the generalized entropic property if and only if it is derived from a commutative monoid, and an inverse semigroup has the generalized entropic property if and only if it is commutative. This presentation reports joint work with Agata Pilitowska (Warsaw University of Technology).

Liskovets, Valery
NAS of Belarus, Minsk, Belarus
Balanced modular edge labellings of graphs, systems of linear congruences and chromatic polynomials

We consider systems of linear congruences that correspond to the cycles of an arbitrary finite graph. The number of invertible (in particular nowhere-zero) solutions of such a system modulo an arbitrary natural number k is investigated as an arithmetic function of k.
Given an orientation of the edges of a graph $G = (V, E)$, we consider edge k-labellings $f : E \to \mathbb{Z}_k^*$, that satisfy Kirchhoff's circuit law, where $\mathbb{Z}_k^*$ is the set of invertible elements (units) of the ring $\mathbb{Z}_k = \mathbb{Z}/k\mathbb{Z}$. Algebraically these restrictions are described in terms of variables $x_e = f(e)$, $e \in E$, by the system of homogeneous linear congruences modulo k that correspond to the (simple, independent) cycles of G, have coefficients 0 and ± 1 (moreover, their matrix is unimodular) and are subject to the 'side condition' that all admissible values of their variables are coprime with k. The solutions with the latter property are called *invertible*. The choices of edge orientations and independent cycles do not influent the resulting system of congruences up to equivalence. Let $R(G, k)$ denote the number of invertible solutions of such a system. It is proved that for any finite graph G, $R(G, k)$ is the *multiplicative* arithmetic function of k which is determined by the formula

$$R(G, p^a) = \chi(G, p)p^{(a-1)(n-c)-c}$$

for every prime p and integer $a \geq 1$, where $\chi(G, z)$ is the chromatic polynomial of G, $n = |V|$ and c is the number of connected components of G. This basic equation shows that $R(G, k)$ is a kind of totient functions [1], which we call a *chromatic totient*. In particular, $R(K_2, k) = \phi(k)$, Euler's totient function, where $K_2 = \bullet\!\!-\!\!\bullet$ is the graph consisting of two vertices and one edge. When G is a cycle, the general formula turns into the well-known formula of Rademacher—Brauer for the number of invertible solutions of the congruence $x_1 + \cdots + x_n \equiv 0 \pmod{k}$ (cf. the concluding remark in [2]).
The proof uses some results of algebraic graph theory, in particular a simple bijection between proper vertex p-colourings of a rooted connected graph and nowhere-zero $\mathbb{Z}_p$-labellings of its edges that satisfy Kirchhoff's second law. Another important ingredient is the familiar fact that every cycle in a fundamental cycle base of G contains an exclusive edge.
The main formula can be generalized so as to embrace non-homogeneous systems of linear congruences when some variables get prescribed values: an appropriate "partial" chromatic polynomial appears instead of $\chi(G, z)$.

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Madarasz, Rozalia
University of Novi Sad, Novi Sad, Serbia
Retraction closure property and entropicity

We say that an algebra $\mathcal{A}$ has the *retraction closure property (RCP)* if the set of all retractions of $\mathcal{A}$ is closed with respect to fundamental operations of $\mathcal{A}$ applied pointwise. The motivation for studying the induced algebraic structure on the set of all retractions of a universal algebra has its roots in theory of groups. Namely, let $\mathcal{G} = \langle G, + \rangle$ be an Abelian group, φ and ψ two endomorphisms of $\mathcal{G}$. It is well known that the mapping $\varphi + \psi$, defined pointwise i.e. for all $x \in G$

$$(\varphi + \psi)(x) := \varphi(x) + \psi(x)$$

is also an endomorphism of $\mathcal{G}$. If $\mathcal{G} = \langle G, \cdot \rangle$ is a group or a groupoid, the set $\mathrm{End}(\mathcal{G})$ need not be closed in respect to the induced operation $\cdot$. This closure property of the induced algebraic structure on the set of all endomorphisms is investigated in the literature also for universal algebras of arbitrary types under the name *endomorphism closure property*.
Another motivation for extending the structure from base set to the set of all retractions comes from lattice theory. If $\mathcal{L} = \langle L, \leq \rangle$ is a lattice, a mapping $\varphi : L \to L$ is a *retraction* of $\mathcal{L}$ if φ is an idempotent monotone mapping of poset $\langle L, \leq \rangle$ into itself. The set $\mathrm{Retr}(\mathcal{L})$ can turn into a poset by letting

$$\varphi \leq \psi \text{ iff } (\forall x \in L)\varphi(x) \leq \psi(x).$$

It well known that the set of all retractions of a lattice is not necessarily a lattice and that $\mathcal{L}$ is a complete lattice iff the set of retractions is a complete lattice.
In this paper we investigate the retraction closure property property of universal algebras, both "locally" (one algebra) and "globally"(in some variety of algebras), specially emphasizing the case of groupoids. We compare the retraction closure property with the endomorphism closure property on both levels and prove that a necessary and sufficient condition for a variety V of algebras to have RCP is that V is a variety of entropic algebras which satisfy the diagonal law.

