Detailed Numerical Results for Several Variants of a Globally Convergent LP-Newton Method *

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1   Introduction

This paper reports in detail on the numerical results obtained for several variants of the globalized LP-Newton method introduced in [3] and of a method which combines a potential reduction method with the (local) LP-Newton method. Details for this hybrid method were given in [1]. All problems used are taken from a test library of generalized Nash equilibrium problems (GNEPs), see [2] and references therein.

In Section 2 we provide results of our computations that are based on the description of the implementation in [3, Section 5]. There, details on starting points and on the setting of parameters being used for the algorithms can be found.

In addition to these computations reported in Section 2, all runs of the algorithms were performed again with a single difference. Instead of giving the solver used for the linear programs (cplexlp from the optimization toolbox CPLEX[®]) the freedom to choose the solution method automatically, we now explicitly forced this solver to use the dual simplex algorithm as solution method. The numerical results corresponding to this explicit setting are presented in Section 3.

A reason for modifying exactly this CPLEX[®] parameter was the observation that the LP solver (with automatic choice of the solution method) often used the dual simplex algorithm. Forcing the dual simplex algorithm to be applied by default may lead to lower computer times and may also save the time needed for choosing an appropriate solution algorithm by cplexlp. In particular, for the large problems, this new setting really helped to significantly decrease computer times for the solution of the subproblems of LP-Newton.

There are further possibilities to choose or to modify the solution algorithm for the LPs in order to further decrease computer times. In this report we do not discuss this question. The results presented in Section 3 just shall show that the modification of CPLEX[®] parameters might have an influence on the performance of the algorithms.

Now, we are going to describe which results are presented in the remainder. For any of the 35 problems in the GNEP test library, one table is presented in Section 2 and in Section 3. Within such a table,

– Algorithm 1 denotes the method LP-N smooth,

– Algorithm 2 denotes the method LP-N nonsmooth,

– Algorithm 3 denotes the method LP-N nonmonotone smooth,

– Algorithm 4 denotes the method LP-N nonmonotone nonsmooth,

– Algorithm 5 denotes the method PRA + LP-N,

see [3, Section 5] for an explanation of the methods' names and details of Algorithms 1–4.

Above the table for any test problem, we first show the name of the problem and its dimensions: the number of players N, the total number of variables n_x, the total number of inequality constraints m_g, and the number n of variables (which is equal to the number of equations) in the constrained systems arising from reformulating the KKT system of a GNEP, see [3, formulas (5.5) and (5.7)]. Note that $n = n_x + 2m_g$ holds.

Within the tables, the following information is presented for each of the 5 algorithms. To obtain this information 20 runs (one for every starting point) were performed for each algorithm.

- number of runs terminated at solutions of the GNEP, where an z^k is regarded as solution if $\|F(z^k)\|_\infty \leq 10^{-8}$ with F defined according to (5.7) in [3],

- number of runs terminated at points z^k with $\varepsilon^* \leq 10^{-8}$, where ε^* denotes the optimal value of the linear program indicated by (5.10) in [3],

- average number of iterations per run (only for runs terminated at solutions),

- average number of evaluations of F per run (only for runs terminated at solutions),

- average number of evaluations of F per run (only for runs terminated at points with $\varepsilon^* \leq 10^{-8}$),

- average number of evaluations of G per run (only for runs terminated at solutions),

- average number of evaluations of G per run (only for runs terminated at points with $\varepsilon^* \leq 10^{-8}$),

- average CPU time per run (only for runs terminated at solutions),

- average CPU time per run (only for runs terminated at points with $\varepsilon^* \leq 10^{-8}$),

- average CPU time needed for the solution of all subproblems (linear programs or linear systems) per run (only for runs terminated at solutions),

- average CPU time needed for line searches per run (only for runs terminated at solutions),

- the percentage of runs for which the very last / the last two / the last three step sizes were equal to 1 (in case of Algorithm 1 – Algorithm 4) or for which the last /the last two / the last three steps were LP-Newton steps (in case of Algorithm 5), in each case only for runs terminated at solutions,

- the percentage of runs for which the ratio of the maximum norm of F at the last but one iterate and the maximum norm of F at the last iterate was greater than or equal to 10^2 / 10^3 / 10^4 (i.e., the maximum norm of F was improved in the last step by a factor which is greater than or equal to 10^2 / 10^3 / 10^4), in each case only for runs terminated at solutions.

The information described in the latter two items is supposed to give some understanding of the local behavior of the algorithms.

2 Results for the Setting Described in [3, Section 5]

In this section, detailed results of the computations described in [3] are presented. Some aspects of these results are summarized in [3, Section 5], e.g., by performance profiles. Concerning the computer time, it can be observed that, for large problems, the variants of the globally convergent LP-Newton method from [3] are much more expensive than the hybrid method described in [1]. However, we refer to Section 3 for a different picture.

The last issues reported in the tables underline the assertions of the very last paragraph of [3, Section 5]: for an absolute majority of runs terminated at a solution, the very last step size was equal to one and a superlinear convergence rate could be observed.

A1

Dimensions: $N = 10$, $n_x = 10$, $m_g = 20$, $\boldsymbol{n = 50}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	19	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	19	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	11.26	9.10	11.50	9.95	14.75
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	12.37	10.60	12.50	11.00	28.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.37	10.60	12.50	11.00	28.75
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	11.26	9.10	11.50	9.95	14.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	11.26	9.10	11.50	9.95	14.75
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.19 s	0.16 s	0.20 s	0.17 s	0.10 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.19 s	0.16 s	0.20 s	0.17 s	0.10 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.01 s	0.01 s	0.01 s	0.03 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	78.9%	75.0%	85.0%	90.0%	20.0%

A2

Dimensions: $N = 10$, $n_x = 10$, $m_g = 24$, $\boldsymbol{n = 58}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	3
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	3
∅ number of iterations per run (for runs terminated at a solution only)	10.80	9.15	11.05	9.15	99.67
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	12.25	10.20	12.05	10.15	360.33
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.25	10.20	12.05	10.15	360.33
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	10.80	9.15	11.05	9.15	98.67
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.80	9.15	11.05	9.15	98.67
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.18 s	0.16 s	0.19 s	0.16 s	0.63 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.18 s	0.16 s	0.19 s	0.16 s	0.63 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.09 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.01 s	0.01 s	0.00 s	0.24 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	95.0%	100.0%	95.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	95.0%	95.0%	95.0%	95.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	60.0%	70.0%	50.0%	70.0%	33.3%

A3

Dimensions: $N = 3$, $n_x = 7$, $m_g = 18$, $\boldsymbol{n = 43}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	16	8	15	8	19
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	19
∅ number of iterations per run (for runs terminated at a solution only)	14.56	11.50	13.60	11.50	37.26
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	17.69	12.50	14.67	12.50	188.58
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.75	8.85	14.45	8.85	188.58
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	14.56	11.50	13.60	11.50	37.26
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	14.55	8.45	13.65	8.45	37.26
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.24 s	0.19 s	0.22 s	0.20 s	0.20 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.24 s	0.14 s	0.22 s	0.14 s	0.20 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.07 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	31.6%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	81.2%	100.0%	66.7%	100.0%	68.4%

