Detailed Numerical Results for Several Variants of a Globally Convergent LP-Newton Method *

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1 Introduction

This paper reports in detail on the numerical results obtained for several variants of the globalized LP-Newton method introduced in [3] and of a method which combines a potential reduction method with the (local) LP-Newton method. Details for this hybrid method were given in [1]. All problems used are taken from a test library of generalized Nash equilibrium problems (GNEPs), see [2] and references therein.

In Section 2 we provide results of our computations that are based on the description of the implementation in [3, Section 5]. There, details on starting points and on the setting of parameters being used for the algorithms can be found.

In addition to these computations reported in Section 2, all runs of the algorithms were performed again with a single difference. Instead of giving the solver used for the linear programs (cplexlp from the optimization toolbox $CPLEX^{(\mathbb{R})}$) the freedom to choose the solution method automatically, we now explicitly forced this solver to use the dual simplex algorithm as solution method. The numerical results corresponding to this explicit setting are presented in Section 3.

A reason for modifying exactly this CPLEX[®] parameter was the observation that the LP solver (with automatic choice of the solution method) often used the dual simplex algorithm. Forcing the dual simplex algorithm to be applied by default may lead to lower computer times and may also save the time needed for choosing an appropriate solution algorithm by cplexlp. In particular, for the large problems, this new setting really helped to significantly decrease computer times for the solution of the subproblems of LP-Newton.

There are further possibilities to choose or to modify the solution algorithm for the LPs in order to further decrease computer times. In this report we do not discuss this question. The results presented in Section 3 just shall show that the modification of CPLEX[®] parameters might have an influence on the performance of the algorithms.

Now, we are going to describe which results are presented in the remainder. For any of the 35 problems in the GNEP test library, one table is presented in Section 2 and in Section 3. Within such a table,

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- Algorithm 1 denotes the method LP-N smooth,
- Algorithm 2 denotes the method LP-N nonsmooth,
- Algorithm 3 denotes the method LP-N nonmonotone smooth,
- Algorithm 4 denotes the method LP-N nonmonotone nonsmooth,
- Algorithm 5 denotes the method PRA + LP-N,

see [3, Section 5] for an explanation of the methods' names and details of Algorithms 1–4.

Above the table for any test problem, we first show the name of the problem and its dimensions: the number of players N, the total number of variables n_x , the total number of inequality constraints m_g , and the number n of variables (which is equal to the number of equations) in the constrained systems arising from reformulating the KKT system of a GNEP, see [3, formulas (5.5) and (5.7)]. Note that $n = n_x + 2m_g$ holds.

Within the tables, the following information is presented for each of the 5 algorithms. To obtain this information 20 runs (one for every starting point) were performed for each algorithm.

- number of runs terminated at solutions of the GNEP, where an z^k is regarded as solution if $||F(z^k)||_{\infty} \leq 10^{-8}$ with F defined according to (5.7) in [3],
- number of runs terminated at points z^k with $\varepsilon^* \leq 10^{-8}$, where ε^* denotes the optimal value of the linear program indicated by (5.10) in [3],
- average number of iterations per run (only for runs terminated at solutions),
- average number of evaluations of F per run (only for runs terminated at solutions),
- average number of evaluations of F per run (only for runs terminated at points with $\varepsilon^* \leq 10^{-8}$),
- average number of evaluations of G per run (only for runs terminated at solutions),
- average number of evaluations of G per run (only for runs terminated at points with $\varepsilon^* \leq 10^{-8}$),
- average CPU time per run (only for runs terminated at solutions),
- average CPU time per run (only for runs terminated at points with $\varepsilon^* \leq 10^{-8}$),
- average CPU time needed for the solution of all subproblems (linear programs or linear systems) per run (only for runs terminated at solutions),
- average CPU time needed for line searches per run (only for runs terminated at solutions),
- the percentage of runs for which the very last / the last two / the last three step sizes were equal to 1 (in case of Algorithm 1 Algorithm 4) or for which the last /the last two / the last three steps were LP-Newton steps (in case of Algorithm 5), in each case only for runs terminated at solutions,
- the percentage of runs for which the ratio of the maximum norm of F at the last but one iterate and the maximum norm of F at the last iterate was greater than or equal to $10^2 / 10^3 / 10^4$ (i.e., the maximum norm of F was improved in the last step by a factor which is greater than or equal to $10^2 / 10^3 / 10^4$), in each case only for runs terminated at solutions.

The information described in the latter two items is supposed to give some understanding of the local behavior of the algorithms.

2 Results for the Setting Described in [3, Section 5]

In this section, detailed results of the computations described in [3] are presented. Some aspects of these results are summarized in [3, Section 5], e.g., by performance profiles. Concerning the computer time, it can be observed that, for large problems, the variants of the globally convergent LP-Newton method from [3] are much more expensive than the hybrid method described in [1]. However, we refer to Section 3 for a different picture.

The last issues reported in the tables underline the assertions of the very last paragraph of [3, Section 5]: for an absolute majority of runs terminated at a solution, the very last step size was equal to one and a superlinear convergence rate could be observed.

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| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|--|--------------------|-------------|---------------|---------|---------|--|--|
| # runs terminated at a solution | 19 | 20 | 20 | 20 | 20 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 20 | 20 | 20 | 20 | | |
| \varnothing number of iterations per run (for | 11.96 | 0.10 | 11 50 | 0.05 | 1475 | | |
| runs terminated at a solution only) | 11.20 | 9.10 | 11.50 | 9.90 | 14.75 | | |
| | Evaluation | is of F | | | | | |
| \varnothing number of evaluations per run (for | 12.37 | 10.60 | 12 50 | 11.00 | 28 75 | | |
| runs terminated at a solution only) | 12.57 | 10.00 | 12.50 | 11.00 | 20.15 | | |
| \varnothing number of evaluations per run (for | 19.37 | 10.60 | 12 50 | 11.00 | 28 75 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.57 | 10.00 | 12.50 | 11.00 | 20.15 | | |
| | Evaluation | ns of G | | | | | |
| \varnothing number of evaluations per run (for | 11.26 | 9.10 | 11 50 | 9.95 | 14 75 | | |
| runs terminated at a solution only) | 11.20 | 5.10 | 11.00 | 9.90 | 14.75 | | |
| \varnothing number of evaluations per run (for | 11.26 | 9.10 | 11 50 | 9.95 | 14 75 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.20 | 5.10 | 11.00 | 9.90 | 14.75 | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.195 | 0.165 | 0.20 s | 0.17s | 0.10 s | | |
| nated at a solution only) | 0.155 | 0.105 | 0.2015 | 0.115 | 0.105 | | |
| \varnothing CPU time per run (for all runs ter- | 0.195 | 0.165 | 0.20 s | 0.17s | 0.10 s | | |
| minated with $\varepsilon^* \le 10^{-8}$) | 0.155 | 0.105 | 0.203 | 0.175 | 0.105 | | |
| \varnothing CPU time for subproblems per run | 0.01 s | 0.01 s | 0.01 s | 0.01 s | 0.03 s | | |
| (for runs term. at a solution only) | 0.015 | 0.015 | 0.015 | 0.015 | 0.005 | | |
| \varnothing CPU time for line search per run (for | 0.00 s | 0.01 s | 0.01 s | 0.01 s | 0.03 s | | |
| runs terminated at a solution only) | 0.0015 | 0.015 | 0.015 | 0.015 | 0.0015 | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | 100.070 | 100.070 | 100.070 | 100.070 | 100.070 | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 2 steps are LPN (hybrid) | 100.070 | 100.070 | 100.070 | 100.070 | 100.070 | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| last 3 steps are LPN (hybrid) | 100.070 | 100.070 | 100.070 | 100.070 | 0.070 | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq \overline{10^4}$ | 78.9% | 75.0% | 85.0% | 90.0% | 20.0% | | |

Dimensions: $N = 10, n_x = 10, m_g = 24, n = 58$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|--------------------|--------------|---------------|--------------|------------------|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 3 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 3 | | |
| \varnothing number of iterations per run (for | 10.80 | 0.15 | 11.05 | 0.15 | 00.67 | | |
| runs terminated at a solution only) | 10.80 | 9.10 | 11.05 | 9.10 | 99.07 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | 12.25 | 10.20 | 12.05 | 10.15 | 360 33 | | |
| runs terminated at a solution only) | 12.20 | 10.20 | 12.00 | 10.15 | 500.55 | | |
| \varnothing number of evaluations per run (for | 12.25 | 10.20 | 12.05 | 10.15 | 360 33 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.20 | 10.20 | 12.00 | 10.10 | 500.55 | | |
| | Evaluation | ns of G | | | | | |
| \varnothing number of evaluations per run (for | 10.80 | 9 15 | 11.05 | 9.15 | 98.67 | | |
| runs terminated at a solution only) | 10.00 | 5.10 | 11.00 | 5.10 | 50.01 | | |
| \varnothing number of evaluations per run (for | 10.80 | 9 15 | 11.05 | 9 15 | 98.67 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.00 | 5.10 | 11.00 | 5.10 | 50.01 | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.18 s | 0.16 s | 0.19s | 0.16 s | $0.63\mathrm{s}$ | | |
| nated at a solution only) | 0.100 | 0.105 | 0.10.5 | 0.10.5 | | | |
| \varnothing CPU time per run (for all runs ter- | 0.18 s | 0.16 s | 0.19 s | 0.16 s | $0.63\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.100 | 0.205 | 0.10.5 | 0.10.5 | 0.000 | | |
| \varnothing CPU time for subproblems per run | 0.01 s | 0.01 s | 0.01 s | 0.01 s | $0.09{ m s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | 0.01 s | 0.01 s | $0.00{ m s}$ | $0.24{ m s}$ | | |
| runs terminated at a solution only) | | 0.015 | 0.015 | | 0.215 | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 3 steps are LPN (hybrid) | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 95.0% | 100.0% | 95.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 95.0% | 95.0% | 95.0% | 95.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 60.0% | 70.0% | 50.0% | 70.0% | 33.3% | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|--------------------|--------------|------------------|------------------|--------------|--|--|
| # runs terminated at a solution | 16 | 8 | 15 | 8 | 19 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 19 | | |
| \varnothing number of iterations per run (for | 14 56 | 11 50 | 12.60 | 11 50 | 27.26 | | |
| runs terminated at a solution only) | 14.50 | 11.50 | 15.00 | 11.50 | 57.20 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | 17.69 | 12 50 | 14.67 | 12 50 | 188 58 | | |
| runs terminated at a solution only) | 17.05 | 12.00 | 14.07 | 12.00 | 100.00 | | |
| \varnothing number of evaluations per run (for | 17 75 | 8 85 | 14 45 | 8 85 | 188 58 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.10 | 0.00 | 14.40 | 0.00 | 100.00 | | |
| | Evaluation | ns of G | | | | | |
| \varnothing number of evaluations per run (for | 14 56 | 11 50 | 13 60 | 11 50 | 37 26 | | |
| runs terminated at a solution only) | 11.00 | 11.00 | 10.00 | 11.50 | 01.20 | | |
| \varnothing number of evaluations per run (for | 14 55 | 8 45 | 13 65 | 8 45 | 37.26 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.00 | 0.10 | 10.00 | 0.10 | 01.20 | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.24{ m s}$ | 0.19s | $0.22\mathrm{s}$ | $0.20\mathrm{s}$ | $0.20{ m s}$ | | |
| nated at a solution only) | 0.210 | 0.20 5 | 0.220 | 0.200 | 0.200 | | |
| \varnothing CPU time per run (for all runs ter- | 0.24 s | 0.14 s | $0.22{ m s}$ | 0.14 s | $0.20{ m s}$ | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.210 | 0.115 | 0.22.5 | 0.115 | 0.200 | | |
| \varnothing CPU time for subproblems per run | 0.01 s | 0.01 s | 0.01 s | 0.01 s | $0.03{ m s}$ | | |
| (for runs term. at a solution only) | | | | 0.02.0 | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.07{ m s}$ | | |
| runs terminated at a solution only) | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 31.6% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| last 3 steps are LPN (hybrid) | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 81.2% | 100.0% | 66.7% | 100.0% | 68.4% | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 18 | 10 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 10 | | | |
| \varnothing number of iterations per run (for | 17 15 | 16.20 | 19.75 | 19 79 | 120 50 | | | |
| runs terminated at a solution only) | 17.15 | 10.20 | 13.75 | 13.70 | 159.50 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 26.30 | 30.85 | 16.30 | 15 33 | 512 30 | | | |
| runs terminated at a solution only) | 20.30 | 30.85 | 10.50 | 10.00 | 512.50 | | | |
| \varnothing number of evaluations per run (for | 26 30 | 30.85 | 16 30 | 1/ 90 | 512 30 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 20.50 | 50.05 | 10.50 | 14.50 | 012.00 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 17 15 | 16 20 | 13 75 | 13 78 | 139 50 | | | |
| runs terminated at a solution only) | 11.10 | 10.20 | 10.10 | 10.10 | 100.00 | | | |
| \varnothing number of evaluations per run (for | 17 15 | 16 20 | 13 75 | 13 45 | 139 50 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.10 | 10.20 | 10.10 | 10.10 | 100.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.28{ m s}$ | 0.26 s | $0.22{ m s}$ | $0.22\mathrm{s}$ | $0.58{ m s}$ | | | |
| nated at a solution only) | | 0.205 | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.28\mathrm{s}$ | 0.26 s | $0.22\mathrm{s}$ | $0.22\mathrm{s}$ | $0.58\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.22\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | 1 | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 95.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 100 007 | 100 000 | 100.007 | 100 00 | 100 00 | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 55.0% | 85.0% | 60.0% | 83.3% | 40.0% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|--------------|---------------|--------------|---------------|--|--|--|
| # runs terminated at a solution | 20 | 19 | 20 | 19 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 19.55 | 0.74 | 19.45 | 0.74 | 14.05 | | | |
| runs terminated at a solution only) | 12.55 | 9.74 | 12.40 | 9.74 | 14.05 | | | |
| | Evaluation | is of F | | | | | | |
| \varnothing number of evaluations per run (for | 14.95 | 10.74 | 13.45 | 10.74 | 41.00 | | | |
| runs terminated at a solution only) | 14.20 | 10.74 | 10.40 | 10.74 | 41.00 | | | |
| \varnothing number of evaluations per run (for | 14 25 | 10 50 | 13.45 | 10 50 | 41.00 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 14.20 | 10.00 | 10.40 | 10.00 | 41.00 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 12.55 | 9 74 | 12 45 | 9 74 | 14.05 | | | |
| runs terminated at a solution only) | 12.00 | 5.14 | 12.10 | 5.14 | 14.00 | | | |
| \varnothing number of evaluations per run (for | 12.55 | 9.55 | 12 45 | 9.55 | 14.05 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.00 | 5.00 | 12.10 | 5.00 | 11.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.21\mathrm{s}$ | 0.16 s | $0.21{ m s}$ | 0.16 s | 0.11s | | | |
| nated at a solution only) | 0.210 | 0.205 | 0.210 | 0.10.5 | 0.115 | | | |
| \varnothing CPU time per run (for all runs ter- | 0.21 s | 0.16 s | 0.21 s | 0.16 s | 0.11 s | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | 0.01 s | 0.01 s | 0.01 s | 0.01 s | $0.05{\rm s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | 0.00 s | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | 0.01 s | | | |
| runs terminated at a solution only) | | | | | 0.02.0 | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | Γ | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 85.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 85.0% | 100.0% | 90.0% | 100.0% | 90.0% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 19 | 19 | 20 | 20 | 18 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 19 | 20 | 20 | 18 | | |
| \varnothing number of iterations per run (for | 48.05 | 19.63 | 17.45 | 16.75 | 74.22 | | |
| runs terminated at a solution only) | | | | | | | |
| Evaluations of F | | | | | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 221.68 | 43.37 | 18.60 | 17.75 | 337.67 | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 221.68 | 43.37 | 18.60 | 17.75 | 337.67 | | |
| | Evaluation | ns of G | I | 1 | I | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 48.05 | 19.63 | 17.45 | 16.75 | 74.22 | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 48.05 | 19.63 | 17.45 | 16.75 | 74.22 | | |
| CPU time | | | | | | | |
| \emptyset CPU time per run (for runs termi- nated at a solution only) | $0.84\mathrm{s}$ | $0.33\mathrm{s}$ | $0.28\mathrm{s}$ | $0.29\mathrm{s}$ | $0.38\mathrm{s}$ | | |
| \varnothing CPU time per run (for all runs ter- minated with $\varepsilon^* \leq 10^{-8}$) | 0.84 s | $0.33\mathrm{s}$ | $0.28\mathrm{s}$ | $0.29\mathrm{s}$ | $0.38\mathrm{s}$ | | |
| \varnothing CPU time for subproblems per run (for runs term. at a solution only) | $0.04\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.05\mathrm{s}$ | | |
| \varnothing CPU time for line search per run (for runs terminated at a solution only) | $0.09\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.17\mathrm{s}$ | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ution) | | | |
| last step size = 1 (glob. LPN) or last step is LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 2 step sizes $= 1$ (glob. LPN) or last 2 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 3 step sizes $= 1$ (glob. LPN) or last 3 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 63.2% | 100.0% | 85.0% | 95.0% | 94.4% | | |

Dimensions: $N = 4, n_x = 20, m_g = 44, n = 108$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 13 | 20 | 13 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 17 70 | 1/1 21 | 13.45 | 1/1 31 | 31.60 | | | |
| runs terminated at a solution only) | 11.10 | 14.01 | 10.40 | 14.01 | 51.00 | | | |
| | | | | | | | | |
| \varnothing number of evaluations per run (for | 29.45 | 15 31 | 14 45 | 15 31 | 138 75 | | | |
| runs terminated at a solution only) | 20.10 | 10.01 | 11.10 | 10.01 | 100.10 | | | |
| \varnothing number of evaluations per run (for | 29 45 | 13 60 | 14 45 | 13 60 | 138 75 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 20.10 | 10.00 | 11.10 | 10.00 | 100.10 | | | |
| | Evaluation | ns of G | 1 | 1 | 1 | | | |
| \varnothing number of evaluations per run (for | 17.70 | 14.31 | 13.45 | 14.31 | 31.60 | | | |
| runs terminated at a solution only) | 11.10 | 11.01 | 10.10 | 11.01 | 01.00 | | | |
| \varnothing number of evaluations per run (for | 17.70 | 12.95 | 13.45 | 12.95 | 31.60 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | | | 10.10 | | 01.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.33\mathrm{s}$ | $0.26\mathrm{s}$ | $0.26\mathrm{s}$ | $0.27\mathrm{s}$ | $0.23\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.33\mathrm{s}$ | $0.24\mathrm{s}$ | $0.26\mathrm{s}$ | $0.24\mathrm{s}$ | $0.23\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.06\mathrm{s}$ | $0.04\mathrm{s}$ | $0.05\mathrm{s}$ | $0.04\mathrm{s}$ | $0.06\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.08\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | [| | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 25.0% | | | |
| Last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{3}$ | 75.0% | 30.8% | 75.0% | 30.8% | 95.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 15.0% | 15.4% | 40.0% | 15.4% | 30.0% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|--------------|---------------|--------------|---------------|--|--|--|
| # runs terminated at a solution | 20 | 19 | 20 | 19 | 10 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 10 | | | |
| \varnothing number of iterations per run (for | 0.25 | 6.05 | 0.25 | 6.05 | 253 60 | | | |
| runs terminated at a solution only) | 9.20 | 0.95 | 9.20 | 0.95 | 200.00 | | | |
| | Evaluation | is of F | | | | | | |
| \varnothing number of evaluations per run (for | 10.25 | 7.05 | 10.25 | 7.05 | 1385-20 | | | |
| runs terminated at a solution only) | 10.25 | 1.95 | 10.25 | 1.90 | 1365.20 | | | |
| \varnothing number of evaluations per run (for | 10.25 | 7 75 | 10.25 | 7 75 | 1385-20 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.20 | 1.10 | 10.25 | 1.15 | 1000.20 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 9.25 | 6.95 | 9.25 | 6 95 | 253.60 | | | |
| runs terminated at a solution only) | 5.20 | 0.55 | 5.20 | 0.50 | 200.00 | | | |
| \varnothing number of evaluations per run (for | 9.25 | 6.80 | 9.25 | 6 80 | 253 60 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.20 | 0.00 | 5.20 | 0.00 | 200.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.15 s | 0.11 s | 0.16 s | 0.11s | $1.09{\rm s}$ | | | |
| nated at a solution only) | 0.10.5 | 0.115 | 0.105 | | 1.00.5 | | | |
| \varnothing CPU time per run (for all runs ter- | 0.15 s | 0.11 s | 0.16 s | 0.11s | $1.09{\rm s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.10.5 | 0.115 | 0.105 | | 1.00.0 | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.05{ m s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.48{ m s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 85.0% | 52.6% | 85.0% | 52.6% | 100.0% | | | |