Marczak, Adam
Wrocław University of Technology, Wrocław, Poland
Some remarks on Cantor identities

Consider algebras $(A; \cdot, ', ^*)$ of type $(2, 1, 1)$ satisfying

$$(xy)' \approx x, \qquad (xy)^* \approx y, \qquad x'x^* \approx x$$

together with a bijection

$$\varphi(x) = (x', x^*) \quad \text{and} \quad \varphi^{-1}\big((x, y)\big) = xy$$

between the set A and its square $A \times A$. These identities have been considered in fifties of the last century by B. Jónsson and A. Tarski in connection with the investigation of algebras with bases of different cardinalities. The same research direction was initiated also in Wrocław by E. Marczewski, who conjectured that abstract algebras having bases with different number of elements have at least one essentially n-ary term operation for every integer $n > 1$. In 1979 year J. Dudek proved that such algebras have at least one proper diagonal operation of every arity. Then, A. Kisielewicz in 1989 presented a much stronger result: clones of such algebras contain infinitely many distinct essentially n-ary term operations for every $n > 1$. Following these two results we prove that the number of distinct diagonal term operations of every arity is also infinite.

Márki, László
Hungarian Acacdemy of Sciences, Budapest, Hungary
Strong Morita equivalence for semigroups with local units

Morita equivalence is a widely used tool for rings with identity. (Two rings are said to be Morita equivalent if the categories of unitary modules over them are equivalent.) For monoids, this notion is not really useful: in most cases it reduces to isomorphism. As the theory of Morita equivalence could be developed for the more general case of rings with local units, and then for idempotent rings, the question arose whether this could lead to a more fruitful development for semigroups. Indeed, in the mid nineties, Talwar could carry over the basic theorems of Morita equivalence to semigroups with local units, showing also the relevance of this notion in the structure theory of semigroups. The theory got stuck at this point, however – for instance, hardly any Morita invariant properties were known. Recently, there has been tremendous progress in this field. Lawson, Laan and Márki have exhibited seven different approaches to Morita equivalence, all equivalent for semigroups with local units. (One of Lawson's approaches makes fundamental use of a recent result of Pécsi.) Laan and Márki have also cleared up the relation of these approaches for factorisable semigroups (those in which every element can be decomposed into a product), which is the limit for Morita equivalence theory. In addition, they have proved Morita invariance of a number of properties for semigroups with various kinds of local units.

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Masulovic, Dragan
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Classifying homomorphism-homogeneous structures

A structure is *homogeneous* if every isomorphism between finite substructures of the structure extends to an automorphism of the structure. The theory of (countable) homogeneous structures gained its momentum in 1953 with the famous theorem of Fraïssé which states that countable homogeneous structures can be recognized by the fact that their collections of finitely induced substructures have the amalgamation property. Nowdays it is a well-established theory with deep consequences in many areas of mathematics.
In their 2006 paper, P. Cameron and J. Nešetřil discuss a variant of homogeneity with respect to various types of morphisms of structures, and in particular introduce the notion of homomorphism-homogeneous structures: a structure is called *homomorphism-homogeneous* if every homomorphism between finite substructures of the structure extends to an endomorphism of the structure.
In this talk we shall present an overview of classification results for somce classes of finite structures including posets, graphs and point-line geometries. We shall also present an overview of a few known results on homomorphism-homogeneous algebras, and reflect on the problem of classifying some countably and uncountably infinite homomorphism-homogeneous structures.

Movsisyan, Yuri
Yerevan State University, Yerevan, Armenia
Hyperidentities of the variety of De Morgan algebras

We give the solution of the problem of B.I. Plotkin on characterization of hyperidentities of the variety of De Morgan algebras and that of the algebras with these hyperidentities.

