Rearrangement-invariant quasi-Banach function spaces and their amalgams

Dalimil Peša

Technische Universität Chemnitz, Fakultät für Mathematik, Chemnitz, Germany

and

Charles University, Faculty of Mathematics and Physics, Praha, Czech Republic

e-mail: `pesa.dalimil@gmail.com`

Abstract classes of function spaces have historically proven extremely useful by providing the necessary frameworks for such fields as operator theory, harmonic analysis or interpolation. Among the many examples, the classes of rearrangement-invariant (r.i.) spaces have always played an important role. However, the existing, well developed theories cover only the normed spaces, leaving many important examples untreated.

In this talk, we present some recent results of our effort to fill this gap. Our focus is on the class of r.i. quasi-Banach function spaces, a natural axiomatisation of Lebesgue spaces that, unlike other approaches, covers also the quasinormed cases, as well as many other important function spaces (with the space $L^{1,\infty}$, also called weak-L^1, being the most significant example). Furthermore, as one of the tools that has proven very useful in studying this class, we introduce the so-called Wiener–Luxemburg amalgam spaces, an abstract framework for constructing rearrangement-invariant amalgams, that is, spaces where the conditions on the local and global behaviour can be prescribed separately.

The talk is based on the first part of the author's dissertation, expanded by some more recent results.

[1] A. Nekvinda and D. Peša, On the Properties of Quasi-Banach Function Spaces, *The Journal of Geometric Analysis*, 2024, 34(8)

[2] D. Peša, Wiener-Luxemburg amalgam spaces, *J. Funct. Anal.*, 2022, 282(1)

[3] A. Musilová, A. Nekvinda, D. Peša, and H. Turčinová, On the properties of rearrangement-invariant quasi-Banach function spaces., *preprint*, ArXiv:2404.00707