A4

Dimensions: $N = 3$, $n_x = 7$, $m_g = 18$, $\boldsymbol{n = 43}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	18	10
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	10
∅ number of iterations per run (for runs terminated at a solution only)	17.15	16.20	13.75	13.78	139.50
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	26.30	30.85	16.30	15.33	512.30
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	26.30	30.85	16.30	14.90	512.30
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	17.15	16.20	13.75	13.78	139.50
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.15	16.20	13.75	13.45	139.50
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.28 s	0.26 s	0.22 s	0.22 s	0.58 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.28 s	0.26 s	0.22 s	0.22 s	0.58 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.07 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.22 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	95.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	55.0%	85.0%	60.0%	83.3%	40.0%

A5

Dimensions: $N = 3$, $n_x = 7$, $m_g = 18$, $\boldsymbol{n = 43}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	19	20	19	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	12.55	9.74	12.45	9.74	14.05
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	14.25	10.74	13.45	10.74	41.00
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	14.25	10.50	13.45	10.50	41.00
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	12.55	9.74	12.45	9.74	14.05
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.55	9.55	12.45	9.55	14.05
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.21 s	0.16 s	0.21 s	0.16 s	0.11 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.21 s	0.16 s	0.21 s	0.16 s	0.11 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.05 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	85.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	85.0%	100.0%	90.0%	100.0%	90.0%

A6

Dimensions: $N = 3$, $n_x = 7$, $m_g = 21$, $\boldsymbol{n = 49}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	19	19	20	20	18
# runs with $\varepsilon^* \leq 10^{-8}$	19	19	20	20	18
∅ number of iterations per run (for runs terminated at a solution only)	48.05	19.63	17.45	16.75	74.22
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	221.68	43.37	18.60	17.75	337.67
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	221.68	43.37	18.60	17.75	337.67
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	48.05	19.63	17.45	16.75	74.22
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	48.05	19.63	17.45	16.75	74.22
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.84 s	0.33 s	0.28 s	0.29 s	0.38 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.84 s	0.33 s	0.28 s	0.29 s	0.38 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.04 s	0.01 s	0.01 s	0.01 s	0.05 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.09 s	0.01 s	0.00 s	0.01 s	0.17 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	63.2%	100.0%	85.0%	95.0%	94.4%

A7

Dimensions: $N = 4$, $n_x = 20$, $m_g = 44$, $\boldsymbol{n = 108}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	13	20	13	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	17.70	14.31	13.45	14.31	31.60
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	29.45	15.31	14.45	15.31	138.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	29.45	13.60	14.45	13.60	138.75
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	17.70	14.31	13.45	14.31	31.60
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.70	12.95	13.45	12.95	31.60
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.33 s	0.26 s	0.26 s	0.27 s	0.23 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.33 s	0.24 s	0.26 s	0.24 s	0.23 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.06 s	0.04 s	0.05 s	0.04 s	0.06 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.01 s	0.00 s	0.00 s	0.08 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	25.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	75.0%	30.8%	75.0%	30.8%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	15.0%	15.4%	40.0%	15.4%	30.0%

A8

Dimensions: $N = 3$, $n_x = 3$, $m_g = 8$, $\boldsymbol{n = 19}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	19	20	19	10
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	10
∅ number of iterations per run (for runs terminated at a solution only)	9.25	6.95	9.25	6.95	253.60
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	10.25	7.95	10.25	7.95	1385.20
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.25	7.75	10.25	7.75	1385.20
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	9.25	6.95	9.25	6.95	253.60
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.25	6.80	9.25	6.80	253.60
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.15 s	0.11 s	0.16 s	0.11 s	1.09 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.15 s	0.11 s	0.16 s	0.11 s	1.09 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.05 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.48 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	85.0%	52.6%	85.0%	52.6%	100.0%

A9a

Dimensions: $N = 7$, $n_x = 56$, $m_g = 63$, $\boldsymbol{n = 182}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	21.80	70.55	30.40	21.75	22.80
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	39.35	323.85	42.35	31.65	125.10
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	39.35	323.85	42.35	31.65	125.10
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	21.80	70.55	30.40	21.75	22.80
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	21.80	70.55	30.40	21.75	22.80
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.75 s	2.66 s	0.98 s	0.67 s	0.48 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.75 s	2.66 s	0.98 s	0.67 s	0.48 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.28 s	0.82 s	0.37 s	0.21 s	0.11 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.06 s	0.65 s	0.05 s	0.04 s	0.27 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	50.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	95.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	95.0%	100.0%	90.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	60.0%	90.0%	85.0%	75.0%	55.0%

A9b

Dimensions: $N = 7$, $n_x = 112$, $m_g = 119$, $\boldsymbol{n = 350}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	24.35	55.05	33.85	21.20	24.70
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	44.25	208.30	44.90	30.95	136.55
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	44.25	208.30	44.90	30.95	136.55
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	24.35	55.05	33.85	21.20	24.70
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	24.35	55.05	33.85	21.20	24.70
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	1.56 s	3.98 s	2.25 s	1.26 s	0.97 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1.56 s	3.98 s	2.25 s	1.26 s	0.97 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.95 s	2.21 s	1.43 s	0.73 s	0.36 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.05 s	0.44 s	0.10 s	0.06 s	0.49 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	95.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	70.0%	70.0%	50.0%	95.0%	50.0%

A10a

Dimensions: $N = 8$, $n_x = 24$, $m_g = 33$, $\boldsymbol{n = 90}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	19	15	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	19	15	20	20	20
⌀ number of iterations per run (for runs terminated at a solution only)	133.37	219.47	26.30	24.50	48.40
Evaluations of F					
⌀ number of evaluations per run (for runs terminated at a solution only)	830.63	1255.33	30.10	26.75	182.75
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	830.63	1255.33	30.10	26.75	182.75
Evaluations of G					
⌀ number of evaluations per run (for runs terminated at a solution only)	133.37	219.47	26.30	24.50	48.40
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	133.37	219.47	26.30	24.50	48.40
CPU time					
⌀ CPU time per run (for runs terminated at a solution only)	3.07 s	4.83 s	0.51 s	0.45 s	0.39 s
⌀ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	3.07 s	4.83 s	0.51 s	0.45 s	0.39 s
⌀ CPU time for subproblems per run (for runs term. at a solution only)	0.34 s	0.49 s	0.05 s	0.05 s	0.06 s
⌀ CPU time for line search per run (for runs terminated at a solution only)	0.56 s	1.00 s	0.01 s	0.02 s	0.19 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	95.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	57.9%	100.0%	85.0%	95.0%	85.0%

A10b

Dimensions: $N = 25$, $n_x = 125$, $m_g = 151$, $\boldsymbol{n = 427}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	17	7	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	18	7	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	161.29	117.86	28.15	28.40	41.90
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	967.18	966.71	39.50	37.35	206.95
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	939.33	966.71	39.50	37.35	206.95
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	161.29	117.86	28.15	28.40	41.50
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	157.50	117.86	28.15	28.40	41.50
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	11.59 s	8.05 s	1.81 s	1.49 s	1.51 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	11.29 s	8.05 s	1.81 s	1.49 s	1.51 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	6.53 s	3.83 s	1.17 s	0.88 s	0.71 s
∅ CPU time for line search per run (for runs terminated at a solution only)	1.08 s	1.70 s	0.05 s	0.07 s	0.51 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	95.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	88.2%	100.0%	80.0%	95.0%	85.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	64.7%	42.9%	70.0%	45.0%	55.0%