Mućka, Anna
Warsaw University of Technology, Warsaw, Poland
Homomorphic images of subdirectly irreducible many-sorted algebras

Relatively few theorems characterizing homomorphic images of subdirectly irreducible single-sorted algebras are known. We now present various examples of many-sorted algebras which are or are not homomorphic images of subdirectly irreducible algebras, and a characterization of such images which is complete for the variety of fibered automata. Furthermore it will be shown that known theorems for single-sorted algebras do not extend to the case of many-sorted algebras.

Mudrinski, Nebojsa
Johannes Kepler University Linz, Linz, Austria
Are all finite congruence uniform Mal'cev algebras expanded groups ?

A uniform algebra is an algebra such that each its congruence has all its congruence classes of the same size. All algebras with a group reduct have this property. We are able to prove that almost all finite congruence permutable uniform algebras with congruence lattice of height at most two are polynomially equivalent to an expanded group.
The results have been obtained in the framework of the research project P24077, FWF Austria.

Muzychuk, Mikhail
Netanya Academic College, Netanya, Israel
A solution of an equivalence problem for semisimple cyclic codes

A linear $(n, k)_q$ code is a k-dimensional subspace of an n-dimensional vector space $\mathbb{F}_q^n$. Two linear codes $C, D \leq \mathbb{F}_q^n$ are called *permutation equivalent* if one of them can be obtained from another by permuting the coordinates. A *cyclic* code is an $(n, k)_q$ code which is invariant under cyclic shift of its coordinates. A cyclic code is called *semisimple* if $gcd(q, n) = 1$. In my talk I'll present a polynomial (in k, n, q) algorithm for equivalence testing of semisimple cyclic codes.

Nurakunov, Anvar
Eurasian National University, Kazakhstan and Institute of T&A Mathematics, Kyrgyzstan, Astana-Bishkek, Kasakhstan-Kyrgyzstan
Notice on classes closed under subdirect products

Let $\mathcal{R}$ be a class of algebras. A congruence θ on an algebra A is called an *$\mathcal{R}$-congruence* provided $A/\theta \in \mathcal{R}$. In event $\mathcal{R}$ is closed under subdirect product the set $Con_{\mathcal{R}}(A)$ of all $\mathcal{R}$-congruences of A forms a complete lattice. A nontrivial algebra A is called *finitely subdirectly $\mathcal{R}$-irreducible* if $A \in \mathcal{R}$ and intersection of any finite set of its nonzero $\mathcal{R}$-congruences is nonzero. Denote the class of all finitely subdirectly $\mathcal{R}$-irreducible algebras by $\mathcal{R}_{FSI}$. We show that the class $\mathcal{R}/\mathcal{R}_{FSI}$ is axiomatizable for any axiomatazable class algebras $\mathcal{R}$ that is closed under subdirect product. We provide some colloraries of this fact, in particular, if $\mathcal{R}_{FSI}$ is infinitely axiomatixable then $\mathcal{R}$ is.

Pálfy, Péter P.
Hungarian Academy of Sciences, Budapest, Hungary
New results on CI-groups

A finite group G has the Cayley Isomorphism property (briefly, G is a CI-group), when any two Cayley graphs of G are isomorphic if and only if there is a group automorphism providing an isomorphism between the two graphs.
At first, Cayley graphs of cyclic groups (i.e., circulant graphs) were investigated. One of the earliest results in this area was due to (among others) Klin and Pöschel establishing the CI property for cyclic groups of order pq (where p and q are distinct primes). Finally, Muzychuk proved that a cyclic group of order n is a CI-group iff either n or $n/2$ is square-free.
In general, the problem of classifying CI-groups is far from being solved. There are severe restrictions on the structure of CI-groups, but only few groups are known to satisfy the CI property.
Two major questions are in the focus of current research.
1. Which elementary abelian groups are CI-groups? Somlai proved that the elementary abelian groups of order p^k are not CI-groups whenever $k \geq 2p + 3$. However, it may be the case that k can be bounded independently of p.
2. Is it true that the direct product of two CI-groups of coprime order is also a CI-group? Somlai characterized all CI-groups of order $8p$ (where $p > 2$ is a prime). In particular, he proved that the direct product of the quaternion group and the cyclic group of order p is a CI-group.

Paseka, Jan
Masaryk University, Brno, Czech Republic
On realization of generalized effect algebras

A well known fact is that there is a finite orthomodular lattice with an order determining set of states which is not order embeddable into the standard quantum logic, the lattice $L(\mathcal{H})$ of all closed subspaces of a separable complex Hilbert space $\mathcal{H}$.
We show that a generalized effect algebra is order embeddable into the generalized effect algebra $\mathcal{G}_D(\mathcal{H})$ of effects of a complex Hilbert space $\mathcal{H}$ iff it has an order determining set of generalized states iff it is order embeddable into the power of $\mathbb{R}_0^+$. This extends the corresponding results for effect algebras of Riečanová and Zajac.

