Iterated Minkowski sums as topological dynamical systems

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In geometric group theory a prominent idea is to associate a Cayley graph to a finitely generated group and study which properties of the group can be recovered from this geometric object. One can then further associate objects like random walks to the Cayley graph and again try to capture properties of the group by the behavior of these objects. Following these ideas, in joint work with Tom Meyerovitch we associate to every Cayley graph a topological dynamical system which is probably the simplest possible model of cluster growth or infection spreading on these graphs.

More precisely, for a given finite generating set A of a group Γ we study the iteration of the map $\Phi_{\Gamma,A} : 2^\Gamma \to 2^\Gamma,\ M \mapsto MA$. In the Abelian group $\mathbb{Z}^n$ this corresponds to iteratively taking the Minkowski sum with A. If we endow 2^Γ with the product topology, we obtain a non-invertible topological dynamical system on a Cantor space. In forward time, the dynamics of this system is rather boring: everything besides the empty set converges to the whole group Γ.

Nevertheless, if we look backwards in time, we can recover important information about the group and the generating set from the isomorphism class of this system. Already in the case of $\Gamma = \mathbb{Z}^n$ some hard questions regarding the classification of these systems remain.