A10c

Dimensions: $N = 37$, $n_x = 222$, $m_g = 260$, $\boldsymbol{n = 742}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	0	0	8	20	0
# runs with $\varepsilon^* \leq 10^{-8}$	5	0	20	20	0
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	–	–	107.00	34.55	–
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	–	–	276.50	48.70	–
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1758.40	–	315.15	48.70	–
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	–	–	107.00	34.55	–
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	326.40	–	131.25	34.55	–
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	–	–	17.83 s	4.31 s	–
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	71.54 s	–	27.01 s	4.31 s	–
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	–	–	14.20 s	3.28 s	–
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	–	–	0.39 s	0.05 s	–
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	0.0%	0.0%	100.0%	100.0%	0.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	0.0%	0.0%	100.0%	100.0%	0.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	0.0%	0.0%	87.5%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	0.0%	0.0%	0.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	0.0%	0.0%	0.0%	85.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	0.0%	0.0%	0.0%	30.0%	0.0%

A10d

Dimensions: $N = 37$, $n_x = 370$, $m_g = 408$, $\boldsymbol{n = 1186}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	8	2	17	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	11	2	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	112.25	91.00	86.76	106.90	52.60
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	487.12	349.50	264.00	377.65	222.80
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	664.91	349.50	298.30	377.65	222.80
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	112.25	91.00	86.76	106.90	51.80
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	136.18	91.00	93.40	106.90	51.80
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	352.25 s	262.60 s	320.42 s	323.65 s	48.68 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	435.77 s	262.60 s	344.12 s	323.65 s	48.68 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	135.63 s	99.85 s	142.06 s	123.50 s	45.26 s
∅ CPU time for line search per run (for runs terminated at a solution only)	2.77 s	1.96 s	1.63 s	2.18 s	1.27 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	75.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	75.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	62.5%	100.0%	76.5%	90.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	25.0%	0.0%	41.2%	10.0%	60.0%

A10e

Dimensions: $N = 48$, $n_x = 576$, $m_g = 625$, $\boldsymbol{n = 1826}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	2	0	11	16	11
# runs with $\varepsilon^* \leq 10^{-8}$	3	0	16	16	11
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	64.50	–	183.00	92.44	57.91
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	209.50	–	720.36	285.06	258.55
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	272.00	–	802.56	285.06	258.55
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	64.50	–	183.00	92.44	57.27
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	79.33	–	196.94	92.44	57.27
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	287.97 s	–	1283.69 s	550.38 s	69.42 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	372.59 s	–	1421.85 s	550.38 s	69.42 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	105.78 s	–	537.39 s	217.04 s	63.02 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	1.59 s	–	5.85 s	2.07 s	2.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	0.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	0.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	0.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	0.0%	90.9%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	50.0%	0.0%	54.5%	93.8%	90.9%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	0.0%	0.0%	9.1%	18.8%	9.1%

A11

Dimensions: $N = 2$, $n_x = 2$, $m_g = 2$, $\boldsymbol{n = 6}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	8.15	5.95	8.15	5.95	10.35
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	9.15	6.95	9.15	6.95	11.35
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.15	6.95	9.15	6.95	11.35
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	8.15	5.95	8.15	5.95	10.35
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.15	5.95	8.15	5.95	10.35
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.13 s	0.10 s	0.13 s	0.10 s	0.06 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.13 s	0.10 s	0.13 s	0.10 s	0.06 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	75.0%	95.0%	75.0%	95.0%	95.0%

A12

Dimensions: $N = 2$, $n_x = 2$, $m_g = 4$, $\boldsymbol{n = 10}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	7.45	5.65	7.45	5.65	10.60
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	8.45	6.65	8.45	6.65	15.00
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.45	6.65	8.45	6.65	15.00
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.45	5.65	7.45	5.65	10.60
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.45	5.65	7.45	5.65	10.60
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.12 s	0.09 s	0.12 s	0.09 s	0.06 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.09 s	0.12 s	0.09 s	0.06 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	45.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	80.0%	70.0%	80.0%	70.0%	95.0%

Dimensions: $N = 3$, $n_x = 3$, $m_g = 6$, $\boldsymbol{n = 15}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	7.85	8.00	7.85	8.00	10.90
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	8.85	9.00	8.85	9.00	14.50
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.85	9.00	8.85	9.00	14.50
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	7.85	8.00	7.85	8.00	10.90
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.85	8.00	7.85	8.00	10.90
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.13 s	0.13 s	0.13 s	0.13 s	0.06 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.13 s	0.13 s	0.13 s	0.13 s	0.06 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	90.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	95.0%	100.0%	95.0%	100.0%	95.0%

A14

Dimensions: $N = 10$, $n_x = 10$, $m_g = 20$, $\boldsymbol{n = 50}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	18	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	18	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	11.00	9.10	11.85	9.20	13.35
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	12.33	10.25	12.95	10.20	19.00
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.33	10.25	12.95	10.20	19.00
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	11.00	9.10	11.85	9.20	13.35
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	11.00	9.10	11.85	9.20	13.35
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.19 s	0.16 s	0.20 s	0.16 s	0.09 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.19 s	0.16 s	0.20 s	0.16 s	0.09 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.04 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	77.8%	90.0%	85.0%	95.0%	20.0%

A15

Dimensions: $N = 3$, $n_x = 6$, $m_g = 12$, $\boldsymbol{n = 30}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
⌀ number of iterations per run (for runs terminated at a solution only)	8.90	7.85	8.90	7.85	13.85
Evaluations of $\boldsymbol{F}$					
⌀ number of evaluations per run (for runs terminated at a solution only)	9.95	8.85	9.90	8.85	40.15
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.95	8.85	9.90	8.85	40.15
Evaluations of $\boldsymbol{G}$					
⌀ number of evaluations per run (for runs terminated at a solution only)	8.90	7.85	8.90	7.85	13.85
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.90	7.85	8.90	7.85	13.85
CPU time					
⌀ CPU time per run (for runs terminated at a solution only)	0.14 s	0.13 s	0.15 s	0.13 s	0.07 s
⌀ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.14 s	0.13 s	0.15 s	0.13 s	0.07 s
⌀ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
⌀ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	20.0%	95.0%	20.0%	95.0%	95.0%

A16a

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	6.85	6.20	6.85	6.20	15.30
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	7.85	7.20	7.85	7.20	41.00
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.85	7.20	7.85	7.20	41.00
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	6.85	6.20	6.85	6.20	15.30
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	6.85	6.20	6.85	6.20	15.30
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.11 s	0.10 s	0.11 s	0.11 s	0.09 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.11 s	0.10 s	0.11 s	0.11 s	0.09 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	55.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	90.0%

A16b

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	17.25
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	8.00	7.95	8.00	7.95	47.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.00	7.95	8.00	7.95	47.75
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	17.25
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.00	6.95	7.00	6.95	17.25
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.12 s	0.12 s	0.12 s	0.12 s	0.10 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.12 s	0.12 s	0.12 s	0.10 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.01 s	0.00 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	65.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	90.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	90.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	85.0%