Pech, Christian
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From ages to generic structures - A journey through different types of homogeneity

In this talk we demonstrate how, using the Baire category theorem, various known Fraïssé-type results can be combined to obtain new such results.

Pech, Maja
University of Novi Sad, Novi Sad, Serbia
The age of homomorphism-homogeneous relational structures

The age of a relational structure **S** is the class of all finite structures that embed into **S**. In this talk we give a characterization of the age of homomorphism-homogeneous relational structures.

Pelea, Cosmin
Babes-Bolyai University, Cluj-Napoca, Romania
Multialgebras, factor multialgebras and universal algebras

One of the most important constructions in multialgebra theory is the formation of factor multialgebras, a topic studied from the outset of this theory. This is not surprising because the first multialgebras were the hypergroups which emerged as a result of the factorization of a group modulo an equivalence relation determined by a subgroup. Later, G. Grätzer proved that any multialgebra can be obtained by an appropriate factorization of a universal algebra modulo an equivalence relation.
Our talk is based on the paper *Multialgebras, universal algebras and identities* (*J. Aust. Math. Soc,* **81**, *2006, 121–139*) which is a joint work with Ioan Purdea. We proved that the multialgebra equivalence relations which determine factor multialgebras which are universal algebras can bring us back in the class of universal algebra in a very natural way. We characterized these relations using term functions and polynomial functions of the universal algebra of the nonempty subsets of the given algebra (A, F). Since many particular multialgebras are defined using identities, we connected these equivalence relations with the identities satisfied by (A, F). Using the model offered by D. Freni in *A new characterization of the derived hypergroup via strongly regular equivalences* (*Comm. Algebra*, **30** *2002, 3977–3989*) for (semi)hypergroups and ommutativity, we characterized, using universal algebra tools, the smallest equivalence relation of (A, F) for which the factor is a universal algebra satisfying a given identity and we connected this kind of factorization with that one from Grätzer's characterization theorem. We applied the obtained results to those hypergroups which are factors of a group modulo an equivalence relation determined by a subgroup in order to give a connection between the derived subgroup of the group and the derived subhypergroup of the hypergroup.

Pervukhina, Tatiana
Ural Federal University, Ekaterinburg, Russia
Structure of finite monoids satisfying $\mathscr{R} = \mathscr{H}$

We consider finite monoids in which Green's relations $\mathscr{R}$ and $\mathscr{H}$ coincide. Our aim is to find a representation for this class of monoids that can be considered as a natural extension of the well-known theorems for $\mathscr{J}$-trivial and $\mathscr{R}$-trivial monoids. Namely, due to Straubing [1], a finite monoid is $\mathscr{J}$-trivial if and only if it divides the monoid of all reflexive binary relations on a set with n elements for some n, as well as some monoid of order preserving extensive transformations of a finite partially ordered set (even of a chain with n elements, due to Pin [2]). It is also known (see, for example, [2]) that any finite $\mathscr{R}$-trivial monoid can be embedded into the monoid $\mathscr{E}_n$ of all extensive transformations on the set $\{1, ..., n\}$ for some n. In this view, our following result obtained as a step to a required representation is of independent interest.

Proposition 1 *Let S be a finite monoid satisfying $\mathscr{R} = \mathscr{H}$, and let $\mathscr{R}^{\#}$ be the smallest congruence containing the relation $\mathscr{R}$. Then, $\mathscr{R}^{\#}$ is contained in $\mathscr{L}$, and the quotient $S/\mathscr{R}^{\#}$ is $\mathscr{R}$-trivial.*

The monoids $\mathscr{E}_n$ can be thought, for our purposes, as monoids of upper-triangular row-monomial $n \times n$-matrices over the trivial group with zero adjoined. Denote now by $TM_n(G)$ the monoid of all upper-triangular row-monomial $n \times n$-matrices over a finite group G with zero adjoined.

Proposition 2 *Any monoid $TM_n(G)$ satisfies $\mathscr{R} = \mathscr{H}$.*

Our main result now follows.

Theorem 1 *Any finite monoid satisfying $\mathscr{R} = \mathscr{H}$ divides $TM_n(G)$ for some finite group G and some n.*

In the proof, the group G, the number n, and the corresponding submonoid of $TM_n(G)$ are given constructively. The construction algorithm is then applied to the series of monoids constructed in [3].

