n-fold positive implicative, and commutative hyper K-ideals

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The theory of hyper compositional structure has been introduced by F. Marty in 1934 during the 8th congress of Scandinavian Mathematicians, where he presented his work [9]. Today the research in the hyper compositional structures field is very vivid. In many universities in the world there are working teams on this theory. In particular M. M. Zahedi and R. A. Borzooei introduced the notions of hyper BCK-algebra and hyper K-algebra in 2000[2]. The concepts of an n-fold positive implicative, commutative, and implicative hyper K-ideals are the generalizations of the concepts of positive implicative, commutative, and implicative hyper K-ideals,respectively, which are related to the concepts of positive implicative, commutative, and implicative ideals of a BCK-algebra[13]. The relationships between positive implicative and commutative hyper K-ideals have been studied by M.M. Zahedi and T. Roodbari[11]. They defined 27 types of positive implicative hyper K-ideals, 9 types of commutative hyper K-ideals, and an implicative hyper K-ideal. They proved some propositions and theorems in this field.

In this paper we are supposed to introduce the definitions of n-fold positive implicative, commutative, and implicative hyper K-ideals. These definitions are the generalizations of the definitions of positive implicative, commutative, and implicative hyper K-ideals, respectively, which have been defined in [11]. We define 9 types of n-fold commutative hyper K-ideals, and we obtain some relationships. Finally we define an n-fold implicative hyper K-ideal and we determine the relationships between n-fold implicative hyper K-ideal and n-fold commutative hyper K-ideals of a hyper K-algebra of order 3, which satisfies the simple condition.

In the first section of this paper we give some theorems an definitions that they all before has been proved. The most important definitions of them are the definitions of a hyper K-algebra, weak hyper K-ideal and hyper K-ideal of a hyper K-algebra, and simple condition.

In the second section of this paper we define the notions of n-fold positive implicative hyper K-ideals of types $1'$,$2'$,$3'$,and $4'$. Then we define 27 other types, and we give many examples to show that these notions are different from each other. Finally we prove some theorems and obtain some related result.

The following theorem is one of the most important theorems in this section:

Theorem. Let I be a non-empty subset of H. Then the following statements hold:

(1) If I is n-fold PIHKI of type 4, then I is n-fold PIHKI of types 1,6,

(2) If I is n-fold PIHKI of type 5, then I is n-fold PIHKI of types 2,6,

(3) If I is n-fold PIHKI of type 6, then I is n-fold PIHKI of type 3,

(4) If I is n-fold PIHKI of type 8, then I is n-fold PIHKI of type 7,

(5) If I is n-fold PIHKI of type 9, then I is n-fold PIHKI of types 7,8,

(6) If I is n-fold PIHKI of type 11, then I is n-fold PIHKI of types 10,12 ,

(7) If I is n-fold PIHKI of type 10, then I is n-fold PIHKI of type 12,

(8) If I is n-fold PIHKI of type 13, then I is n-fold PIHKI of types 14,15,

(9) If I is n-fold PIHKI of type 14, then I is n-fold PIHKI of 15,

(10) If I is n-fold PIHKI of type 18, then I is n-fold PIHKI of 16, 17,

(11) If I is n-fold PIHKI of type 17, then I is n-fold PIHKI of type 16,

(12) If I is n-fold PIHKI of type 20, then I is n-fold PIHKI of type 3,

(13) If I is n-fold PIHKI of type 21, then I is n-fold PIHKI of type 19,

(14) If I is n-fold PIHKI of type 24, then I is n-fold PIHKI of type 23,

(15) If I is n-fold PIHKI of type 22, then I n-fold PIHKI of type 24,

(16) If I is n-fold PIHKI of type 27 , then I is n-fold PIHKI of type 26.

Also we give an example to show that the converse of the statements of this Theorem are not true in general.

In the third section of this paper we define 9 types of n-fold commutative hyper K-ideals and n-fold implicative hyper K-ideal. Then we determine the relationships between n-fold implicative hyper K-ideal and n-fold commutative hyper K-ideals of a hyper K-algebra of order 3, which satisfies the simple condition.

the following theorem is the most important result in this section : **Theorem.**

Let $H = 0, 1, 2$ be a hyper K-algebra of order 3 that satisfies the simple condition and $0 \neq I \subseteq H$. If I is an n-fold implicative hyper K-ideal, then I is an n-fold commutative hyper K-ideals of types 1, 2, 3, 4, 5, 6, 7, 8, and 9.

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