A16c

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	18.70
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	8.00	7.95	8.00	7.95	51.40
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.00	7.95	8.00	7.95	51.40
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	18.70
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.00	6.95	7.00	6.95	18.70
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.12 s	0.12 s	0.12 s	0.12 s	0.10 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.12 s	0.12 s	0.12 s	0.10 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.00 s	0.03 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	25.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	75.0%

A16d

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	7.40	7.85	7.40	7.85	15.90
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	8.40	8.85	8.40	8.85	45.30
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.40	8.85	8.40	8.85	45.30
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.40	7.85	7.40	7.85	15.90
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.40	7.85	7.40	7.85	15.90
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.13 s	0.13 s	0.12 s	0.13 s	0.10 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.13 s	0.13 s	0.12 s	0.13 s	0.10 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	75.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	90.0%

A17

Dimensions: $N = 2$, $n_x = 3$, $m_g = 7$, $\boldsymbol{n = 17}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	19
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	19
∅ number of iterations per run (for runs terminated at a solution only)	9.65	6.60	9.65	6.60	15.53
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	10.65	7.60	10.65	7.60	24.63
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.65	7.60	10.65	7.60	24.63
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	9.65	6.60	9.65	6.60	15.53
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.65	6.60	9.65	6.60	15.53
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.16 s	0.11 s	0.15 s	0.11 s	0.11 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.16 s	0.11 s	0.15 s	0.11 s	0.11 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.07 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.01 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	68.4%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	80.0%	95.0%	80.0%	95.0%	5.3%

A18

Dimensions: $N = 2$, $n_x = 12$, $m_g = 28$, $\boldsymbol{n = 68}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	9	20	9	19
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	12.65	9.67	12.15	9.67	17.95
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	16.40	10.67	13.15	10.67	97.05
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	16.40	8.30	13.15	8.30	96.85
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	12.65	9.67	12.15	9.67	17.95
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.65	7.85	12.15	7.85	18.05
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.23 s	0.19 s	0.21 s	0.18 s	0.14 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.23 s	0.14 s	0.21 s	0.14 s	0.14 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.02 s	0.01 s	0.02 s	0.01 s	0.06 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.00 s	0.00 s	0.00 s	0.04 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	55.0%	55.6%	85.0%	55.6%	100.0%

Harker

Dimensions: $N = 2$, $n_x = 2$, $m_g = 6$, $\boldsymbol{n = 14}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	9.30	6.90	9.30	6.90	13.45
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	10.30	7.90	10.30	7.90	28.25
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.30	7.90	10.30	7.90	28.25
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	9.30	6.90	9.30	6.90	13.45
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.30	6.90	9.30	6.90	13.45
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.15 s	0.11 s	0.15 s	0.11 s	0.07 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.15 s	0.11 s	0.15 s	0.11 s	0.07 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.04 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	90.0%	95.0%	90.0%	95.0%	95.0%

Heu

Dimensions: $N = 2$, $n_x = 10$, $m_g = 20$, $\boldsymbol{n = 50}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	10	20	15	19
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	17.00	12.80	14.95	15.60	32.79
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	22.20	13.90	16.10	16.60	124.95
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	22.20	25.25	16.10	20.15	124.00
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	17.00	12.80	14.95	15.60	32.79
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.00	15.85	14.95	19.15	32.70
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.28 s	0.21 s	0.24 s	0.25 s	0.17 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.28 s	0.26 s	0.24 s	0.31 s	0.16 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.04 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.00 s	0.00 s	0.00 s	0.05 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	36.8%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	85.0%	100.0%	85.0%	93.3%	89.5%

Lob

Dimensions: $N = 50$, $n_x = 50$, $m_g = 50$, $\boldsymbol{n = 150}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	0	20	0	0	19
# runs with $\varepsilon^* \leq 10^{-8}$	0	20	0	0	20
∅ number of iterations per run (for runs terminated at a solution only)	–	24.70	–	–	22.05
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	–	36.50	–	–	55.16
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	–	36.50	–	–	150.95
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	–	24.70	–	–	22.05
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	–	24.70	–	–	25.65
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	–	0.66 s	–	–	0.43 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	–	0.66 s	–	–	0.73 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	–	0.15 s	–	–	0.13 s
∅ CPU time for line search per run (for runs terminated at a solution only)	–	0.08 s	–	–	0.17 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	0.0%	100.0%	0.0%	0.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	0.0%	100.0%	0.0%	0.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	0.0%	100.0%	0.0%	0.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	0.0%	100.0%	0.0%	0.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	0.0%	100.0%	0.0%	0.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	0.0%	100.0%	0.0%	0.0%	100.0%

NTF1

Dimensions: $N = 2$, $n_x = 2$, $m_g = 4$, $\boldsymbol{n = 10}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	7.50	5.45	7.50	5.45	10.20
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	8.50	6.45	8.50	6.45	11.25
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.50	6.45	8.50	6.45	11.25
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.50	5.45	7.50	5.45	10.20
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.50	5.45	7.50	5.45	10.20
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.12 s	0.09 s	0.12 s	0.09 s	0.07 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.09 s	0.12 s	0.09 s	0.07 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.04 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.01 s	0.00 s	0.00 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	80.0%	100.0%	80.0%	100.0%	95.0%

NTF2

Dimensions: $N = 2$, $n_x = 2$, $m_g = 4$, $\boldsymbol{n = 10}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	9.10	26.65	9.15	9.55	13.60
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	10.25	130.35	10.15	10.60	19.20
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.25	130.35	10.15	10.60	19.20
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	9.10	26.65	9.15	9.55	13.60
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.10	26.65	9.15	9.55	13.60
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.15 s	0.45 s	0.15 s	0.15 s	0.07 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.15 s	0.45 s	0.15 s	0.15 s	0.07 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.04 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.03 s	0.00 s	0.00 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	95.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	95.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	95.0%

Spam

Dimensions: $N = 101$, $n_x = 2020$, $m_g = 4040$, $\boldsymbol{n = 10100}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	19	20	20	20	18
# runs with $\varepsilon^* \leq 10^{-8}$	19	20	20	20	19
∅ number of iterations per run (for runs terminated at a solution only)	14.68	16.10	65.55	38.05	14.94
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	25.84	50.95	88.20	48.70	35.44
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	25.84	50.95	88.20	48.70	35.58
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	14.68	16.10	65.55	38.05	14.94
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	14.68	16.10	65.55	38.05	15.05
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	1403.94 s	1063.12 s	3732.40 s	2254.92 s	253.31 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1403.94 s	1063.12 s	3732.40 s	2254.92 s	241.05 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	228.91 s	186.93 s	608.66 s	398.27 s	236.83 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.78 s	1.56 s	2.94 s	1.81 s	1.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	77.8%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	0.0%	0.0%	5.0%	0.0%	27.8%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	0.0%	0.0%	0.0%	0.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	0.0%	0.0%	0.0%	0.0%	0.0%

