Geometric modelling is needed in mechanical engineering to calculate the geometry of worm gears, threads, and similar mechanical elements that can be described by helicoids or surfaces of revolution, and that are to be manufactured by milling, grinding, or whirling. In the past, the tasks of calculating surfaces of revolution (representing a form cutter or grinding wheel) for milling a given helicoid (representing a worm gear), and calculating ehe enveloped helicoid for a given surface of revolution were solved separately. Here it will be shown that the two results can be achieved by a more general approach.

Furthermore, in this paper we consider not only the computation of points of interesting curves of enveloped helicoids but also the computation of tangent lines and osculating circles of such curves. Therefore, some results of differential geometry of helicoids are provided. At first, we calculate the normal curvature of an arbitrary plane section through a helicoid that is defined by an arbitrary generating curve. Then this result is specialised to transverse and axial sections. Furthermore, it is shown that the classification of points of the helicoid as elliptic, parabolic, or hyperbolic can be based on a constructive criterion in the transverse section plane. Finally, the example of an elliptical helicoid and its envelope is used to illustrate the theory.
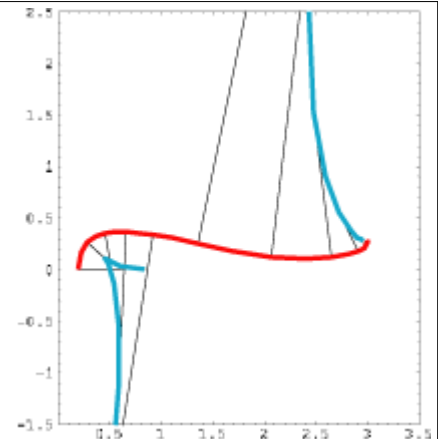
Axial section of an enveloped helical surface along with the evolute.
(The evolute is the locus of the centers of curvature and the envelope of the axial section's normals.)