A9a

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for runs terminated at a solution only) | 21.80 | 70.55 | 30.40 | 21.75 | 22.80 | | | |
| | Evaluation | ns of F | | | | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 39.35 | 323.85 | 42.35 | 31.65 | 125.10 | | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 39.35 | 323.85 | 42.35 | 31.65 | 125.10 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 21.80 | 70.55 | 30.40 | 21.75 | 22.80 | | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 21.80 | 70.55 | 30.40 | 21.75 | 22.80 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- nated at a solution only) | $0.75\mathrm{s}$ | $2.66\mathrm{s}$ | $0.98\mathrm{s}$ | $0.67\mathrm{s}$ | $0.48\mathrm{s}$ | | | |
| \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$) | $0.75\mathrm{s}$ | $2.66\mathrm{s}$ | $0.98\mathrm{s}$ | $0.67\mathrm{s}$ | $0.48\mathrm{s}$ | | | |
| \varnothing CPU time for subproblems per run (for runs term. at a solution only) | $0.28\mathrm{s}$ | $0.82\mathrm{s}$ | $0.37\mathrm{s}$ | $0.21\mathrm{s}$ | $0.11\mathrm{s}$ | | | |
| \varnothing CPU time for line search per run (for runs terminated at a solution only) | $0.06\mathrm{s}$ | $0.65\mathrm{s}$ | $0.05\mathrm{s}$ | $0.04\mathrm{s}$ | $0.27\mathrm{s}$ | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or laststep is LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 step sizes $= 1$ (glob. LPN) or last 2 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 step sizes $= 1$ (glob. LPN) or last 3 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 50.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 95.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 95.0% | 100.0% | 90.0% | 100.0% | 95.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 60.0% | 90.0% | 85.0% | 75.0% | 55.0% | | | |

A9b

Dimensions: $N = 7, n_x = 112, m_g = 119, n = 350$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 24.35 | 55.05 | 33.85 | 21.20 | 24 70 | | | |
| runs terminated at a solution only) | 24.55 | 00.00 | 55.65 | 21.20 | 24.70 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 44.25 | 208-30 | 44.90 | 30.95 | 136 55 | | | |
| runs terminated at a solution only) | 11.20 | 200.50 | 44.50 | 50.55 | 130.00 | | | |
| \varnothing number of evaluations per run (for | 44 25 | 208.30 | 44 90 | 30.95 | 136 55 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.20 | 200.00 | 11.50 | 00.00 | 100.00 | | | |
| | Evaluation | ns of G | 1 | 1 | 1 | | | |
| \varnothing number of evaluations per run (for | 24 35 | 55 05 | 33 85 | 21.20 | $24\ 70$ | | | |
| runs terminated at a solution only) | 21.00 | | | 21.20 | 21.10 | | | |
| \varnothing number of evaluations per run (for | 24.35 | 55.05 | 33.85 | 21.20 | 24.70 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | - 1.00 | | 00.00 | | | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $1.56\mathrm{s}$ | $3.98\mathrm{s}$ | $2.25\mathrm{s}$ | $1.26\mathrm{s}$ | $0.97\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $1.56\mathrm{s}$ | $3.98\mathrm{s}$ | $2.25\mathrm{s}$ | $1.26\mathrm{s}$ | $0.97\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.95\mathrm{s}$ | $2.21\mathrm{s}$ | $1.43\mathrm{s}$ | $0.73\mathrm{s}$ | $0.36\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.05\mathrm{s}$ | $0.44\mathrm{s}$ | $0.10\mathrm{s}$ | $0.06\mathrm{s}$ | $0.49\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{DN} | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or $I = I = I = I$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| $1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^{3}$ | 100.0% | 100.0% | 95.0% | | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 70.0% | 70.0% | 50.0% | 95.0% | 50.0% | | | |

A10a

Dimensions: $N = 8, n_x = 24, m_g = 33, n = 90$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 19 | 15 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 15 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for runs terminated at a solution only) | 133.37 | 219.47 | 26.30 | 24.50 | 48.40 | | | |
| | Evaluation | ns of F | | | | | | |
| Ø number of evaluations per run (for | | | | | | | | |
| runs terminated at a solution only) | 830.63 | 1255.33 | 30.10 | 26.75 | 182.75 | | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 830.63 | 1255.33 | 30.10 | 26.75 | 182.75 | | | |
| | Evaluation | ns of G | L | | | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 133.37 | 219.47 | 26.30 | 24.50 | 48.40 | | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* < 10^{-8}$) | 133.37 | 219.47 | 26.30 | 24.50 | 48.40 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- nated at a solution only) | $3.07\mathrm{s}$ | 4.83 s | $0.51\mathrm{s}$ | $0.45\mathrm{s}$ | $0.39\mathrm{s}$ | | | |
| \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$) | $3.07\mathrm{s}$ | 4.83 s | $0.51\mathrm{s}$ | $0.45\mathrm{s}$ | $0.39\mathrm{s}$ | | | |
| \varnothing CPU time for subproblems per run (for runs term. at a solution only) | $0.34\mathrm{s}$ | $0.49\mathrm{s}$ | $0.05\mathrm{s}$ | $0.05\mathrm{s}$ | $0.06\mathrm{s}$ | | | |
| Ø CPU time for line search per run (for runs terminated at a solution only) | $0.56\mathrm{s}$ | 1.00 s | $0.01\mathrm{s}$ | $0.02\mathrm{s}$ | $0.19\mathrm{s}$ | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last step is LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 step sizes $= 1$ (glob. LPN) or last 2 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 step sizes $= 1$ (glob. LPN) or last 3 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 95.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 57.9% | 100.0% | 85.0% | 95.0% | 85.0% | | | |

A10b

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|-------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 17 | 7 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 18 | 7 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 161 20 | 117.86 | 28.15 | 28.40 | 41.00 | | | |
| runs terminated at a solution only) | 101.29 | 117.00 | 20.10 | 20.40 | 41.90 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 067 18 | 966 71 | 30.50 | 37 35 | 206.95 | | | |
| runs terminated at a solution only) | 501.10 | 500.11 | 55.50 | 01.00 | 200.55 | | | |
| \varnothing number of evaluations per run (for | 939 33 | 966 71 | 39 50 | 37 35 | 206 95 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 555.55 | 500.11 | 00.00 | 01.00 | 200.00 | | | |
| | Evaluation | ns of G | 1 | 1 | 1 | | | |
| \varnothing number of evaluations per run (for | 161 29 | 117 86 | 28 15 | 28 40 | 41 50 | | | |
| runs terminated at a solution only) | 101.20 | 111.00 | 20.10 | 20.10 | | | | |
| \varnothing number of evaluations per run (for | 157.50 | 117.86 | 28.15 | 28.40 | 41.50 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 101100 | 11 | -0.10 | | 11.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $11.59\mathrm{s}$ | $8.05\mathrm{s}$ | $1.81\mathrm{s}$ | $1.49\mathrm{s}$ | $1.51\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $11.29\mathrm{s}$ | $8.05\mathrm{s}$ | $1.81\mathrm{s}$ | $1.49\mathrm{s}$ | $1.51\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \bigotimes CPU time for subproblems per run | $6.53\mathrm{s}$ | $3.83\mathrm{s}$ | $1.17\mathrm{s}$ | $0.88\mathrm{s}$ | $0.71\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $1.08\mathrm{s}$ | $1.70\mathrm{s}$ | $0.05\mathrm{s}$ | $0.07\mathrm{s}$ | $0.51\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{PN} | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or last 2 steps are LDN (help id) | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| $\begin{array}{c c} \text{Iast 5 steps are LPN (nybrid)} \\ \hline \\ \text{Iast improvement of } \ E \ & \text{is } > 10^2 \\ \hline \end{array}$ | 100.007 | 100.007 | 05.007 | 100.007 | 100.007 | | | |
| $\begin{array}{c c} \text{iast improvement of } \ F\ _{\infty} \text{ is } \geq 10^2 \\ \hline \\ \text{last improvement of } \ F\ _{\infty} \text{ is } \geq 10^3 \\ \hline \end{array}$ | 100.0% | 100.0% | 90.0% | 100.0% | 100.0% | | | |
| $\begin{array}{c c} \text{1ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^{6} \\ \hline 1 \text{ is } is $ | 88.2% | 100.0% | 80.0% | 95.0% | 85.0% | | | |
| 1 ast improvement of $ F _{\infty}$ is $\geq 10^4$ | 64.7% | 42.9% | 70.0% | 45.0% | 55.0% | | | |

A10c

Dimensions: $N = 37, n_x = 222, m_g = 260, n = 742$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | | |
|---|-------------------|--------------|-------------------|------------------|--------|--|--|--|--|--|
| # runs terminated at a solution | 0 | 0 | 8 | 20 | 0 | | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 5 | 0 | 20 | 20 | 0 | | | | | |
| \varnothing number of iterations per run (for | | | 107.00 | 24 55 | | | | | | |
| runs terminated at a solution only) | _ | _ | 107.00 | 34.55 | _ | | | | | |
| Evaluations of F | | | | | | | | | | |
| \varnothing number of evaluations per run (for | | | 276 50 | 48 70 | | | | | | |
| runs terminated at a solution only) | _ | | 270.50 | 40.70 | | | | | | |
| \varnothing number of evaluations per run (for | 1758 40 | | 315 15 | 48 70 | _ | | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1756.40 | | 515.15 | 40.70 | | | | | | |
| | Evaluation | ns of G | | | | | | | | |
| \varnothing number of evaluations per run (for | | _ | 107.00 | 34 55 | _ | | | | | |
| runs terminated at a solution only) | | | 107.00 | 54.00 | | | | | | |
| \varnothing number of evaluations per run (for | $326\ 40$ | _ | 131 25 | 3455 | _ | | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 020.10 | | 101.20 | 04.00 | | | | | | |
| CPU time | | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | _ | _ | 17 83 s | 4 31 s | _ | | | | | |
| nated at a solution only) | | | 11.005 | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $71.54\mathrm{s}$ | _ | $27.01\mathrm{s}$ | 4 31 s | _ | | | | | |
| minated with $\varepsilon^* \le 10^{-8}$) | 11.010 | | 21.015 | 1.015 | | | | | | |
| \varnothing CPU time for subproblems per run | _ | _ | $14.20\mathrm{s}$ | $3.28\mathrm{s}$ | _ | | | | | |
| (for runs term. at a solution only) | | | 1 | 0.200 | | | | | | |
| \varnothing CPU time for line search per run (for | | _ | $0.39\mathrm{s}$ | $0.05{\rm s}$ | _ | | | | | |
| runs terminated at a solution only) | | | 0.000 | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | | |
| last step size $= 1$ (glob. LPN) or last | 0.0% | 0.0% | 100.0% | 100.0% | 0.0% | | | | | |
| step is LPN (hybrid) | / 0 | | | | / 0 | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 0.0% | 0.0% | 100.0% | 100.0% | 0.0% | | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 0.0% | 0.0% | 87.5% | 100.0% | 0.0% | | | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 0.0% | 0.0% | 0.0% | 100.0% | 0.0% | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 0.0% | 0.0% | 0.0% | 85.0% | 0.0% | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 0.0% | 0.0% | 0.0% | 30.0% | 0.0% | | | | | |

A10d

Dimensions: $N = 37, n_x = 370, m_g = 408, n = 1186$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|--------------------|--------------------|--------------------|-------------------|--|--|--|
| # runs terminated at a solution | 8 | 2 | 17 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 11 | 2 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 119.95 | 01.00 | 86 76 | 106.00 | 52.60 | | | |
| runs terminated at a solution only) | 112.20 | 91.00 | 80.70 | 100.90 | 52.00 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 487 12 | 3/19/50 | 264.00 | 377.65 | 222.80 | | | |
| runs terminated at a solution only) | 407.12 | 049.00 | 204.00 | 511.05 | 222.00 | | | |
| \varnothing number of evaluations per run (for | 664 91 | 349 50 | 298.30 | 377.65 | 222.80 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 001.01 | 015.00 | 230.00 | 011.00 | 222.00 | | | |
| | Evaluation | ns of G | 1 | 1 | 1 | | | |
| \varnothing number of evaluations per run (for | 112 25 | 91.00 | 86 76 | 106 90 | 51.80 | | | |
| runs terminated at a solution only) | 112.20 | 01.00 | | 100.00 | 01.00 | | | |
| \varnothing number of evaluations per run (for | 136.18 | 91.00 | 93.40 | 106.90 | 51.80 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 100.10 | 01.00 | 00110 | 100.00 | 01.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $352.25\mathrm{s}$ | $262.60\mathrm{s}$ | $320.42\mathrm{s}$ | $323.65\mathrm{s}$ | $48.68\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $435.77\mathrm{s}$ | $262.60\mathrm{s}$ | $344.12\mathrm{s}$ | $323.65\mathrm{s}$ | $48.68\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \bigotimes CPU time for subproblems per run | $135.63\mathrm{s}$ | $99.85\mathrm{s}$ | $142.06\mathrm{s}$ | $123.50\mathrm{s}$ | $45.26\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $2.77\mathrm{s}$ | $1.96\mathrm{s}$ | $1.63\mathrm{s}$ | $2.18\mathrm{s}$ | $1.27\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{PN} | 75.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or | 75.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| $\begin{bmatrix} \text{Iast 5 steps are LFN (nybrid)} \\ \text{Iast improvement of } \ F\ & \text{is } > 10^2 \end{bmatrix}$ | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% 60.507 | | | | 100.0% | | | |
| $1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^{6}$ | 02.5% | | (0.5%) | 90.0% | 95.0% | | | |
| 1 last improvement of $ F _{\infty}$ is $\geq 10^4$ | 25.0% | 0.0% | 41.2% | 10.0% | 60.0% | | | |

A10e

Dimensions: $N = 48, n_x = 576, m_g = 625, n = 1826$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|--------------|---------------------|--------------------|-------------------|--|--|--|--|
| # runs terminated at a solution | 2 | 0 | 11 | 16 | 11 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 3 | 0 | 16 | 16 | 11 | | | | |
| \varnothing number of iterations per run (for | 64 50 | | 182.00 | 02.44 | 57.01 | | | | |
| runs terminated at a solution only) | 04.50 | _ | 185.00 | 92.44 | 57.91 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 200 50 | | 720.36 | 285.06 | 258 55 | | | | |
| runs terminated at a solution only) | 209.00 | | 120.30 | 200.00 | 200.00 | | | | |
| \varnothing number of evaluations per run (for | 272.00 | _ | 802 56 | 285.06 | 258 55 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 212.00 | | 002.00 | 200.00 | 200.00 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 64 50 | | 183.00 | 92.44 | 57 27 | | | | |
| runs terminated at a solution only) | 01.00 | | 100.00 | 52.11 | 01.21 | | | | |
| \varnothing number of evaluations per run (for | 79.33 | _ | 196 94 | 92.44 | 57 27 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.00 | | 100.01 | 02.11 | 01.21 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $287.97\mathrm{s}$ | | $1283.69\mathrm{s}$ | $550.38\mathrm{s}$ | $69.42\mathrm{s}$ | | | | |
| nated at a solution only) | | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $372.59\mathrm{s}$ | | $1421.85\mathrm{s}$ | $550.38\mathrm{s}$ | $69.42\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.2.000 | | 1121.000 | | 001120 | | | | |
| \varnothing CPU time for subproblems per run | $105.78\mathrm{s}$ | _ | $537.39\mathrm{s}$ | 217.04 s | $63.02\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | 100.100 | | | | 001020 | | | | |
| \varnothing CPU time for line search per run (for | $1.59{ m s}$ | _ | $5.85\mathrm{s}$ | $2.07{ m s}$ | $2.00{ m s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 0.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 0.0% | 100.0% | 100.0% | 100.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 0.0% | 100.0% | 100.0% | 100.0% | | | | |
| last 3 steps are LPN (hybrid) | 100.070 | 0.070 | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 0.0% | 90.9% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 50.0% | 0.0% | 54.5% | 93.8% | 90.9% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 0.0% | 0.0% | 9.1% | 18.8% | 9.1% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|---------------|--------------|---------------|--------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 8 15 | 5.05 | 8 15 | 5.05 | 10.35 | | | | |
| runs terminated at a solution only) | 0.10 | 0.90 | 0.15 | 0.90 | 10.55 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 0.15 | 6.95 | 9.15 | 6.95 | 11 35 | | | | |
| runs terminated at a solution only) | 9.10 | 0.35 | 9.10 | 0.90 | 11.55 | | | | |
| \varnothing number of evaluations per run (for | 9.15 | 6.95 | 9.15 | 6 95 | 11 35 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 5.10 | 0.55 | 5.10 | 0.50 | 11.55 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 8 15 | 5 95 | 8 15 | 5 95 | 10.35 | | | | |
| runs terminated at a solution only) | 0.10 | 0.50 | 0.10 | 0.00 | 10.00 | | | | |
| \varnothing number of evaluations per run (for | 8 15 | 5 95 | 8 15 | 5 95 | 10.35 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.10 | 0.00 | 0.10 | 0.00 | 10.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.13 s | 0.10 s | 0.13 s | 0.10 s | $0.06\mathrm{s}$ | | | | |
| nated at a solution only) | 0.100 | 0.105 | 0.10.5 | 0.105 | 0.000 | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.13 s | 0.10 s | 0.13 s | 0.10 s | $0.06{ m s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.100 | 0.105 | 0.10.5 | 0.105 | | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03{ m s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | 0.01 s | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 1001070 | 1000070 | 100.070 | 1001070 | 0.070 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 75.0% | 95.0% | 75.0% | 95.0% | 95.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|------------------|---------------|------------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 7.45 | 5.65 | 7.45 | 5.65 | 10.60 | | | | |
| runs terminated at a solution only) | 1.40 | 0.00 | 1.40 | 0.00 | 10.00 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 8.45 | 6 65 | 8.45 | 6 65 | 15.00 | | | | |
| runs terminated at a solution only) | 0.40 | 0.05 | 0.40 | 0.05 | 15.00 | | | | |
| \varnothing number of evaluations per run (for | 8 45 | 6 65 | 8 45 | 6 65 | 15.00 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.40 | 0.00 | 0.40 | 0.00 | 10.00 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 7 45 | 5 65 | 7 45 | 5 65 | 10.60 | | | | |
| runs terminated at a solution only) | 1.10 | 0.00 | 1.10 | 0.00 | 10.00 | | | | |
| \varnothing number of evaluations per run (for | 7 45 | 5 65 | 7 45 | 5.65 | 10.60 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.10 | 0.00 | 1.10 | 0.00 | 10.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | $0.12{\rm s}$ | $0.09\mathrm{s}$ | $0.06\mathrm{s}$ | | | | |
| nated at a solution only) | 0.12.0 | 0.000 | 0.120 | 0.000 | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.12 s | $0.09{ m s}$ | $0.12{ m s}$ | $0.09{ m s}$ | $0.06{ m s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.12.5 | 0.000 | 0.12.5 | 0.000 | | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00\mathrm{s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 45.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 100.007 | 100.00 | 100.00 | 100.00 | 07.007 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 80.0% | 70.0% | 80.0% | 70.0% | 95.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|---------------|--------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 7.95 | 8.00 | 7.85 | 8.00 | 10.00 | | | |
| runs terminated at a solution only) | 1.65 | 8.00 | 1.65 | 8.00 | 10.90 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 8 85 | 0.00 | 8 85 | 0.00 | 14 50 | | | |
| runs terminated at a solution only) | 0.00 | 9.00 | 0.00 | 9.00 | 14.00 | | | |
| \varnothing number of evaluations per run (for | 8 85 | 9.00 | 8 85 | 9.00 | 14 50 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 3.00 | 0.00 | 9.00 | 14.00 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 7 85 | 8.00 | 7 85 | 8.00 | 10.90 | | | |
| runs terminated at a solution only) | 1.00 | 0.00 | 1.00 | 0.00 | 10.50 | | | |
| \varnothing number of evaluations per run (for | 7 85 | 8.00 | 7 85 | 8 00 | 10.90 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 0.00 | 1.00 | 0.00 | 10.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.13 s | 0.13 s | $0.13{ m s}$ | $0.13{ m s}$ | $0.06\mathrm{s}$ | | | |
| nated at a solution only) | 0.10.5 | 0.10.5 | 0.10.5 | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.13 s | 0.13 s | $0.13\mathrm{s}$ | $0.13\mathrm{s}$ | $0.06\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.10.5 | 0.10.5 | 0.10.5 | | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03{ m s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.01{ m s}$ | $0.00{ m s}$ | $0.01{ m s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 90.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 100.004 | 100.004 | 100.007 | 100.007 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 95.0% | 100.0% | 95.0% | 100.0% | 95.0% | | | |