Tr1a

Dimensions: $N = 6$, $n_x = 18$, $m_g = 72$, $\boldsymbol{n = 162}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	15	12	20	16	17
# runs with $\varepsilon^* \leq 10^{-8}$	19	20	20	20	17
∅ number of iterations per run (for runs terminated at a solution only)	57.47	27.42	17.40	19.81	91.82
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	256.00	82.42	22.95	24.50	369.18
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1039.11	66.20	22.95	22.80	369.18
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	57.47	27.42	17.40	19.81	91.82
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	79.74	24.10	17.40	18.40	91.82
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	1.39 s	0.63 s	0.36 s	0.41 s	1.06 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	3.66 s	0.53 s	0.36 s	0.37 s	1.06 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.09 s	0.06 s	0.04 s	0.05 s	0.09 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.34 s	0.10 s	0.05 s	0.02 s	0.60 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	93.3%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	86.7%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	73.3%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	40.0%	91.7%	65.0%	100.0%	5.9%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	33.3%	83.3%	55.0%	93.8%	0.0%

Tr1b

Dimensions: $N = 6$, $n_x = 60$, $m_g = 228$, $\boldsymbol{n = 516}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	14	3	19	18	3
# runs with $\varepsilon^* \leq 10^{-8}$	18	19	20	20	3
∅ number of iterations per run (for runs terminated at a solution only)	143.50	151.67	42.42	70.06	180.00
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	760.29	738.00	52.11	94.94	986.67
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	698.06	1188.74	54.10	140.75	986.67
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	143.50	151.67	42.42	70.06	170.67
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	135.89	253.11	42.25	81.05	170.67
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	8.75 s	7.75 s	2.24 s	3.87 s	6.99 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.11 s	11.75 s	2.22 s	4.30 s	6.99 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	3.57 s	2.57 s	1.16 s	2.22 s	2.59 s
∅ CPU time for line search per run (for runs terminated at a solution only)	2.31 s	2.59 s	0.12 s	0.22 s	3.50 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	85.7%	100.0%	89.5%	94.4%	66.7%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	71.4%	66.7%	68.4%	77.8%	33.3%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	35.7%	0.0%	21.1%	44.4%	0.0%

Tr1c

Dimensions: $N = 7$, $n_x = 80$, $m_g = 304$, $\boldsymbol{n = 688}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	14	1	20	20	1
# runs with $\varepsilon^* \leq 10^{-8}$	19	14	20	20	1
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	143.36	162.00	80.60	82.25	115.00
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	654.07	880.00	108.60	108.40	692.00
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	637.58	2045.14	108.60	108.40	692.00
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	143.36	162.00	80.60	82.25	113.00
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	138.37	367.36	80.60	82.25	113.00
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	11.31 s	13.16 s	8.69 s	6.78 s	4.92 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.89 s	24.28 s	8.69 s	6.78 s	4.92 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	5.36 s	5.74 s	6.27 s	4.43 s	0.93 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	2.14 s	2.74 s	0.32 s	0.33 s	3.10 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	92.9%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	85.7%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	85.7%	100.0%	80.0%	80.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	57.1%	100.0%	65.0%	55.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	14.3%	100.0%	25.0%	40.0%	0.0%

3 Results with Modification of a CPLEX® Parameter

As described in the Introduction, the runs of the algorithms were performed again, where the LP solver from the optimization toolbox CPLEX® was forced to use the dual simplex algorithm as solution method. It turned out that, for the large examples and on average, the CPU time for the (successful) runs seriously decreased. One reason might be that the solver saves the time for choosing an appropriate algorithm. Apart from CPU time, there are not many differences compared to the results presented in Section 2. In fact, the only test problems where issues like number of runs terminated at a solution or function evaluations appreciably differ are A10d, A10e, and Spam. Probably, the reason is that, for the latter three examples, the LP solver (with automatic choice of the solution method) did not always use the dual simplex algorithm for the solution of the subproblems. This might result in different solutions of the linear programs. The results show that the modification of parameters for the solution method of the subproblems may significantly influence the performance of the algorithms.

A1

Dimensions: $N = 10$, $n_x = 10$, $m_g = 20$, $\boldsymbol{n = 50}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	19	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	19	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	11.26	9.10	11.50	9.95	14.75
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	12.37	10.60	12.50	11.00	28.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.37	10.60	12.50	11.00	28.75
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	11.26	9.10	11.50	9.95	14.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	11.26	9.10	11.50	9.95	14.75
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.20 s	0.15 s	0.19 s	0.17 s	0.10 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.20 s	0.15 s	0.19 s	0.17 s	0.10 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.04 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.00 s	0.01 s	0.01 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	78.9%	75.0%	85.0%	90.0%	20.0%

A2

Dimensions: $N = 10$, $n_x = 10$, $m_g = 24$, $\boldsymbol{n = 58}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	20	3
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	3
∅ number of iterations per run (for runs terminated at a solution only)	10.80	9.15	11.05	9.15	99.67
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	12.25	10.20	12.05	10.15	360.33
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.25	10.20	12.05	10.15	360.33
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	10.80	9.15	11.05	9.15	98.67
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.80	9.15	11.05	9.15	98.67
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.19 s	0.15 s	0.18 s	0.15 s	0.58 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.19 s	0.15 s	0.18 s	0.15 s	0.58 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.15 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.00 s	0.01 s	0.00 s	0.19 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	95.0%	100.0%	95.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	95.0%	95.0%	95.0%	95.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	60.0%	70.0%	50.0%	70.0%	33.3%

A3

Dimensions: $N = 3$, $n_x = 7$, $m_g = 18$, $\boldsymbol{n = 43}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	16	8	15	8	19
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	19
∅ number of iterations per run (for runs terminated at a solution only)	14.56	11.50	13.60	11.50	37.26
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	17.69	12.50	14.67	12.50	188.58
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.75	8.85	14.45	8.85	188.58
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	14.56	11.50	13.60	11.50	37.26
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	14.55	8.45	13.65	8.45	37.26
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.24 s	0.19 s	0.22 s	0.19 s	0.18 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.24 s	0.14 s	0.22 s	0.14 s	0.18 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.02 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.01 s	0.07 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	31.6%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	81.2%	100.0%	66.7%	100.0%	68.4%

A4

Dimensions: $N = 3$, $n_x = 7$, $m_g = 18$, $\boldsymbol{n = 43}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	18	10
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	10
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	17.15	16.20	13.75	13.78	139.50
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	26.30	30.85	16.30	15.33	512.30
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	26.30	30.85	16.30	14.90	512.30
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	17.15	16.20	13.75	13.78	139.50
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.15	16.20	13.75	13.45	139.50
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.27 s	0.26 s	0.22 s	0.22 s	0.54 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.27 s	0.26 s	0.22 s	0.21 s	0.54 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.05 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.01 s	0.00 s	0.01 s	0.19 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	95.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	55.0%	85.0%	60.0%	83.3%	40.0%

A5

Dimensions: $N = 3$, $n_x = 7$, $m_g = 18$, $\boldsymbol{n = 43}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	19	20	19	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	12.55	9.74	12.45	9.74	14.05
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	14.25	10.74	13.45	10.74	41.00
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	14.25	10.50	13.45	10.50	41.00
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	12.55	9.74	12.45	9.74	14.05
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.55	9.55	12.45	9.55	14.05
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.20 s	0.16 s	0.20 s	0.16 s	0.10 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.20 s	0.15 s	0.20 s	0.15 s	0.10 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.05 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	85.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	85.0%	100.0%	90.0%	100.0%	90.0%