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Pilitowska, Agata
Warsaw University of Technology, Warsaw, Poland
Commuting operations in aggregation

The aggregation refers to the process of combining and merging several (most often numerical) values into a single one.
Commuting is an important property in any two-step information merging procedure where the results should not depend on the order in which the single steps are performed. Very often, the commuting property is instrumental in the preservation of some property during an aggregation process.
The entropicity property expresses the condition that aggregation of all the elements of any matrix can be performed first on the rows, then on the columns, or conversely.
The entropicity is a special case of the so called rectangular generalized bisymmetry equation which plays a key role in microeconomic models.
During the talk some characterization (in particular, the role of unary operations and idempotency) of entropic and rectangular generalized bisymmetric functions will be presented.

Pilz, Günter
Johannes Kepler University Linz, Linz, Austria
Infinite planar near-rings

A near-ring N is called planar if each equation $ax = bx + c$ has a unique solution provided that ax is not always equal to bx. If this is the case then the collection of all mappings from N to N of the form $x \to ax$ forms a fixed-point-free automorphism group Φ of $(N, +)$, and the pair (N, Φ) is a Frobenius group. Conversely, every Frobenius group gives rise to a planar near-ring (in the infinite case, one needs a slight modification). Finite planar near-rings yield lots of balanced incomplete block designs and are useful in the design of statistical experiments. In a joint effort with W.F. Ke, H. Kiechle, and G. Wendt, we study infinite planar near-rings, more precisely those where $(N, +)$ is a finite dimensional real vector space and determine which of them are topological near-rings. Continuous representations play a major role.

Pinsker, Michael
Paris 7, Paris, France
Lattices of subgroups of the symmetric group

We show which lattices can be embedded into the subgroup lattice of the symmetric group on an infinite set X. We compare this result with earlier similar results for the lattice of transformation monoids and the lattice of clones on X. Moreover, we examine what happens when one requires the groups, monoids, and clones to be locally closed, i.e., closed sets with respect to the natural ("convergence") topology on the set of finitary functions on X: life becomes very hard.

Pinus, Alexsandr
Novosibirsk State Technical University, Novosibirsk, Russia
Algebraic Geometry of Universal Algebras, Infinite Quasiidentities and Implicit Operations on Universal Algebras

Notions of algebraic geometry of universal algebras have been introduced in the works by B. Plotkin in 1990-s and it was related to the problem of classification of universal algebras, particularly, in terms of their lattices of algebraic sets (which are the sets of solutions of the systems, finite or infinite, of termal equations). For basic notions of this theory see, for example, in [1, 2]. For any variety V of universal algebras two V-algebras $\mathfrak{A}_1$, $\mathfrak{A}_2$ defined as *geometrically equivalent* iff pairs of systems of termal equations that define the same algebraic sets on the algebra $\mathfrak{A}_1$ and on the algebra $\mathfrak{A}_2$ are coinside (see [3]).
The concept of implicit operation (on pseudovarieties) belongs to S. Eilenberg and M. P .Schutzenberger [4]. The implicit operation on the algebra $\mathfrak{A} = \langle A; \sigma \rangle$ is the function $f : A^n \to A$ which commutes with the inner homomorphisms of the algebra $\mathfrak{A}$ (homomorphisms between some subalgebras of the algebra $\mathfrak{A}$) and such that the subalgebras of the algebra $\mathfrak{A}$ are closed under f.
Any implicit operation $f(x_1, \ldots, x_n)$ on the algebra $\mathfrak{A}$ can be defined on $\mathfrak{A}$ with the help of some ∞-positive-conditional term $t(x_1, \ldots, x_n)$ (the definition see, for example, in [5]). The V-algebras $\mathfrak{A}_1$ and $\mathfrak{A}_2$ are *syntactically implicitly equivalent* iff any ∞-positive-conditional term for the algebra $\mathfrak{A}_1$ are same for the algebra $\mathfrak{A}_2$ and any two ∞-positive-conditional terms which define on $\mathfrak{A}_1$ the same implicit operation define on the same implicit operation on $\mathfrak{A}_2$ and vice versa (see [6]).
The ∞-*quasiidentity* is the $L_{\infty,\omega}$-formula of the type

$$\forall \overline{x} \Big(\mathop{\&}_{i \in I} s_i(\overline{x}) = t_i(\overline{x}) \to s(\overline{x}) = t(\overline{x}) \Big),$$

where $s_i(\overline{x})$, $t_i(\overline{x})$, $s(\overline{x})$, $t(\overline{x})$ terms from variables $\overline{x} = x_1, \ldots x_n$.
The ∞-quasiequational theory of the algebra $\mathfrak{A}$ is a collection of all ∞-quasiidentities which are thru on $\mathfrak{A}$.

