

RESIDUATED MULTILATTICES: A FIRST GLIMPSE INTO THEIR STRUCTURES

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ABSTRACT. We prove that when divisibility is added to a residuated multilattice, this causes the multilattice structure to collapse down to a residuated lattice. This motivates the study of semi-divisibility and regularity on residuated multilattices. The ordinal sum construction is also applied to residuated multilattices as a way to construct new examples of both residuated multilattices and consistent filters. Finally, it is also established that the smallest residuated multilattice that is not a residuated lattice has order 7.

Key words: Multilattice, residuated multilattice, consistent filter, ordinal sum, semi-divisible residuated sublattice, Girard-monoid residuated lattice.

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