A6

Dimensions: $N = 3$, $n_x = 7$, $m_g = 21$, $\boldsymbol{n = 49}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	19	19	20	20	18
# runs with $\varepsilon^* \leq 10^{-8}$	19	19	20	20	18
∅ number of iterations per run (for runs terminated at a solution only)	48.05	19.63	17.45	16.75	74.22
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	221.68	43.37	18.60	17.75	337.67
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	221.68	43.37	18.60	17.75	337.67
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	48.05	19.63	17.45	16.75	74.22
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	48.05	19.63	17.45	16.75	74.22
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.79 s	0.32 s	0.28 s	0.26 s	0.35 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.79 s	0.32 s	0.28 s	0.26 s	0.35 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.04 s	0.01 s	0.01 s	0.01 s	0.05 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.08 s	0.02 s	0.01 s	0.01 s	0.13 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	63.2%	100.0%	85.0%	95.0%	94.4%

A7

Dimensions: $N = 4$, $n_x = 20$, $m_g = 44$, $\boldsymbol{n = 108}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	13	20	13	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	17.70	14.31	13.45	14.31	31.60
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	29.45	15.31	14.45	15.31	138.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	29.45	13.60	14.45	13.60	138.75
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	17.70	14.31	13.45	14.31	31.60
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.70	12.95	13.45	12.95	31.60
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.34 s	0.26 s	0.25 s	0.25 s	0.21 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.34 s	0.23 s	0.25 s	0.23 s	0.21 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.06 s	0.04 s	0.05 s	0.04 s	0.06 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.01 s	0.00 s	0.01 s	0.07 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	25.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	75.0%	30.8%	75.0%	30.8%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	15.0%	15.4%	40.0%	15.4%	30.0%

A8

Dimensions: $N = 3$, $n_x = 3$, $m_g = 8$, $\boldsymbol{n = 19}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	19	20	19	10
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	10
∅ number of iterations per run (for runs terminated at a solution only)	9.25	6.95	9.25	6.95	253.60
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	10.25	7.95	10.25	7.95	1385.20
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.25	7.75	10.25	7.75	1385.20
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	9.25	6.95	9.25	6.95	253.60
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.25	6.80	9.25	6.80	253.60
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.15 s	0.11 s	0.14 s	0.11 s	0.97 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.15 s	0.11 s	0.14 s	0.11 s	0.97 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.07 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.41 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	85.0%	52.6%	85.0%	52.6%	100.0%

A9a

Dimensions: $N = 7$, $n_x = 56$, $m_g = 63$, $\boldsymbol{n = 182}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	21.80	70.55	30.40	21.75	22.80
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	39.35	323.85	42.35	31.65	125.10
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	39.35	323.85	42.35	31.65	125.10
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	21.80	70.55	30.40	21.75	22.80
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	21.80	70.55	30.40	21.75	22.80
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.71 s	2.60 s	0.96 s	0.65 s	0.46 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.71 s	2.60 s	0.96 s	0.65 s	0.46 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.27 s	0.81 s	0.37 s	0.21 s	0.09 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.07 s	0.59 s	0.08 s	0.05 s	0.25 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	50.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	95.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	95.0%	100.0%	90.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	60.0%	90.0%	85.0%	75.0%	55.0%

A9b

Dimensions: $N = 7$, $n_x = 112$, $m_g = 119$, $\boldsymbol{n = 350}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	24.35	55.05	33.85	21.20	24.70
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	44.25	208.30	44.90	30.95	136.55
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	44.25	208.30	44.90	30.95	136.55
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	24.35	55.05	33.85	21.20	24.70
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	24.35	55.05	33.85	21.20	24.70
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	1.54 s	3.89 s	2.20 s	1.23 s	0.95 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1.54 s	3.89 s	2.20 s	1.23 s	0.95 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.94 s	2.20 s	1.42 s	0.73 s	0.33 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.14 s	0.62 s	0.13 s	0.10 s	0.42 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	95.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	70.0%	70.0%	50.0%	95.0%	50.0%

A10a

Dimensions: $N = 8$, $n_x = 24$, $m_g = 33$, $n = 90$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	19	15	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	19	15	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	133.37	219.47	26.30	24.50	48.40
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	830.63	1255.33	30.10	26.75	182.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	830.63	1255.33	30.10	26.75	182.75
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	133.37	219.47	26.30	24.50	48.40
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	133.37	219.47	26.30	24.50	48.40
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	2.93 s	4.62 s	0.47 s	0.43 s	0.34 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	2.93 s	4.62 s	0.47 s	0.43 s	0.34 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.34 s	0.49 s	0.05 s	0.05 s	0.05 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.62 s	0.89 s	0.02 s	0.02 s	0.15 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	95.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	57.9%	100.0%	85.0%	95.0%	85.0%

A10b

Dimensions: $N = 25$, $n_x = 125$, $m_g = 151$, $\boldsymbol{n = 427}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	17	7	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	18	7	20	20	20
⌀ number of iterations per run (for runs terminated at a solution only)	161.29	117.86	28.15	28.40	41.90
Evaluations of $\boldsymbol{F}$					
⌀ number of evaluations per run (for runs terminated at a solution only)	967.18	966.71	39.50	37.35	206.95
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	939.33	966.71	39.50	37.35	206.95
Evaluations of $\boldsymbol{G}$					
⌀ number of evaluations per run (for runs terminated at a solution only)	161.29	117.86	28.15	28.40	41.50
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	157.50	117.86	28.15	28.40	41.50
CPU time					
⌀ CPU time per run (for runs terminated at a solution only)	11.27 s	7.67 s	1.74 s	1.47 s	1.39 s
⌀ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	11.00 s	7.67 s	1.74 s	1.47 s	1.39 s
⌀ CPU time for subproblems per run (for runs term. at a solution only)	6.52 s	3.81 s	1.16 s	0.88 s	0.63 s
⌀ CPU time for line search per run (for runs terminated at a solution only)	1.78 s	1.78 s	0.08 s	0.07 s	0.38 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	95.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	88.2%	100.0%	80.0%	95.0%	85.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	64.7%	42.9%	70.0%	45.0%	55.0%

A10c

Dimensions: $N = 37$, $n_x = 222$, $m_g = 260$, $\boldsymbol{n = 742}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	0	0	8	20	0
# runs with $\varepsilon^* \leq 10^{-8}$	5	0	20	20	0
∅ number of iterations per run (for runs terminated at a solution only)	–	–	107.00	34.55	–
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	–	–	276.50	48.70	–
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1758.40	–	315.15	48.70	–
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	–	–	107.00	34.55	–
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	326.40	–	131.25	34.55	–
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	–	–	17.50 s	4.20 s	–
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	71.05 s	–	26.57 s	4.20 s	–
∅ CPU time for subproblems per run (for runs term. at a solution only)	–	–	14.01 s	3.21 s	–
∅ CPU time for line search per run (for runs terminated at a solution only)	–	–	0.75 s	0.14 s	–
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	0.0%	0.0%	100.0%	100.0%	0.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	0.0%	0.0%	100.0%	100.0%	0.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	0.0%	0.0%	87.5%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	0.0%	0.0%	0.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	0.0%	0.0%	0.0%	85.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	0.0%	0.0%	0.0%	30.0%	0.0%