A14

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|--|
| # runs terminated at a solution | 18 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 18 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 11.00 | 0.10 | 11.95 | 0.20 | 12.25 | | | | |
| runs terminated at a solution only) | 11.00 | 9.10 | 11.65 | 9.20 | 15.55 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 19 33 | 10.25 | 12.95 | 10.20 | 10.00 | | | | |
| runs terminated at a solution only) | 12.00 | 10.20 | 12.30 | 10.20 | 13.00 | | | | |
| \varnothing number of evaluations per run (for | 12.33 | 10.25 | 12.95 | 10.20 | 19.00 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.00 | 10.20 | 12.50 | 10.20 | 15.00 | | | | |
| | Evaluation | ns of G | r | r | r | | | | |
| \varnothing number of evaluations per run (for | 11.00 | 9 10 | 11.85 | 9 20 | 13 35 | | | | |
| runs terminated at a solution only) | 11.00 | 0.10 | 11.00 | 0.20 | 10.00 | | | | |
| \varnothing number of evaluations per run (for | 11.00 | 9.10 | 11.85 | 9.20 | 13.35 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.00 | 0.10 | | 0.20 | 10:00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.19\mathrm{s}$ | 0.16 s | $0.20\mathrm{s}$ | $0.16\mathrm{s}$ | $0.09\mathrm{s}$ | | | | |
| nated at a solution only) | | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.19\mathrm{s}$ | 0.16 s | $0.20\mathrm{s}$ | $0.16\mathrm{s}$ | $0.09\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.04\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | 1 | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 77.8% | 90.0% | 85.0% | 95.0% | 20.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|---------------|--------------|---------------|--------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 8.00 | 7.85 | 8.00 | 7.85 | 13.85 | | | | |
| runs terminated at a solution only) | 0.90 | 1.00 | 8.90 | 1.00 | 15.65 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 0.05 | 8 85 | 9 90 | 8 85 | 40.15 | | | | |
| runs terminated at a solution only) | 9.90 | 0.00 | 9.90 | 0.00 | 40.15 | | | | |
| \varnothing number of evaluations per run (for | 9.95 | 8 85 | 9 90 | 8 85 | 40.15 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 5.50 | 0.00 | 5.50 | 0.00 | 40.10 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 8 90 | 7 85 | 8 90 | 7 85 | 13.85 | | | | |
| runs terminated at a solution only) | 0.50 | 1.00 | 0.50 | 1.00 | 10.00 | | | | |
| \varnothing number of evaluations per run (for | 8 90 | 7 85 | 8 90 | 7 85 | 13.85 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 1.00 | 0.00 | 1.00 | 10.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.14 s | 0.13 s | $0.15{\rm s}$ | 0.13 s | $0.07{ m s}$ | | | | |
| nated at a solution only) | 0.110 | 0.10.5 | 0.10.5 | 0.10.5 | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.14 s | 0.13 s | $0.15{ m s}$ | 0.13 s | $0.07\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.110 | 0.10.5 | 0.200 | 0.10.5 | | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | 0.01 s | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 1001070 | 100.070 | 100.070 | 100.070 | 0.070 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 20.0% | 95.0% | 20.0% | 95.0% | 95.0% | | | | |

A16a

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|--------------|---------------|--------------|---------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 6.85 | 6.20 | 6.85 | 6.20 | 15.20 | | | | |
| runs terminated at a solution only) | 0.85 | 0.20 | 0.85 | 0.20 | 10.00 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 7.85 | 7.20 | 7.85 | 7.20 | 41.00 | | | | |
| runs terminated at a solution only) | 1.00 | 1.20 | 1.00 | 1.20 | 41.00 | | | | |
| \varnothing number of evaluations per run (for | 7 85 | 7 20 | 7.85 | 7 20 | 41.00 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 1.20 | 1.00 | 1.20 | 41.00 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 6.85 | 6.20 | 6.85 | 6.20 | 15.30 | | | | |
| runs terminated at a solution only) | 0.00 | 0.20 | 0.00 | 0.20 | 10.00 | | | | |
| \varnothing number of evaluations per run (for | 6.85 | 6 20 | 6.85 | 6 20 | 15 30 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 0.20 | 0.00 | 0.20 | 10.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.11 s | 0.10 s | 0.11 s | 0.11s | $0.09{\rm s}$ | | | | |
| nated at a solution only) | 0.110 | 0.105 | 0.115 | 0.115 | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.11 s | 0.10 s | 0.11 s | 0.11s | $0.09{\rm s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.110 | 0.105 | 0.115 | 0.115 | 0.000 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03{ m s}$ | | | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.005 | 0.000 | 0.000 | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| runs terminated at a solution only) | | 0.000 | | | 0.02.5 | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 55.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 100 004 | 100 007 | 100 007 | 100 004 | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 90.0% | | | | |

A16b

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|------------------|------------------|--------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 7.00 | 6.05 | 7.00 | 6.05 | 17.95 | | | | |
| runs terminated at a solution only) | 7.00 | 0.95 | 1.00 | 0.95 | 17.20 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 8.00 | 7.95 | 8.00 | 7.95 | 47 75 | | | | |
| runs terminated at a solution only) | 0.00 | 1.30 | 0.00 | 1.30 | 41.10 | | | | |
| \varnothing number of evaluations per run (for | 8.00 | 7 95 | 8.00 | 7 95 | 47.75 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 1.50 | 0.00 | 1.50 | 1.10 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 7.00 | 6 95 | 7.00 | 6 95 | 17.25 | | | | |
| runs terminated at a solution only) | 1.00 | 0.50 | 1.00 | 0.50 | 11.20 | | | | |
| \varnothing number of evaluations per run (for | 7.00 | 6 95 | 7.00 | 6 95 | $17\ 25$ | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 0.50 | 1.00 | 0.50 | 11.20 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | 0.12 s | $0.10{\rm s}$ | | | | |
| nated at a solution only) | 0.120 | 0.120 | 0.120 | 0.120 | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | 0.12 s | $0.10{\rm s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.120 | 0.125 | 0.125 | 0.125 | | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.000 | 0.000 | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | 0.01 s | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| runs terminated at a solution only) | | 0.015 | | | 0.02.5 | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 65.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 100 004 | 100 007 | 100 007 | 100 004 | 00.00 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 90.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 90.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 85.0% | | | | |

A16c

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|------------------|------------------|--------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 7.00 | 6.05 | 7.00 | 6.05 | 18 70 | | | | |
| runs terminated at a solution only) | 7.00 | 0.95 | 7.00 | 0.95 | 18.70 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 8.00 | 7.95 | 8.00 | 7.95 | 51.40 | | | | |
| runs terminated at a solution only) | 0.00 | 1.30 | 0.00 | 1.30 | 01.40 | | | | |
| \varnothing number of evaluations per run (for | 8.00 | 7 95 | 8.00 | 7 95 | 51 40 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 1.50 | 0.00 | 1.50 | 01.40 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 7.00 | 6 95 | 7.00 | 6 95 | 18 70 | | | | |
| runs terminated at a solution only) | 1.00 | 0.50 | 1.00 | 0.50 | 10.10 | | | | |
| \varnothing number of evaluations per run (for | 7.00 | 6 95 | 7.00 | 6 95 | 18 70 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 0.50 | 1.00 | 0.50 | 10.10 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | 0.12 s | 0.10 s | | | | |
| nated at a solution only) | 0.120 | 0.120 | 0.120 | 0.125 | 0.10.5 | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | $0.12\mathrm{s}$ | 0.12 s | 0.10 s | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.120 | 0.125 | 0.125 | 0.125 | 0.10.5 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.005 | 0.000 | 0.02.0 | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | 0.01 s | $0.00{ m s}$ | $0.03\mathrm{s}$ | | | | |
| runs terminated at a solution only) | | 0.000 | 0.015 | | 0.000 | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 25.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 100 004 | 100 007 | 100 007 | 100 004 | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 75.0% | | | | |

A16d

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|--------------|------------------|--------------|--------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 7.40 | 7.85 | 7.40 | 7.85 | 15.00 | | | | |
| runs terminated at a solution only) | 1.40 | 1.00 | 7.40 | 1.00 | 10.90 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 8.40 | 8 85 | 8.40 | 8 85 | 45 30 | | | | |
| runs terminated at a solution only) | 0.40 | 0.00 | 0.40 | 0.00 | 40.00 | | | | |
| \varnothing number of evaluations per run (for | 8 40 | 8 85 | 8 40 | 8 85 | 45 30 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.40 | 0.00 | 0.40 | 0.00 | 40.00 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 7 40 | 7 85 | 7 40 | 7 85 | 15 90 | | | | |
| runs terminated at a solution only) | 1.10 | 1.00 | 1.10 | 1.00 | 10.50 | | | | |
| \varnothing number of evaluations per run (for | 7 40 | 7 85 | 7 40 | 7 85 | 15 90 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.10 | 1.00 | 1.10 | 1.00 | 10.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.13 s | 0.13 s | $0.12{\rm s}$ | 0.13 s | 0.10 s | | | | |
| nated at a solution only) | 0.100 | 0.105 | 0.125 | 0.10.5 | 0.10.5 | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.13 s | 0.13 s | $0.12\mathrm{s}$ | 0.13 s | 0.10 s | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.100 | 0.105 | 0.125 | 0.10.5 | 0.10.5 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03{ m s}$ | | | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.005 | 0.005 | 0.000 | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| runs terminated at a solution only) | | 0.000 | 0.000 | | 0.02.5 | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 75.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 100 004 | 100 007 | 100 007 | 100 004 | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 90.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 19 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 19 | | |
| \varnothing number of iterations per run (for | 0.65 | 6 60 | 0.65 | 6 60 | 15 52 | | |
| runs terminated at a solution only) | 9.05 | 0.00 | 9.05 | 0.00 | 10.00 | | |
| | Evaluation | is of F | | | | | |
| \varnothing number of evaluations per run (for | 10.65 | 7.60 | 10.65 | 7.60 | 24.63 | | |
| runs terminated at a solution only) | 10.05 | 7.00 | 10.05 | 7.00 | 24.03 | | |
| \varnothing number of evaluations per run (for | 10.65 | 7.60 | 10.65 | 7.60 | 24.63 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.05 | 1.00 | 10.05 | 7.00 | 24.05 | | |
| | | | | | | | |
| \varnothing number of evaluations per run (for | 9.65 | 6 60 | 9.65 | 6 60 | 1553 | | |
| runs terminated at a solution only) | 5.00 | 0.00 | 5.00 | 0.00 | 10.00 | | |
| \varnothing number of evaluations per run (for | 9.65 | 6 60 | 9.65 | 6 60 | 1553 | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) and the second | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.16 s | 0.11 s | $0.15{ m s}$ | 0.11 s | 0.11 s | | |
| nated at a solution only) | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.16\mathrm{s}$ | $0.11\mathrm{s}$ | $0.15\mathrm{s}$ | $0.11\mathrm{s}$ | $0.11\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.07\mathrm{s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or L_{LPN} | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{5}$ | 100.0% | 100.0% | 100.0% | 100.0% | 08.4% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 80.0% | 95.0% | 80.0% | 95.0% | 5.3% | | |

Dimensions: $N = 2, n_x = 12, m_g = 28, n = 68$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 20 | 9 | 20 | 9 | 19 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | |
| \varnothing number of iterations per run (for | 12.65 | 0.67 | 19.15 | 0.67 | 17.05 | | |
| runs terminated at a solution only) | 12.05 | 9.07 | 12.10 | 9.07 | 17.95 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | 16.40 | 10.67 | 13 15 | 10.67 | 97.05 | | |
| runs terminated at a solution only) | 10.40 | 10.07 | 10.10 | 10.07 | 91.00 | | |
| \varnothing number of evaluations per run (for | 16 40 | 8 30 | 13 15 | 8 30 | 96.85 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.40 | 0.00 | 10.10 | 0.00 | 50.00 | | |
| Evaluations of G | | | | | | | |
| \varnothing number of evaluations per run (for | 12.65 | 9.67 | 12 15 | 9.67 | 17 95 | | |
| runs terminated at a solution only) | 12.00 | 5.01 | 12.10 | 5.01 | 11.50 | | |
| \varnothing number of evaluations per run (for | 12.65 | 7.85 | 12.15 | 7.85 | 18.05 | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) 12.00 1.00 12.10 1.00 10.00 | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.23{ m s}$ | $0.19{ m s}$ | $0.21{ m s}$ | 0.18 s | 0.14 s | | |
| nated at a solution only) | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.23{ m s}$ | 0.14 s | $0.21{ m s}$ | 0.14 s | 0.14 s | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.02\mathrm{s}$ | $0.01\mathrm{s}$ | $0.02\mathrm{s}$ | $0.01\mathrm{s}$ | $0.06\mathrm{s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.04\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | | 0.0 | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | Γ | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| last 3 steps are LPN (hybrid) | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 55.0% | 55.6% | 85.0% | 55.6% | 100.0% | | |

Harker

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|------------------|------------------|---------------|---------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 9.30 | 6.90 | 9.30 | 6.90 | 13.45 | | | |
| runs terminated at a solution only) | 5.50 | 0.50 | 5.50 | 0.50 | 10.10 | | | |
| | Evaluation | ns of F | - | - | - | | | |
| \varnothing number of evaluations per run (for | 10.30 | 7 90 | 10.30 | 7 90 | 28.25 | | | |
| runs terminated at a solution only) | 10.00 | 1.50 | 10.00 | 1.50 | 20.20 | | | |
| \varnothing number of evaluations per run (for | 10.30 | 7 90 | 10.30 | 7 90 | 28 25 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.00 | 1.50 | 10.00 | 1.50 | 20.20 | | | |
| Evaluations of G | | | | | | | | |
| \varnothing number of evaluations per run (for | 9 30 | 6 90 | 9 30 | 6 90 | 13 45 | | | |
| runs terminated at a solution only) | 0.00 | 0.50 | 5.50 | 0.50 | 10.10 | | | |
| \varnothing number of evaluations per run (for | 9.30 | 6 90 | 9 30 | 6 90 | 13 45 | | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) all runs terminated with $\varepsilon^* \le 10^{-8}$) | | | | | | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.15 s | 0.11 s | 0.15 s | 0.11 s | 0.07s | | | |
| nated at a solution only) | 0.105 | 0.115 | 0.105 | 0.115 | 0.01.5 | | | |
| \varnothing CPU time per run (for all runs ter- | $0.15{\rm s}$ | 0.11 s | $0.15{\rm s}$ | 0.11s | $0.07{\rm s}$ | | | |
| minated with $\varepsilon^* \le 10^{-8}$) | 0.105 | 0.115 | 0.105 | 0.115 | 0.01.5 | | | |
| \varnothing CPU time for subproblems per run | $0.00{\rm s}$ | $0.00{\rm s}$ | $0.00{\rm s}$ | $0.00{\rm s}$ | 0.04 s | | | |
| (for runs term. at a solution only) | 0.000 | 0.002 | 0.000 | 0.005 | 0.015 | | | |
| \varnothing CPU time for line search per run (for | $0.00{\rm s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00{\rm s}$ | 0.01 s | | | |
| runs terminated at a solution only) | 0.005 | 0.005 | 0.005 | 0.005 | 0.015 | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | 1 | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| step is LPN (hybrid) | 1000070 | 1000070 | 100.070 | 100.070 | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last 2 steps are LPN (hybrid) | 100.070 | 100.070 | 100.070 | 100.070 | 50.070 | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 100.070 | 100.070 | 100.070 | 100.070 | 0.070 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 90.0% | 95.0% | 90.0% | 95.0% | 95.0% | | | |

Heu

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 20 | 10 | 20 | 15 | 19 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | |
| \varnothing number of iterations per run (for | 17.00 | 12.80 | 14.05 | 15.60 | 32 70 | | |
| runs terminated at a solution only) | 17.00 | 12.80 | 14.95 | 15.00 | 52.19 | | |
| | Evaluation | ns of F | | - | - | | |
| \varnothing number of evaluations per run (for | 22.20 | 13.00 | 16 10 | 16.60 | 124.05 | | |
| runs terminated at a solution only) | 22.20 | 10.50 | 10.10 | 10.00 | 124.30 | | |
| \varnothing number of evaluations per run (for | 22.20 | 25.25 | 16 10 | 20.15 | 124.00 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 22.20 | 20.20 | 10.10 | 20.10 | 124.00 | | |
| Evaluations of G | | | | | | | |
| \varnothing number of evaluations per run (for | 17.00 | 12.80 | 14 95 | 15.60 | 32 79 | | |
| runs terminated at a solution only) | 11.00 | 12.00 | 11.00 | 10.00 | 02.10 | | |
| \varnothing number of evaluations per run (for | 17.00 | 15.85 | 14.95 | 19.15 | 32.70 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-\circ}$) | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.28\mathrm{s}$ | $0.21\mathrm{s}$ | $0.24\mathrm{s}$ | $0.25\mathrm{s}$ | $0.17\mathrm{s}$ | | |
| nated at a solution only) | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.28\mathrm{s}$ | $0.26\mathrm{s}$ | $0.24\mathrm{s}$ | $0.31\mathrm{s}$ | $0.16\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \bigotimes CPU time for subproblems per run | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.04\mathrm{s}$ | | |
| (for runs term. at a solution only) \sim (for runs term. (f | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.05\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | | | | |
| | Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size = 1 (glob. LPN) or last $(1, 1, 2)$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (nybrid) | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{PN} | 100.0% | 100.0% | 100.0% | 100.0% | 36.8% | | |
| last 2 steps are LPN (nybrid) | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| $\begin{bmatrix} \text{Iast 5 steps are LFN (nybrid)} \\ \text{Iast improvement of } \ F\ & \text{is } > 10^2 \end{bmatrix}$ | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| $\frac{13st \text{ improvement of } \ F\ _{\infty} \text{ is } \geq 10^{3}}{1 \text{ s} \text{ s} \text{ s} \text{ s} \text{ s} \text{ s}^{4}}$ | 100.0% | 100.0% | | 100.0% | | | |
| 1 last improvement of $ F _{\infty}$ is $\geq 10^4$ | 85.0% | 100.0% | 85.0% | 93.3% | 89.5% | | |

