



# "GEOMETRIETAG 2014"

4.-5. Dezember 2014 in Dresden

Stand: 01.12.2014

#### Organisatoren

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### Hauptvortragende:

JProf. Dr. Wolfgang STEIMLE (Leipzig) und

Prof. Dr. Christian SEVENHECK (Chemnitz)

### Programmübersicht:

Do 04.12.2014 Raum: WIL A 317		
11.00-12.00	Hauptvortrag Wolfgang STEIMLE	
Mittagspause		
13.30-14.15	Nadine GROSSE	
14.25-14.45	Thomas JAHN	
14.50-15.10	Daniel LORDICK	
Kaffeepause		
15.40-16.25	Victor VINNIKOV	
16.35-17.20	Markus SCHWEIGHOFER	
17.30-17.50	Charu GOEL	
17.55-18.15	Kai KELLNER	
19.30	Abendessen	

Fr 05.12.20	14 Raum: WIL C 307
09.00-09.20	Rolf SCHNEIDER
09.25-09.45	Enno KESSLER
09.55-10.40	Jan KRÜMPELMANN
Kaffeepause	
11.00-12.00	Hauptvortrag Christian SEVENHECK
Mittagspause	
13.30-14.15	Ivan IZMESTIEV
14.25-14.45	Christian RICHTER
Kaffeepause	
15.15-16.00	Undine LEOPOLD



## Programm

Time	Time Donnerstag, 04.12.2014 Raum: WIL A 317	
11.00-12.00	Wolfgang STEIMLE The space of metrics with positive scalar curvature	Hauptvortrag
	Mittagspause	
13.30-14.15	Nadine GROSSE (Spinorial) Yamabe-type constans, bordism bounds and positive scalar curvature	re
14.25-14.45	Thomas JAHN Bifocal Curves in Generalized Minkowski Planes	
14.50-15.10       Daniel LORDICK         Make Haste Slowly: Discretization of a Curve by Means of Parametric Modeling		5
	Kaffeepause	
15.40-16.25	Victor VINNIKOV Determinantal representations of hyperbolic and stable polynomials	
16.35-17.20	Markus SCHWEIGHOFER Inclusion of spectrahedra, free spectrahedra and coin tossing (joint work with Bill Helton, Igor Klep and Scott McCullough)	
17.30-17.50	Charu GOEL A general principle for extending Hilbert's 1888 Theorem to even symmetric fo	rms
17.55-18.15	Kai KELLNER Positivstellensatz certificates for containment of H-polytopes in V-polytopes	
19.30	Abendessen (Sächsisch-Böhmisches Bierhaus »Altmarktkeller«, Altmarkt 4)	

Time	Freitag, 05.12.2014 Raum: WIL C 307	
09.00-09.20	Rolf SCHNEIDER Affine diameters of convex bodies	
09.25-09.45	Enno KESSLER A Superconformal Action Functional for Super Riemann Surfaces	
09.55-10.40	Jan KRÜMPELMANN Extending lattice-free integral polytopes in dimension 3	
	Kaffeepause	
11.00-12.00	Christian SEVENHECK Cohomological invariants of isolated and non-isolated singularities	Hauptvortrag
	Mittagspause	
13.30-14.15	Ivan IZMESTIEV Matching centroids by projective transformations	
14.25-14.45	Christian RICHTER Tiling convex polygons with congruent equilateral triangles	
	Kaffeepause	
15.15-16.00	Undine LEOPOLD Vertex-Transitive Polyhedra of Higher Genus in 3-Space	

### Abstracts

Charu GOEL	04.12.14 (Do)
A general principle for extending Hilbert's 1888 Theorem to even symmetric forms	17.30-17.50 Uhr
In 1888 Hilbert proved that:	

 $\mathcal{P}_{n,2d}$  \$ = \$ $\Sigma_{n,2d}$  if and only if n = 2, d = 1, or (n,2d) = (3,4), where  $\mathcal{P}_{n,2d}$  and  $\Sigma_{n,2d}$  are respectively the cones of positive semidefinite (psd) and sum of squares (sos) forms (real homogenous polynomials) of degree 2d in n variables. In order to establish that  $\Sigma_{n,2d} \subsetneq \mathcal{P}_{n,2d}$ , he demonstrated that  $\Sigma_{3,6} \subsetneq \mathcal{P}_{3,6}$ ,  $\Sigma_{4,4} \subsetneq \mathcal{P}_{4,4}$ , and then used an argument to increase the number of variables and degree of a given psd not sos form while simultaneously preserving the psd not sos property.

