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Contents

Affine Completeness in Expanded Groups

E. Aichinger, J. Ecker

We call a universal algebra $\mathbf{A}$ *k-affine complete* if every k-ary congruence preserving function on $\mathbf{A}$ is polynomial; we call it *affine complete* if it is k-affine complete for all k in $\mathbb{N}$. For each k in $\mathbb{N}$, the algebra $\mathbf{R}_k :=< \mathbb{Z}_4 \times \mathbb{Z}_2; +, f_k >$ with $f_k(x_1, x_2, ..., x_k) := 2x_1x_2...x_k$ is k-affine complete, but not affine complete. It is not known if there is an algorithm that decides whether a given finite algebra (of finite type, with given operation tables for all operations) is affine complete. However, we will show that it is decidable whether a finite nilpotent group (given by its multiplication table) is affine complete: having computed its nilpotent class k, it is sufficient to check whether the group is $(k+1)$-affine complete.

Johannes Kepler University Linz, Austria
Email: erhard@algebra.uni-linz.ac.at
Fachhochschule Hagenberg, Austria

Actions of Finite Quantum Groups on Quantum Polynomials

V. A. Artamonov

A noncommutative analog of a polynomial algebra over a field k is a quantum polynomial algebra Λ generated by elements

$$X_1, X_1^{-1}, \ldots, X_r, X_r^{-1}, X_{r+1}, \ldots, X_n$$

subject to defining relations $X_i X_i^{-1} = X_i^{-1} X_i = 1$ if $1 \leq i \leq r$ and $X_i X_j = q_{ij} X_j X_i$ if $1 \leq i, j \leq n$. Elements $q_{ij} \in k^*$ satisfy the relations $q_{ii} = q_{ij} q_{ji} = 1$ for all $1 \leq i, j \leq n$, and r is an integer such that $0 \leq r \leq n$. The algebra Λ can be considered as a coordinate algebra of a direct product of quantum torus $\mathbb{T}^r$ and a quantum affine space $\mathbb{A}^{n-r}$ [1].

The aim of this study is a classification of actions of finite quantum groups on $\mathbb{T}^r \times \mathbb{A}^{n-r}$ in terms of H-module structure on the algebra Λ where H is a finite dimensional pointed Hopf algebra [2].

There are some examples of actions of some special pointed finite dimensional Hopf algebras on Λ. Namely let U be a subgroup of a finite index in $\mathbb{Z}^n$ and $[k(\mathbb{Z}^n/U)]^*$ the dual Hopf algebra of the group algebra $k(\mathbb{Z}^n/U)$ of the finite factorgroup $\mathbb{Z}^n/U$. For any $f \in [k(\mathbb{Z}^n/U)]^*$ we put $f \circ X^v = f(v+U)X^v$ for any monomial X^v, where $v \in \mathbb{Z}^n$ is a multi-index. Hopf algebra $[k(\mathbb{Z}^n/U)]^*$ is provided with an involutive automorphism $f \mapsto \tilde{f}$ where $\tilde{f}(v) = f(-v)$. So we can form a smash product $[k(\mathbb{Z}^n/U)]^* \sharp k\langle\xi\rangle$ with the cyclic group $\langle\xi\rangle$ of order 2. If $r = n$ then the action of $[k(\mathbb{Z}^n/U)]^*$ on Λ can be extended to an action of the smash product where $\xi \circ X^v = X^{-v}$ for all $v \in \mathbb{Z}^n$.

Theorem 0.1: Suppose that q_{ij}, $1 \leq i < j \leq n$, are independent in the multiplicative group k^* of the field k and H is a pointed finite dimensional Hopf algebra H such that if $r = n = 2$ then $\dim H$ is not divisible neither by 4 nor by 3. Let Λ be a left H-module algebra. Then there exists a subgroup U in $\mathbb{Z}^n$ of a finite index and a Hopf algebra homomorphism $\Psi : H \to [k(\mathbb{Z}^n/U)]^* \sharp k\langle\xi\rangle$ such that the action of H in Λ is a product of Ψ and the mentioned action of $[k(\mathbb{Z}^n/U)]^* \sharp k\langle\xi\rangle$ on Λ. If Ψ is not surjective then its image is equal to $[k(\mathbb{Z}^n/U)]^*$. It is always the case if $r < n$.

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Moscow State University,
Email: artamon@mech.math.msu.su

Dimension of Topological Spaces and Local Homologies

D. Artamonov

The dimension of a topological space is characterized by local homologies. In the trivial case of a polyhedra the local homologies are free abelian groups. In the talk I will give a topological characterization of a space with free local homologies: it will be shown that in some natural class of spaces this property is eqvivalent to the property to be a dimensionally full valued space. It is discussed whether local homologies can have torsion. Also it will be considered a relationship between “generalized polyhedras” - spaces with free local homologies - and generalized (homological) manyfolds.

Moscow State University
Email: artamon@mech.math.msu.su

An Algorithm How to Determine the Numbers of Endomorphisms on Paths

Sr. Arworn, U. Knauer

An endomorphism of a graph is a function on that graph which preserves edges of the graph. In this paper we show an algorithm to determine the formula of the cardinalities of endomorphism monoids of undirected paths.

Theorem 0.2: The cardinalities $|End(P_m)|$ of the endomorphism monoids of undirected paths P_m with m points are as follows
$|End(P_{2n})| = 2\sum_{r=0}^{n-1} e(P_{2n}, r)$ and
$|End(P_{2n+1})| = 2\sum_{r=0}^{n-1} e(P_{2n+1}, r) + e(P_{2n+1}, n), n \in N$, where

- $e(P_{2n}, 0) = \binom{2n-1}{n-1}$
- $e(P_{2n}, 1) = \binom{2n-1}{n-1} + \binom{2n-1}{n} - \binom{2n-1}{0}$
- $e(P_{2n}, 2) = \binom{2n-1}{n-2} + \binom{2n-1}{n-1} + \binom{2n-1}{n} - 2\binom{2n-1}{0}$
- $e(P_{2n}, 2t) = \sum_{j=t}^{n+t-1} \binom{2n-1}{j} - \sum_{j=0}^{t-1} \binom{2n-1}{j} - \sum_{j=0}^{n-t-2} \binom{2n-1}{j}$ for $2 \leq t \leq n/2$
- $e(P_{2n}, 2t-1) = \sum_{j=t}^{n+t-1} \binom{2n-1}{j} - \sum_{j=0}^{t-2} \binom{2n-1}{j} - \sum_{j=0}^{n-t-1} \binom{2n-1}{j}$ for $2 \leq t \leq n/2$
- $e(P_{2n+1}, 0) = \binom{2n}{n}$
- $e(P_{2n+1}, 1) = \binom{2n}{n-1} + \binom{2n}{n} - \binom{2n}{0}$
- $e(P_{2n+1}, 2) = \binom{2n}{n-1} + \binom{2n}{n} + \binom{2n}{n+1} - 2\binom{2n}{0}$

- $e(P_{2n+1}, 2t) = \sum_{j=t}^{n+t} \binom{2n}{j} - \sum_{j=0}^{t-1} \binom{2n}{j} - \sum_{j=0}^{n-t-1} \binom{2n}{j}$ for $2 \leq t \leq n/2$
- $e(P_{2n+1}, 2t-1) = \sum_{j=t}^{n+t-1} \binom{2n}{j} - \sum_{j=0}^{t-2} \binom{2n}{j} - \sum_{j=0}^{n-t-1} \binom{2n}{j}$ for $2 \leq t \leq n/2$.

Chiangmai University, Chiangmai, Thailand
C.v. Ossietzky University
Oldenburg, Germany
E-mail:srichan28@yahoo.com

On Decomposable Finite Groups

A. R. Ashrafi

Let G be a finite group and A be a normal subgroup of G. We denote by $ncc(A)$ the number of $G-$conjugacy classes of A and A is called $n-$decomposable, if $ncc(A) = n$. Set

$$\mathcal{K}_G = \{ncc(A) \mid \texttt{A} \text{ is a normal sub group of } \texttt{G}\}.$$

Let X be a non-empty subset of positive integers. A group G is called $X-$decomposable, if $\mathcal{K}_G = X$.
In this talk, we report on the problem of classifying $X-$decomposable finite groups, for some finite subset X of positive integers.

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Department of Mathematics, University of Kashan
Kashan, Iran
Email: ashrafi@kashanu.ac.ir

Galois closed sets of strongly invariant relations

F. Börner

An n–ary relation $\varrho \subseteq A^n$ on a base set A is called *strongly invariant* for a permutation $g \in \mathrm{Sym}(A)$ if $g[\varrho] = \{g(\underline{a}) \mid \underline{a} \in \varrho\} = \varrho$. This relationship defines a Galois connection sInv–Aut between sets of permutations and sets of relations on A. The relation sets R with $R = \mathsf{sInv}\,\mathsf{Aut}\,R$ are called *Galois closed*. We ask for a characterization of these relation sets. The answer depends on the cardinality of the base set A. If A is finite, then the Galois closed sets are exactly the Krasneralgebras, and also for countable A there exists a good characterization.
In the general case, it was conjectured that a relation set is Galois closed if it is closed under arbitrary intersections and under all *invariant operations*, i.e. operations which commute with all permutations. Surprisingly, this conjecture is not true, as a counterexample by M. Goldstern and S. Shelah shows.