Theorem 1 *The following conditions are equivalent:*
a) algebras $\mathfrak{A}$, $\mathfrak{B}$ are syntactically implicitly equivalent;
b) the ∞-quasiequational theories of algebras $\mathfrak{A}$ and $\mathfrak{B}$ coinside;
c) algebras $\mathfrak{A}$, $\mathfrak{B}$ are geometrically equivalent;
d) a finitely generated subalgebra of the algebra $\mathfrak{A}$ is isomorphically embedded in some direct power of the algebra $\mathfrak{B}$ and vice versa.

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Ploscica, Miroslav
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Congruence-maximal varieties

A variety $\mathcal{V}$ is called congruence FD-maximal, if for every finite distributive lattice L the following two conditions are equivalent:

(i) L is isomorphic to Con A for some $A \in \mathcal{V}$;

(ii) for every meet-irreducible $x \in L$, the lattice $\uparrow x$ is isomorphic to Con T for some subdirectly irreducible $T \in \mathcal{V}$.

(Notice that (i) always implies (ii).) We consider some special types of congruence distributive varieties and present a criterion for them, characterizing the congruence FD-maximality.

Plotkin, Eugene
Bar Ilan University, Givat Shmuel, Israel

Equations over algebras: how the logical geometry appears

The aim of the talk is to describe the ideas of logical geometry in more or less plain words. We will trace how some algebraic/model-theoretic problems arise in a geometrical way. We also compare methods of solutions of equations over simple algebras with the ones in free algebras. If time permits, we discuss the Borel-type theorems for word maps over simple Lie algebras and distinguish the cases of Engel maps and quasi-Engel maps. Recall that according to the theorem of Borel a non-trivial word map $G^n \to G$, where G is a connected semi-simple algebraic group, is dominant in the Zariski topology.

Plotkin, Tanya
Bar Ilan University, Givat Shmuel, Israel

Skeletons of knowledge bases

The talk is devoted to algebraic theory of knowledge bases. Recall that a subcategory Λ_1 of a category Λ is a skeleton of Λ if

- the inclusion functor is an equivalence,
- no two objects of Λ_1 are isomorphic.

We will discuss the following theorem:

Theorem 1 *Two knowledge bases $KB_1 = (D_1, \Phi_1, F_1)$ and $KB_2 = (D_2, \Phi_2, F_2)$ are informationally equivalent if and only if their skeletons are isomorphic.*

Here D are data algebras, Φ are the sets of relation names, and F are the sets of feasible states. From the perspective of applications this theorem is related to the elimination of redundancy in knowledge bases. We use logical geometry methods for the description of a knowledge base.

Pongrácz, András
Central European University, Budapest, Hungary
Structures definable from the generic partial order

In model theory, it is of general interest to describe the structures that has a first-order definition in a given structure F. The cases when F is the dense linear order or the random graph were settled by Cameron ('76) and Thomas ('91), respectively. It turned out that up to first-order interdefinability, in both cases there are five structures definable by F. In my talk I am going to present the analogous result for the generic partial order. By implementing a technique of Bodirsky and Pinsker, we have shown that there are five closed permutation groups that contain the automorphism group of the random partial order. Equivalently, there are five essentially different structures definable by the generic partial order, as well. One of these structures give rise to a new operation on partially ordered sets, which can be considered as the generalization of cyclic permutations of totally ordered sets.

Radeleczki, Sándor
Miskolc, Miskolc, Hungary
Tolerance lattice of tolerance factors