Lob

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|-------------|------------------|---------------|--------|------------------|--|--|
| # runs terminated at a solution | 0 | 20 | 0 | 0 | 19 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 0 | 20 | 0 | 0 | 20 | | |
| \varnothing number of iterations per run (for | | 24.70 | | | 22.05 | | |
| runs terminated at a solution only) | _ | 24.70 | _ | _ | 22.05 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | | 36 50 | _ | | 55 16 | | |
| runs terminated at a solution only) | | 50.50 | | | 55.10 | | |
| \varnothing number of evaluations per run (for | _ | 36.50 | _ | | 150.95 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | | 50.50 | | | 100.00 | | |
| | | | | | | | |
| \varnothing number of evaluations per run (for | _ | 24 70 | _ | _ | 22.05 | | |
| runs terminated at a solution only) | | 21.10 | | | | | |
| \varnothing number of evaluations per run (for | | 24.70 | _ | | 25.65 | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | _ | $0.66\mathrm{s}$ | _ | _ | $0.43\mathrm{s}$ | | |
| nated at a solution only) | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | _ | $0.66\mathrm{s}$ | _ | _ | $0.73\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \bigotimes CPU time for subproblems per run | _ | $0.15\mathrm{s}$ | _ | _ | $0.13\mathrm{s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| \varnothing CPU time for line search per run (for | _ | $0.08\mathrm{s}$ | _ | _ | $0.17\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size = 1 (glob. LPN) or last | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | |
| step is LPN (nybrid) | | | | | | | |
| 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 step sizes = 1 step sizes = 1 (glob. LPN) or 1 step sizes = 1 (glob. LPN) or 1 step sizes = 1 (glob. LPN) or 1 step sizes = 1 | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | |
| last 2 steps are LPN (nybrid) | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | |
| $\begin{bmatrix} \text{rast 5 steps are LFN (hydrid)} \\ \text{last improvement of } \ E\ & \text{is } > 10^2 \end{bmatrix}$ | 0.007 | 100.007 | 0.007 | 0.007 | 100.007 | | |
| last improvement of $ F _{\infty}$ is $\geq 10^{-1}$ | 0.0% | 100.0% | 0.0% | 0.070 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^{\circ}$ | 0.0% | 100.07 | 0.070 | 0.070 | | | |
| as improvement of $ F _{\infty}$ is $\geq 10^{-1}$ | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | |

NTF1

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 7 50 | 5.45 | 7 50 | 5.45 | 10.20 | | | |
| runs terminated at a solution only) | 1.50 | 0.40 | 1.50 | 0.40 | 10.20 | | | |
| | Evaluation | ns of F | | | | | | |
| \varnothing number of evaluations per run (for | 8 50 | 6 45 | 8 50 | 6 45 | 11.25 | | | |
| runs terminated at a solution only) | 0.00 | 0.10 | 0.00 | 0.10 | 11.20 | | | |
| \varnothing number of evaluations per run (for | 8 50 | 6 45 | 8 50 | 6 45 | 11 25 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 0.10 | 0.00 | 0.10 | 11.20 | | | |
| Evaluations of G | | | | | | | | |
| \varnothing number of evaluations per run (for | 7.50 | 5.45 | 7.50 | 5.45 | 10.20 | | | |
| runs terminated at a solution only) | | | | | 10.20 | | | |
| \varnothing number of evaluations per run (for | 7.50 | 5.45 | 7.50 | 5.45 | 10.20 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \bigotimes CPU time for subproblems per run | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.04\mathrm{s}$ | | | |
| (for runs term. at a solution only) $\sim CDU + \frac{1}{2} \int \frac{1}{2} \frac{1}{2$ | | | | | | | | |
| © CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| | e Rate (for i | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last step is LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| step is LPN (nybrid) | | | | | | | | |
| 1 last 2 step sizes = 1 (glob. LPN) of 1 last 2 steps are LDN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last 2 steps are LFN (hybrid) | | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or last 2 steps are LDN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last improvement of $ F = i_{\rm S} > 10^2$ | 100.0% | 100.0% | 100.00% | 100.00% | 05.0% | | | |
| last improvement of $ F \approx 15 \ge 10$ | 100.070 | 100.070 | 100.070 | 100.070 | 95.070 | | | |
| last improvement of $ F _{\infty}$ is ≥ 10 | 80.0% | 100.070 | 80.0% | 100.070 | 05.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 80.0% | 100.0% | 80.0% | 100.0% | 95.0% | | | |

NTF2

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|--|--------------------|------------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | |
| \varnothing number of iterations per run (for | 9.10 | 26.65 | 9.15 | 9.55 | 13.60 | | |
| runs terminated at a solution only) | | 6.5 | | | | | |
| | Evaluation | ns of F' | | | | | |
| \emptyset number of evaluations per run (for runs terminated at a solution only) | 10.25 | 130.35 | 10.15 | 10.60 | 19.20 | | |
| Ø number of evaluations per run (for | | | | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.25 | 130.35 | 10.15 | 10.60 | 19.20 | | |
| Evaluations of G | | | | | | | |
| \varnothing number of evaluations per run (for | 9.10 | 26.65 | 9.15 | 9.55 | 13.60 | | |
| runs terminated at a solution only) | 3.10 | 20.05 | 3.10 | 3.00 | 15.00 | | |
| \varnothing number of evaluations per run (for | 9 10 | 26.65 | 9 15 | 9 55 | 13 60 | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) and $\varepsilon^* \le 10^{-8}$ | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.15{ m s}$ | $0.45\mathrm{s}$ | $0.15{ m s}$ | $0.15{ m s}$ | $0.07{ m s}$ | | |
| nated at a solution only) | 0.100 | 0.10.5 | 0.105 | 0.10.5 | | | |
| \varnothing CPU time per run (for all runs ter- | $0.15\mathrm{s}$ | $0.45\mathrm{s}$ | $0.15\mathrm{s}$ | $0.15\mathrm{s}$ | $0.07\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.04\mathrm{s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.03\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | I | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 95.0% | 100.0% | 100.0% | 95.0% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 95.0% | 100.0% | 100.0% | 0.0% | | |
| last 3 steps are LPN (hybrid) | 100.00 | 100.007 | 100.007 | 100.007 | 05.007 | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | |

Spam

Dimensions: $N = 101, n_x = 2020, m_g = 4040, n = 10100$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|--|---------------------|---------------------|---------------------|---------------------|--------------------|--|--|
| # runs terminated at a solution | 19 | 20 | 20 | 20 | 18 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 20 | 20 | 20 | 19 | | |
| \varnothing number of iterations per run (for | 14.69 | 16 10 | 65 55 | 28.05 | 14.04 | | |
| runs terminated at a solution only) | 14.00 | 10.10 | 05.55 | 38.05 | 14.94 | | |
| | Evaluation | is of F | | | | | |
| \varnothing number of evaluations per run (for | 25.84 | 50.05 | 88.20 | 48 70 | 35.44 | | |
| runs terminated at a solution only) | 20.04 | 50.95 | 88.20 | 40.70 | 55.44 | | |
| \varnothing number of evaluations per run (for | 25.84 | 50.95 | 88.20 | 48 70 | 35 58 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 20.04 | 00.00 | 00.20 | 40.10 | 30.00 | | |
| | | | | | | | |
| \varnothing number of evaluations per run (for | 14 68 | 16 10 | 65 55 | 38.05 | 14 94 | | |
| runs terminated at a solution only) | 11.00 | 10.10 | 00.00 | 00.00 | 11.51 | | |
| \varnothing number of evaluations per run (for | 14 68 | 16 10 | 65 55 | 38.05 | 15.05 | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $1403.94\mathrm{s}$ | $1063.12\mathrm{s}$ | $3732.40\mathrm{s}$ | $2254.92\mathrm{s}$ | $253.31\mathrm{s}$ | | |
| nated at a solution only) | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $1403.94\mathrm{s}$ | $1063.12\mathrm{s}$ | $3732.40{ m s}$ | $2254.92\mathrm{s}$ | $241.05\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \varnothing CPU time for subproblems per run | $228.91\mathrm{s}$ | $186.93\mathrm{s}$ | $608.66\mathrm{s}$ | $398.27\mathrm{s}$ | $236.83\mathrm{s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.78{ m s}$ | $1.56\mathrm{s}$ | $2.94\mathrm{s}$ | $1.81\mathrm{s}$ | $1.01\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 77.8% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or 1 last 3 step sizes = 1 | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| last 3 steps are LPN (hybrid) | 0.00 | 0.004 | F 007 | 0.004 | 27.004 | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 0.0% | 0.0% | 5.0% | 0.0% | 21.8% | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{3}$ | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | | |

Tr1a

Dimensions: $N = 6, n_x = 18, m_g = 72, n = 162$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|--------------------|------------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 15 | 12 | 20 | 16 | 17 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 20 | 20 | 20 | 17 | | |
| \varnothing number of iterations per run (for | 57.47 | 27 42 | 17.40 | 10.81 | 01.82 | | |
| runs terminated at a solution only) | 51.41 | 21.42 | 17.40 | 13.01 | 51.62 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | 256.00 | 82 42 | 22.95 | 24 50 | 369 18 | | |
| runs terminated at a solution only) | 230.00 | 02.42 | 22.30 | 24.00 | 505.10 | | |
| \varnothing number of evaluations per run (for | 1039 11 | 66 20 | 22.95 | 22.80 | 369 18 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1005.11 | 00.20 | 22.50 | 22.00 | 000.10 | | |
| | Evaluation | ns of G | I | 1 | 1 | | |
| \varnothing number of evaluations per run (for | 57 47 | 27 42 | 17 40 | 19.81 | 91.82 | | |
| runs terminated at a solution only) | 0111 | 21112 | | 10:01 | 01:02 | | |
| \varnothing number of evaluations per run (for | 79.74 | 24.10 | 17.40 | 18.40 | 91.82 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $1.39\mathrm{s}$ | $0.63\mathrm{s}$ | $0.36\mathrm{s}$ | $0.41\mathrm{s}$ | $1.06\mathrm{s}$ | | |
| nated at a solution only) | 1.595 | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $3.66\mathrm{s}$ | $0.53\mathrm{s}$ | $0.36\mathrm{s}$ | $0.37\mathrm{s}$ | $1.06\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \bigotimes CPU time for subproblems per run | $0.09\mathrm{s}$ | $0.06\mathrm{s}$ | $0.04\mathrm{s}$ | $0.05\mathrm{s}$ | $0.09\mathrm{s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| © CPU time for line search per run (for | $0.34\mathrm{s}$ | $0.10\mathrm{s}$ | $0.05\mathrm{s}$ | $0.02\mathrm{s}$ | $0.60\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | <u> </u> | | | |
| | Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{PN} | 93.3% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 86.7% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 3 steps are LPN (hybrid) | 79.907 | 100.007 | 100.007 | 100.007 | 100.007 | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 13.3% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^3$ | 40.0% | 91.7% | 65.0% | 100.0% | 5.9% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 33.3% | 83.3% | 55.0% | 93.8% | 0.0% | | |

Tr1b

Dimensions: $N = 6, n_x = 60, m_g = 228, n = 516$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|--|------------------|-------------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 14 | 3 | 19 | 18 | 3 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 18 | 19 | 20 | 20 | 3 | | |
| \varnothing number of iterations per run (for | 1/3 50 | 151.67 | 19 19 | 70.06 | 180.00 | | |
| runs terminated at a solution only) | 145.50 | 101.07 | 42.42 | 70.00 | 180.00 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | 760 29 | 738.00 | 52 11 | 94.94 | 986 67 | | |
| runs terminated at a solution only) | 100.25 | 130.00 | 02.11 | 54.54 | 500.01 | | |
| \varnothing number of evaluations per run (for | 698.06 | 1188 74 | 54 10 | 140 75 | 986 67 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 030.00 | 1100.14 | 01.10 | 140.10 | 500.01 | | |
| Evaluations of G | | | | | | | |
| \varnothing number of evaluations per run (for | 143 50 | 151 67 | 42.42 | 70.06 | 170.67 | | |
| runs terminated at a solution only) | 110.00 | 101.01 | 12.12 | 10.00 | 110.01 | | |
| \varnothing number of evaluations per run (for | 135.89 | 253.11 | 42.25 | 81.05 | 170.67 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $8.75\mathrm{s}$ | $7.75\mathrm{s}$ | $2.24\mathrm{s}$ | $3.87\mathrm{s}$ | $6.99\mathrm{s}$ | | |
| nated at a solution only) | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $8.11\mathrm{s}$ | $11.75\mathrm{s}$ | $2.22\mathrm{s}$ | $4.30\mathrm{s}$ | $6.99\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | |
| \bigotimes CPU time for subproblems per run | $3.57\mathrm{s}$ | $2.57\mathrm{s}$ | $1.16\mathrm{s}$ | $2.22\mathrm{s}$ | $2.59\mathrm{s}$ | | |
| (for runs term. at a solution only) | | | | | | | |
| \varnothing CPU time for line search per run (for | $2.31\mathrm{s}$ | $2.59\mathrm{s}$ | $0.12\mathrm{s}$ | $0.22\mathrm{s}$ | $3.50\mathrm{s}$ | | |
| runs terminated at a solution only) | | | | | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or L_{DN} (1 l : 1) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last 3 steps are LPN (hybrid) | | 100.007 | 00 F 07 | 04.407 | CC 7 07 | | |
| $1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^2$ | 85.7% | 100.0% | 89.5% | 94.4% | 00.7% | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{5}$ | 71.4% | 00.7% | 08.4% | 11.8% | <u>33.3%</u> | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 35.7% | 0.0% | 21.1% | 44.4% | 0.0% | | |
Tr1c

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|-------------------|-------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 14 | 1 | 20 | 20 | 1 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 14 | 20 | 20 | 1 | | | |
| \varnothing number of iterations per run (for | 142.26 | 162.00 | 80.60 | 82.25 | 115.00 | | | |
| runs terminated at a solution only) | 145.50 | 102.00 | 80.00 | 82.20 | 115.00 | | | |
| | Evaluation | ns of F | | | | | | |
| \varnothing number of evaluations per run (for | 654.07 | 880.00 | 108.60 | 108.40 | 602.00 | | | |
| runs terminated at a solution only) | 004.07 | 880.00 | 108.00 | 100.40 | 092.00 | | | |
| \varnothing number of evaluations per run (for | 637 58 | 2045 14 | 108 60 | 108.40 | 692.00 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 001.00 | 2045.14 | 100.00 | 100.40 | 032.00 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 143 36 | 162.00 | 80.60 | 82.25 | 113.00 | | | |
| runs terminated at a solution only) | 140.00 | 102.00 | 00.00 | 02.20 | 115.00 | | | |
| \varnothing number of evaluations per run (for | 138.37 | 367 36 | 80.60 | 82.25 | 113.00 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 100.01 | 001.00 | 00.00 | 02.20 | 110.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 11.31 s | 13.16 s | 8.69 s | 6.78 s | $4.92\mathrm{s}$ | | | |
| nated at a solution only) | | 10.105 | | | 1.0 - 5 | | | |
| \varnothing CPU time per run (for all runs ter- | $10.89\mathrm{s}$ | $24.28\mathrm{s}$ | 8.69 s | 6.78 s | $4.92\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 10.000 | | 0.00 2 | | 1.0 - 5 | | | |
| \varnothing CPU time for subproblems per run | 5.36 s | $5.74\mathrm{s}$ | $6.27\mathrm{s}$ | $4.43\mathrm{s}$ | $0.93\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | 2.14 s | $2.74\mathrm{s}$ | 0.32 s | 0.33 s | 3.10 s | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 92.9% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 85.7% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 steps are LPN (hybrid) | | | 20.00 | 00.00 | 0.004 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 85.7% | 100.0% | 80.0% | 80.0% | 0.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 57.1% | 100.0% | 65.0% | 55.0% | 0.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 14.3% | 100.0% | 25.0% | 40.0% | 0.0% | | | |

3 Results with Modification of a CPLEX[®] Parameter

As described in the Introduction, the runs of the algorithms were performed again, where the LP solver from the optimization toolbox CPLEX[®] was forced to use the dual simplex algorithm as solution method. It turned out that, for the large examples and on average, the CPU time for the (successful) runs seriously decreased. One reason might be that the solver saves the time for choosing an appropriate algorithm. Apart from CPU time, there are not many differences compared to the results presented in Section 2. In fact, the only test problems where issues like number of runs terminated at a solution or function evaluations appreciably differ are A10d, A10e, and Spam. Probably, the reason is that, for the latter three examples, the LP solver (with automatic choice of the solution method) did not always use the dual simplex algorithm for the solution of the subproblems. This might result in different solutions of the linear programs. The results show that the modification of parameters for the solution method of the subproblems may significantly influence the performance of the algorithms.