In 1976 Choi and Lam gave an analogue of Hilbert's 1888 Theorem for symmetric forms. By assuming the existence of psd not sos symmetric *n*-ary quartics for  $n \ge 5$ , they showed that  $S\mathcal{P}_{n,2d} = S\Sigma_{n,2d}$  if and only if n = 2, d = 1, or (n, 2d) = (3, 4), where  $S\mathcal{P}_{n,2d}$  and  $S\Sigma_{n,2d}$  are respectively the cones of symmetric psd and symmetric sos *n*-ary 2*d*-ics.

More precisely, using a trick that increases the degree – however not the number of variables – of a given psd not sos symmetric form by simultaneously preserving the psd not sos symmetric property, it reduces to show that  $S\Sigma_{3,6} \subsetneq S\mathcal{P}_{3,6}$  and  $S\Sigma_{n,4} \subsetneq S\mathcal{P}_{n,4}$  for  $n \ge 4$  in order to establish that  $S\Sigma_{n,2d} \subsetneq S\mathcal{P}_{n,2d}$  for all  $n \ge 3, 2d \ge 4$  and  $(n,2d) \ne (3,4)$ .

In this talk, we explain the algebraic geometric ideas behind these reductions and complete their proof by constructing explicitly psd not sos symmetric *n*-ary quartics for  $n \ge 5$ . We then extend these methods to investigate *even symmetric forms* (i.e. forms invariant under the action of the group  $S_n \times \mathbb{Z}_2^n$ ). We first review known results by Choi, Lam, Reznick and Harris relating even symmetric psd and sos forms for the pairs  $(n,4)_{n\ge4}, (n,6)_{n\ge3}, (3,8), (3,10), (4,8)$  and conjecture that

 $S\mathcal{P}_{n,2d}^e = S\Sigma_{n,2d}^e$  if and only if n = 2, d = 1 or  $(n, 2d) = (n, 4)_{n \ge 4}$ , (3,8).

We demonstrate that it is enough to establish  $S\Sigma_{n,2d}^e \subseteq S\mathcal{P}_{n,2d}^e$  for the pairs  $(n,8)_{n\geq5}$  and  $(n,2d)_{n\geq4,d=5,6}$  to get  $S\Sigma_{n,2d}^e \subseteq S\mathcal{P}_{n,2d}^e$  for all  $n\geq3, 2d\geq8$  and  $(n,2d)\neq(3,8),(3,10)$ ; where  $S\mathcal{P}_{n,2d}^e$  and  $S\Sigma_{n,2d}^e$  are respectively the cones of even symmetric psd and even symmetric sos *n*-ary 2*d*-ics. For proving this we present a *degree jumping principle* that increases the degree of a given psd not sos even symmetric form while simultaneously preserving the psd not sos even symmetric property.

We further give a construction of explicit psd not sos even symmetric *n*-ary 2*d*-ics for the pairs  $(n,8)_{n\geq 5}$ , (4,10) and (5,10). Thus to complete our conjecture it remains to establish that  $S\Sigma_{n,2d}^e \subseteq S\mathcal{P}_{n,2d}^e$  for the pairs  $(n,10)_{n\geq 6}$  and  $(n,12)_{n\geq 4}$ .

This is an ongoing joint work with S. Kuhlmann and B. Reznick.

Nadine GROSSE	04.12.14 (Do)
(Spinorial) Yamabe-type constans, bordism bounds and positive scalar curvature	13.30-14.15 Uhr
We give an introduction in the Yamabe constant and their syblings. The smooth Yamabe in whether a closed Riemannian maniolf posesses a metric of positive scalar curvature. Unfor value is only known for a few manifolds. In particular, one knows no manifold of dimensior and not simply-connected with Yamabe invariant not being zero or the one of the standarc An important first step in finding out whether such manifolds can actually exist is to find be Yamabe invariant. For that a spinorial brother of the Yamabe invariant and a surgery ineque Yamabe invariant might be helpful. But the surgery inequalities contain certain threshold c type constants of noncompact model spaces. We examine those (spinorial) Yamabe-type c applications to closed manifold. This is joint work with Bernd Ammann, Regensburg.	tunately its explicite n bigger or equal 5 I sphere. Dunds on the alities for the onstants - Yamabe-

Ivan IZMESTIEV	05.12.14 (Fr)
Matching centroids by projective transformations	13.30-14.15 Uhr
Let K and L be two subsets of $\mathbb{R}^d$ . Does there exist a projective transformation f such that the centroid $f(K)$ and $f(L)$ coincide? We allow each of K and L to be a point, a finite set of points, or a d-dimensional set of points.	

body, and find in each case a functional whose critical points correspond to solutions. Under certain assumptions the transformation *f* is unique modulo post-composition with affine transformations. Connections arise with the algebraic polarity, Moebius centering of polytopes, Santalo points, and Hilbert geometry.

