Ordered Banach Spaces, Positive Operators and Applications

Workshop, March, 27 - 29, 2019

Program and **Abstracts**

DFG-Projekt: Ordnungserhaltende Operatoren in Problemen der optimalen Steuerung und in der Theorie der partiellen Differentialgleichungen
CH 1282/5-1

Wednesday, March 27

9:00 – 9:40

Witold WNUK:

Finitely generated vector sublattices

For a subset S of an Archimedean vector lattice E, we let $vlt(E,S)$ denote the vector sublattice of E generated by S, that is, the smallest vector sublattice of E containing S. We are particularly interested in the case, where the set S is finite; then the sublattice vlt(E,S) is said to be finitely generated. We show a kind of "distributive law" for the operator vlt, namely that $vlt(X;Y\cup Z)=vlt(X,Y)+vlt(X,Z)$ if Y and Z are orthogonal, and derive a number of its consequences. We characterize those sets in a vector lattice that generate n-dimensional sublattices, and show that a finite set generates a finite-dimensional sublattice if so does each pair of its elements. We also show that in a uniformly complete vector lattice, every principal ideal of infinite dimension contains pairs of positive elements generating countable as well as continuum dimensional sublattices. The special case of lattices $C(K)$ is also treated in this respect. Moreover, for a compact set K in an n-dimensional space with a nonempty interior, it is shown that the minimal number of functions in $C(K)$ generating a dense sublattice is $n+1$. We additionally prove that in a discrete and σ-Dedekind complete separable F-lattice, one can always find a pair of positive elements generating a dense sublattice.

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10:00 – 10:40

Faruk POLAT:

On some algebraic Properties of narrow orthogonally additive Operators

The aim of my talk is to present some new results concern- ing narrow orthogonally additive operators. We say that an orthogonally additive operator T from a vector lattice E to a normed space X is narrow if for any $e\in E$ and $\varepsilon>0$ there exist a disjoint decomposition $e=e_1+e_2$ such that $\|Te_1-Te_2\|<\varepsilon$. We prove that the sum of two orthogonally additive operators $S+T$ defined on a vector lattice and taking values in a Banach space, where S is a narrow operator and T is a laterally-to-order continuous C-compact operator, is a narrow operator as well.

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12:00 – 13:30 Lunch break

13:30 – 14:10

Zalina A. KUSRAEVA:

Representation and Classification of Homogeneous Polynomials in Vector Lattices

The talk is devoted to the order properties of homogeneous polynomials in vector and Banach lattices. Despite the fact that the algebraic and linear-topological study of polynomials has a long history and is well represented in literature, the study of the properties of polynomials that depend on the interaction between the norm and the order relation has begun only recently. One of the central results states that each bounded orthogonally additive homogeneous polynomial defined on a vector lattice, under some additional assumptions, can be represented as the composition of a bounded linear operator and a special homogeneous polynomial which plays the role of the exponentiation absent in an arbitrary vector lattice. Applications of this fact are given to the problems of dominance and integral representation of homogeneous polynomials. Particular attention is paid to the problem of a complete description of disjointness preserving homogeneous polynomials. As an auxiliary tool, factorization of order bounded disjointness preserving multilinear operators is used.

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14:30 – 15:10

Martin R. WEBER:

On almost interior points in ordered Banach spaces and some applications

Almost interior points in ordered Banach spaces replace the strong requirement of the existence of interior points in the cone in many applications and, so are much more appropriate both for theoretical investigation and applications. Such points exist in ordered Banach spaces if the space is separable and the cone is total. Those points generalize the notions of "functions which are strictly positive almost everywhere" on L^p-spaces and of "quasi-interior points" in Banach lattices. Many properties of almost interior points are scattered throughout the literature. In this talk we gather an overview of their basic properties, their relations to other concepts and their appearance in operator theoretic context. We discuss several examples and open problems which we consider to be central to their theory. Almost interior points play a essential role both in the spectral theory of positive operators and in the possibility of extending positive functionals. If a positive operator semigroup $(T_t)_{t\in J}$ on an ordered Banach space X has the property that for each vector $0 \leq x$ there is a time $t_x \in J$ such that $T_{t_x}x$ is an almost interior point then the relative compactness of each orbit $\{T_t x\colon t \in J\},\ x \in X$ guarantees the strong convergence of $(T_t)_{t\in J}$. Somewhat stronger conditions guarentee even the uniform on the semigroup.