$\mathbf{A1}$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 19 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 20 | 20 | 20 | 20 | | | |
| Ø number of iterations per run (for | 11.96 | 0.10 | 11.50 | 0.05 | 14.75 | | | |
| runs terminated at a solution only) | 11.20 | 9.10 | 11.50 | 9.95 | 14.75 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 19 37 | 10.60 | 12 50 | 11.00 | 28 75 | | | |
| runs terminated at a solution only) | 12.57 | 10.00 | 12.50 | 11.00 | 20.15 | | | |
| \varnothing number of evaluations per run (for | 19 37 | 10.60 | 12 50 | 11.00 | 28 75 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.07 | 10.00 | 12.00 | 11.00 | 20.10 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 11.26 | 9 10 | 11 50 | 9 95 | 14 75 | | | |
| runs terminated at a solution only) | 11.20 | 0.10 | 11.00 | 0.00 | 11.10 | | | |
| \varnothing number of evaluations per run (for | 11.26 | 9 10 | 11 50 | 9 95 | 14 75 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.20 | 0.10 | 11.00 | 0.00 | 11110 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.20\mathrm{s}$ | $0.15\mathrm{s}$ | $0.19{ m s}$ | $0.17{ m s}$ | 0.10 s | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.20\mathrm{s}$ | $0.15\mathrm{s}$ | $0.19{ m s}$ | $0.17{ m s}$ | 0.10 s | | | |
| minated with $\varepsilon^* \leq 10^{-\circ}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.04\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.02\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | 1 | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| $\frac{1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^{3}}{1 \text{ st improvement of } \ F\ _{\infty} \text{ is } \geq 10^{4}}$ | 100.0% | | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 78.9% | 75.0% | 85.0% | 90.0% | 20.0% | | | |

Dimensions: $N = 10, n_x = 10, m_q = 20, n = 50$

Dimensions: $N = 10, n_x = 10, m_g = 24, n = 58$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|---------------|--------------|---------------|---------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 3 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 3 | | | |
| \varnothing number of iterations per run (for | 10.80 | 0.15 | 11.05 | 0.15 | 00.67 | | | |
| runs terminated at a solution only) | 10.80 | 9.10 | 11.05 | 9.15 | 99.07 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 12.25 | 10.20 | 12.05 | 10.15 | 360 33 | | | |
| runs terminated at a solution only) | 12.20 | 10.20 | 12.00 | 10.15 | 000.00 | | | |
| \varnothing number of evaluations per run (for | 12.25 | 10.20 | 12.05 | 10.15 | 360 33 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.20 | 10.20 | 12.00 | 10.10 | 500.55 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 10.80 | 9 15 | 11.05 | 9.15 | 98.67 | | | |
| runs terminated at a solution only) | 10.00 | 5.10 | 11.00 | 5.10 | 50.01 | | | |
| \varnothing number of evaluations per run (for | 10.80 | 9 15 | 11.05 | 9 15 | 98.67 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.00 | 5.10 | 11.00 | 5.10 | 50.01 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.19 s | 0.15 s | 0.18s | $0.15{\rm s}$ | $0.58{ m s}$ | | | |
| nated at a solution only) | 0.10 5 | 0.100 | | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.19 s | 0.15 s | 0.18 s | 0.15 s | $0.58\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.10 5 | 0.205 | 0.105 | 0.10.5 | 0.000 | | | |
| \varnothing CPU time for subproblems per run | 0.01 s | 0.01 s | 0.01 s | 0.01 s | $0.15{ m s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | 0.01 s | $0.00{ m s}$ | 0.01 s | $0.00{ m s}$ | $0.19{ m s}$ | | | |
| runs terminated at a solution only) | | 0.000 | 0.015 | | 0.10.5 | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 95.0% | 100.0% | 95.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 95.0% | 95.0% | 95.0% | 95.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 60.0% | 70.0% | 50.0% | 70.0% | 33.3% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|--------------|------------------|--------------|--------------|--|--|--|--|
| # runs terminated at a solution | 16 | 8 | 15 | 8 | 19 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 19 | | | | |
| \varnothing number of iterations per run (for | 14 56 | 11 50 | 13.60 | 11 50 | 37 96 | | | | |
| runs terminated at a solution only) | 14.00 | 11.50 | 15.00 | 11.00 | 51.20 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 17.69 | 12 50 | 14.67 | 12 50 | 188 58 | | | | |
| runs terminated at a solution only) | 11.05 | 12.00 | 14.07 | 12.00 | 100.00 | | | | |
| \varnothing number of evaluations per run (for | 17 75 | 8 85 | 14 45 | 8 85 | 188 58 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.10 | 0.00 | 14.40 | 0.00 | 100.00 | | | | |
| | Evaluation | ns of G | r | r | r | | | | |
| \varnothing number of evaluations per run (for | 14 56 | 11 50 | 13 60 | 11 50 | 37.26 | | | | |
| runs terminated at a solution only) | 11.00 | 11.00 | 10.00 | 11.00 | 01.20 | | | | |
| \varnothing number of evaluations per run (for | 14 55 | 8 45 | 1365 | 8 45 | 37.26 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.00 | 0.10 | 10.00 | 0.10 | 01.20 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.24 s | 0.19 s | $0.22\mathrm{s}$ | $0.19{ m s}$ | 0.18 s | | | | |
| nated at a solution only) | | 0.105 | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.24 s | 0.14 s | $0.22\mathrm{s}$ | 0.14 s | 0.18 s | | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | | |
| \varnothing CPU time for subproblems per run | 0.01 s | 0.01 s | 0.01 s | 0.01 s | $0.02{ m s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | 0.01 s | 0.01 s | $0.07{ m s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 31.6% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 81.2% | 100.0% | 66.7% | 100.0% | 68.4% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|--------------|--------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 18 | 10 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 10 | | | |
| \varnothing number of iterations per run (for | 17 15 | 16.20 | 12 75 | 12 78 | 120 50 | | | |
| runs terminated at a solution only) | 11.10 | 10.20 | 15.75 | 13.70 | 159.50 | | | |
| | Evaluation | ns of F | | | | | | |
| \varnothing number of evaluations per run (for | 26.30 | 30.85 | 16.30 | 15 33 | 512 30 | | | |
| runs terminated at a solution only) | 20.50 | 50.05 | 10.50 | 10.00 | 012.00 | | | |
| \varnothing number of evaluations per run (for | 26.30 | 30.85 | 16.30 | 14 90 | 512 30 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 20.00 | 50.05 | 10.50 | 14.50 | 012.00 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 17 15 | 16 20 | 13 75 | 13 78 | 139 50 | | | |
| runs terminated at a solution only) | 11.10 | 10.20 | 10.10 | 10.10 | 100.00 | | | |
| \varnothing number of evaluations per run (for | 17 15 | 16 20 | 13 75 | 13 45 | 139 50 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.10 | 10.20 | 10.10 | 10.10 | 100.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.27\mathrm{s}$ | $0.26\mathrm{s}$ | $0.22\mathrm{s}$ | $0.22{ m s}$ | $0.54{ m s}$ | | | |
| nated at a solution only) | 0.21.0 | 0.202 | 0.220 | 0.220 | 0.010 | | | |
| \varnothing CPU time per run (for all runs ter- | $0.27{ m s}$ | $0.26{ m s}$ | $0.22{ m s}$ | 0.21 s | $0.54{ m s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.21 0 | 0.202 | 0.22.5 | 0.210 | 0.010 | | | |
| \varnothing CPU time for subproblems per run | 0.01 s | 0.01 s | 0.01 s | 0.01 s | $0.05{ m s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | 0.01 s | 0.01 s | $0.00{ m s}$ | 0.01 s | 0.19 s | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 95.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 55.0% | 85.0% | 60.0% | 83.3% | 40.0% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|------------------|------------------|------------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 19 | 20 | 19 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 19.55 | 0.74 | 12.45 | 0.74 | 14.05 | | | | |
| runs terminated at a solution only) | 12.00 | 5.74 | 12.40 | 9.14 | 14.05 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 14.95 | 10.74 | 13.45 | 10.74 | 41.00 | | | | |
| runs terminated at a solution only) | 14.20 | 10.74 | 15.45 | 10.74 | 41.00 | | | | |
| \varnothing number of evaluations per run (for | 14 25 | 10.50 | 13 45 | 10.50 | 41.00 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 14.20 | 10.00 | 10.40 | 10.00 | 41.00 | | | | |
| | Evaluation | ns of G | r | r | r | | | | |
| \varnothing number of evaluations per run (for | 12 55 | 9 74 | 12.45 | 9 74 | 14 05 | | | | |
| runs terminated at a solution only) | 12.00 | 0.11 | 12.10 | 0.11 | 11.00 | | | | |
| \varnothing number of evaluations per run (for | 12.55 | 9.55 | 12.45 | 9.55 | 14.05 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.00 | 0.00 | 12.10 | 0.00 | 11.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.20 s | $0.16\mathrm{s}$ | $0.20\mathrm{s}$ | $0.16\mathrm{s}$ | $0.10\mathrm{s}$ | | | | |
| nated at a solution only) | | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.20\mathrm{s}$ | $0.15\mathrm{s}$ | $0.20\mathrm{s}$ | $0.15\mathrm{s}$ | $0.10\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.05\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.02\mathrm{s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | [| | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 85.0% | | | | |
| last 3 steps are LPN (hybrid) | 100.007 | 100.00 | 100 00 | 100.00 | 100 00 | | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 85.0% | 100.0% | 90.0% | 100.0% | 90.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 19 | 19 | 20 | 20 | 18 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 19 | 20 | 20 | 18 | | | |
| \varnothing number of iterations per run (for | 48.05 | 10.62 | 17.45 | 16 75 | 74.99 | | | |
| runs terminated at a solution only) | 40.00 | 19.05 | 17.40 | 10.75 | 14.22 | | | |
| | Evaluation | is of F | | | | | | |
| \varnothing number of evaluations per run (for | 221.68 | 43.37 | 18 60 | 17 75 | 337 67 | | | |
| runs terminated at a solution only) | 221.00 | 40.07 | 10.00 | 11.10 | 001.01 | | | |
| \varnothing number of evaluations per run (for | 221.68 | <i>1</i> 3 37 | 18.60 | 17 75 | 337.67 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 221.00 | 40.01 | 10.00 | 11.10 | 001.01 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 48.05 | 19.63 | 17 45 | 16 75 | 74.22 | | | |
| runs terminated at a solution only) | 40.00 | 15.00 | 11.10 | 10.10 | 11.22 | | | |
| \varnothing number of evaluations per run (for | 48.05 | 19.63 | 17 45 | 16 75 | 74.22 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.00 | 10.00 | 11110 | 10.110 | 1 1.22 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.79{ m s}$ | $0.32\mathrm{s}$ | $0.28\mathrm{s}$ | $0.26\mathrm{s}$ | $0.35\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.79{ m s}$ | $0.32\mathrm{s}$ | $0.28{ m s}$ | $0.26\mathrm{s}$ | $0.35\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.04\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.05\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.08\mathrm{s}$ | $0.02\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.13\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 100.004 | 100.004 | 100.007 | 100.007 | 100.007 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 63.2% | 100.0% | 85.0% | 95.0% | 94.4% | | | |

Dimensions: $N = 4, n_x = 20, m_g = 44, n = 108$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 13 | 20 | 13 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 17 70 | 1/1 21 | 13.45 | 1/1 31 | 31.60 | | | |
| runs terminated at a solution only) | 17.70 | 14.51 | 15.45 | 14.51 | 51.00 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 20.45 | 15 31 | 14 45 | 15 31 | 138 75 | | | |
| runs terminated at a solution only) | 23.40 | 10.01 | 14.40 | 10.01 | 130.15 | | | |
| \varnothing number of evaluations per run (for | 29.45 | 13.60 | 14 45 | 13.60 | 138 75 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 20.40 | 10.00 | 11.10 | 10.00 | 100.10 | | | |
| | Evaluation | ns of G | r | r | | | | |
| \varnothing number of evaluations per run (for | 17 70 | 14 31 | 13 45 | 14 31 | $31\ 60$ | | | |
| runs terminated at a solution only) | 11.10 | 11.01 | 10.10 | 11.01 | 01.00 | | | |
| \varnothing number of evaluations per run (for | 17.70 | 12.95 | 13.45 | 12.95 | 31.60 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.10 | 12.00 | 10.10 | 12.00 | 01.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.34{ m s}$ | $0.26\mathrm{s}$ | $0.25\mathrm{s}$ | $0.25\mathrm{s}$ | $0.21\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.34{ m s}$ | $0.23\mathrm{s}$ | $0.25\mathrm{s}$ | $0.23\mathrm{s}$ | $0.21\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.06\mathrm{s}$ | $0.04\mathrm{s}$ | $0.05\mathrm{s}$ | $0.04\mathrm{s}$ | $0.06\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 25.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 75.0% | 30.8% | 75.0% | 30.8% | 95.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 15.0% | 15.4% | 40.0% | 15.4% | 30.0% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|--------------|---------------|--------------|---------------|--|--|--|--|
| # runs terminated at a solution | 20 | 19 | 20 | 19 | 10 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 10 | | | | |
| \varnothing number of iterations per run (for | 0.25 | 6.05 | 0.25 | 6.05 | 253 60 | | | | |
| runs terminated at a solution only) | 9.20 | 0.95 | 9.20 | 0.95 | 200.00 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 10.25 | 7.95 | 10.25 | 7.05 | 1385-20 | | | | |
| runs terminated at a solution only) | 10.20 | 1.30 | 10.25 | 1.50 | 1000.20 | | | | |
| \varnothing number of evaluations per run (for | 10.25 | 7 75 | 10.25 | 7 75 | 1385-20 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.20 | 1.10 | 10.25 | 1.10 | 1000.20 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 9.25 | 6 95 | 9.25 | 6 95 | 253 60 | | | | |
| runs terminated at a solution only) | 0.20 | 0.50 | 5.20 | 0.55 | 200.00 | | | | |
| \varnothing number of evaluations per run (for | 9.25 | 6.80 | 9 25 | 6 80 | $253\ 60$ | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.20 | 0.00 | 0.20 | 0.00 | 200.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.15 s | 0.11 s | 0.14 s | 0.11s | $0.97{\rm s}$ | | | | |
| nated at a solution only) | 0.100 | 0.110 | | | 0.01.5 | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.15 s | 0.11 s | $0.14{ m s}$ | 0.11 s | $0.97{ m s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.100 | 0.115 | | | 0.01.5 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.07{ m s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | 0.41 s | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 85.0% | 52.6% | 85.0% | 52.6% | 100.0% | | | | |

A9a

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 21.80 | 70.55 | 30.40 | 21.75 | 22.80 | | | |
| runs terminated at a solution only) | | <u> </u> | | | | | | |
| Evaluations of <i>F</i> | | | | | | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 39.35 | 323.85 | 42.35 | 31.65 | 125.10 | | | |
| Ø number of evaluations per run (for | | | | | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 39.35 | 323.85 | 42.35 | 31.65 | 125.10 | | | |
| | Evaluation | is of G | | | | | | |
| \varnothing number of evaluations per run (for | 91.90 | 70 55 | 20.40 | 01.75 | 22.80 | | | |
| runs terminated at a solution only) | 21.60 | 70.55 | 30.40 | 21.75 | 22.80 | | | |
| \varnothing number of evaluations per run (for | 21.80 | 70.55 | 30.40 | 21 75 | 22.80 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 21.00 | 10.00 | 30.40 | 21.75 | 22.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.71s | $2.60\mathrm{s}$ | 0.96 s | 0.65 s | 0.46s | | | |
| nated at a solution only) | 0.115 | 2.005 | 0.005 | 0.0015 | 0.105 | | | |
| \varnothing CPU time per run (for all runs ter- | $0.71{ m s}$ | $2.60{ m s}$ | $0.96\mathrm{s}$ | $0.65\mathrm{s}$ | $0.46\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 015 | | | 0.000 | 0.10.5 | | | |
| \varnothing CPU time for subproblems per run | $0.27{ m s}$ | 0.81 s | $0.37{ m s}$ | 0.21 s | $0.09{ m s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.07\mathrm{s}$ | $0.59\mathrm{s}$ | $0.08\mathrm{s}$ | $0.05\mathrm{s}$ | $0.25\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | 1 | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or $I = I = I = I$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or $I = I = I = I$ | 100.0% | 100.0% | 100.0% | 100.0% | 50.0% | | | |
| Last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 05 007 | 100.007 | 100.007 | | | |
| $\begin{array}{c c} 1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^2 \\ \hline 1 \text{ , } \text$ | 100.0% | 100.0% | 95.0% | 100.0% | 100.0% | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{5}$ | 95.0% | 100.0% | 90.0% | 100.0% | 95.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 60.0% | 90.0% | 85.0% | 75.0% | 55.0% | | | |

A9b

Dimensions: $N = 7, n_x = 112, m_g = 119, n = 350$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 24.25 | 55.05 | 22.95 | 21.20 | 24 70 | | | |
| runs terminated at a solution only) | 24.35 | 55.05 | 55.65 | 21.20 | 24.70 | | | |
| | Evaluation | ns of F | | | | | | |
| \varnothing number of evaluations per run (for | 44.95 | 208-30 | 44.00 | 30.05 | 136 55 | | | |
| runs terminated at a solution only) | 44.20 | 200.50 | 44.90 | 50.95 | 130.33 | | | |
| \varnothing number of evaluations per run (for | 44.25 | 208-30 | 44.90 | 30.95 | 136 55 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.20 | 200.50 | 44.50 | 30.55 | 100.00 | | | |
| | Evaluation | ns of G | T | r | r | | | |
| \varnothing number of evaluations per run (for | 24 35 | 55.05 | 33.85 | 21 20 | 24 70 | | | |
| runs terminated at a solution only) | 21.00 | 55.05 | 00.00 | 21.20 | 24.10 | | | |
| \varnothing number of evaluations per run (for | 24 35 | 55,05 | 33 85 | 21 20 | 24 70 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 21.00 | 00.00 | 00.00 | 21.20 | 21.10 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $1.54\mathrm{s}$ | $3.89\mathrm{s}$ | $2.20\mathrm{s}$ | $1.23\mathrm{s}$ | $0.95\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $1.54{ m s}$ | $3.89\mathrm{s}$ | $2.20{ m s}$ | $1.23\mathrm{s}$ | $0.95{ m s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.94\mathrm{s}$ | $2.20\mathrm{s}$ | $1.42\mathrm{s}$ | $0.73\mathrm{s}$ | $0.33\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.14\mathrm{s}$ | $0.62\mathrm{s}$ | $0.13\mathrm{s}$ | $0.10\mathrm{s}$ | $0.42\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | r | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 steps are LPN (hybrid) | 100.004 | 100.004 | 100.004 | 100.007 | 100.007 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 95.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 70.0% | 70.0% | 50.0% | 95.0% | 50.0% | | | |

A10a

Dimensions: $N = 8, n_x = 24, m_g = 33, n = 90$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 19 | 15 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 15 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for runs terminated at a solution only) | 133.37 | 219.47 | 26.30 | 24.50 | 48.40 | | | |
| Tuns terminated at a solution only) | Further of F | | | | | | | |
| a number of evoluctions non-mup (for | | | | | | | | |
| runs terminated at a solution only) | 830.63 | 1255.33 | 30.10 | 26.75 | 182.75 | | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 830.63 | 1255.33 | 30.10 | 26.75 | 182.75 | | | |
| | Evaluation | is of G | 1 | 1 | 1 | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 133.37 | 219.47 | 26.30 | 24.50 | 48.40 | | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* < 10^{-8}$) | 133.37 | 219.47 | 26.30 | 24.50 | 48.40 | | | |
| CPU time | | | | | | | | |
| \emptyset CPU time per run (for runs termi- | 2.93 s | $4.62\mathrm{s}$ | $0.47\mathrm{s}$ | $0.43\mathrm{s}$ | $0.34\mathrm{s}$ | | | |
| \varnothing CPU time per run (for all runs ter- minated with $\varepsilon^* \le 10^{-8}$) | 2.93 s | 4.62 s | $0.47\mathrm{s}$ | $0.43\mathrm{s}$ | $0.34\mathrm{s}$ | | | |
| \varnothing CPU time for subproblems per run (for runs term. at a solution only) | $0.34\mathrm{s}$ | 0.49 s | $0.05\mathrm{s}$ | $0.05\mathrm{s}$ | $0.05\mathrm{s}$ | | | |
| \varnothing CPU time for line search per run (for runs terminated at a solution only) | $0.62\mathrm{s}$ | $0.89\mathrm{s}$ | $0.02\mathrm{s}$ | $0.02\mathrm{s}$ | $0.15\mathrm{s}$ | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size $= 1$ (glob. LPN) or last step is LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 step sizes $= 1$ (glob. LPN) or last 2 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 step sizes $= 1$ (glob. LPN) or last 3 steps are LPN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 95.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 57.9% | 100.0% | 85.0% | 95.0% | 85.0% | | | |