A10d

Dimensions: $N = 37$, $n_x = 370$, $m_g = 408$, $\boldsymbol{n = 1186}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	10	0	15	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	12	0	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	138.40	–	102.20	70.70	52.90
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	1243.20	–	323.20	155.70	223.15
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1121.83	–	283.85	155.70	223.15
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	138.40	–	102.20	70.70	52.05
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	135.33	–	94.00	70.70	52.05
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	52.53 s	–	54.07 s	20.15 s	15.79 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	51.96 s	–	48.53 s	20.15 s	15.79 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	43.23 s	–	49.15 s	17.04 s	12.83 s
∅ CPU time for line search per run (for runs terminated at a solution only)	4.78 s	–	1.25 s	0.62 s	0.87 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	0.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	90.0%	0.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	90.0%	0.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	0.0%	100.0%	95.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	80.0%	0.0%	80.0%	90.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	60.0%	0.0%	40.0%	15.0%	65.0%

A10e

Dimensions: $N = 48$, $n_x = 576$, $m_g = 625$, $\boldsymbol{n = 1826}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	8	1	17	16	14
# runs with $\varepsilon^* \leq 10^{-8}$	10	1	20	16	14
∅ number of iterations per run (for runs terminated at a solution only)	164.88	239.00	207.71	106.00	55.14
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	834.00	1240.00	835.35	287.06	247.79
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	720.40	1240.00	768.35	287.06	247.79
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	164.88	239.00	207.71	106.00	54.64
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	147.60	239.00	195.50	106.00	54.64
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	151.98 s	200.59 s	324.51 s	79.52 s	28.97 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	137.59 s	200.59 s	301.19 s	79.52 s	28.97 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	138.93 s	181.68 s	308.06 s	71.43 s	23.49 s
∅ CPU time for line search per run (for runs terminated at a solution only)	3.81 s	5.66 s	3.85 s	1.45 s	1.34 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	94.1%	93.8%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	87.5%	100.0%	58.8%	93.8%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	25.0%	100.0%	17.6%	12.5%	35.7%

A11

Dimensions: $N = 2$, $n_x = 2$, $m_g = 2$, $\boldsymbol{n = 6}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	8.15	5.95	8.15	5.95	10.20
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	9.15	6.95	9.15	6.95	11.20
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.15	6.95	9.15	6.95	11.20
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	8.15	5.95	8.15	5.95	10.20
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.15	5.95	8.15	5.95	10.20
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.13 s	0.09 s	0.13 s	0.14 s	0.06 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.13 s	0.09 s	0.13 s	0.14 s	0.06 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	75.0%	95.0%	75.0%	95.0%	100.0%

A12

Dimensions: $N = 2$, $n_x = 2$, $m_g = 4$, $n = 10$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	7.45	5.65	7.45	5.65	10.45
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	8.45	6.65	8.45	6.65	14.85
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.45	6.65	8.45	6.65	14.85
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.45	5.65	7.45	5.65	10.45
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.45	5.65	7.45	5.65	10.45
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.12 s	0.09 s	0.12 s	0.10 s	0.05 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.09 s	0.12 s	0.10 s	0.05 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	45.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	80.0%	70.0%	80.0%	70.0%	100.0%

A13

Dimensions: $N = 3$, $n_x = 3$, $m_g = 6$, $\boldsymbol{n = 15}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	7.85	8.00	7.85	8.00	10.75
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	8.85	9.00	8.85	9.00	14.35
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.85	9.00	8.85	9.00	14.35
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	7.85	8.00	7.85	8.00	10.75
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.85	8.00	7.85	8.00	10.75
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.13 s	0.13 s	0.13 s	0.13 s	0.06 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.13 s	0.13 s	0.13 s	0.13 s	0.06 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	95.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	95.0%	100.0%	95.0%	100.0%	100.0%

A14

Dimensions: $N = 10$, $n_x = 10$, $m_g = 20$, $\boldsymbol{n = 50}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	18	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	18	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	11.00	9.10	11.85	9.20	13.15
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	12.33	10.25	12.95	10.20	18.80
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.33	10.25	12.95	10.20	18.80
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	11.00	9.10	11.85	9.20	13.15
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	11.00	9.10	11.85	9.20	13.15
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.18 s	0.15 s	0.20 s	0.16 s	0.09 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.18 s	0.15 s	0.20 s	0.16 s	0.09 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.04 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.01 s	0.01 s	0.01 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	77.8%	90.0%	85.0%	95.0%	20.0%

A15

Dimensions: $N = 3$, $n_x = 6$, $m_g = 12$, $\boldsymbol{n = 30}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	8.90	7.85	8.90	7.85	13.70
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	9.95	8.85	9.90	8.85	40.00
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.95	8.85	9.90	8.85	40.00
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	8.90	7.85	8.90	7.85	13.70
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.90	7.85	8.90	7.85	13.70
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.15 s	0.12 s	0.14 s	0.13 s	0.06 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.15 s	0.12 s	0.14 s	0.13 s	0.06 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	20.0%	95.0%	20.0%	95.0%	100.0%

A16a

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	6.85	6.20	6.85	6.20	15.15
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.85	7.20	7.85	7.20	40.85
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.85	7.20	7.85	7.20	40.85
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	6.85	6.20	6.85	6.20	15.15
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	6.85	6.20	6.85	6.20	15.15
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.11 s	0.10 s	0.11 s	0.10 s	0.08 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.11 s	0.10 s	0.11 s	0.10 s	0.08 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	55.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	95.0%

A16b

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	17.10
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	8.00	7.95	8.00	7.95	47.60
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.00	7.95	8.00	7.95	47.60
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	17.10
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.00	6.95	7.00	6.95	17.10
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.12 s	0.11 s	0.11 s	0.11 s	0.09 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.11 s	0.11 s	0.11 s	0.09 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	65.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	95.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	90.0%

A16c

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	18.55
Evaluations of $\boldsymbol{F}$					
∅ number of evaluations per run (for runs terminated at a solution only)	8.00	7.95	8.00	7.95	51.25
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.00	7.95	8.00	7.95	51.25
Evaluations of $\boldsymbol{G}$					
∅ number of evaluations per run (for runs terminated at a solution only)	7.00	6.95	7.00	6.95	18.55
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.00	6.95	7.00	6.95	18.55
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.12 s	0.11 s	0.11 s	0.12 s	0.09 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.11 s	0.11 s	0.12 s	0.09 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.02 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	30.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	80.0%

A16d

Dimensions: $N = 5$, $n_x = 5$, $m_g = 10$, $\boldsymbol{n = 25}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	7.40	7.85	7.40	7.85	15.75
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	8.40	8.85	8.40	8.85	45.15
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.40	8.85	8.40	8.85	45.15
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.40	7.85	7.40	7.85	15.75
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.40	7.85	7.40	7.85	15.75
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.12 s	0.13 s	0.12 s	0.13 s	0.09 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.13 s	0.12 s	0.13 s	0.09 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.00 s	0.02 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	80.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	95.0%

A17

Dimensions: $N = 2$, $n_x = 3$, $m_g = 7$, $\boldsymbol{n = 17}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	9.65	6.60	9.65	6.60	15.50
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	10.65	7.60	10.65	7.60	24.30
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.65	7.60	10.65	7.60	24.30
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	9.65	6.60	9.65	6.60	15.50
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.65	6.60	9.65	6.60	15.50
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.15 s	0.11 s	0.15 s	0.11 s	0.10 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.15 s	0.11 s	0.15 s	0.11 s	0.10 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.06 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.01 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	70.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	80.0%	95.0%	80.0%	95.0%	5.0%