The tolerances of a lattice L are reflexive, symmetric relations $T \subseteq L^2$ compatible with the operations of L. They form a lattice denoted by $\mathrm{Tol}(L)$ with respect to the inclusion $\subseteq$. A block of a tolerance $T \in \mathrm{Tol}(L)$ is a maximal set $X \subseteq L$ satisfying $X^2 \subseteq T$. The blocks are convex sublattices of L, and it was shown by G. Czédli that they form a lattice denoted by L/T, called the factor lattice of L modulo T. Although this construction generalise the notion of a factor lattice L/φ defined by the mean of a congruence $\varphi \in \mathrm{Con}(L)$, its properties are significantly different. For instance, L/φ belongs to the same equational class as L, however, the lattice L/T does not belong to it, in general. There are also some other properties, which are generally valid for the congruence factors of any algebra, but they are not true in general for factor lattices: It is known that for any $\varphi \in \mathrm{Con}(L)$, the congruence lattice of the factor lattice L/φ is isomorphic to the principal filter $[\varphi)$ of φ in $\mathrm{Con}(L)$ (homomorphism theorem); Moreover, any $\psi \in \mathrm{Con}(L)$, $\psi \geq \varphi$ induces a congruence ψ/φ on the factor lattice L/φ, such that $(L/\varphi)/(\psi/\varphi) \cong L/\psi$ holds (second isomorphism theorem). In this paper we formulate analogous results for tolerance factors, defining a new partial order $\sqsubseteq$ on the lattice $\mathrm{Tol}(L)$, such that for any $S \in \mathrm{Tol}(L)$, with $T \sqsubseteq S$, a tolerance S/T is induced on the factor lattice L/T. Although it is proved that the new relation $\sqsubseteq$ is involved in similar theorems as $\leq$, the poset $(\mathrm{Tol}(L), \sqsubseteq)$ is not a lattice in general - we prove that it has the structure of a particular commutative join-directoid (see e.g. [3]).

[1] Chajda, I. and Nieminen, J. *Direct decomposability of tolerances on lattices, semilattices and quasilattices*, Czech. Math. J. **32** (1982), 110-115.

[2] Czédli, G.: *Factor lattices by tolerances*, Acta Sci. Math. (Szeged) **44** (1982), 35-42.

[3] Ježek, J. and Quackenbush, R.: *Directoids: algebraic models of updirected sets*, Algebra Universalis **27** (1990), 49-69.

Reichard, Sven
—, Essen, Germany
Constructive classification of coherent configurations

Coherent configurations are objects that capture the combinatorial properties of finite group actions. They were introduced independently by Higman and Weisfeiler-Leman, generalising the earlier notion of association schemes. They form a language for the unified description of various objects in Algebraic Combinatorics, such as distance-regular graphs, partial geometries, and certain block designs. Whereas association schemes of small size have been classified by Hanaki and Miyamoto, no such catalogues existed for coherent configurations until recently. We will discuss enumeration strategies, report our classification results, and compare them with work done independently by Nagatomo and Shigezumi.
This project was prepared under the supervision of Misha Klin of Beer Sheva, and was partially supported by the Center for Advanced Studies in Mathematics at Ben-Gurion University in Beer Sheva, Israel.

Romanowska, Anna
Warsaw University of Technology, Warsaw, Poland
Classifying the intervals of the line $\mathbb{Z}[1/p]$

Algebraic convex sets over the ring $\mathbb{Z}[1/p]$, for a prime number p, are described as certain subreducts of affine spaces over this ring. Among them, geometric convex sets are described as the intersections of convex subsets of real affine spaces with affine subspaces over $\mathbb{Z}[1/p]$. We will examine the properties of such convex sets, with special emphasis on the classification of certain one-dimensional convex sets, namely closed intervals in the line $\mathbb{Z}[1/p]$.

Ruzicka, Pavel
Charles University in Prague, Prague, Czech republic
Abelian groups with a minimal generating set

We will characterize abelian groups possessing a minimal generating set: Let A be an infinitely generating abelian group of size $\varkappa$ with its maximal torsion subgroup denoted by τA. Then the group A has a minimal generating set iff any of the following conditions is satisfied:

1. $\dim A/pA = \dim A/qA = \varkappa$, for two different primes p, q;
2. $\dim(\tau A/p\tau A) = \varkappa$ for some prime number p;
3. For every finitely generated subgroup B of A,

$$\sum\{\dim(A/(pA+B)) \mid p \text{ is a prime and } \dim(A/pA) < \varkappa\} = \varkappa.$$

Moreover, if the group A is uncountable, property (3) can be simplified to

(3′) $\sum\{\dim(A/pA) \mid p \text{ is a prime and } \dim(A/pA) < \varkappa\} = \varkappa$.

and if the size of the group A has uncountable cofinality, then A has a minimal generating set iff any of properties (1) and (2) is satisfied.

Sacarea, Christian
Babes-Bolyai University, Cluj-Napoca, Romania
On some categorical aspects of topological and pseudometric contexts

Formal Concept Analysis is well known for applications in data analysis and artificial intelligence. Nevertheless, there have been some fundamental studies about topological contexts and contexts with pseudometrics. This paper continues these studies using category theory. We prove that there is a categorical duality between the category of standard topological contexts and the category of 0-1 lattices. The category of contexts of pseudometrics is studied and some categorical equivalences are proved.