A10b

| $\begin{array}{c c c c c c c c c c c c c c c c c c c $ |
|---|
| $\begin{array}{c c c c c c c c c c c c c c c c c c c $ |
| $ \begin{array}{ c c c c c } \hline \varnothing & \text{number of iterations per run (for runs terminated at a solution only)} & 161.29 & 117.86 & 28.15 & 28.40 & 41.90 \\ \hline & \textbf{Evaluations of } F \\ \hline & \blacksquare & \textbf{Summer of evaluations per run (for runs terminated at a solution only)} & 967.18 & 966.71 & 39.50 & 37.35 & 206.95 \\ \hline & \square & \blacksquare & \square & \square$ |
| runs terminated at a solution only)101.29111.3023.1526.4041.50 \varXi number of evaluations per run (for runs terminated at a solution only)967.18966.7139.5037.35206.95 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)939.33966.7139.5037.35206.95 \varXi number of evaluations per run (for runs terminated at a solution only)161.29117.8628.1528.4041.50 \varnothing number of evaluations per run (for runs terminated at a solution only)161.29117.8628.1528.4041.50 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)157.50117.8628.1528.4041.50 \varnothing CPU time per run (for runs termi- nated at a solution only)11.27 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs ter- minated with $\varepsilon^* \leq 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time for subproblems per run (for runs term. at a solution only)6.52 s3.81 s1.16 s0.88 s0.63 s \varnothing CPU time for line search per run (for runs terminated at a solution only)1.78 s1.78 s0.08 s0.07 s0.38 s |
| Evaluations of F \varnothing number of evaluations per run (for runs terminated at a solution only)967.18966.7139.5037.35206.95 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)939.33966.7139.5037.35206.95 \varnothing number of evaluations per run (for runs terminated at a solution only)161.29117.8628.1528.4041.50 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)157.50117.8628.1528.4041.50 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)11.27 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for runs terminated at a solution only)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time for subproblems per run (for runs term. at a solution only)6.52 s3.81 s1.16 s0.88 s0.63 s \varnothing CPU time for line search per run (for runs terminated at a solution only)1.78 s1.78 s0.08 s0.07 s0.38 s |
| |
| runs terminated at a solution only) 501.10 500.11 500.11 500.11 500.00 501.50 200.50 \varnothing number of evaluations per run (for runs terminated at a solution only) 939.33 966.71 39.50 37.35 206.95 \varnothing number of evaluations per run (for runs terminated at a solution only) 161.29 117.86 28.15 28.40 41.50 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$) 157.50 117.86 28.15 28.40 41.50 \circlearrowright CPU time \circlearrowright CPU time per run (for runs terminated at a solution only) 11.27 s 7.67 s 1.74 s 1.47 s 1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$) 11.00 s 7.67 s 1.74 s 1.47 s 1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$) 6.52 s 3.81 s 1.16 s 0.88 s 0.63 s \circlearrowright CPU time for subproblems per run (for runs term. at a solution only) 6.52 s 3.81 s 1.16 s 0.88 s 0.63 s \circlearrowright CPU time for line search per run (for runs terminated at a solution only) 1.78 s 1.78 s 0.08 s 0.07 s 0.38 s |
| |
| all runs terminated with $\varepsilon^* \le 10^{-8}$)sourcesourcesourcesource \varnothing number of evaluations per run (for runs terminated at a solution only)161.29117.8628.1528.4041.50 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$)157.50117.8628.1528.4041.50 \varnothing CPU time per run (for runs terminated at a solution only)157.50117.8628.1528.4041.50 \oiint CPU time per run (for runs terminated at a solution only)11.27 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time for subproblems per run (for runs term. at a solution only)6.52 s3.81 s1.16 s0.88 s0.63 s \varnothing CPU time for line search per run (for runs terminated at a solution only)1.78 s1.78 s0.08 s0.07 s0.38 s |
| Evaluations of G \varnothing number of evaluations per run (for runs terminated at a solution only)161.29117.8628.1528.4041.50 \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)157.50117.8628.1528.4041.50CPU time \varnothing CPU time per run (for runs terminated at a solution only)11.27 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time for subproblems per run (for runs term. at a solution only)6.52 s3.81 s1.16 s0.88 s0.63 s \varnothing CPU time for line search per run (for runs terminated at a solution only)1.78 s1.78 s0.08 s0.07 s0.38 s |
| |
| runs terminated at a solution only)101101110010110< |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$ |
| all runs terminated with $\varepsilon^* \leq 10^{-6}$)DenoteDenoteDenoteDenoteDenote \varnothing CPU time per run (for runs terminated at a solution only)11.27 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time for subproblems per run (for runs term. at a solution only)6.52 s3.81 s1.16 s0.88 s0.63 s \varnothing CPU time for line search per run (for runs terminated at a solution only)1.78 s1.78 s0.08 s0.07 s0.38 s |
| \varnothing CPU time per run (for runs terminated at a solution only)11.27 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time for subproblems per run (for runs term. at a solution only)6.52 s3.81 s1.16 s0.88 s0.63 s \varnothing CPU time for line search per run (for runs terminated at a solution only)1.78 s1.78 s0.08 s0.07 s0.38 s |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$ |
| nated at a solution only)Image: CPU time per run (for all runs terminated with $\varepsilon^* \le 10^{-8}$)11.00 s7.67 s1.74 s1.47 s1.39 s \varnothing CPU time for subproblems per run (for runs term. at a solution only) $6.52 s$ $3.81 s$ $1.16 s$ $0.88 s$ $0.63 s$ \varnothing CPU time for line search per run (for runs terminated at a solution only) $1.78 s$ $1.78 s$ $0.08 s$ $0.07 s$ $0.38 s$ |
| $ \begin{array}{ c c c c c c c c } & \varnothing & CPU & \text{time per run (for all runs terminated with } \varepsilon^* \leq 10^{-8}) & 11.00 \text{s} & 7.67 \text{s} & 1.74 \text{s} & 1.47 \text{s} & 1.39 \text{s} \\ \hline & \varnothing & CPU & \text{time for subproblems per run} & 6.52 \text{s} & 3.81 \text{s} & 1.16 \text{s} & 0.88 \text{s} & 0.63 \text{s} \\ \hline & \varnothing & CPU & \text{time for line search per run (for runs terminated at a solution only)} & 1.78 \text{s} & 1.78 \text{s} & 0.08 \text{s} & 0.07 \text{s} & 0.38 \text{s} \\ \hline & \text{Local Convergence Bate (for runs terminated at a solution)} & \end{array} $ |
| minated with $\varepsilon^* \leq 10^{-6}$)6.52 s3.81 s1.16 s0.88 s0.63 sØ CPU time for subproblems per run (for runs term. at a solution only)6.52 s3.81 s1.16 s0.88 s0.63 sØ CPU time for line search per run (for runs terminated at a solution only)1.78 s1.78 s0.08 s0.07 s0.38 s |
| |
| (for runs term, at a solution only) Ø 0.08 s 0.07 s 0.38 s Ø CPU time for line search per run (for runs terminated at a solution only) 1.78 s 1.78 s 0.08 s 0.07 s 0.38 s Local Convergence Bate (for runs terminated at a solution) 0.08 s 0.07 s 0.38 s |
| \square CPU time for line search per run (for runs terminated at a solution only) 1.78 s 1.78 s 0.08 s 0.07 s 0.38 s Local Convergence Bate (for runs terminated at a solution) |
| Local Convergence Bate (for runs terminated at a solution) |
| \mathbf{L} |
| last star size 1 (slab LDN) an last |
| $\begin{bmatrix} \text{nast step size} = 1 \text{ (glob. LPN) or nast} \\ \text{step is LPN} \text{ (hybrid)} \end{bmatrix} 100.0\% \begin{bmatrix} 100.0\% \\ 100.0\% \end{bmatrix} 100.0\% \begin{bmatrix} 100.0\% \\ 100.0\% \end{bmatrix}$ |
| legt 2 step sizes = 1 (slob L DN) or |
| $\begin{bmatrix} 1351 & 2 & \text{step sizes} = 1 & (\text{glob. LFIV}) & 100.0\% & 100.0\% & 100.0\% & 100.0\% & 95.0\% \\ \end{bmatrix}$ |
| last 2 steps are LFN (hybrid) $ $ |
| $\begin{array}{c c c c c c c c c c c c c c c c c c c $ |
| Inst 5 steps are LL R (hybrid) last improvement of $ F $ is > 10 ² 100.0% 05.0% 100.0% |
| Instrumptovement of $ F _{100}$ is $> 10^3$ 88.9% 100.0% 95.0% 100.0% 100.0% last improvement of $ F _{100}$ is $> 10^3$ 88.9% 100.0% 80.0% 95.0% 100.0% |
| Instrumptorement of $ F _{\infty}$ is $> 10^4$ 64.7% 42.0% 70.0% 45.0% 55.0% |

A10c

Dimensions: $N = 37, n_x = 222, m_g = 260, n = 742$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|---------------|--------------|-------------------|---------|--------|--|--|--|--|
| # runs terminated at a solution | 0 | 0 | 8 | 20 | 0 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 5 | 0 | 20 | 20 | 0 | | | | |
| \varnothing number of iterations per run (for | | | 107.00 | 24.55 | | | | | |
| runs terminated at a solution only) | _ | _ | 107.00 | 34.55 | | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | | | 276 50 | 48 70 | | | | | |
| runs terminated at a solution only) | _ | _ | 270.30 | 40.70 | | | | | |
| \varnothing number of evaluations per run (for | 1758 40 | | 215 15 | 48 70 | | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1756.40 | _ | 515.15 | 40.70 | _ | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | | | 107.00 | 34 55 | | | | | |
| runs terminated at a solution only) | | | 107.00 | 04.00 | | | | | |
| \varnothing number of evaluations per run (for | 326.40 | | 131.25 | 34 55 | _ | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 520.40 | | 131.25 | 04.00 | | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | | _ | $17.50\mathrm{s}$ | 4 20 5 | | | | | |
| nated at a solution only) | | | 11.005 | 1.205 | | | | | |
| \varnothing CPU time per run (for all runs ter- | 71.05 s | _ | 26.57 s | 4 20 5 | | | | | |
| minated with $\varepsilon^* \le 10^{-8}$) | 11.005 | | 20.015 | 1.205 | | | | | |
| \varnothing CPU time for subproblems per run | _ | _ | 14 01 s | 3 21 s | _ | | | | |
| (for runs term. at a solution only) | | | 111015 | 0.215 | | | | | |
| \varnothing CPU time for line search per run (for | _ | _ | 0.75 s | 0.14 s | _ | | | | |
| runs terminated at a solution only) | | | 0.105 | 0.115 | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 0.0% | 0.0% | 100.0% | 100.0% | 0.0% | | | | |
| step is LPN (hybrid) | 0.070 | 0.070 | 100.070 | 100.070 | 0.070 | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 0.0% | 0.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 2 steps are LPN (hybrid) | 0.070 | 0.070 | 100.070 | 100.070 | 0.070 | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 0.0% | 0.0% | 87.5% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 0.070 | 0.070 | 01.070 | 100.070 | 0.070 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 0.0% | 0.0% | 0.0% | 100.0% | 0.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 0.0% | 0.0% | 0.0% | 85.0% | 0.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 0.0% | 0.0% | 0.0% | 30.0% | 0.0% | | | | |

A10d

Dimensions: $N = 37, n_x = 370, m_g = 408, n = 1186$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|--|-------------------|-------------|-------------------|-------------------|-------------------|--|--|--|--|
| # runs terminated at a solution | 10 | 0 | 15 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 12 | 0 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 138.40 | | 102.20 | 70.70 | 52.00 | | | | |
| runs terminated at a solution only) | 130.40 | | 102.20 | 10.10 | 52.90 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 1243 20 | _ | 323.20 | 155 70 | 223 15 | | | | |
| runs terminated at a solution only) | 1240.20 | | 525.20 | 100.10 | 223.10 | | | | |
| \varnothing number of evaluations per run (for | 1121.83 | | 283 85 | 155 70 | 223 15 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1121.00 | | 200.00 | 100.10 | 220.10 | | | | |
| | Evaluation | ns of G | T | 1 | r | | | | |
| \varnothing number of evaluations per run (for | 138 40 | _ | 102 20 | 70 70 | 52.05 | | | | |
| runs terminated at a solution only) | 100.10 | | 102.20 | 10.10 | 02.00 | | | | |
| \varnothing number of evaluations per run (for | 135.33 | | 94.00 | 70.70 | 52.05 | | | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) 100.00 100.00 100.00 02.00 | | | | | | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $52.53\mathrm{s}$ | _ | $54.07{ m s}$ | $20.15\mathrm{s}$ | $15.79\mathrm{s}$ | | | | |
| nated at a solution only) | | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $51.96\mathrm{s}$ | _ | $48.53\mathrm{s}$ | $20.15\mathrm{s}$ | $15.79\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | | |
| \varnothing CPU time for subproblems per run | $43.23\mathrm{s}$ | _ | $49.15\mathrm{s}$ | $17.04\mathrm{s}$ | $12.83\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $4.78\mathrm{s}$ | _ | $1.25\mathrm{s}$ | $0.62\mathrm{s}$ | $0.87\mathrm{s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 0.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 90.0% | 0.0% | 100.0% | 100.0% | 95.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or $I = I = I = I$ | 90.0% | 0.0% | 100.0% | 100.0% | 95.0% | | | | |
| last 3 steps are LPN (hybrid) | 100.007 | 0.007 | 100.007 | | 100.007 | | | | |
| $\begin{array}{c c} 1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^2 \\ \hline 1 \text{ , } \text$ | 100.0% | 0.0% | 100.0% | 95.0% | 100.0% | | | | |
| $1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^{3}$ | 80.0% | 0.0% | 80.0% | 90.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 60.0% | 0.0% | 40.0% | 15.0% | 65.0% | | | | |

A10e

Dimensions: $N = 48, n_x = 576, m_g = 625, n = 1826$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|--------------------|--------------------|-------------------|-------------------|--|--|--|
| # runs terminated at a solution | 8 | 1 | 17 | 16 | 14 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 10 | 1 | 20 | 16 | 14 | | | |
| \varnothing number of iterations per run (for | 164.88 | 220.00 | 207 71 | 106.00 | 55.14 | | | |
| runs terminated at a solution only) | 104.00 | 239.00 | 201.11 | 100.00 | 55.14 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 834.00 | 1240.00 | 835 35 | 287.06 | 247 79 | | | |
| runs terminated at a solution only) | 004.00 | 1240.00 | 000.00 | 201.00 | 241.13 | | | |
| \varnothing number of evaluations per run (for | 720.40 | 1240.00 | 768 35 | 287.06 | 247.79 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 120.10 | 1240.00 | 100.00 | 201.00 | 241.15 | | | |
| | Evaluation | ns of G | 1 | 1 | 1 | | | |
| \varnothing number of evaluations per run (for | 164 88 | 239.00 | 207 71 | 106.00 | 54.64 | | | |
| runs terminated at a solution only) | 101.00 | 200.00 | 201.11 | 100.00 | 01.01 | | | |
| \varnothing number of evaluations per run (for | 147.60 | 239.00 | 195.50 | 106.00 | 54.64 | | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) all runs terminated with $\varepsilon^* \le 10^{-8}$. | | | | | | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $151.98\mathrm{s}$ | $200.59\mathrm{s}$ | $324.51\mathrm{s}$ | $79.52\mathrm{s}$ | $28.97\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $137.59\mathrm{s}$ | $200.59\mathrm{s}$ | $301.19\mathrm{s}$ | $79.52\mathrm{s}$ | $28.97\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \bigotimes CPU time for subproblems per run | $138.93\mathrm{s}$ | $181.68\mathrm{s}$ | $308.06\mathrm{s}$ | $71.43\mathrm{s}$ | $23.49\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| & CPU time for line search per run (for | $3.81\mathrm{s}$ | $5.66\mathrm{s}$ | $3.85\mathrm{s}$ | $1.45\mathrm{s}$ | $1.34\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| | e Rate (for i | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (nybrid) | | | | | | | | |
| 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 last 2 step sizes = 1 (glob. LPN) or 1 step sizes = 1 step sizes = 1 (glob. LPN) or 1 step sizes = 1 (glob. LPN) or 1 step sizes = 1 (glob. LPN) or 1 step sizes = 1 | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (nybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or last 2 steps are LDN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 04.107 | 02.007 | 100.007 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% 97 E07 | | 94.1% 50.007 | 93.8% 02.0% | 100.0% | | | |
| $\begin{array}{c c} \text{ last improvement of } \ F\ _{\infty} \text{ is } \geq 10^{\circ} \\ \hline \\ \text{ last improvement of } \ F\ _{\infty} \text{ is } \geq 10^{4} \\ \hline \end{array}$ | 81.3% 25.0% | 100.0% | | 95.8% | | | | |
| 1 ast improvement of $ F _{\infty}$ is $\geq 10^4$ | 25.0% | 100.0% | 17.0% | 12.5% | 35.7% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|------------------|---------------|--------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | Q 15 | 5.05 | 9 15 | 5.05 | 10.20 | | | | |
| runs terminated at a solution only) | 0.15 | 0.90 | 0.15 | 0.90 | 10.20 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 0.15 | 6.05 | 0.15 | 6.05 | 11.20 | | | | |
| runs terminated at a solution only) | 9.10 | 0.95 | 9.10 | 0.95 | 11.20 | | | | |
| \varnothing number of evaluations per run (for | 9.15 | 6.95 | 9.15 | 6 95 | 11.20 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 3.10 | 0.90 | 9.10 | 0.90 | 11.20 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 8 15 | 5 95 | 8 15 | 5 95 | 10.20 | | | | |
| runs terminated at a solution only) | 0.10 | 0.50 | 0.10 | 0.00 | 10.20 | | | | |
| \varnothing number of evaluations per run (for | 8 15 | 5 95 | 8 15 | 5 95 | 10.20 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.10 | 0.50 | 0.10 | 0.00 | 10.20 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.13 s | $0.09\mathrm{s}$ | 0.13 s | 0.14 s | $0.06\mathrm{s}$ | | | | |
| nated at a solution only) | 0.10.5 | 0.00 5 | 0.10.5 | | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.13 s | $0.09\mathrm{s}$ | 0.13 s | 0.14 s | $0.06\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.10.5 | 0.005 | 0.10.5 | 0.110 | 0.000 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 1000070 | 100.070 | 100.070 | 100.070 | 0.070 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 75.0% | 95.0% | 75.0% | 95.0% | 100.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 7.45 | 5.65 | 7.45 | 5 65 | 10.45 | | | | |
| runs terminated at a solution only) | 1.40 | 0.00 | 1.40 | 5.05 | 10.45 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 8.45 | 6 65 | 8.45 | 6 65 | 1/1.85 | | | | |
| runs terminated at a solution only) | 0.40 | 0.00 | 0.40 | 0.00 | 14.00 | | | | |
| \varnothing number of evaluations per run (for | 8 45 | 6 65 | 8 45 | 6 65 | 14 85 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.40 | 0.00 | 0.10 | 0.00 | 14.00 | | | | |
| | Evaluation | ns of G | 1 | | 1 | | | | |
| \varnothing number of evaluations per run (for | 7 45 | 5 65 | 7 45 | 5.65 | 10.45 | | | | |
| runs terminated at a solution only) | | 0.00 | | | 10.10 | | | | |
| \varnothing number of evaluations per run (for | 7.45 | 5.65 | 7.45 | 5.65 | 10.45 | | | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) all runs terminated with $\varepsilon^* \le 10^{-8}$. | | | | | | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12{ m s}$ | $0.09\mathrm{s}$ | $0.12\mathrm{s}$ | $0.10\mathrm{s}$ | $0.05\mathrm{s}$ | | | | |
| nated at a solution only) | | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.12{ m s}$ | $0.09\mathrm{s}$ | $0.12\mathrm{s}$ | $0.10\mathrm{s}$ | $0.05\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.02\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | [| | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{DN} | 100.0% | 100.0% | 100.0% | 100.0% | 45.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or $1 + 2 + 1 + 1$ | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| Last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 100.007 | 100 007 | 100.007 | | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{3}$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 80.0% | 70.0% | 80.0% | 70.0% | 100.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 7 85 | 8.00 | 7 85 | 8.00 | 10 75 | | | |
| runs terminated at a solution only) | 1.00 | 0.00 | 1.00 | 0.00 | 10.10 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 8 85 | 9.00 | 8 85 | 9.00 | 14 35 | | | |
| runs terminated at a solution only) | 0.00 | 5.00 | 0.00 | 5.00 | 14.00 | | | |
| \varnothing number of evaluations per run (for | 8 85 | 9.00 | 8 85 | 9.00 | $14\ 35$ | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 5.00 | 0.00 | 5.00 | 11.00 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 7 85 | 8.00 | 7 85 | 8.00 | 10.75 | | | |
| runs terminated at a solution only) | 1.00 | 0.00 | 1.00 | 0.00 | 10.110 | | | |
| \varnothing number of evaluations per run (for | 7.85 | 8.00 | 7.85 | 8.00 | 10.75 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 0.00 | 1.00 | 0.00 | 10.10 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.13 s | 0.13 s | 0.13 s | 0.13 s | $0.06\mathrm{s}$ | | | |
| nated at a solution only) | 0.100 | 0.105 | 0.10.5 | 0.10.5 | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.13\mathrm{s}$ | $0.13\mathrm{s}$ | $0.13\mathrm{s}$ | $0.13\mathrm{s}$ | $0.06{ m s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.10.5 | 0.10.5 | 0.10.5 | 0.10.5 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 95.0% | 100.0% | 95.0% | 100.0% | 100.0% | | | |