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Universität Potsdam
E-mail: fboerner@rz.uni-potsdam.de

Simulation and Universal Machines

J. Buls

We investigate a specific three sorted algebra $\langle Q, A, B, \circ, *\rangle$, a so-called Mealy machine: Q, A, B are finite, nonempty sets; $Q \times A \xrightarrow{\circ} Q$ is a function and $Q \times A \xrightarrow{*} B$ is a surjective function. Let $V = \langle Q, A, B\rangle$, $'V = \langle 'Q, 'A, 'B\rangle$ be Mealy machines. A map $\mu = (\mu_1, \mu_2, \mu_3) : (Q, A, B) \longrightarrow ('Q, 'A, 'B)$ is called a homomorphism $\mu : V \longrightarrow 'V$ if $\mu_1(q \circ a) = \mu_1(q) \circ \mu_2(a)$ and $\mu_3(q * a) = \mu_1(q) * \mu_2(a)$ for all $(q, a) \in Q \times A$. A homomorphism μ is called a homomorphism in input signals if $Q = 'Q$, $B = 'B$ and μ_1, μ_3 are identity maps of sets Q and B respectively. Let $T(Q)$ denotes the semigroup of all transformations on the set Q and let $Fun(Q, B)$ denotes the set of all maps from Q to B. On the set $S(Q, B) = T(Q) \times Fun(Q, B)$ define the multiplication by

$$(\alpha_1, \beta_1)(\alpha_2, \beta_2) = (\alpha_1\alpha_2, \alpha_1\beta_2);\ \alpha_1, \alpha_2 \in T(Q),\ \beta_1, \beta_2 \in Fun(Q, B).$$

Under this operation $S(Q, B)$ is easily seen to be a semigroup.

Proposition[1]. Let $\mathfrak{S}(Q, B) = \langle Q, S(Q, B), B\rangle$ be a Mealy machine, where $q \circ (\alpha, \beta) = \alpha(q)$ and $q * (\alpha, \beta) = \beta(q)$ for all $q \in Q$, $(\alpha, \beta) \in S(Q, B)$. For every Mealy machine $V = \langle Q, A, B\rangle$ there exists a unique homomorphism in input signals $\mu : V \longrightarrow \mathfrak{S}(Q, B)$.

Let A^* be a free monoid generated by A. We say that $'V$ *simulates* V if there exist maps

$$Q \xrightarrow{h_1} 'Q, \quad A \xrightarrow{h_2} 'A, \quad 'B \xrightarrow{h_3} B$$

such that the diagram

$$\begin{array}{ccccc} Q & \times & A^* & \xrightarrow{*} & B^* \\ h_1\downarrow & & \downarrow h_2 & & \uparrow h_3 \\ 'Q & \times & 'A^* & \xrightarrow{*} & 'B^* \end{array}$$

commutes. That is, if

$$q * u = h_3(h_1(q) * h_2(u)) \qquad for\ all \quad (q,u) \in Q \times A^*.$$

Corollary. $\mathfrak{S}(Q,B)$ simulates every Mealy machine $\langle Q, A, B \rangle$.

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University of Latvia
Email: buls@fmf.lu.lv

Strong Regular n-full Varieties of Partial Algebras

S. Busaman

A strongly n-full identity in the partial algebra $\mathcal{A}$ is pair of n-full terms of type τ such that the induced term operations are equal as partial operations. This concept will be generalized to strongly n-full hyperidentities and to strongly n-full regular hyperidentities in partial algebras. Moreover, we will characterize strong varieties of partial algebras where every regular n-full identity is a strongly n-full regular hyperidentity.

Universität Potsdam, Institut für Mathematik
E-mail:bsaofee@bunga.pn.psu.ac.th

Characterization of Clones by Hyperidentities

R. Butkote

Let $C \subseteq O^{(n)}(A)$ be a set of n-ary operations defined on the non-empty set A. We define an operation $f^A \in C$ to satisfy an equation $s \approx t$, where s, t are terms of type (n), if $s \approx t$ is satisfied as an identity in the algebra $(A; f^A)$. This relation defines a Galois connection between sets of n-ary operations and equations of type (n). We denote the Galois-closed sets on the side of operations by $FMod^{(n)}\Sigma$ for a set Σ of equations of type (n) and prove some properties of these classes with the aim to get an algebraic characterization. Especially we prove that $FMod^{(n)}\Sigma$ is a union of clones iff every equation from Σ is hypersatisfied by all operations from $FMod^{(n)}\Sigma$. Finally, we present several examples.

Universität Potsdam, Institut für Mathematik
14415 Potsdam
Am Neuen Palais
E-Mail: unique1st@hotmail.com

On Algebraic Interior Systems

I. Chajda

We get an interrelation between an algebraic closure system and its conjugated interior system. We introduce the concept of algebraic interior system and we get its algebraic representation.

Palacky University Olomouc
Czech Republic
Email: CHAJDA@INF.UPOL.CZ

Forests in Connected Cubic Graphs

A. Chantasartrassmee, N. Punnim

For a graph G and $F \subseteq V(G)$, if $G[F]$ is acyclic, then F is called an induced forest of G. We denote by $t(G)$ the cardinality of a maximum induced forest of a graph G. It is easy to see that if G is a cubic graph of order n and F is a maximum induced forest of G, then $G - F$ is also an induced forest. Thus for any cubic graph G of order n, we have $t(G) \geq \frac{n}{2}$. This bound can be improved if we restrict to the class of connected graphs. Zheng and Lu (1990) proved that $t(G) \geq \frac{2n}{3}$ for any connected cubic graph G of order n without triangle, except for two cubic graphs with $n = 8$ and $t(G) = 5$. Liu and Zhao (1996) proved that $t(G) \geq \frac{5}{8}n - \frac{1}{4}$ for any connected cubic graph G of order n. They also characterized all such graphs with equality holds. We prove some significant interpolation results on the graph parameter t with respect to the class of connected cubic graphs and other related subclasses. In particular we prove that $t(G) \geq \frac{2n}{3}$ for any connected cubic K_4'-free graph G of order $n \neq 8$, where K_4' is a graph obtained from K_4 and a subdivision. This is an extended result of Zheng and Lu (1990).

Department of Mathematics
The University of the Thai Chamber of Commerce
Bangkok 10400,Thailand
E-mail:avapa@hotmail.com

When is a quasi-p-injective module continuous ?

S. Chotchaisthit

It is well-known that every quasi-p-injective module has C2-condition. In this note, it is shown that for a quasi-p-injective module M which is a self-generator, if M is projective, duo and semiperfect, then M is continuous. As a special case we re-obtain a result of Puninski-Wisbauer-Yousif saying that, a semiperfect ring R is right continuous if it is right duo, right p-injective.
Key words: quasi-p-injective, duo, semiperfect module and ring, continuous module,
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Department of Mathematics
Khon Kaen University
Thailand
Email:somchit@kku.ac.th

A Triple Construction of Skew Nearlattices

J. Cīrulis

A *right normal skew nearlattice* (rnsn-lattice, for short) is an algebra $(L, \vee, \odot)$, where $(L, \odot)$ is a right normal band possessing the upper bound property w.r.t. its natural ordering, and $\vee$ is the corresponding partial join operation. An rnsn-lattice with $\odot$ commutative is known as a nearlattice. Such partial algebras arise, e.g., in computer science – see [1,2].

A triple is a system (T, V, θ), where T is a nearlattice, V is a set, and θ is a homomorphism of T into the dual of the lattice of equivalences on V.

We shall present two constructions, one of which transforms an rnsn-lattice into a triple, while the other transforms a triple into an rnsn-lattice, and discuss conditions on rnsn-lattices and triples under which these transformations are mutually inverse.

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University of Latvia
Email: jc@lanet.lv

Maximal Subsemigroups of the Semigroup of All Isotone Transformations

I. Dimitrova, I. Gyudzhenov

In this paper we consider the finite set $X(<) = \{1 < 2 < \cdots < n\}$ – ordered in the standard way. The full transformation α of $X(<)$ is *isotone* if $i \leq j \Longrightarrow i\alpha \leq j\alpha$.
We denoted by O_n the semigroup of all isotone transformations α of the finite set $X = \{1, \ldots, n\}$ under the operation of composition of transformations, by $\hat{J}_k = \{\alpha \in O_n : |X\alpha| = k\}, (1 \leq k \leq n-1)$ the J-class, and by $\hat{I}_k = \bigcup\limits_{i=1}^{k} \hat{J}_i$ the ideal of the semigroup O_n.
We give a description of the maximal subsemigroups of the class $\hat{J}_{n-2}$ and of the ideal I_{n-2} of the semigroup of all isotone transformations of a finite linearly ordered set X. The obtained results are based on previously proved propositions stating that elements of every $\hat{J}_k$-class, and therefore of every ideal $\hat{I}_k$, can be represented as products of idempotents of the same $\hat{J}_k$-class.
We prove the following theorem.
Theorem: *Let $n \geq 4$. Then each one of the following types of subsemigroups is a maximal subsemigroup of the ideal $\hat{I}_{n-2}$.*

(A) $A_{kl} = \hat{I}_{n-3} \cup (\hat{J}_{n-2} \setminus L_{k,l})$,
for $1 \leq k < l \leq n$.

