Asymptotic behaviour of biharmonic heat equations on unbounded domains

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(Joint work with Daniel Daners and Jochen Glück)

It is well-known that one cannot expect the positivity preserving property to hold for higher-order parabolic equations. Nevertheless, it seems that positivity is "almost" preserved in some sense. Gazzola and Grunau have shown that the biharmonic heat equation $u_t + (-\Delta)^2 u = 0$ on $\mathbb{R}^n$ displays *local eventual positivity.* Roughly speaking, this means that given positive initial datum u_0, for every compact set $K \subset \mathbb{R}^n$ there exists a time $T > 0$ such that the solution $u = u(t, x)$ is positive on K for all $t \geq T$. Intuitively, this phenomenon occurs as a result of the oscillatory behaviour of the fundamental solution, and so far, local eventual positivity for these equations has been studied via explicit analysis of such biharmonic heat kernels.

In this talk, I will present some recent work on the asymptotic behaviour of solutions to the biharmonic heat equation on 'infinite cylinders' of the form $\mathbb{R} \times \Omega$, where $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary. As a consequence of our results, we recover the local eventual positivity of solutions qualitatively (i.e. without use of explicit heat kernels), and for a larger class of initial data than was previously considered. The analysis on the infinite cylinder uses various properties of a family of fourth-order eigenvalue problems, which may be of independent interest. I will also comment on some connections with the theory of *eventually positive semigroups* in infinite dimensions, which was first developed systematically by Daners, Glück and Kennedy in 2016, and recently extended by Arora to treat local eventual positivity.