A14

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|--|--|
| # runs terminated at a solution | 18 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 18 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 11.00 | 0.10 | 11.85 | 0.20 | 13.15 | | | | |
| runs terminated at a solution only) | 11.00 | 9.10 | 11.05 | 9.20 | 15.15 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 12 33 | 10.25 | 12.95 | 10.20 | 18.80 | | | | |
| runs terminated at a solution only) | 12.00 | 10.20 | 12.50 | 10.20 | 10.00 | | | | |
| \varnothing number of evaluations per run (for | 12.33 | 10.25 | 12.95 | 10.20 | 18.80 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.00 | 10.20 | 12.50 | 10.20 | 10.00 | | | | |
| | Evaluation | ns of G | 1 | 1 | 1 | | | | |
| \varnothing number of evaluations per run (for | 11.00 | 9 10 | 11.85 | 9 20 | 13 15 | | | | |
| runs terminated at a solution only) | 11.00 | 0.10 | | 0.20 | 10.10 | | | | |
| \varnothing number of evaluations per run (for | 11.00 | 9.10 | 11.85 | 9.20 | 13.15 | | | | |
| all runs terminated with $\varepsilon^* \le 10^{-8}$) all runs terminated with $\varepsilon^* \le 10^{-8}$. | | | | | | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.18\mathrm{s}$ | $0.15\mathrm{s}$ | $0.20\mathrm{s}$ | $0.16\mathrm{s}$ | $0.09\mathrm{s}$ | | | | |
| nated at a solution only) | | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.18\mathrm{s}$ | $0.15\mathrm{s}$ | $0.20\mathrm{s}$ | $0.16\mathrm{s}$ | $0.09\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.04\mathrm{s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | | | | |
| runs terminated at a solution only) | | | | | | | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or 1 last 3 step sizes = 1 | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{3}$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 77.8% | 90.0% | 85.0% | 95.0% | 20.0% | | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|---|------------------|---------------|--------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 8.00 | 7.85 | 8.00 | 7.85 | 12 70 | | | | |
| runs terminated at a solution only) | 0.90 | 1.00 | 8.90 | 1.00 | 15.70 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 0.05 | 8 85 | 9 90 | 8 85 | 40.00 | | | | |
| runs terminated at a solution only) | 9.90 | 0.00 | 9.90 | 0.00 | 40.00 | | | | |
| \varnothing number of evaluations per run (for | 9.95 | 8 85 | 9 90 | 8 85 | 40.00 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 5.50 | 0.00 | 5.50 | 0.00 | 40.00 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 8 90 | 7 85 | 8 90 | 7 85 | 13 70 | | | | |
| runs terminated at a solution only) | 0.50 | 1.00 | 0.50 | 1.00 | 10.10 | | | | |
| \varnothing number of evaluations per run (for | 8 90 | 7 85 | 8 90 | 7 85 | 13 70 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | $* \le 10^{-8}$) 0.50 1.05 0.50 1.05 10.10 | | | | | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.15 s | $0.12\mathrm{s}$ | 0.14 s | 0.13 s | $0.06\mathrm{s}$ | | | | |
| nated at a solution only) | 0.100 | 0.120 | 0.110 | 0.10.5 | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.15 s | $0.12{ m s}$ | 0.14 s | 0.13 s | $0.06{ m s}$ | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.100 | 0.125 | 0.110 | 0.10.5 | 0.000 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| (for runs term. at a solution only) | | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | | | |
| runs terminated at a solution only) | | 0.000 | | | 0.02.5 | | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | / 0 | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 20.0% | 95.0% | 20.0% | 95.0% | 100.0% | | | | |

A16a

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|--------------------|--------------|---------------|--------------|------------------|--|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | | |
| \varnothing number of iterations per run (for | 6.85 | 6.20 | 6.85 | 6.20 | 15.15 | | | | |
| runs terminated at a solution only) | 0.00 | 0.20 | 0.00 | 0.20 | 10.10 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | 7.85 | 7.20 | 7.85 | 7.20 | 40.85 | | | | |
| runs terminated at a solution only) | 1.00 | 1.20 | 1.00 | 1.20 | 40.00 | | | | |
| \varnothing number of evaluations per run (for | 7 85 | 7 20 | 7.85 | 7 20 | 40.85 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 1.20 | 1.00 | 1.20 | 40.00 | | | | |
| | Evaluation | ns of G | | | | | | | |
| \varnothing number of evaluations per run (for | 6.85 | 6.20 | 6.85 | 6.20 | 15 15 | | | | |
| runs terminated at a solution only) | 0.00 | 0.20 | 0.00 | 0.20 | 10.10 | | | | |
| \varnothing number of evaluations per run (for | 6.85 | 6 20 | 6.85 | 6 20 | 15 15 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 0.20 | 0.00 | 0.20 | 10.10 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.11 s | 0.10 s | 0.11 s | 0.10 s | $0.08\mathrm{s}$ | | | | |
| nated at a solution only) | 0.110 | 0.105 | 0.115 | 0.105 | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.11 s | 0.10 s | 0.11 s | 0.10 s | 0.08 s | | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.110 | 0.105 | 0.115 | 0.105 | 0.000 | | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03{ m s}$ | | | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.005 | 0.005 | 0.000 | | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | 0.01 s | | | | |
| runs terminated at a solution only) | | 0.005 | 0.000 | | 0.015 | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| step is LPN (hybrid) | | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 55.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | | |
| last 3 steps are LPN (hybrid) | 100 004 | 100 007 | 100 007 | 100 007 | 100.004 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | | |

A16b

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|------------------|---------------|------------------|------------------|------------------|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | |
| \varnothing number of iterations per run (for | 7.00 | 6.05 | 7.00 | 6.05 | 17 10 | | |
| runs terminated at a solution only) | 7.00 | 0.95 | 7.00 | 0.95 | 17.10 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | 8.00 | 7.95 | 8.00 | 7.95 | 47.60 | | |
| runs terminated at a solution only) | 0.00 | 1.30 | 0.00 | 1.30 | 47.00 | | |
| \varnothing number of evaluations per run (for | 8.00 | 7.95 | 8.00 | 7.95 | 47.60 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 1.50 | 0.00 | 1.50 | 41.00 | | |
| | Evaluation | ns of G | | | | | |
| \varnothing number of evaluations per run (for | 7.00 | 6.95 | 7.00 | 6.95 | 17 10 | | |
| runs terminated at a solution only) | 1.00 | 0.50 | 1.00 | 0.50 | 17.10 | | |
| \varnothing number of evaluations per run (for | 7.00 | 6 95 | 7.00 | 6 95 | 17 10 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 0.50 | 1.00 | 0.50 | 11.10 | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.12 s | 0.11 s | 0.11 s | 0.11 s | 0.095 | | |
| nated at a solution only) | 0.125 | 0.115 | 0.115 | 0.115 | 0.005 | | |
| \varnothing CPU time per run (for all runs ter- | $0.12\mathrm{s}$ | 0.11 s | 0.11 s | 0.11 s | $0.09\mathrm{s}$ | | |
| minated with $\varepsilon^* \le 10^{-8}$) | 0.125 | 0.115 | 0.115 | 0.115 | 0.005 | | |
| \varnothing CPU time for subproblems per run | $0.00{\rm s}$ | $0.00{\rm s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.03{\rm s}$ | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.005 | 0.005 | 0.000 | | |
| \varnothing CPU time for line search per run (for | $0.00{\rm s}$ | $0.00{\rm s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.02{\rm s}$ | | |
| runs terminated at a solution only) | | 0.000 | 0.000 | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | 1 | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | 1000070 | 1000070 | 100.070 | 1000070 | 100.070 | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 65.0% | | |
| last 2 steps are LPN (hybrid) | 100.070 | 100.070 | 100.070 | 100.070 | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| last 3 steps are LPN (hybrid) | 100.070 | 100.070 | 100.070 | 100.070 | 0.070 | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 90.0% | | |

A16c

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | |
|---|------------------|--------------|---------------|------------------|------------------|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | |
| \varnothing number of iterations per run (for | 7.00 | 6.05 | 7.00 | 6.05 | 18 55 | | |
| runs terminated at a solution only) | 1.00 | 0.35 | 7.00 | 0.35 | 10.00 | | |
| | Evaluation | ns of F | | | | | |
| \varnothing number of evaluations per run (for | 8.00 | 7.95 | 8.00 | 7.95 | 51.25 | | |
| runs terminated at a solution only) | 0.00 | 1.30 | 0.00 | 1.30 | 01.20 | | |
| \varnothing number of evaluations per run (for | 8.00 | 7 95 | 8.00 | 7 95 | 51 25 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 1.50 | 0.00 | 1.50 | 01.20 | | |
| | Evaluation | ns of G | | | | | |
| \varnothing number of evaluations per run (for | 7.00 | 6 95 | 7.00 | 6 95 | 18 55 | | |
| runs terminated at a solution only) | 1.00 | 0.50 | 1.00 | 0.50 | 10.00 | | |
| \varnothing number of evaluations per run (for | 7.00 | 6 95 | 7.00 | 6 95 | 18 55 | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.00 | 0.50 | 1.00 | 0.50 | 10.00 | | |
| CPU time | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12\mathrm{s}$ | 0.11 s | 0.11 s | $0.12{\rm s}$ | $0.09\mathrm{s}$ | | |
| nated at a solution only) | 0.120 | 0.115 | 0.115 | 0.120 | 0.000 | | |
| \varnothing CPU time per run (for all runs ter- | 0.12 s | 0.11 s | 0.11 s | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.120 | 0.115 | 0.115 | 0.120 | 0.005 | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.005 | 0.005 | 0.025 | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | $0.01{ m s}$ | $0.00{ m s}$ | $0.02{ m s}$ | | |
| runs terminated at a solution only) | | 0.000 | 0.015 | | 0.025 | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| step is LPN (hybrid) | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 30.0% | | |
| last 2 steps are LPN (hybrid) | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | |
| last 3 steps are LPN (hybrid) | 100.070 | | 200.070 | | 0.070 | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 80.0% | | |

A16d

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|--------------|------------------|--------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 7.40 | 7.85 | 7.40 | 7.85 | 15 75 | | | |
| runs terminated at a solution only) | 1.40 | 1.00 | 1.40 | 1.00 | 10.70 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 8.40 | 8 85 | 8 40 | 8 85 | 45.15 | | | |
| runs terminated at a solution only) | 0.40 | 0.00 | 0.40 | 0.00 | 40.10 | | | |
| \varnothing number of evaluations per run (for | 8 40 | 8 85 | 8 40 | 8 85 | 45 15 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.40 | 0.00 | 0.40 | 0.00 | 40.10 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 7 40 | 7 85 | 7 40 | 7 85 | 15 75 | | | |
| runs terminated at a solution only) | 1.10 | 1.00 | 1.10 | 1.00 | 10.10 | | | |
| \varnothing number of evaluations per run (for | 7 40 | 7 85 | 7 40 | 7 85 | 15 75 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1.10 | 1.00 | 1.10 | 1.00 | 10.10 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12\mathrm{s}$ | 0.13 s | $0.12\mathrm{s}$ | 0.13 s | $0.09\mathrm{s}$ | | | |
| nated at a solution only) | 0.120 | 0.105 | 0.120 | 0.105 | 0.000 | | | |
| \varnothing CPU time per run (for all runs ter- | 0.12 s | 0.13 s | $0.12\mathrm{s}$ | 0.13 s | $0.09\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-8}$) | 0.120 | 0.105 | 0.125 | 0.105 | 0.005 | | | |
| \varnothing CPU time for subproblems per run | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.00{ m s}$ | $0.03{ m s}$ | | | |
| (for runs term. at a solution only) | 0.002 | 0.005 | 0.005 | 0.005 | 0.000 | | | |
| \varnothing CPU time for line search per run (for | $0.00{ m s}$ | $0.00{ m s}$ | 0.01 s | $0.00{ m s}$ | $0.02{ m s}$ | | | |
| runs terminated at a solution only) | | 0.005 | 0.015 | | 0.025 | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size $= 1$ (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 80.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 95.0% | | | |

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 0.65 | 6 60 | 0.65 | 6 60 | 15 50 | | | |
| runs terminated at a solution only) | 9.05 | 0.00 | 9.05 | 0.00 | 15.50 | | | |
| | Evaluation | ns of F | | | | | | |
| \varnothing number of evaluations per run (for | 10.65 | 7.60 | 10.65 | 7.60 | 24 30 | | | |
| runs terminated at a solution only) | 10.05 | 7.00 | 10.05 | 7.00 | 24.50 | | | |
| \varnothing number of evaluations per run (for | 10.65 | 7 60 | 10.65 | 7 60 | 24 30 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.05 | 1.00 | 10.00 | 1.00 | 24.50 | | | |
| | Evaluation | ns of G | r | r | 1 | | | |
| \varnothing number of evaluations per run (for | 9.65 | 6 60 | 9.65 | 6 60 | 15 50 | | | |
| runs terminated at a solution only) | 5.00 | 0.00 | 5.00 | 0.00 | 10.00 | | | |
| \varnothing number of evaluations per run (for | 9.65 | 6 60 | 9.65 | 6 60 | 15 50 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 5.00 | 0.00 | 5.00 | 0.00 | 10.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.15 s | 0.11 s | $0.15{ m s}$ | 0.11 s | 0.10 s | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.15 s | 0.11 s | $0.15{ m s}$ | 0.11 s | 0.10 s | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.06\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | 0.02.0 | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 steps are LPN (hybrid) | 100.004 | 100.004 | 100.007 | 100.007 | 100.007 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 70.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 80.0% | 95.0% | 80.0% | 95.0% | 5.0% | | | |

Dimensions: $N = 2, n_x = 12, m_g = 28, n = 68$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 9 | 20 | 9 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 12.65 | 0.67 | 19.15 | 0.67 | 17.00 | | | |
| runs terminated at a solution only) | 12.05 | 9.07 | 12.10 | 9.07 | 17.90 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 16.40 | 10.67 | 13 15 | 10.67 | 96 70 | | | |
| runs terminated at a solution only) | 10.40 | 10.07 | 10.10 | 10.07 | 30.10 | | | |
| \varnothing number of evaluations per run (for | 16 40 | 8 30 | 13 15 | 8 30 | 96 70 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.40 | 0.00 | 10.10 | 0.00 | 50.10 | | | |
| | Evaluation | ns of G | r | r | | | | |
| \varnothing number of evaluations per run (for | 12.65 | 9.67 | 12 15 | 9.67 | 17 90 | | | |
| runs terminated at a solution only) | 12.00 | 5.01 | 12.10 | 5.01 | 11.50 | | | |
| \varnothing number of evaluations per run (for | 12.65 | 7.85 | 12.15 | 7.85 | 17.90 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 12.00 | 1.00 | 12.10 | 1.00 | 11.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | 0.21 s | $0.17{ m s}$ | $0.20{ m s}$ | $0.17{ m s}$ | $0.13\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | 0.21 s | 0.13 s | $0.20{ m s}$ | 0.13 s | $0.13\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.02\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.05\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.04\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | 0.0 | | | |
| Local Convergence | e Rate (for 1 | runs termina | ted at a solu | ition) | F | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | | | | | 01070 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 55.0% | 55.6% | 85.0% | 55.6% | 100.0% | | | |

Harker

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 0.30 | 6.00 | 0.30 | 6.00 | 13.35 | | | |
| runs terminated at a solution only) | 9.30 | 0.90 | 9.50 | 0.90 | 15.55 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 10.30 | 7 90 | 10.30 | 7 90 | 28.15 | | | |
| runs terminated at a solution only) | 10.50 | 1.50 | 10.50 | 1.50 | 20.10 | | | |
| \varnothing number of evaluations per run (for | 10.30 | 7 90 | 10.30 | 7 90 | 28.15 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.00 | 1.50 | 10.00 | 1.50 | 20.10 | | | |
| | Evaluation | ns of G | 1 | 1 | | | | |
| \varnothing number of evaluations per run (for | 9.30 | 6 90 | 9 30 | 6 90 | 13 35 | | | |
| runs terminated at a solution only) | 0.00 | 0.00 | 0.00 | 0.00 | 10.00 | | | |
| \varnothing number of evaluations per run (for | 9.30 | 6.90 | 9.30 | 6.90 | 13.35 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 0.00 | 0.00 | 0.000 | 10100 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.15\mathrm{s}$ | $0.11\mathrm{s}$ | $0.14\mathrm{s}$ | $0.11\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.15\mathrm{s}$ | $0.11\mathrm{s}$ | $0.14\mathrm{s}$ | $0.11\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \bigotimes CPU time for subproblems per run | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.03\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{PN} | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or L_{1} | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| $\frac{1}{1} \frac{1}{1} \frac{1}$ | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | | 100.0% | | | | | | |
| $1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^{6}$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| 1 last improvement of $ F _{\infty}$ is $\geq 10^4$ | 90.0% | 95.0% | 90.0% | 95.0% | 100.0% | | | |