(B) $B_{jm} = \hat{I}_{n-3} \cup (\hat{J}_{n-2} \setminus R_{j,m})$,
for $j = 1,\ 3 \leq m \leq n-2$, and for $2 \leq j \leq n-3$, $j < m \leq n-1$.

$$(C) \quad C_{jm} = \hat{I}_{n-3} \cup [\bigcup_{s=1}^{j-2} \bigcup_{t=s+1}^{n-1} R_{s,t}] \cup [\bigcup_{t=j}^{m-1} R_{j-1,t}] \cup [\bigcup_{l=m+1}^{n} L_{j,l}] \cup$$

$[\bigcup_{k=j+1}^{n-1} \bigcup_{l=k+1}^{n} L_{k,l}]$,
for $j = 2$, $m = n$, and for $3 \leq j \leq n-2$, $j < m \leq n$.

(D) $D_{kl} = \hat{I}_{n-3} \cup [\bigcup_{p=1}^{k-1} \bigcup_{q=p+1}^{n} L_{p,q}] \cup [\bigcup_{q=k+1}^{l-1} L_{k,q}] \cup [\bigcup_{m=l}^{n-1} R_{k,m}] \cup [\bigcup_{j=k+1}^{n-2} \bigcup_{m=j+1}^{n-1} R_{j,m}]$, for $1 \leq k < l \leq n-1$, and for $k = 1$, $l > 3$.

(E) $E_{kl;pq} = \hat{I}_{n-3} \cup [\hat{J}_{n-2} \setminus (L_{k,l} \cup \cdots \cup L_{p,q})] \cup R_{k,l} \cup \cdots \cup R_{p-1,q-1}$, for $1 \leq k \leq n-3$, $k < l \leq n-1$, $2 < l$, $k+2 \leq p \leq n-2$, $p < q \leq n$.

(F) $F_{jm;st} = \hat{I}_{n-3} \cup [\hat{J}_{n-2} \setminus (R_{j,m} \cup \cdots \cup R_{s,t})] \cup (\bigcup_{l=m+1}^{n} L_{k,l}) \cup L_{j+1,m+1} \cup \cdots \cup L_{s,q}$,
for $1 \leq j < s-1 \leq n-4$, $j < m \leq n-1$, $s < t \leq n-1$ $3 \leq m \leq t+1$;

(G) $G_{kl;pq} = \hat{I}_{n-3} \cup [\hat{J}_{n-2} \setminus (\bigcup_{u=k}^{p} \bigcup_{v=l}^{q} L_{u,v})] \cup (\bigcup_{j=k}^{p-1} \bigcup_{m=l}^{q-1} R_{j,m})$,
for $1 \leq k < p < l < q \leq n$.

(H) $H_{jm;st} = \hat{I}_{n-3} \cup [\hat{J}_{n-2} \setminus (\bigcup_{u=j}^{s} \bigcup_{v=m}^{t} R_{u,v})] \cup (\bigcup_{k=r}^{s} \bigcup_{l=m+1}^{q} L_{k,l})$,
for $1 \leq j \leq n-4$, $j < m < t \leq n-1$, $r \leq s \leq t-1$, also the case when $m = t = n-1$, where
Conversely, every maximal subsemigroup S of the ideal $\hat{I}_{n-2}$ belongs to one of types (A) $-$ (H).
Maximal semigroups up to uniqueness for the ideal I_{n-1} of the semigroup of all isotone transformations of a finite linearly ordered set are given in the work of Yang Xiuliang, Communications in Algebra, 28(3), (2000), 1503-15132. Our main result is a continuation of this paper, but the proofs are obtained by different methods.

South-West University "Neofit Rilski"
Blagoevgrad, Bulgaria
Email: ilinka-dimitrova@yahoo.com

Weighted Automata and Quantitative Logics

M. Droste

In automata theory, a classical result of Büchi states that the recognizable languages are precisely the ones definable by sentences of monadic second order logic. We will present a generalization of this result to the context of weighted automata. A weighted automaton is a classical nondeterministic automaton in which each transition carries a weight describing e.g. the ressources used for its execution, the length of time needed, or its reliability. The behaviour (language) of such a weighted automaton is a function associating to each word the weight of its execution. We develop syntax and semantics of a quantitative logics; the semantics counts 'how often' a formula is true for a given word. Our main result shows that if the weights are taken in an arbitrary commutative semiring, then the functions associated to weighted automata are precisely the ones definable by sentences of our quantitative logic. (Joint work with P. Gastin (Paris).)

Leipzig University, Fachbereich Informatik
E-Mail:droste@informatik.uni-leipzig.de

Bounded Lattices with Antitone Involutions and Properties of MV-algebras

P. Emanovský, I. Chajda

We introduce a bounded lattice L, where for each p from L there exists an antitone involution on the interval [p,1]. We show that there exists a binary operation "." on L such that L is term equivalent to an algebra A(L)= (L,.,0) (the assigned algebra to L) and we characterize A(L) by simple axioms similar to that of Abbott's implication algebra. We define two new operations on A(L) which satisfy some of the axioms of MV-algebra. Finally we show what properties must be satisfied by L or A(L) to obtain all axioms of MV-algebra.

Department of Algebra and Geometry, Palack University, Faculty of Sciences
Tomkova 40, 779 00 Olomouc
Czech Republic
Email: emanovsky@inf.upol.cz

On Irreducible and Solvable Polynomials

J. Forstner

We consider polynomials f over the ring $(\mathbb{Z}/n\mathbb{Z},+,.) =: \mathbb{Z}_n$,and let $n = p_1^{t_1} * .... * p_k^{t_k}$ be the primefactorization of n.It is a well-known result that the decomposition of $\mathbb{Z}_n$ into the direct product $\mathbb{Z}_{p_1^{t_1}} x ... x \mathbb{Z}_{p_k^{t_k}}$ induces a decomposition of the corresponding polynomial rings $\mathbb{Z}_n[x]$ into $\mathbb{Z}_{p_1^{t_1}}[x] x ... x \mathbb{Z}_{p_k^{t_k}}[x]$. We will see that a polynomial f over $\mathbb{Z}_n$ is irreducible in $\mathbb{Z}_n[x]$ if and only if f is irreducible in $\mathbb{Z}_{p_i^{t_i}}[x]$ for all i in $\{1,...,k\}$ and at most one component of the direct product is not a unit. The equation $f = 0$ is called solvable if there exists a solution in $\mathbb{Z}_n$ or in an extension ring of $\mathbb{Z}_n$. We prove that the equation $f = 0$ is not solvable if and only if f is of the form $f = k * u$, where k is a constant not equal to 0 and u is a unit in $\mathbb{Z}_n[x]$.

Johannes Kepler Universitt, Linz
Austria,
Email: Joerg.Forstner@gmx.at

Matrix Theory of Symmetric Functions

A. S. Gasparyan

For a given permutation group $G \subseteq S_n$ we define the module $M^G_{n,k}$ of G-symmetric matrices. A k-dimensional cubical matrix $A = \|a_{i_1 \dots i_k}\|^n_1$ we call G-symmetric if its elements satisfy the condition $a_{i_1 \dots i_k} = a_{\sigma(i_1), \dots \sigma(i_k)}$ for any $\sigma \in G$..
The module $M^G_n = \bigoplus_k M^G_{n,k}$ is closed under all the operations generated by $(0,0)$- ,$(0,1)$- and $(1,0)$-multiplications that we call tensor multiplication, index-convolution and index-contraction correspondingly. Resulting algebraic system — G-symmetric matrix algebra of order n — is the background of a new theory of symmetric functions. The algebraic system of newely defined G-symmetric functions is more rich than usual algebra of symmetric functions that corresponds to $G = S_n$, and generated, in fact, only by $(0,0)$-multiplication and addition.
Then we introduce more general algebra, the algebra of Γ-symmetric functions of several blocks of variables, where Γ is a subgroup of $S_{n_1} \times \dots \times S_{n_d}$. Corresponding base matrix algebra — Γ-symmetric matrix algebra — consists of (nonnecessarily cubical) multidimensional matrices that may have different symmetries in different directions, being e.g. $G^{(\alpha)}$-symmetric in (α)-block direction.

Program Systems Institute of RAS
Pereslavl-Zalesskii, Russia
E-mail: armen@math.botik.ru

On Irreducible and Solvable Polynomials

J. Forstner

We consider polynomials f over the ring $(\mathbb{Z}/n\mathbb{Z}, +, .) =: \mathbb{Z}_n$,and let $n = p_1^{t_1} * .... * p_k^{t_k}$ be the primefactorization of n.It is a well-known result that the decomposition of $\mathbb{Z}_n$ into the direct product $\mathbb{Z}_{p_1^{t_1}} x ... x \mathbb{Z}_{p_k^{t_k}}$ induces a decomposition of the corresponding polynomial rings $\mathbb{Z}_n[x]$ into $\mathbb{Z}_{p_1^{t_1}}[x]x...x\mathbb{Z}_{p_k^{t_k}}[x]$. We will see that a polynomial f over $\mathbb{Z}_n$ is irreducible in $\mathbb{Z}_n[x]$ if and only if f is irreducible in $\mathbb{Z}_{p_i^{t_i}}[x]$ for all i in $\{1, ..., k\}$ and at most one component of the direct product is not a unit. The equation $f = 0$ is called solvable if there exists a solution in $\mathbb{Z}_n$ or in an extension ring of $\mathbb{Z}_n$. We prove that the equation $f = 0$ is not solvable if and only if f is of the form $f = k * u$, where k is a constant not equal to 0 and u is a unit in $\mathbb{Z}_n[x]$.