The talk is based on the arxiv preprint 1409.6176.

Thomas JAHN	04.12.14 (Do)
Bifocal Curves in Generalized Minkowski Planes	14.25-14.45 Uhr
We present some results and observations concerning ellipses and Cassini curves in planes where the	

distances are measured by gauges.

Kai KELLNER	04.12.14 (Do)
Positivstellensatz certificates for containment of H-polytopes in V-polytopes	17.55-18.15 Uhr

Given an H-polytope P and a V-polytope Q, the decision problem whether P is contained in Q is co-NPcomplete. This hardness remains if P is restricted to be a standard cube and Q is restricted to be (the affine image of) a cross polytope. Since this hardness classification by Freund and Orlin (see also Gritzmann and Klee), there seems to be only limited progress on that problem so far.

We formulate the H-in-V containment problem in terms of a bilinear feasibility problem and characterize its geometric properties. The application of Handelman's Positivstellensatz and Putinar's Positivstellensatz yields hierarchies of linear programs and semidefinite programs, respectively, to decide the containment problem. As a main result, we show that under mild preconditions the Putinar representation converges in finitely many steps.

(joint work with Thorsten Theobald)

Enno KESSLER A Superconformal Action Functional for Super Riemann Surfaces	05.12.14 (Fr) 09.25-09.45 Uhr
The harmonic action functional on Riemann surfaces is a tool to study the moduli space of Riemann surfaces.	
In 2D-Supergravity and string theory one considers a supersymmetric extension of the action functional,	
where both the field and the metric get a superpartner. The appropriate geometric setting for this action	
functional is supported and in particular super Discourse suffered. In this talk we are asian to see how the	

functional is supergeometry and in particular super Riemann surfaces. In this talk we are going to see how the geometric properties of super Riemann surfaces lead to the known symmetries of the supersymmetric action functional and that the supersymmetric action functional is also a functional on the moduli space of super Riemann surfaces. It will be shown that the gravitino, which is the superpartner of the metric, encodes information about the odd dimensions of this moduli space, similar to the idea of Teichmüller spaces, where metrics encode information on the moduli space of Riemann surfaces

Jan KRÜMPELMANN	05.12.14 (Fr)
Extending lattice-free integral polytopes in dimension 3	09.55-10.40 Uhr

We say that a *d*-dimensional polytope *P* with integral vertices is lattice-free if it contains no integral point in its interior. Our focus is on the following question: given a real point r not in *P* such that the convex hull of *P* and *r* is still lattice-free, is it always true that there also exists an integral point z not in P such that the convex hull of *P* and *z* is also lattice-free? The answer to this question is "yes" in dimensions one and two and was shown to be "no" by Nill and Ziegler (2011) for dimension at least four, but the case d = 3 was still open. In this talk, we answer the question for d = 3.

This is joint work with Gennadiy Averkov and Stefan Weltge (both of OvGU Magdeburg).

Undine LEOPOLD	05.12.14 (Fr)
Vertex-Transitive Polyhedra of Higher Genus in 3-Space	15.15-16.00 Uhr

Regular and chiral maps successfully generalize the combinatorial structure of the Platonic solids. While it is hopeless for their higher genus examples to exist as symmetric polyhedra in 3-space (with flat, non-self-intersecting faces tiling an embedded surface), an interesting related problem is that of combinatorially uniform, or geometrically vertex-transitive, polyhedra. Indeed, an infinite family of genus one and a few higher genus examples of these highly symmetric polyhedra exist in 3-space (e.g., the combinatorially regular Grünbaum polyhedron), but the completeness of the list has never been established. I will present an overview of recent advances on this question and outline the solution for the case of tetrahedral rotation symmetry.

Daniel LORDICK	04.12.14 (Do)
Make Haste Slowly: Discretization of a Curve by Means of Parametric Modeling	14.50-15.10 Uhr
»Make Haste Slowly« is the title of an art project developed in collaboration with Hansjörg this project an industrial robot situated at the company Artis in Berlin had to span a carbon over a frame. The straight lines, thus generated, compose a given motion path as an envelo- the French scientist Étienne-Jules Marey (1830-1904) photographically recorded in the late During the layout of the discrete set of tangents for the envelope it became evident that co approximation routines for splines, as they are implemented in various graphic software pa	fiber rather slowly ope – a path, which 19th century. ommon linear ackages, do not

satisfy the requirements of a sophisticated and visual elegant design. This talk will show, how by means of parametric modeling a regular and aesthetically pleasing discretization can be achieved and generalized.

