Lyapunov couples and regularisation of semigroups

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In the talk we consider functional inequalities such as

$$\varphi(T_t u) + \int_0^t \psi(T_s u)\,\mathrm{d}s \leq \varphi(u),$$

where $(T_t)_{t\geq 0}$ denotes a (nonlinear) semigroup on a closed subset D of a Banach space X and $\varphi, \psi\colon X \to \mathbb{R} \cup \{\infty\}$. Here, we call a pair (φ, ψ) *Lyapunov couple* for $(T_t)_{t\geq 0}$ if the above inequality is satisfied for all $(u,t) \in D \times [0,\infty)$. In particular, we use Lyapunov couples to derive regularisation effects of the form

$$\psi(T_t u) \lesssim t^{-1}\varphi(u).$$

Using Sobolev type inequalities, we see examples of Lyapunov couples for the semigroup $(e^{-t\Delta_p^D})_{t\geq 0}$ generated by the Dirichlet p-Laplacian Δ_p^D and study its regularity effects and asymptotic behaviour.