Shtrakov, Slavcho
South West University, Blagoevgrad, Bulgaria
Stable varieties of semigroups

The paper deals with $\Sigma-$composition and Σ-essential composition of terms, which lead to stable and s-stable varieties of algebras. We obtain a full description of all stable varieties of semigroups, commutative and idempotent groupoids. We investigate essential and term reductions of terms. We introduce an abstract reduction system which simplifies the presentations of terms of type $\tau = (2)$ down to the corresponding normal forms. We consider the s-stable varieties of groupoids as an alternating of the stable ones, aiming to highlight replacing the subterms of a term in a deductive systems instead of the usual replacing of the variables with terms.

Supaporn, Worakrit
Potsdam University, Potsdam, Germany
Category Equivalences of $Pol_A Q$

A classification of clones on a finite set is frequently asked in clone theory. For clones on finite set with at least three elements, a complete classification is not yet known. We want to use the concept of category equivalence of clone to classify clones. This concept was introduced by K. Denecke and O. Lüders in 1995. In fact, each clone on a finite set A can be written in the form $Pol_A Q$ for some collection Q of relations on A. In this presentation, we consider clones of the form $Pol_A Q$ where Q is a collection of unary relations on a finite set A and show a method that allows us to find a clone on a set of smaller cardinality which is category equivalent to $Pol_A Q$.

Turhan, Anni-Yasmin
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Reasoning in Description Logics – the case of $\mathcal{EL}$

Description Logics (DLs) are a family of logic-based knowledge representation formalisms, which can be used to represent the conceptual knowledge of an application domain in a structured and formally well-understood way. DLs are closely related to modal and dynamic logics.
Based on the formal semantics, many reasoning problems have been defined and algorithms to compute their solution have been investigated for a range of DLs. Ontology languages for the Semantic Web such as the OWL dialects are based on DLs and thus have greatly increased interest in DLs and their related reasoning techniques in recent years.
This talk will provide an introduction to knowledge representation with Description Logics. The main focus is on reasoning procedures for the light-weight Description Logic $\mathcal{EL}$, which offers limited expressiveness, but allows for efficient reasoning procedures. We consider reasoning procedures motivated by different aspects of practical applications.

Vargas Garcia, Edith M.
Universidad Autónoma de la Ciudad de México. UACM, Mexico, Mexico
Which maximal clones can be maximal C-clones

A restricted version of the GALOIS connection between polymorphisms and invariants, called $Pol - CInv$, is studied, where the invariant relations are restricted to so called clausal relations. Moreover, the relationship of maximal C-clones and maximal clones is investigated. It is shown that, with the exception of one special case occurring for $|D| = 2$, maximal C-clones are never maximal clones.

Vincekova, Elena
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A note on abelian extensions of partial monoids

We introduce a notion of a partial monoid and define an extension of partially ordered partial monoid by partially ordered abelian group (problem of group extensions for non-abelian structures), using the notion of a cocycle. We state a condition when the extension exists and give some illustrative examples.

Volkov, Mikhail
Ural Federal University, Ekaterinburg, Russia
A Quest for Short Identities - Which questions does automata theory ask algebra over and over again (but gets no answers so far)?

I shall demonstrate that several popular and apparently hard questions in the theory of finite automata and formal languages have a very transparent algebraic meaning. Namely, these questions amount to ask for short (in some natural sense) identities in certain algebras. For instance, the famous Černý conjecture about synchronizing automata can be reformulated in purely algebraic terms as follows: if a unary algebra whose carrier set has n elements satisfies an identity of the form $f(x) = g(y)$, then the algebra satisfies such an identity in which the length of the term $f(x)$ is at most $(n-1)^2$.
It would be quite naïve to expect that a hard problem of the Černý conjecture's caliber can be solved by being merely retold in algebraic terms. However, I think that the fact that there are many important problems that admit algebraic reformulations of similar flavor may be interesting for algebraists and should stimulate a systematic study of shortest identities in various algebraic structures.

Zamojska-Dzienio, Anna
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On idempotent factors of power algebras

In a natural way we can "lift" any operation defined on a set A to an operation on the set of all non-empty subsets of A and obtain from any algebra (A, Ω) its *power algebra* of subsets. Power algebras of non-empty subsets with one additional semilattice operation are known as *extended power algebras*. We consider extended power algebras of modes (entropic and idempotent algebras). We describe some congruence relations on these algebras such that factor algebras are idempotent. Such congruences determine some class of non-trivial subvarieties of the variety of all semilattice ordered modes.