Heu

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|--------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 10 | 20 | 15 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 17.00 | 12.80 | 14.95 | 15.60 | 32.55 | | | |
| runs terminated at a solution only) | | | | | | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 22.20 | 13.90 | 16.10 | 16.60 | 123.85 | | | |
| <i>C</i> number of evaluations non-mup (for | | | | | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$ | 22.20 | 25.25 | 16.10 | 20.15 | 123.85 | | | |
| | Evaluation | is of G | | | 1 | | | |
| \varnothing number of evaluations per run (for | 17.00 | 10.00 | 14.05 | 15 00 | 22 55 | | | |
| runs terminated at a solution only) | 17.00 | 12.80 | 14.95 | 15.60 | 32.55 | | | |
| \varnothing number of evaluations per run (for | 17.00 | 15 85 | 14 95 | 19 15 | 3255 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 11.00 | 10.00 | 11.00 | 10.10 | 02.00 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.27\mathrm{s}$ | $0.20{ m s}$ | $0.24\mathrm{s}$ | $0.25\mathrm{s}$ | $0.15{ m s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.27\mathrm{s}$ | $0.25\mathrm{s}$ | $0.24\mathrm{s}$ | $0.32\mathrm{s}$ | $0.15\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.01\mathrm{s}$ | $0.03\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.00\mathrm{s}$ | $0.01\mathrm{s}$ | $0.05\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | (;) | | | | |
| | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (nybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or last 2 steps are LDN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 35.0% | | | |
| last 2 steps are LPN (nybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or last 2 steps are LDN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 5 steps are LFN (hybrid) | 100.007 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| last improvement of $ F _{\infty}$ is ≥ 10 | 100.070 | 100.070 | 100.070 | 100.070 | 100.070 | | | |
| $\frac{1351 \text{ improvement of } \ F\ _{\infty} \text{ is } \geq 10^{4}}{104}$ | 100.070 | 100.07 | 100.07 | 100.07 | | | | |
| as improvement of $ F _{\infty}$ is $\geq 10^{12}$ | 89.0% | 100.0% | 85.0% | 93.3% | 90.0% | | | |

Lob

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | | |
|---|-------------|------------------|---------------|--------|------------------|--|--|--|--|
| # runs terminated at a solution | 0 | 20 | 0 | 0 | 20 | | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 0 | 20 | 0 | 0 | 20 | | | | |
| \varnothing number of iterations per run (for | | 24.70 | | | 22.05 | | | | |
| runs terminated at a solution only) | _ | 24.70 | | _ | 22.05 | | | | |
| Evaluations of F | | | | | | | | | |
| \varnothing number of evaluations per run (for | | 36 50 | | | 55 15 | | | | |
| runs terminated at a solution only) | | 50.50 | | | 55.15 | | | | |
| \varnothing number of evaluations per run (for | | 36 50 | _ | _ | 55 15 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | | 50.50 | | | 55.15 | | | | |
| | Evaluation | ns of G | - | - | - | | | | |
| \varnothing number of evaluations per run (for | _ | 24 70 | _ | | 22.05 | | | | |
| runs terminated at a solution only) | | 24.10 | | | 22.00 | | | | |
| \varnothing number of evaluations per run (for | _ | 24 70 | | | 22.05 | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | | 24.10 | | | 22.00 | | | | |
| CPU time | | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | _ | 0.59 s | _ | _ | 0.35 s | | | | |
| nated at a solution only) | | 0.005 | | | 0.0015 | | | | |
| \varnothing CPU time per run (for all runs ter- | _ | $0.59\mathrm{s}$ | _ | _ | $0.35\mathrm{s}$ | | | | |
| minated with $\varepsilon^* \le 10^{-8}$) | | 0.005 | | | 0.0015 | | | | |
| \varnothing CPU time for subproblems per run | _ | 0.14 s | _ | _ | 0.13 s | | | | |
| (for runs term. at a solution only) | | 0.115 | | | 0.105 | | | | |
| \varnothing CPU time for line search per run (for | _ | 0.06 s | _ | _ | 0.11 s | | | | |
| runs terminated at a solution only) | | 0.005 | | | 0.115 | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | tion) | | | | | |
| last step size $= 1$ (glob. LPN) or last | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | | | |
| step is LPN (hybrid) | 0.070 | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | | | |
| last 3 steps are LPN (hybrid) | 0.070 | 1000070 | 0.070 | 0.070 | 100.070 | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^2$ | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 0.0% | 100.0% | 0.0% | 0.0% | 100.0% | | | | |

NTF1

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for | 7 50 | 5.45 | 7 50 | 5.45 | 10.05 | | | |
| runs terminated at a solution only) | 1.00 | 0.10 | 1.00 | 0.40 | 10.00 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 8 50 | 6 45 | 8 50 | 6 45 | 11 10 | | | |
| runs terminated at a solution only) | 0.00 | 0.10 | 0.00 | 0.10 | 11.10 | | | |
| \varnothing number of evaluations per run (for | 8 50 | 6 45 | 8 50 | 6 45 | 11 10 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.00 | 0.10 | 0.00 | 0.10 | 11.10 | | | |
| | Evaluation | ns of G | 1 | 1 | 1 | | | |
| \varnothing number of evaluations per run (for | 7.50 | 5.45 | 7.50 | 5.45 | 10.05 | | | |
| runs terminated at a solution only) | | | | | 10100 | | | |
| \varnothing number of evaluations per run (for | 7.50 | 5.45 | 7.50 | 5.45 | 10.05 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | | | | | | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | $0.12\mathrm{s}$ | $0.10{ m s}$ | $0.07\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $0.12\mathrm{s}$ | $0.09\mathrm{s}$ | $0.12\mathrm{s}$ | $0.10\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.05\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | $0.00\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 100.00 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F' _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 80.0% | 100.0% | 80.0% | 100.0% | 100.0% | | | |

NTF2

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|--------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 20 | 20 | 20 | 20 | 20 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 20 | 20 | 20 | 20 | 20 | | | |
| \varnothing number of iterations per run (for runs terminated at a solution only) | 9.10 | 26.65 | 9.15 | 9.55 | 13.45 | | | |
| Tuns terminated at a solution only) | Evaluation | s of F | | | | | | |
| Ø number of evaluations per run (for | | | | | | | | |
| runs terminated at a solution only) | 10.25 | 130.35 | 10.15 | 10.60 | 19.05 | | | |
| \varnothing number of evaluations per run (for all runs terminated with $\varepsilon^* < 10^{-8}$) | 10.25 | 130.35 | 10.15 | 10.60 | 19.05 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 0.10 | 2.2.27 | 0.1.7 | | 10.17 | | | |
| runs terminated at a solution only) | 9.10 | 26.65 | 9.15 | 9.55 | 13.45 | | | |
| \varnothing number of evaluations per run (for | 9 10 | 26.65 | 9 15 | 9 55 | 13 45 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 0.10 | 20.00 | 0.10 | 0.00 | 10.10 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $0.14\mathrm{s}$ | $0.42\mathrm{s}$ | $0.14\mathrm{s}$ | $0.15\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- minated with $\varepsilon^* \le 10^{-8}$) | $0.14\mathrm{s}$ | $0.42\mathrm{s}$ | $0.14\mathrm{s}$ | $0.15\mathrm{s}$ | $0.07\mathrm{s}$ | | | |
| \varnothing CPU time for subproblems per run | 0.00 a | 0.00 a | 0.00 a | 0.00 a | 0.02 - | | | |
| (for runs term. at a solution only) | 0.00 s | 0.00 s | 0.00 s | 0.00 s | 0.05 S | | | |
| \varnothing CPU time for line search per run (for | 0.00 s | 0.03 s | 0.00 s | 0.00 s | 0.01s | | | |
| runs terminated at a solution only) | 0.003 | 0.005 | 0.005 | 0.003 | 0.015 | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ution) | 1 | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 95.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or 1 + 2 + (1 + 1) 1 + 1 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 | 100.0% | 95.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 100.004 | 100.004 | 100.004 | 100.004 | 100.007 | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^3$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |

Spam

Dimensions: $N = 101, n_x = 2020, m_g = 4040, n = 10100$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|---|--------------------|--------------------|--------------------|--------------------|-------------------|--|--|--|
| # runs terminated at a solution | 19 | 20 | 20 | 20 | 19 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 20 | 20 | 20 | 19 | | | |
| \varnothing number of iterations per run (for | 15.05 | 14.40 | 75.00 | 52 70 | 14.05 | | | |
| runs terminated at a solution only) | 15.95 | 14.40 | 75.90 | 55.70 | 14.95 | | | |
| | Evaluation | is of F | | | | | | |
| \varnothing number of evaluations per run (for | 32.68 | 26 10 | 103 25 | 75.45 | 35.47 | | | |
| runs terminated at a solution only) | 52.00 | 20.10 | 105.25 | 10.40 | 55.47 | | | |
| \varnothing number of evaluations per run (for | 32.68 | 26 10 | 103 25 | 75 45 | 3547 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 02.00 | 20.10 | 100.20 | 10.10 | 50.11 | | | |
| | Evaluation | ns of G | T | r | 1 | | | |
| \varnothing number of evaluations per run (for | 15 95 | 14 40 | 75 90 | 53 70 | 14 95 | | | |
| runs terminated at a solution only) | 10.00 | 11.10 | 10.00 | | 11.50 | | | |
| \varnothing number of evaluations per run (for | 15.95 | 14.40 | 75.90 | 53.70 | 14.95 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10:00 | | | | 1100 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $193.09\mathrm{s}$ | $157.12\mathrm{s}$ | $656.70\mathrm{s}$ | $528.40\mathrm{s}$ | $51.32\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $193.09\mathrm{s}$ | $157.12\mathrm{s}$ | $656.70\mathrm{s}$ | $528.40\mathrm{s}$ | $51.32\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $177.65\mathrm{s}$ | $142.77\mathrm{s}$ | $588.02\mathrm{s}$ | $476.45\mathrm{s}$ | $34.44\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.60\mathrm{s}$ | $0.47\mathrm{s}$ | $1.92\mathrm{s}$ | $1.41\mathrm{s}$ | $0.94\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes = 1 (glob. LPN) or L_{PN} | 100.0% | 100.0% | 100.0% | 100.0% | 73.7% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| 1 last 3 step sizes = 1 (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 0.0% | | | |
| last 3 steps are LPN (hybrid) | 0.007 | 0.007 | F 007 | 00.007 | 21 007 | | | |
| 1ast improvement of $ F _{\infty}$ is $\geq 10^2$ | 0.0% | 0.0% | 0.0% | 20.0% | 31.0% 0.0% | | | |
| $\begin{array}{c c} \text{Iast improvement of } \ F\ _{\infty} \text{ is } \geq 10^{6} \\ \hline 1 \text{ is } \text{ is } \text{ is } 10^{4} \\ \end{array}$ | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | | | |

Tr1a

Dimensions: $N = 6, n_x = 18, m_g = 72, n = 162$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|--------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 15 | 12 | 20 | 16 | 18 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 20 | 20 | 20 | 18 | | | |
| \varnothing number of iterations per run (for | 57.47 | 27.42 | 17.40 | 19.81 | 89.78 | | | |
| runs terminated at a solution only) | | | | | | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for runs terminated at a solution only) | 256.00 | 82.42 | 22.95 | 24.50 | 363.56 | | | |
| Ø number of evaluations per run (for | | | | | | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 1039.11 | 66.20 | 22.95 | 22.80 | 363.56 | | | |
| | Evaluation | ns of G | I | 1 | 1 | | | |
| \varnothing number of evaluations per run (for | 57 47 | 97.49 | 17 40 | 10.91 | 20.79 | | | |
| runs terminated at a solution only) | 57.47 | 27.42 | 17.40 | 19.81 | 89.78 | | | |
| \varnothing number of evaluations per run (for | 79 74 | 24 10 | 17 40 | 18 40 | 89.78 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 10.11 | 21110 | 11110 | 10.10 | 00.10 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $1.35{ m s}$ | $0.60{ m s}$ | $0.35\mathrm{s}$ | 0.41 s | $0.97{ m s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $3.53\mathrm{s}$ | $0.52\mathrm{s}$ | $0.35\mathrm{s}$ | $0.38\mathrm{s}$ | $0.97\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $0.09\mathrm{s}$ | $0.06\mathrm{s}$ | $0.04\mathrm{s}$ | $0.05\mathrm{s}$ | $0.09\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $0.37\mathrm{s}$ | $0.11\mathrm{s}$ | $0.03\mathrm{s}$ | $0.03\mathrm{s}$ | $0.55\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | uns termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 93.3% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes $= 1$ (glob. LPN) or | 86.7% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 3 steps are LPN (hybrid) | 79.907 | 100.007 | 100.007 | 100.007 | 100.007 | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^2$ | 73.3% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^{5}$ | 40.0% | 91.7% | 65.0% | 100.0% | 5.6% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 33.3% | 83.3% | 55.0% | 93.8% | 0.0% | | | |

Tr1b

Dimensions: $N = 6, n_x = 60, m_g = 228, n = 516$

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|------------------|------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 14 | 3 | 19 | 18 | 3 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 18 | 19 | 20 | 20 | 3 | | | |
| \varnothing number of iterations per run (for | 142 50 | 151.67 | 49.49 | 70.06 | 180.00 | | | |
| runs terminated at a solution only) | 143.30 | 151.07 | 42.42 | 70.00 | 180.00 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 760.20 | 738.00 | 59.11 | 04.04 | 086.67 | | | |
| runs terminated at a solution only) | 100.23 | 130.00 | 52.11 | 94.94 | 360.07 | | | |
| \varnothing number of evaluations per run (for | 698.06 | 1188 74 | 54 10 | 140 75 | 986.67 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 050.00 | 1100.74 | 94.10 | 140.10 | 500.01 | | | |
| | Evaluation | ns of G | | | | | | |
| \varnothing number of evaluations per run (for | 143 50 | 151.67 | 42.42 | 70.06 | 170.67 | | | |
| runs terminated at a solution only) | 140.00 | 101.01 | 12.12 | 10.00 | 110.01 | | | |
| \varnothing number of evaluations per run (for | 135 89 | 253 11 | 42.25 | 81.05 | 170.67 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 100.00 | 200.11 | 12:20 | 01.00 | 110.01 | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $8.64{ m s}$ | $7.65{ m s}$ | $2.22\mathrm{s}$ | $3.86\mathrm{s}$ | $6.98\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \varnothing CPU time per run (for all runs ter- | $8.02{ m s}$ | $11.57{ m s}$ | $2.20\mathrm{s}$ | $4.28{ m s}$ | $6.98\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \varnothing CPU time for subproblems per run | $3.57\mathrm{s}$ | $2.57\mathrm{s}$ | $1.17\mathrm{s}$ | $2.22\mathrm{s}$ | $2.41\mathrm{s}$ | | | |
| (for runs term. at a solution only) | | | | | | | | |
| \varnothing CPU time for line search per run (for | $2.44\mathrm{s}$ | $2.33\mathrm{s}$ | $0.17\mathrm{s}$ | $0.30\mathrm{s}$ | $3.15\mathrm{s}$ | | | |
| runs terminated at a solution only) | | | | | | | | |
| Local Convergence | Rate (for 1 | runs termina | ted at a solu | ition) | | | | |
| last step size = 1 (glob. LPN) or last | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| step is LPN (hybrid) | | | | | | | | |
| last 2 step sizes $= 1$ (glob. LPN) or | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LPN) or $I = I = I = I$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| Last 3 steps are LPN (hybrid) | | 100.007 | 00 507 | 04 407 | CC 7 07 | | | |
| $\begin{array}{c c} 1 \text{ ast improvement of } \ F\ _{\infty} \text{ is } \geq 10^2 \\ \hline 1 \text{ , } \text$ | 85.7% | 100.0% | 89.5% | 94.4% | 00.7% | | | |
| Last improvement of $ F _{\infty}$ is $\geq 10^{3}$ | 71.4% | 00.7% | 08.4% | 11.8% | 33.3% | | | |
| last improvement of $ F _{\infty}$ is $\geq 10^4$ | 35.7% | 0.0% | 21.1% | 44.4% | 0.0% | | | |

Tr1c

| | Alg. 1 | Alg. 2 | Alg. 3 | Alg. 4 | Alg. 5 | | | |
|--|------------------------|-------------------|------------------|------------------|------------------|--|--|--|
| # runs terminated at a solution | 14 | 1 | 20 | 20 | 1 | | | |
| # runs with $\varepsilon^* \leq 10^{-8}$ | 19 | 14 | 20 | 20 | 1 | | | |
| \varnothing number of iterations per run (for | 1/13/36 | 162.00 | 80.60 | 82.25 | 115.00 | | | |
| runs terminated at a solution only) | 140.00 | 102.00 | 00.00 | 02.20 | 110.00 | | | |
| Evaluations of F | | | | | | | | |
| \varnothing number of evaluations per run (for | 654.07 | 880.00 | 108.60 | 108 40 | 692.00 | | | |
| runs terminated at a solution only) | 001.01 | 000.00 | 100.00 | 100.40 | 052.00 | | | |
| \varnothing number of evaluations per run (for | 637 58 | 2045 14 | 108 60 | 108 40 | 692.00 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | 001.00 | 2010.11 | 100.00 | 100.10 | 002.00 | | | |
| | Evaluation | ns of G | I | 1 | | | | |
| \varnothing number of evaluations per run (for | 143.36 | 162.00 | 80.60 | 82.25 | 113.00 | | | |
| runs terminated at a solution only) | 110100 | 102.00 | | | | | | |
| \varnothing number of evaluations per run (for | 138.37 | 367.36 | 80.60 | 82.25 | 113.00 | | | |
| all runs terminated with $\varepsilon^* \leq 10^{-8}$) | | | | | | | | |
| CPU time | | | | | | | | |
| \varnothing CPU time per run (for runs termi- | $11.05\mathrm{s}$ | $12.79\mathrm{s}$ | $8.73\mathrm{s}$ | $6.68\mathrm{s}$ | $4.88\mathrm{s}$ | | | |
| nated at a solution only) | | | | | | | | |
| \bigotimes CPU time per run (for all runs ter- | $10.65\mathrm{s}$ | $23.91\mathrm{s}$ | $8.73\mathrm{s}$ | $6.68\mathrm{s}$ | $4.88\mathrm{s}$ | | | |
| minated with $\varepsilon^* \leq 10^{-6}$) | | | | | | | | |
| \bigotimes CPU time for subproblems per run | $5.29\mathrm{s}$ | $5.59\mathrm{s}$ | $6.33\mathrm{s}$ | $4.38\mathrm{s}$ | $0.83\mathrm{s}$ | | | |
| (for runs term. at a solution only) \sim CDU4: (for runs term. (for runs term.) | | | | | | | | |
| © CPU time for line search per run (for | $2.72\mathrm{s}$ | $3.58\mathrm{s}$ | $0.45\mathrm{s}$ | $0.45\mathrm{s}$ | $2.90\mathrm{s}$ | | | |
| runs terminated at a solution only) | $\mathbf{D} = 1 + 1 1$ | · · · | | | | | | |
| | e Rate (for i | runs termina | ted at a solu | ition) | | | | |
| last step size = I (glob. LPN) or last step is I DN (hybrid) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 stop sizes 1 (rich LDN) or | | | | | | | | |
| 1ast 2 step sizes = 1 (glob. LFN) or last 2 steps are LDN (hybrid) | 92.9% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| last 2 steps are LPN (hybrid) | | | | | | | | |
| last 3 step sizes = 1 (glob. LFN) or $ $ last 3 steps are LPN (hybrid) | 85.7% | 100.0% | 100.0% | 100.0% | 100.0% | | | |
| $\begin{array}{c c} 1ast & 0 & \text{steps are L1 IV (Hydrid)} \\ \hline \\ 1ast & \text{improvement of } \ F\ & \text{is } > 10^2 \\ \hline \end{array}$ | 85 707 | 100.0% | 80.002 | 80.002 | 0.00% | | | |
| $\frac{1}{1} \frac{1}{1} \frac{1}$ | 57.1% | 100.070 | 65.0% | 55.0% | 0.070 | | | |
| $\frac{1}{2} \frac{1}{2} \frac{1}$ | 1/ 20% | 100.070 | 25.0% | 40.0% | 0.070 | | | |
| ast improvement of $ F _{\infty}$ is $\geq 10^{-1}$ | 14.370 | 100.0% | 23.0% | 40.0% | 0.0% | | | |
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