Johannes Kepler Universitt, Linz
Austria,
Email: Joerg.Forstner@gmx.at

Q-independent Subsets in Stone Algebras and Double Stone Algebras

A. Chwastyk, K. Głazek

A notion of independece with repect to a family Q of mappings (defined on subsets of A) into A, Q-independence for short, is a common way of defining almost all known notions of independence. A non-empty subset X of the carrier A of an algebra A is called Q-*independent* if the equality of two term operations f and g of the algebra A on any finite system of elements $a_1, \ldots, a_n$ of X implies $f(p(a_1), \ldots, p(a_n)) = g(p(a_1), \ldots, p(a_n))$ for any mapping $p \in Q$. We investigate independent subsets in Stone algebras and double Stone algebras for some specified families Q of mappings (e.g. M, S, S_0, and A_1).

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Technical University of Opole
University of Zielona Góra
E-mail:

Power Hypersubstitutions

P. Glubudom

Unitary Menger algebras of rank n are algebras of type $(n+1, 0, \ldots, 0)$ and were introduce by K. Menger ([Men;]). The $(n+1)$-ary operation S^n and the nullary operations $\lambda_1, \ldots, \lambda_n$ satisfy the following axioms:

(C1) $\S^n(T, \S^n(F_1, T_1, \ldots, T_n), \ldots, \S^n(F_n, T_1, \ldots, T_n))$
$\approx \S^n(\S^n(T, F_1, \ldots, F_n), T_1, \ldots, T_n), n \in \mathtt{N}^+$.

(C2) $\S^n(T, \lambda_1, \ldots, \lambda_n) = T, n \in \mathtt{N}^+$.

(C3) $\S^n(\lambda_i, T_1, \ldots, T_n) = T_i$ for $1 \leq i \leq n, n \in \mathtt{N}^+$.

Let $W_\tau(X)$ be the set of all terms of type τ and let $\mathcal{P}(W_\tau(X))$ be its power set. Then we define an $n+1)$-ary operation $\hat{S}^n$ on $\mathcal{P}(W_\tau(X))$ and show that the algebra $(\mathcal{P}(W_\tau(X_n)); \hat{S}^n, \{x_1\}, \ldots, \{x_n\})$ satisfies (C1), (C2), (C3). Power hypersubstitutions are mappings which assign to each n-ary operation symbol of the type set of n-ary terms of this type. Using the operations $\hat{S}^n$ we extend power hyper substitutiuons to mappings defined on $\mathcal{P}(W_\tau(X))$ and show that these extensions are precisely the endomorphisms of $(\mathcal{P}(W_\tau(X_n)); \hat{S}^n, \{x_1\}, \ldots, \{x_n\})$.

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Chiangmai University,
Chiangmai, Thailand,
Department of Mathematics,
E-mail:

M-hyperquasivarieties

E. Graczyńska, D. Schweigert

The aim of the talk is to generalize teh notions of quasiidentity and quasivariety of [6] and [9] to the notion of M-hyperquasiidentity and M-hyperquasivariety. A quasi-identity e is *satisfied in a class* V of algebras of a given type, if and only if it is satisfied in all algebras $\mathbf{A}$ belonging to V.

DEFINITION 1. In the sequel, a *hyperquasi-identity* e is formally the same as a quasi-identity, for a given monoid M of hypersubstitutions of a given type. We define an M-hypersatisfaction. and say then, that e is an M-hyper-quasi-identity of $\mathbf{A}$. A hyper-quasi-identity e is M-hyper-satisfied (holds) in a class V if and only if it is M-hypersatisfied in any algebra of V. We recall the notion of a *M-derived algebra* and the *M-derived class* of algebras of a given type. For a class K of algebras of type τ we denote by $\mathbf{D_M}(K)$ the class of all M-derived algebras (of type τ) of K.
A class K is called M-*derivably closed* if and only if $\mathbf{D_M}(K)$ is included in K, for a given monoid M of hypersubstitutions of type τ. For a given algebra $\mathbf{A}$ we denote by $QId(\mathbf{A})$ and $MHQId(\mathbf{A})$ the set of all quasi-identities and hyper-quasi-identities satisfied (M-hypersatisfied) in $\mathbf{A}$, respectively. Similarly for a class K, $QId(K)$ and $MHQId(K)$ denote the set of all quasi-identities and hyper-quasi-identities satisfied (M-hypersatisfied) in K, respectively. By $\mathbf{Q}(K)$ and $MH\mathbf{Q}(K)$ we denote the class of all algebras of a given type satisfying (M-hypersatisfying) all the quasi-identities and M-hyper-quasi-identities of K, respectively.

PROPOSITION 1. Given a class K of algebras of type τ. Then the following equality holds: $MHQId(K) = QId(\mathbf{D_M})(K)$. We reformulate the notion of *quasivariety* invented by A. I. Mal'cev [13] [p. 210] and *hyperquasivariety* of [9] for the case of *M-hyper-quasivariety* of a given type in a natural way.

THEOREM 1. A quasivariety K of algebras given type is a M-hyperquasivariety if and only if it is M-deriverably closed.

THEOREM2. A class K of algebras of a given type is an M-hyperquasivariety if and only if K
i) is ultraclosed;
ii) is heraditery;
iii) is multiplicatively closed;
iv) contains a trivial system;
v) is M-deriverably closed.

THEOREM 3. For every class K of algebras of a given type we have

$$MH\mathbf{Q}(K) = \mathbf{SP_rD_M}(K_0)$$

Quasi equational logic was considered by many authors. We use the notions of [6]. We reformulate his Theorem 2.3.5 of [6] in the following way:

THEOREM 4. A set Σ is a set of M-hyperquasi-identities of a class K of algebras of type τ if and only if it is closed under the substitution rule, M-hypersubstitution rule, the cut rule and the extension rule.
By MHQ we denote the M-hyperquasi-identty logic based on the rules above. For a set Σ of hyperquasi-identities of a given type τ, $MHQMod(\Sigma)$ denotes the class of all algebras $\mathbf{A}$ which M-hypersatisfy all the elements of Σ. Then the completeness theorem may be expressed in the following way:

THEOREM 5. A hyperquasi-identity is M-hypersatisfied in $MHMod(\Sigma)$ if and only if it is deriverable from σ in MHQ.

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Institute of Mathematics
Technical University of Opole, Poland
Email: egracz@po.opole.pl

On extensions of ideals in posets

R. Halaš

Every ideal in a poset can be extended in a natural way to a subset being always an order ideal. If for a given ideal its extensions are again ideals, then such an ideal is called perfect. I shall present how this notion is related to other notions such as distributivity, prime property etc.

Dept. of Algebra and Geometry
Palacky University Olomouc
Email: halas@inf.upol.cz

Symmetry Properties of Tetraammine Platinum(II) with C_{2v} and C_{4v} Point Groups

M. Hamadanian, A, R. Ashrafi

Let G be a weighted graph with the adjacency matrix $A = [a_{ij}]$. An Euclidean graph associated to a molecule is defined by a weighted graph with the adjacency matrix $D = [d_{ij}]$, where for i, j, d_{ij} is the Euclidean distance between the nuclei i and j. In this matrix d_{ii} can be taken as zero if all the nuclei are equivalent. Otherwise, one may introduce different weights for different nuclei. Balasubramanian computed the Euclidean graphs and its automorphism groups for benzene, eclipsed and staggered forms of ethane and eclipsed and staggered forms of ferrocene (see Chem. Phys. Letters 232(1995), 415-423). In this work a simple method is described, by means of which it is possible to calculate the automorphism group of weighted graphs. We apply this method to compute the symmetry of tetraammine platinum(II) with C_{2v} and C_{4v} point groups.

Keywords and Phrases: Weighted graph, Euclidean graph, tetraammine platinum(II).

Department of Chemistry, University of Kashan
Kashan, Iran
Department of Mathematics, University of Kashan
Kashan, Iran
E-mail:hamadani@kashanu.ac.ir

Large Varieties of Lattice-Ordered Groups and MV-Algebras

W. C. Holland

We will discuss the close connection between lattice-ordered groups and generalized MV-algebras. Every variety (equationally defined class) of lattice-ordered groups gives rise to a corresponding variety of generalized MV-algebras. But because the language of MV-algebras is stronger than that of lattice-ordered groups, there are many more varieties of generalized MV-algebras. In particular, there is a unique maximal proper variety of lattice-ordered groups, the normal valued variety. But there are infinitely many varieties of generalized MV-algebras which contain the normal valued variety.

Bowling Green State University
Email: chollan@bgnet.bgsu.edu

Interval subalgebras of BL-algebras

M. Hyčko

We define a BL-algebraic structure on subintervals $[a, b]$ of BL-algebras. First, we restrict the operations to subintervals of the $[a, 1]$ type and then to $[0, a]$ type. The desired result, for $[a, b]$, is obtained as the combination of the previous two types. Although, the case $[0, a]$ imposes a special condition, it turns out, after the analysis of the structure theorems of BL-algebras, that each BL-algebra satisfies it. Similar results with additional assumptions can be proved for pseudo BL-algebras, (commutative) bounded residuated ℓ-monoids. Obtained operations, when restricted to (pseudo) MV-algebras, coincide with the results of J. Jakubík [1] and Chajda, Kühr [2, 3].