Jochen Glück and Martin R. Weber: Almost Interior Points in Ordered Banach Spaces and the Long–Term Behaviour of Strongly Positive Operator Semigroups. arXiv1901.03306, 2019.

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15:30 Discussion in the coffee room

Thursday, March 28

9:00 – 9:40

Anke KALAUCH:
Pre-Riesz Spaces I

Pre-Riesz spaces are partially ordered vector spaces that embed order densely into Riesz spaces. Concepts from Riesz space theory such as disjointness, ideals, and bands are extended to pre-Riesz spaces, as well as disjointness preserving operators and related notions. The analysis of these concepts revolves around embedding techniques into vector lattice covers, in particular into the functional representation.

Anke Kalauch, Onno v. Gaans: Pre-Riesz Spaces. Walter de Gruyter GmbH, Berlin/Boston. 2018.

The presentation is divided into two parts.

I. We give a gentle introduction into the above mentioned notions and techniques. Every directed Archimedean partially ordered vector space is a pre-Riesz space, hence every ordered Banach space with generating closed cone is as well. We give examples of results from the theory of Banach lattices, for instance concerning disjointness preserving operator semigroups, that can smoothly be generalized to ordered Banach spaces. On the other hand, we give instances where the vector lattice theory and the theory of pre-Riesz spaces differ essentially.

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10:00 – 10:40

Onno van GAANS:
Pre-Riesz Spaces II

II. The inverse of a disjointness preserving bijection between Banach lattices is again disjointness preserving. We discuss a few extensions of this result to pre-Riesz spaces. In finite dimensions, the statement relies on a bound on the number of bands that only depends on the dimension. In infinite dimensions, an appropriate extension of norms to a vector lattice cover is needed.

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12:00 – 13:30 Lunch break

13:30 – 14:30

Marat PLIEV:

Orthogonally additive Operators in Vector Lattices and lattice-normed Spaces

The talk is devoted to some new results of the theory of orthogonally additive operators in vector lattices and lattice-normed spaces. The main topics are: a problem of an analytic representation of an orthogonally additive operator, a problem of an extension of a orthogonally additive map, a domination problem for different types of orthogonally additive operators.

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14:50–15:30

Mr. Yang DENG:

Topological bicommutant theorems in vector lattices

Suppose that F is a vector lattice. Then we let τ_0 be the topology on F generated by the subsets $O_{f,n^{-1},y}(x_0) := \{x \in E : f(|x - x_0| \wedge y) < n^{-1}\}$ of F for all $n \in \mathbb{N}$, $x_0 \in F$, $y \in F^+$, and positive order continuous functions $f : F \to \mathbb{R}$. Let E and F be vector lattices, where F is Dedekind complete. For $x \in E^+$, define $\varphi_x : \mathcal{L}_b(E, F) \to F$ by setting $\varphi_x(T) := Tx$. Let τ' be the coarsest topology on $\mathcal{L}_b(E, F)$ such that φ_x is continuous for all $x \in E^+$, when F is supplied with the τ_0-topology.
Suppose that E is a Dedekind complete vector lattice. Then we let $\mathcal{L}_n(E)$ denote the Riesz algebra of all order continuous operators on E. We say that a subalgebra $\mathcal{A}$ of $\mathcal{L}_n(E)$ is a *band algebra* if $\mathcal{A}$ is also a band in $\mathcal{L}_n(E)$. We say that a band B in E is $\mathcal{A}$-*invariant* if $Tx \in B$ for all $x \in B$ and $T \in \mathcal{A}$, and that B is $\mathcal{A}$-*reducing* if both B and B^d are $\mathcal{A}$-invariant. A band algebra $\mathcal{A}$ in $\mathcal{L}_n(E)$ is said to have the $*$-*property* if every $\mathcal{A}$-invariant band in E is also $\mathcal{A}$-reducing. Let $\mathcal{A}' = \{S \in \mathcal{L}_n(E) : ST = TS \text{ for all } T \text{ in } \mathcal{A}\}$. Then we have the following topological bicommutant theorem in vector lattices, which is analogous to the topological version of the von Neumann's bicommutant theorem in Hilbert spaces.
Theorem. Let E be a Dedekind complete vector lattice such that, for all $x \in E^+$, there exists a strictly positive order continuous function f_x on $[0, x]$. Suppose that $\mathcal{A} \subseteq \mathcal{L}_n(E)$ is a unital band algebra with the $*$-property. Then $\mathcal{A}'' = \overline{\mathcal{A}}^{\tau'}$. If E is an order continuous Banach lattice, then $\mathcal{A}'' = \overline{\mathcal{A}}^{sot}$. If E is atomic, then $\mathcal{A}'' = \mathcal{A}$. This supplements earlier results by Björn de Rijk and Marcel de Jeu on the ordered analogue of the representation-theoretical version of the classical bicommutant theorem.

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15:30 Discussion in the coffee room

17:30 ⋆ *Conference Dinner* ⋆
Sindbad Restaurant

Friday, March 29

9:00–10:00

Anatolij G. KUSRAEV:

Injective Banach Lattices of Operators

An injective Banach lattice is an injective object in the category of Banach lattices with the positive contractions as morphisms. The talk deals with some aspects of bounded linear operators on injective Banach lattices. Under consideration is the question under which conditions various classes of regular linear operators between Banach lattices form themselves injective Banach lattices. Boolean valued analysis framework for this problem is also discussed. The talk is based on the joint paper with A. W. Wickstead.

A.G. Kusraev and A.W. Wickstead: Some Problems Concerning Operators on Banach Lattices. Preprint / SMI VSC RAS; No. 1. Vladikavkaz, 28 p., 2016.

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10:10–10:50

Mohamed Amine BEN AMOR:

Compact-like Operators in lattice-normed spaces: Answers to open questions

In Aydin and al. [1], the authors introduce new notions of Compact operators in Lattice-Normed spaces like p-compact, rp-compact and p-semi-compact operators, as a generalization of the Banach classical case. They leave several open questions. In this talk, we will give answers to all the open problems in [1]. We will prove among other facts that every p-compact operator is p-bounded. Finally, we will investigate the case of orthogonally-additive compact-like operators.

[1] A. Aydin, E. Yu. Emelyanov, N. Erkur sun Özcan, and M. A. A. Marabeh. Compact- like operators in lattice-normed spaces. Indag. Math. (N.S.), 29(2):633–656, 2018.

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11:00 –12:00

Koratti Ch. SIVAKUMAR:
Positivity Aspects of Matrices and the Interval Property For real square matrices F, G of the same size with $F \leq G$ (where the order is defined element wise), let $[F, G] := \{X : F \leq X \leq G\}$ denote an interval of matrices. Consider the following, referred to as interval property: Suppose that F and G belong to a given matrix class. When does each $X \in [F, G]$ belong to that class? This question has been investigated for a number of matrix classes. For instance, it is well known that F and G are invertible M -matrices if, and only if, any matrix in [F, G] is an invertible M -matrix. The aim of this talk will be to present a survey of some interesting results, wherein F and G belong to certain specific matrix positivity classes, including inverse M -matrices. Some new unpublished results will be presented for the class of H-matrices.

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12:20 – 13:50 Lunch break

14:00 Final Discussion