Christian RICHTER	05.12.14 (Fr)
Tiling convex polygons with congruent equilateral triangles	14.25-14.45 Uhr
We study the sets $T_v = \{m \in \{1, 2,\}$ : there is a convex polygon in $\mathbb{R}^2$ that has $v$ vertices and can be tiled with $m$ congruent equilateral triangles}, $v = 3, 4, 5, 6$ . $T_3$ , $T_4$ and $T_6$ can be quoted completely. The complement $\{1, 2,\} \setminus T_5$ of $T_5$ turns out to be a subset of Euler's <i>numeri idonei</i> . As a consequence, $\{1, 2,\} \setminus T_5$ can be characterized with up to two exceptions, and a complete characterization is given under the assumption of the Generalized Riemann Hypothesis.	

Rolf SCHNEIDER	05.12.14 (Fr)
Affine diameters of convex bodies	09.00-09.20 Uhr
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Markus SCHWEIGHOFER Inclusion of spectrahedra, free spectrahedra and coin tossing (joint work with Bill Helton, Igor Klep and Scott McCullough)	04.12.14 (Do) 16.35-17.20 Uhr
Spectrahedra are generalizations of (convex) polyhedra sharing many of the good algorithm polyhedra but allowing for roundness in their shapes. Given two spectrahedra in form of a inequality, it is in general hard to decide whether one contains the other. To the linear mat can however associate not only scalar solutions but also matrix solutions. This gives rise to spectrahedra. Inclusion of free spectrahedra is in many cases easy to decide, e.g., by using the Gram matrix method and a linear Positivstellensatz for symmetric linear matrix polyno Klep and McCullough. The natural question arises how the inclusion of two free spectrahedra inclusion of the corresponding classical spectrahedra. Surprisingly, this question is related to trivial properties of Binomial and Beta distributions some of which are known and some of	linear matrix trix inequalities one so called free a generalization of mials due to Helton, dra relates to the to very subtle non-

Cohomological invariants of isolated and non-isolated singularities	

05.12.14 (Fr) 11.00-12.00 Uhr

In this talk I would like to discuss some Hodge-theoretic invariants for hypersurface singularities. I will start by explaining a classical construction due to Steenbrink, Varchenko and other of a mixed Hodge structure on the vanishing cohomology of an isolated hypersurface singularity. Associated to such a Hodge structure is a set of rational numbers, called spectrum. Under some additional hypotheses, the spectrum can be related to some other classical invariant of any holomorphic of algebraic function, namely, the Bernstein polynomial. In the second part of the talk, I will explain how these invariants can be understood (and in some cases calculated) for a special class of non-isolated hypersurface singularities, the so-called free divisors.

Wolfgang STEIMLE	04.12.14 (Do)
The space of metrics with positive scalar curvature	11.00-12.00 Uhr

We study the topology of the space of positive scalar curvature metrics on high dimensional spheres and other spin manifolds. Our main result provides elements in higher homotopy and homology groups of these spaces, which, in contrast to previous approaches, are of infinite order and survive in the (observer) moduli space of such metrics.

Along the way we construct smooth fiber bundles over spheres whose total spaces have non-vanishing A-hatgenera, thus establishing the non-multiplicativity of the A-hat-genus in fiber bundles with simply connected base.

Victor VINNIKOV	04.12.14 (Do)
Determinantal representations of hyperbolic and stable polynomials	15.40-16.25 Uhr

Representing a multivariable polynomial as a determinant of a matrix with linear entries is an old problem in algebraic geometry that has been worked on from different perspectives during the last hundred plus years. A more recent problem assumes some localization constraints on the zeroes of the polynomial, and asks for determinantal representations with constrains – structural or metric – on the coefficient matrices that imply these zeroes localization constraints.

Two important examples are

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- a) positive selfadjoint determinantal representations of real hyperbolic polynomials, that are related to the problem of representing hyperbolic cones as spectrahedra [a prominent topic in convex algebraic geometry];
- b) contractive structured determinantal representations of complex polynomials that are stable [i.e., zero free] on a domain in  $C^d$  described by certain polynomial inequalities [closely related to the von Neumann inequality on this domain, a prominent topic in multivariable operator theory].

I will briefly survey the area, including possible connections between items a) and b). I will then discuss a very recent positive result on item b) allowing for an extra factor and assuming a genericity condition [strict stability] of the polynomial. The proof uses a Positivstellensatz of Putinar for sums of hermitian squares as well as some techniques ["lurking contraction argument"] of operator theory.

## Teilnehmer

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