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Mathematical Institute, Slovak Academy of Sciences
Štefánikova 49, SK-814 73 Bratislava Slovakia
E-mail:hycko@mat.savba.sk

A Characterization of $PSU(29, q)$

A.Iranmanesh

For an integer n let $\pi(n)$ be the set of prime divisors of n. If G is a finite group, then $\pi(G)$ is defined to be $\pi(|G|)$. The prime graph $\Gamma(G)$ of a group G is a graph whose vertex set is $\pi(G)$, and two distinct promes p and q are lined by an edge iff G contains an element of order pq. Let $\pi_i, i = 1, 2, \ldots, t(G)$ be the connected components of $\Gamma(G)$. For $|G|$ even, π_1 will be the connected component containing 2. Then $|G|$ can be expressed as a product of some positive integers $m_i, i = 1, 2, \ldots, t(G)$ with $\pi(m_i) = \pi_i$. The integers m_i are called the order components of G. The set of all order components of G will be denoted by $OC(G)$. If the order of G is even, we will assume that m_1 is the even order component and $m_2, \ldots, m_{t(G)}$ will be the odd order components of G.
In this paper we prove that $PSU(29, q)$ are also uniquely determined by their order components, that is, we have;

Main Theorem. Let G be a finite group and $M = PSU(29, q)$. Then $OC(G) = OC(M)$ if and only if $G \cong M$.
As a corollary of this result, the validity of a conjecture of J. G. Thompson on $PSU(29, q)$ is obtained:

Thompson's conjecture: If G is a finite group with $Z(G) = 1$ and M is a non-abelian simple group satisfying $N(G) = N(M)$ where $N(G) = \{n \mid G \text{ has a conjugacy class of size } n\}$, then $G \cong M$.

Department of Mathematics, Tarbiat Moarres University
P.O.Box 14115-137 Tehran, Iran

E-Mail:iranmana@modares.ac.ir

Automata and Groups

M. Ito

An *automaton* $\mathcal{A} = (S, X, \delta)$ consists of the following data: (1) S is a finite nonempty set, called a *state set*. (2) X is a finite nonempty set, called an *alphabet*. (3) δ is a function, called a *state transition function* of $S \times X$ into S.

Let $\mathcal{A} = (S, X, \delta)$ be an automaton and let ρ be a mapping of S into itself. If $\rho(\delta(s, a)) = \delta(\rho(s), a)$ holds for any $s \in S$ and any $a \in X$, then ρ is called an *endomomorphism* of $\mathcal{A}$. If ρ is a bijection, then ρ is called an *automorphism* of $\mathcal{A}$. The set $E(\mathcal{A})$ of all endomorphisms of $\mathcal{A}$ forms a monoid and the set $G(\mathcal{A})$ of all automorphisms of $\mathcal{A}$ forms a group.

An automaton $\mathcal{A} = (S, X, \delta)$ is said to be *strongly connected* if for any pair of states $s, t \in S$ there exists an element $x \in X^*$ such that $\delta(s, x) = t$.

It can be easily seen that an endomorphism of a strongly connected automaton is surjective and hence bijective. Thus $E(\mathcal{A}) = G(\mathcal{A})$ for a strongly connected automaton $\mathcal{A}$.

In the lecture, we provide several properties of automorphism groups of automata, especially strongly connected automata. We also consider an inverse problem, i.e. we will determine all strongly connected automata whose automorphism groups are isomorphic to a given finite group.

Faculty of Science, Kyoto Sangyo University
Kyoto 603-8555, Japan

E-mail: ito@ksuvx0.kyoto-su.ac.jp

Globals of Unary Algebras

D. Jakubíková-Studenovská ,J. Herchl

For an algebraic structure $\mathcal{A}$ we denote by $P(\mathcal{A})$ the global of $\mathcal{A}$. This notion was studied for several types of algebraic structures; for a comprehensive list of papers cf. Bošnjak, Madarász (Algebra and Discrete Math., 2003). The question on the reconstructability of a finite unary algebra from its global is dealt with. Drápal (Czechoslovak Math. J., 1985) proved that the class of all finite monounary algebras is globally determined. We prove that the class of all finite unary algebras is not globally determined. Particularly, 7-element cyclic algebras $\mathcal{A}_1$, $\mathcal{A}_2$ with two unary operations are described such that $\mathcal{A}_1$, $\mathcal{A}_2$ are not isomorphic and $P(\mathcal{A}_1)$,$P(\mathcal{A}_2)$ are isomorphic.

Institute of Mathematics, P. J. Šafárik University
Jesenná 5, 041 54 Košice
Slovak Republic
Email:studenovska@science.upjs.sk

Local Polynomial Functions on the Ring of Integers

M. Kapl

For an algebra A and a natural number k, we take $P_k(A)$ to be the set of all k-ary polynomial functions on A. For a natural number m, we let $L_m P_k(A)$ be the set of all functions from A^k to A that can be interpolated by a k-ary polynomial function at each collection of no more than m points of A^k. These functions are called *k-ary local polynomial functions with respect to m points*. We will study the case that A is the ring of integers, $I =< \mathbb{Z}; +, \cdot >$. We give an explicit description of all functions in $LP_k(I) = \bigcap_{m \in \mathbb{N}} L_m P_k(I)$. Moreover we will show that the absolute values of every function $f \in L_m P_1(I)$, which is not a polynomial, will increase faster than the exponential function $b(n) = (\frac{1}{4}(e^s + (-4 + e^{2s})^{\frac{1}{2}})^2)^n$ with $s = \sum_{i=1}^{m} \frac{1}{i}$. (For $m = 2$, $b(n) \approx 5.19^n$.)

Institut fuer Algebra, Universität Linz
4040 Linz, Austria
Email: kapl@algebra.uni-linz.ac.at

On Flatness Properties of S-posets

M. Kilp

Let S be a partially ordered monoid, or briefly, pomonoid. A right S-poset (often denoted A_S) is a poset A together with a right S-action $(a, s) \mapsto as$ that is monotone in both arguments and that satisfies the conditions $a(st) = (as)t$ and $a1 = 1$ for all $a \in A$, $s, t \in S$. Left S-posets ${}_SB$ are defined analogously, and the left or right S-posets form categories, S-POS and POS-S, whose morphisms are the monotone maps that preserve the S-action. A tensor product $A_S \otimes {}_SB$ exists (a poset) that has the customary universal property with respect to balanced, bi-monotone maps from $A \times B$ into posets. Various flatness properties of A_S can be defined in terms of the functor $A_S \otimes -$ from S-POS into POS. More specifically, an S-poset A_S is called *flat* if the induced morphism $A_S \otimes {}_SB \to A_S \otimes {}_SC$ is injective whenever ${}_SB \to {}_SC$ is an embedding in S-POS : this means that, for all ${}_SB$ and all $a, a' \in A$ and $b, b' \in B$, if $a \otimes b = a' \otimes b'$ in $A \otimes B$ then the same equality holds in $A \otimes (Sb \cup Sb')$. A_S is called *(principally) weakly* flat if the induced morphism above is injective for all embeddings of (principal) left ideals into ${}_SS$. Similarly, A_S is called *po-flat* if the functor $A_S \otimes -$ preserves embeddings: for this definition, replace $=$ by $\leq$ in the description above. Weak and principally weak versions of po-flatness are defined in an obvious way. We present examples that distinguish between various types of flatness and the corresponding, generally stronger, notions of po-flatness.
A study of absolute flatness for pomonoids is initiated: a monoid (resp. pomonoid) S is called right absolutely flat if all right S-acts (resp. S-posets) are flat. The findings for absolute flatness of pomonoids are markedly different from the corresponding unordered results. For example, every inverse monoid is absolutely flat. In contrast, however, a semilattice with identity, considered as a pomonoid, is absolutely flat if, and only if, it has at most two elements.

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University of Tartu, Department of Mathematics
Email: Mati.Kilp@ut.ee

Complete Multilattices

J. Klimes

Our aim in this talk is to introduce a kind of completeness notion for partially ordered sets, so called complete multilattices. Various interesting examples show that complete multilattices arise quite natural in the theory of partially ordered sets and are generalizations of complete lattices. We have studied some special classes of complete multilattices with respect to the fixed point property. We prove that a complete multilattice has the fixed point property for almost comparable order-preserving mappings. On the other hand, we discuss characterizations of multilattice completeness which are connected with the existence of fixed points for mappings. Especially, we show that a multilattice is complete if it has the fixed point property for almost comparable order-preserving mappings. Thus the fixed point property for these mappings characterize multilattice completeness whereas order-preserving mappings do not. If time permits, we would like to present a description of all retracts of these mappings on a complete multilattice.

University of T.Bata in Zlin
Czech Republic
Email: klimes@ft.utb.cz

Near-Semirings in Generalized Linear Sequential Machines

K. V. Krishna, N. Chatterjee

Holcombe used the theory of near-rings to study linear sequential machines. There a radical of near-rings has been introduced to test the minimality of linear sequential machines. The construction of the radical is motivated by his result:
Let $M = (Q, A, B, F, G)$ *be a linear sequential machine. If* M *is minimal then there is no proper nonzero* N*-submodule* K *of* Q *such that* $G_0(K) = \{0\}$*, where* N *is the syntactic near-ring of* M *and* $G_0(q) = G(q, 0)\ \forall q \in Q$.
However, with the hypothesis of the above result, one can observe that there is no proper nonzero N-submodule of Q. This observation enables us to introduce a new radical in a more general setup. We carry out this work for generalized linear sequential machines, that are obtained by replacing modules with semimodules in linear sequential machines. Even in this generalization, without losing much information, we could get all the results that have been obtained by Holcombe for linear sequential machines. Moreover the radical obtained is much simpler.
In this work, first we study the properties of near-semirings to introduce the radical. Then we observe the role of near-semirings in generalized linear sequential machines, and we test the minimality through the above radical.

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Department of Mathematics, Indian Institute of Technology
Delhi, Hauz Khas
New Delhi - 110 016, India
Email: kv.krishna@member.ams.org

Order Affine Completeness of Lattices with Boolean Congruence Lattices

V. Kuchmei, K. Kaarli

A lattice L is called (locally) order affine complete, if all order and congruence preserving functions on L are (local) polynomial functions of that lattice. The following result was proved by R. Wille for the finite case and generalized by K. Kaarli and A. F. Pixley to lattices of finite height.

Theorem A lattice L of finite height is locally order affine complete iff every finitely generated tolerance of L is congruence generated. It was also proved by K. Kaarli that the characterization of locally order affine complete lattices of finite height can be reduced to subdirect products of two SI lattices. In view of these results we prove the following

Theorem Let $\mathbf{L}_1$ and $\mathbf{L}_2$ be simple lattices of finite height and $\mathbf{L}$ be a nontrivial subdirect product in $\mathbf{L}_1 \times \mathbf{L}_2$. Then $\mathbf{L}$ is locally order affine complete if and only if $\mathbf{L}_1$ and $\mathbf{L}_2$ are tolerance trivial and $\mathbf{L}$ is of the one of the following two types:

1. $\mathbf{L}$ is maximal in $\mathbf{L}_1 \times \mathbf{L}_2$;
2. $L = K_1 \cap K_2$, where lattices $\mathbf{K}_1$ and $\mathbf{K}_2$ are maximal in $\mathbf{L}_1 \times \mathbf{L}_2$ and $(0, 1) \notin L$, $(1, 0) \notin L$.

Further we study maximal subdirect products in $\mathbf{L}_1 \times \mathbf{L}_2$, where $\mathbf{L}_1$ and $\mathbf{L}_2$ are simple modular lattices of finite height.

Institute of Pure Mathematics, University of Tartu
Tartu, Estonia
Email: kucmei@math.ut.ee

Commutative pseudo BCK-algebras

J. Kühr

A pseudo BCK-algebra is an algebra A of type (2,2,0) satisfying certain quasi-identities; if the binary operations coincide then A is a classical BCK-algebra. Thus pseudo BCK-algebras generalize BCK-algebras in the same way in which e.g. pseudo MV-algebras generalize MV-algebras. Commutative pseudo BCK-algebras are a natural extension of commutative BCK-algebras. We show that they form a variety and characterize representable commutative pseudo BCK-algebras and those with the relative cancellation property. We prove that the lattice of all deductive systems of such a pseudo BCK-algebra is isomorphic to the lattice of all convex l-subgroups of some lattice-ordered group.

Palacky University Olomouc
Czech Republic
Email: kuhr@inf.upol.cz

Categories of Fuzzy BCK-algebras

C. Lele

The notion of fuzzy subsets of a set was introduced by Zadeh in 1965 . Since then, the study of fuzzy subsets and its applications to various mathematical settings has given rise to what is nowadays known as fuzzy mathematics.
In the first paper on fuzzy BCK-algebra in 1991 X.Ougen only speaks, based on Zadeh's definition, of fuzzy algebraic sub-structures but not of fuzzy objects in their own right. In this note, we define a fuzzy BCK-algebras as a pair $\underline{X} = (X, f_X)$ where X is a BCK-algebras and $f_X : X \to [0, 1]$ is a function from X to the interval $[0, 1]$. Moreover, we define a *fuzzy BCK-homomorphism* $f : \underline{X} \to \underline{Y}$ $(\underline{X} = (X, f_X),\ \underline{Y} = (Y, f_Y)$ fuzzy BCK-algebras) to be a BCK- homomorphism $f : X \to Y$ such that $f_X(x) \le f_Y(f(x))$ holds for all $x \in X$. Fuzzy BCK-algebra are defined, and maps ("homomorphisms") between them. So we have objects and maps, this make a category. The purpose of this paper ist to initiate the study of some properties of this Category.

Department of Mathematics,
University of Dschang,
BP 67, Cameroon,

On the Category of Hypermodules

A. Madanshekaf

The theory of hyperstructures has been introduced by Marty in 1934 during the 8^{th} congress of the Scandinavian Mathematicians [2]. Marty introduced the notion of a hypergroup and then many researchers have been worked on this new topic of modern algebra and developed it. The notion of a hyperfield and a hyperring was studied first by Krasner [1] and then some authors followed him, for example see [6]. The canonical hypergroups are a special type of hypergroups. Initially they derived from the additive part of the hyerfield and hyperring. The name canonical has been given to these hypergroups by J. Mittas, who is the first one that studied them extensively [5]. Again in the context of canonical hypergroup some mathematicians(e.g. [3]) studied hypermodules whose additive structure is just a canonical hypergroup.
Considering the class of hypermodules over a fixed hyperring R and the class of all homomorphisms among hypermodules together with the composition of the mappings, knowing that the composite of two homomorphisms is again a homomorphism and for any hypermodules A over the hyperring R, $id_A : A \to A, id(a) = a$ is a homomorphism among hypermodules, we can construct a category which is denoted by **Hmod**.
In this talk some aspects of hypermodules are studied, We will show that the category **Hmod** is exact in the sense that it is normal and conormal with kernels and cokernels in which every arrow f has a factorization $f = \nu q$, with ν a monomorphism and q an epimorphism.(See [4]) Two of the most used results of the paper are those which state that the monomorphisms of **Hmod**(in the categorical sense) are the one to one homomorphisms and the epimorphisms of **Hmod** are the onto homomorphisms.
AMS *subject classification*: 20N20
Key words: Hypermodules, Normal and Conormal Category, Exact

Category.

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Department of Mathematics, Semnan University
Semnan, Iran

Rational and Recognizable Picture Series

I. Maeurer

Given an alphabet A, one can consider the set of all two-dimensional rectangular arrays of letters in A. In the literature there are several automata models to define a robust set of "recognizable" picture languages; it coincides with the set of languages corresponding to projections of regular expressions of a certain type (Kleene-like theorem). As an extension, we define picture series mapping pictures to elements of a semiring. It could be interpreted e.g. as the intensity of light of a picture. Now, it can be shown that series defined by "weighted automata" over pictures and several other devices with weights (e.g. weighted tiling systems) coincide with projections of rational picture series (Kleene-Schuetzenberger theorem).

Universitaet Leipzig, Institut fuer Informatik
Augustusplatz 10-11
04109 Leipzig
Email: maeurer@informatik.uni-leipzig.de

Exactness property of an endofunctor in the category of coalgebras

J.-P. Mavoungou

Given a category $\mathcal{C}$ and $T : \mathcal{C} \to \mathcal{C}$ an endofunctor. It is known that if $\mathcal{C}$ has pullbacks and if T (weakly) preserves these pullbacks, then the category $\mathcal{C}_T$ has pullbacks. We establish the existence of (generalized) pullbacks under the hypotheses which are different of that mentionned above. Under these new hypotheses, we show that if $\mathcal{C}$ has ihe exactness property then so has $\mathcal{C}_T$.

Université de Yaoundé I
Faculté des Sciences, Département de Mathématiques
B.P 812 Yaoundé-Cameroun
Email:jpmavoungou@yahoo.fr

Nilpotent Elements and Idempotents in Commutative Group Rings

N. A. Nachev

Let R and L be commutative rings with identity and $R \subseteq L$. Then we say that L is an extension of R. Let $\alpha \in L$. Denote by $R[\alpha]$ the smallest subring of L containing R and α. In this paper we consider an extension $R[\alpha]$ when α is an algebraic element over R and the minimal polynomial $f(x)$ of α is monic. In this direction the following two problems arise in a natural way:
(i) If R does not contain nilpotent elements, does it follow that this property holds for $R[\alpha]$?
(ii) If R does not have nontrivial idempotents, does it follow, that it is fulfilled for $R[\alpha]$.
We give a positive answer to this two problems under some conditions of the polynomial $f(x)$.

University of Plovdiv
Bulgaria
Email: nachev@pu.acad.bg

Analytical Derivatives of Molecular Integrals for Calculation of Energy Components

M. Oftadeh

Chemical calculations are based on using an estimated basis set and then omptimizing parameters including the basis set in order to obtain the best possible results such as energy, force constant, structure electric and magnetic properties. Using the method, which could calculate such properties with more accuracy and less time consuming by computers, is very interesting for most of the quantum chemists. From optimizing the parameter point of view, applying the subroutines in which analytical derivatives instead of numerical derivatives in addition to decreasing the number of iteration steps, it leads to better orientation through which one can obtain direct optimizing process. In the most of quantum chemical problems, the total energy function for a given system by a desired method is selected for optimization. The explicit expression of the energy for the most of existing methods in quantum chemistry including FSGO method can be derived according to the number of parameters. With the help of explicit expression of total energy in FSGO method which is based on one- and two-electronic integrals, the total first and second derivatives of each of the existing elements in this expression with respect to all existing parameters can be obtains which are comparable with numerical methods such as finite difference (FD) method. From speed of calculation first and second derivatives and independence of the results on any kind of parameters similar to selected increments in FD method point of view, this comparison is of outmost importance. All first and second derivatives of each components of total energy expression with respect to all parameters for FSGO method have been derived analytically and programmed in FORTRAN for PC computers.

Isfahan Payam-E-Noor University, Department of Chemistry
Kohandej Street
Isfahan, Iran
Email: moftadeh@pnu3.ac.ir

Generalized Landau and Brauer-Rademacher Identities

*

N. Pabhapote, V. Laohakosol and P. Ruengsinsub

The following two arithmetical identities are known as Landau and Brauer-Rademacher identities, respectively,

$$\sum_{d|r} \frac{\mu^2(d)}{\phi(d)} = \frac{r}{\phi(r)}$$

$$\phi(r) \sum_{\substack{d|r \\ (d,n)=1}} \frac{(d)}{\phi(d)} \mu(\frac{r}{d}) = \mu(r) \sum_{d|(n,r)} d\mu(\frac{r}{d}) \qquad (r, n \in I\!N),$$

where ϕ is the Euler function, and μ the Möbius function . There have appeared several generalizations of these two identities (see, e.g., [2, 3, 8, 10 , 11]). We propose here to give another generalization replacing, in the identities above, the usual Möbuis function by use of its generalizations, known as the Hsu's generalized Möbuis fucntion μ_α, which is defined by

$$\mu_\alpha(n) = \prod_{p|n} \binom{\alpha}{\nu_p(n)} (-1)^{\nu_p(n)},$$

where $\alpha \in I\!R$, and $\nu_p(n)$ is the highest power of the prime number p dividing n.

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The University of the Thai Chamber of Commerce
Bangkok 10400, Thailand
email: pab@utcc.ac.th

Department of Mathematics, Faculty of Science
Kasetsart University, Bangkok 10900, Thailand
email: fscivil@ku.ac.th

M-solid Pseudovarieties and Galois Connections

B. Pibaljommee

The concept of a solid pseudovariety, indroduced by Graczynska, Poeschel and Volkov, can be easily generalized to M-solid pseudovarieties. Using identity filters we will give a characterization of M-solid pseudovarieties by Galois connections. Finally we prove that the lattice of all M-solid pseudovarieties forms a complete sublattice of the lattice of all pseudovarieties.

Universitt Potsdam Institut fr Mathematik
D-14415 Potsdam, Germany
PF 601553
E-Mail:bunbip@rz.uni-potsdam.de

On the Scales of Computability Potentials of Finite Algebras: Results and Problems

A.G.Pinus

On the basis of the concept of a conditional term the concept of the computability potential of universal algebra and the concept of the scale of computability potentials of n-element algebras are defined. Some results on the structures of these scales are formulated and some natural problems of the theory of the scales of computability potentials of finite universal algebras are posed.

Novosibirsk State Technical University
Email: algebra@nstu.ru

Test Elements and the Retract Theorem for Monounary Algebras

J. Pócs, D. Jakubíková-Studenovská

An element t of a structure $\mathcal{A}$ is said to be a test element if for any endomorphism ϕ of $\mathcal{A}$, $\phi(t) = t$ implies that ϕ is an automorphism. The term “the Retract Theorem” has been applied in the literature in connection with group theory. We prove that the Retract Theorem is valid (i) for each finite structure, and (ii) for each monounary algebra. On the other hand, we show that this theorem fails to be valid, in general, for algebras of the form $\mathcal{A} = (A, F)$, where each $f \in F$ is unary and card $F > 1$.

Mathematical Institute, Slovak Academy of Science
Grešákova 6, 04001 Košice
Slovak Republic
Email: pocs@saske.sk

Unary Operations with long Pre-periods

Ch. Ratanaprasert

It is well-known that the congruence lattice $Con\mathcal{A}$ of an algebra $\mathcal{A}$ is uniquely determined by the unary polynomial operations of $\mathcal{A}$. Let $\mathcal{A}$ be a finite algebra with $|A| = n$. If $Imf = A$ or $|Imf| = 1$ for every unary polynomial operation f of $\mathcal{A}$, then $\mathcal{A}$ is called a permutation algebra. Permutation algebras play an important role in tame congruence theory. If $f : A \to A$ is not a permutation then $A \supset Imf$ and there is a least natural number $\lambda(f)$ with $Imf^{\lambda(f)} = Imf^{\lambda(f)+1}$. We consider unary operations with $\lambda(f) = n-1$ for $n \geq 2$ and $\lambda(f) = n-2$ for $n \geq 3$ and ask for equivalence relations on A which are invariant under such unary operations. As application we show that every finite group which has a unary polynomial operation with one of these properties is simple or has only normal subgroups of index 2.

Silpakorn University, Department of Mathematics
Nakhon Pathom, Thailand E-Mail:ratach@su.ac.th

n-distributivity in Formal Concept Analysis

H. Reppe

The n-distributivity was introduced by A. Huhn and is a generalization of the distributivity law for lattices. We restrict our attention to the finite case and establish via Formal Concept Analysis another characterization. This can be seen in the maximal size of the premise in implications with proper premise. Analogeously, the dual n-distributivity strongly relates to minimal non-trivial join-covers in lattices. In my talk I will describe the n-distributivity from the equational theory point of view as well as lattice theory and FCA.

Institut f. Algebra, TU Dresden
Email: reppe@math.tu-dresden.de

Ordered Groups - Recent Results

A. Rhemtulla

An ordered group is a group G together with a total order relation $\leq$ on the set G such that the order relation is preserved under group operation. In other words, for all a, b, x, y in G, $a \leq b$ implies $xay \leq xby$. Examples of ordered groups include: The additive group of real numbers.; The group of unitriangular matrices over ordered number systems such as the integers or rational numbers or real numbers; Torsion free nilpotent groups; free soluble groups and free groups. We will present the basic structure of ordered groups followed by recent results and open questions.
Some of these are:
(1) orderable groups that can not be embedded in divisible orderable groups.
(2) Structure of orderable groups in which every order is central.
There are groups which admit only finitely many different total orders. For instance, there are precisely two ways to totally order the infinite cyclic group. We pose the question "which groups admit only a finite number of orders?" For some classes of groups the answer is known but not in general. There is a related question, "Can the order on a group G be uniquely determined by a given order on a finite subset of G?" The answer to this question is yes for those groups which admit only a finite number of orders. Are these the only groups where the answer to the second question is in the affirmative?
Find structural results for orderable groups having additional properties such as finiteness of rank or Engel condition.

University of Alberta, Edmonton

E-mail: ar@ualberta.ca

Ramanujan Sums via Generalized Möbius Functions and Applications

P. Ruengsinsub, V. Laohakosol, N. Phahapote

A generalized Ramanujan sum is defined by replacing the usual Möbius function with the Souriau-Hsu-Möbius function. Using this generalized Ramanujan sum, a characteristic of completely multiplicative function is derived extending a similar work of Aleksander Ivic.

Department of Mathematics, Kasetsart University, Thailand
Email: fscipar@ku.ac.th

On Transitive Cayley Graphs of Clifford Semigroups

S. Panma

With this research we continue the discussion of Cayley graphs of semigroups from [2], considering now conditions for automorphism vertex transitivity, using strongly the results of [1]. We investigate Cayley graphs of Clifford semigroups. We show under which conditions they satisfy the property of automorphism vertex transitivity in analogy to Cayley graphs of groups.

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Chiangmai University, Department of Mathematics
Chiangmai, Thailand
E-mail:panmayan@yahoo.com

Ideal Congruences of Some Relation Systems

D. Schweigert

The talk is the continuation of the paper "Congruence relations of multialgebra", Discrete Mathematics, 53, 1985, 249-253.

FB Mathematik, TU Kaiserslautern
Email: Dietmar.Schweigert@web.de

Coloured Terms and Multi-hypersubstitutions

S. Shtrakov

We consider the concepts of coloured terms and multi-hypersubstitutions. Let $Q \subset N$ be an non-empty set of natural numbers, called set of colours. Let $t \in W_\tau(X)$ be a term of type τ. Any function $\alpha_t : Sub(t) \to Q$ is called coloring function or coloration of the term t. The set $W_\tau^c(X)$ of coloured terms consists of all possible pairs $\langle t, \alpha_t \rangle$. The set of fundamental coloured terms is $F^c = \{\langle f, q \rangle | f \in F, q \in Q\}$. Hypersubstitutions of type τ are mappings which assign to each operation symbol $f \in F$ terms $\sigma(f)$ of type τ, which have the same arity as the operation symbol f. The set of all hypersubstitutions of type τ is denoted by $Hyp(\tau)$. Let ρ be a mapping of Q into $Hyp(\tau)$ i.e. $\rho : Q \to Hyp(\tau)$. Any such mapping is called multi-hypersubstitution of type τ. The set of all multi-hypersubstitutions of type τ is denoted by $Hyp^Q(\tau)$. A multi-hypersubstitution ρ is a mapping of the set of fundamental coloured terms into the set $W_\tau^c(X)$. The extension $\hat{\rho} : W_\tau^c(X) \to W_\tau^c(X)$ of a multi-hypersubstitution ρ on the set of all coloured terms, is defined and some internal properties of the monoid $Hyp^Q(\tau)$ are studied.

South-West University, Blagoevgrad
Email: shtrakov@aix.swu.bg

Two-generated Graded Algebras

E. Shirikov

Let $A_0 = \mathbb{K}$ be a field and $A = \bigoplus_{n=0}^{\infty} A_n$ the associative graded algebra generated over $\mathbb{K}$ by elements $X, Y \in A_1$. Suppose that $\dim A_2 = 3$. We prove that if $\mathbb{K}$ has no quadratic extensions, A is a domain and either $\dim A_{n+1} = n + 1$ or A is a central algebra then A is either the algebra of quantum polynomials in two variables

$$\Lambda_1(\mathbb{K}, \lambda) = {}^{\mathbb{K}\langle X, Y\rangle}\!/\!_{(YX - \lambda XY)}, \quad \lambda \in \mathbb{K}^*,$$

or Jordanian plane

$$\Lambda_2(\mathbb{K}) = {}^{\mathbb{K}\langle X, Y\rangle}\!/\!_{(YX - XY - Y^2)}.$$

In the case when $\dim A_{n+1} = n + 1$ we also find a criterion for A to be a domain. We also study some properties of Jordanian plane. We describe its center, derivations and the Lie algebra of outer derivations for an arbitrary field $\mathbb{K}$. In the case $char\mathbb{K} = 0$ we describe prime spectrum, the group of automorphisms, the endomorphisms with non-trivial kernels. Note that similar problems for quantum polynomials have been considered by V. A. Artamonov [1], [2], [3] and some properties of quantum polynomials are also studied in details in [4].

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Moscow State University,

Tensor Product in the Category JCPos

S. Solovjovs, H. Herrlich

The binary tensor product in the category of modules over a commutative ring is widely known in algebra. However, the notion can be considered in any category as a left adjoint to the internal *Hom*-functor. We are interested in the construct (**JCPos**, $|-|$) of complete lattices and join-preserving maps. It is possible to get an explicit description of the tensor product functor for this category."s"s

University of Latvija
University of Bremen
Email: sergejs@lu.lv

Tame Order-primal Algebras

C. Rattanaprasert and R. Srithus

Let $h: \underline{A} \to \underline{B}$ be a non-constant homomorphism. It is well-known that $h(\underline{A})$ is a subalgebra of $\underline{B}$ whose cardinality is greater than 2. If $\underline{A}$ is a tame algebra, we give a necessary and sufficient condition that all images of $\underline{A}$ under homomorphisms are tame. An order-primal algebra $\underline{A}$ corresponding to an ordered set $(A; \leq)$ is an algebra such that $T(\underline{A}) = Pol(A)$. It was shown in **[3]** that if $(A; \leq)$ is either an antichain or connected, then $\underline{A}$ is simple. We can prove that an antichain or a connected property is a necessary and sufficient condition for $\underline{A}$ to be tame. Moreover, we can prove that every simple algebra is tame. By these results, we can characterize all order-primal algebras which are tame; that is, an order-primal algebra $\underline{A}$ is tame if and only if $(A; \leq)$ is either an antichain or connected. Furthermore, we can prove that every order-primal algebra is non-abelian and it has a minimal set whose cardinality is 2. In Tame Congruence Theorey, they charactrize all minimal algebras by assigning type 1-5 and every tame algebra $\underline{A}$ has at least one type which is the same type of its minimal algebra. In **[5]**, they characterize all algebras to be abelian by using properties of types of an algebra; that is, an algebra $\underline{A}$ is abelian if and only if type $\underline{A} \in \{1, 2\}$. By this result, every type of an order-primal algebra is not type 1 or 2 which implies that type $\{\underline{A}\} \subseteq \{3, 4, 5\}$. We can characterize all types of a tame order-primal algebra $\underline{A}$ by the property of the ordered set $(A; \leq)$ corresponding to $\underline{A}$; that is, $(A; \leq)$ is an antichain if and only if type $\{\underline{A}\} = \{3\}$ and $(A; \leq)$ is connected if and only if type $\{\underline{A}\} = \{4\}$. Moreover, we can find all possible types of a variety generated by an order-primal algebra $\underline{A}$ and give some sufficient conditions that $V(\underline{A})$ is residually small.

Silpakorn University
Nakhon Pathom
Thailand
E-Mail: ratanasrithus@yahoo.com

Forcing Linearity Numbers for Multiplication Modules

J. Sanwong

In this paper we prove that for any multiplication module M, the forcing linearity number of M, $fln(M)$, is less than or equal two and if M is finitely generated then $fln(M) = 0$. Also, the forcing linearity numbers of multiplication modules over some special rings are given. We also show that every multiplication module is semi-endomorphal.

Department of Mathematics, Faculty of Science
Chiang Mai University
Chiang Mai, 50200, THAILAND

Homomorphic Images of Subdirectly Irreducible Algebras

D. Stanovsky

Homomorphic Images of Subdirectly Irreducible Algebras Abstract: A couple of papers about homomorphic images of subdirectly irreducible algebras appeared recently. Let me mention, for instance, that an algebra with at least one at least binary operation is a factor of a subdirectly irreducible algebra (over its monolith), if and only if it has non-empty intersection of ideals. A similar result was obtained also for finite unary algebras. I'll provide an overview of results and a particular attention will be given to a recent proof (obtained with Ralph McKenzie) that every quasigroup is isomorphic to a factor of a subdirectly irreducible quasigroup over its monolith.

Charles University in Prague
Czech Republic
Email: stanovsk@karlin.mff.cuni.cz

Varieties of Recognizable Subsets

M. Steinby

The Variety Theorem of S. Eilenberg (1976) establishes a correspondence between all varieties of finite monoids and certain families of regular languages called $*$-varieties, or alternatively, between all varieties of finite semigroups and the so-called $+$-varieties of regular languages. By describing precisely the families of regular languages that can be characterized by syntactic monoids or by syntactic semigroups, it forms a generally accepted common framework for the algebraic classification theory of regular languages. The original variety theory has been extended, generalized and adapted in many ways. In particular, there are a few different variety theorems for regular tree languages. On the other hand, one can define the recognizable subsets of an arbitrary algebra in such a way that regular languages and regular tree languages are obtained as special cases. In this lecture we outline a variety theorem for recognizable subsets of finitely generated free algebras over a given variety $\mathbf{V}$ of a finite type. It shows how varieties of recognizable sets, varieties of finite algebras contained in $\mathbf{V}$ and varieties of finite congruences on the free algebras correspond to each other. The two forms of Eilenberg's theorem as well as a variety theorem for tree languages are special instances of this general theorem. Finally, we shall note a few recent developments that mostly concern varieties of tree languages.

Department of Mathematics, University of Turku
FIN-20014 Turku, Finland
Email: steinby@utu.fi

Tree-Based Numbers: Higher Arithmetic Operations and Division Trees

H. Trappmann

I introduce my research regarding tree-based number systems. Tree-based means that instead of beginning the number constructions (natural to fractional to real) with the natural numbers we begin them with (specific) binary trees. This leads to interesting constructions (like "division trees") and opens a variety of research possibilities. It also sheds light on how to uniquely extend the definitions of hyperpowers to the continuous case.

Kameruner Str. 9
13351 Berlin
Email: bo198214@yahoo.de

Symmetric Groupoids and Cores

A. Vanzurova

Left symmetric, left distributive and idempotent groupoids, SID, arise e.g. as cores of Bol loops. But the class of Bol loop cores is not closed under subalgebras, hence is no variety, even no quasi-variety. It appears that cores of groups generate the variety SID, while abelian group cores generate its medial (entropic) subvariety. A core of a group is medial iff the group is nilpotent of class at most two. A necessary and sufficient condition is formulated for a Bol loop to have a medial core.

Palacky University, Olomouc
Email: vanzurov@inf.upol.cz

IA-Automorphisms of the Free Solvable S_p-permutable Algebra of a Finite Rank

P. B. Zhdanovich

Let $S = S_0 * S_p \cup \{1\}$ be a free product of semigroups S_0 and S_p with adjoined unit element 1. By S_p-permutable algebra we mean an algebra A of type $\langle S, p \rangle$ such that S acts on A and ternary operation p satisfies two Mal'cev identities $p(x, x, y) = p(y, x, x) = y$. We also assume that $p(s(x), s(y), s(z)) = s(p(x, y, z))$ for all s in S_p. By Mal'cev's theorem the variety V of all S_p-permutable algebras is congruence permutable. Thus we can apply the theory of commutators of congruences [2]. We consider the variety V_k of k-step solvable V-algebras for each positive k. In [3] we used S_p-permutable algebras for a construction of free abelian extensions and free solvable algebras in an arbitrary congruence-permutable variety V. Let F_k be a free V_k-algebra of a finite rank n. An automorphism of F_k is an *IA-automorphism* if it induces an identical automorphism of F_{k-1}. We explore some properties of IA-automorphisms of F_K including their Jacobian matrix over a preadditive category associated with F_{k-1} [1]. We also define the elementary automorphism of F_k and investigate conditions under which an IA-automorphism of F_k is *tame*, i.e. it is a product of elementary automorphisms.

References

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Volgograd State Pedagogical University
Volgograd
E-mail: comlab@vspu.ru