A18

Dimensions: $N = 2$, $n_x = 12$, $m_g = 28$, $n = 68$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	9	20	9	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	12.65	9.67	12.15	9.67	17.90
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	16.40	10.67	13.15	10.67	96.70
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	16.40	8.30	13.15	8.30	96.70
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	12.65	9.67	12.15	9.67	17.90
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	12.65	7.85	12.15	7.85	17.90
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.21 s	0.17 s	0.20 s	0.17 s	0.13 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.21 s	0.13 s	0.20 s	0.13 s	0.13 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.02 s	0.01 s	0.01 s	0.01 s	0.05 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.01 s	0.01 s	0.01 s	0.00 s	0.04 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	55.0%	55.6%	85.0%	55.6%	100.0%

Harker

Dimensions: $N = 2$, $n_x = 2$, $m_g = 6$, $\boldsymbol{n = 14}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	9.30	6.90	9.30	6.90	13.35
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	10.30	7.90	10.30	7.90	28.15
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.30	7.90	10.30	7.90	28.15
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	9.30	6.90	9.30	6.90	13.35
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.30	6.90	9.30	6.90	13.35
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.15 s	0.11 s	0.14 s	0.11 s	0.07 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.15 s	0.11 s	0.14 s	0.11 s	0.07 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.01 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	90.0%	95.0%	90.0%	95.0%	100.0%

Heu

Dimensions: $N = 2$, $n_x = 10$, $m_g = 20$, $\boldsymbol{n = 50}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	10	20	15	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	17.00	12.80	14.95	15.60	32.55
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	22.20	13.90	16.10	16.60	123.85
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	22.20	25.25	16.10	20.15	123.85
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	17.00	12.80	14.95	15.60	32.55
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	17.00	15.85	14.95	19.15	32.55
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.27 s	0.20 s	0.24 s	0.25 s	0.15 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.27 s	0.25 s	0.24 s	0.32 s	0.15 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.01 s	0.01 s	0.01 s	0.01 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.01 s	0.00 s	0.01 s	0.05 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	35.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	85.0%	100.0%	85.0%	93.3%	90.0%

Lob

Dimensions: $N = 50$, $n_x = 50$, $m_g = 50$, $\boldsymbol{n = 150}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	0	20	0	0	20
# runs with $\varepsilon^* \leq 10^{-8}$	0	20	0	0	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	–	24.70	–	–	22.05
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	–	36.50	–	–	55.15
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	–	36.50	–	–	55.15
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	–	24.70	–	–	22.05
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	–	24.70	–	–	22.05
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	–	0.59 s	–	–	0.35 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	–	0.59 s	–	–	0.35 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	–	0.14 s	–	–	0.13 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	–	0.06 s	–	–	0.11 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	0.0%	100.0%	0.0%	0.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	0.0%	100.0%	0.0%	0.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	0.0%	100.0%	0.0%	0.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	0.0%	100.0%	0.0%	0.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	0.0%	100.0%	0.0%	0.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	0.0%	100.0%	0.0%	0.0%	100.0%

NTF1

Dimensions: $N = 2$, $n_x = 2$, $m_g = 4$, $\boldsymbol{n = 10}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	7.50	5.45	7.50	5.45	10.05
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	8.50	6.45	8.50	6.45	11.10
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.50	6.45	8.50	6.45	11.10
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	7.50	5.45	7.50	5.45	10.05
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	7.50	5.45	7.50	5.45	10.05
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	0.12 s	0.09 s	0.12 s	0.10 s	0.07 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.12 s	0.09 s	0.12 s	0.10 s	0.07 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.05 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.00 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	80.0%	100.0%	80.0%	100.0%	100.0%

NTF2

Dimensions: $N = 2$, $n_x = 2$, $m_g = 4$, $\boldsymbol{n = 10}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	20	20	20	20	20
# runs with $\varepsilon^* \leq 10^{-8}$	20	20	20	20	20
∅ number of iterations per run (for runs terminated at a solution only)	9.10	26.65	9.15	9.55	13.45
Evaluations of F					
∅ number of evaluations per run (for runs terminated at a solution only)	10.25	130.35	10.15	10.60	19.05
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.25	130.35	10.15	10.60	19.05
Evaluations of G					
∅ number of evaluations per run (for runs terminated at a solution only)	9.10	26.65	9.15	9.55	13.45
∅ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	9.10	26.65	9.15	9.55	13.45
CPU time					
∅ CPU time per run (for runs terminated at a solution only)	0.14 s	0.42 s	0.14 s	0.15 s	0.07 s
∅ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	0.14 s	0.42 s	0.14 s	0.15 s	0.07 s
∅ CPU time for subproblems per run (for runs term. at a solution only)	0.00 s	0.00 s	0.00 s	0.00 s	0.03 s
∅ CPU time for line search per run (for runs terminated at a solution only)	0.00 s	0.03 s	0.00 s	0.00 s	0.01 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	95.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	95.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	100.0%	100.0%	100.0%	100.0%	100.0%

Spam

Dimensions: $N = 101$, $n_x = 2020$, $m_g = 4040$, $\boldsymbol{n = 10100}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	19	20	20	20	19
# runs with $\varepsilon^* \leq 10^{-8}$	19	20	20	20	19
⌀ number of iterations per run (for runs terminated at a solution only)	15.95	14.40	75.90	53.70	14.95
Evaluations of F					
⌀ number of evaluations per run (for runs terminated at a solution only)	32.68	26.10	103.25	75.45	35.47
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	32.68	26.10	103.25	75.45	35.47
Evaluations of G					
⌀ number of evaluations per run (for runs terminated at a solution only)	15.95	14.40	75.90	53.70	14.95
⌀ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	15.95	14.40	75.90	53.70	14.95
CPU time					
⌀ CPU time per run (for runs terminated at a solution only)	193.09 s	157.12 s	656.70 s	528.40 s	51.32 s
⌀ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	193.09 s	157.12 s	656.70 s	528.40 s	51.32 s
⌀ CPU time for subproblems per run (for runs term. at a solution only)	177.65 s	142.77 s	588.02 s	476.45 s	34.44 s
⌀ CPU time for line search per run (for runs terminated at a solution only)	0.60 s	0.47 s	1.92 s	1.41 s	0.94 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	73.7%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	0.0%	0.0%	5.0%	20.0%	31.6%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	0.0%	0.0%	0.0%	0.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	0.0%	0.0%	0.0%	0.0%	0.0%

Tr1a

Dimensions: $N = 6$, $n_x = 18$, $m_g = 72$, $\boldsymbol{n = 162}$

	Alg. 1	Alg. 2	Alg. 3	Alg. 4	Alg. 5
# runs terminated at a solution	15	12	20	16	18
# runs with $\varepsilon^* \leq 10^{-8}$	19	20	20	20	18
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	57.47	27.42	17.40	19.81	89.78
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	256.00	82.42	22.95	24.50	363.56
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	1039.11	66.20	22.95	22.80	363.56
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	57.47	27.42	17.40	19.81	89.78
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	79.74	24.10	17.40	18.40	89.78
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	1.35 s	0.60 s	0.35 s	0.41 s	0.97 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	3.53 s	0.52 s	0.35 s	0.38 s	0.97 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	0.09 s	0.06 s	0.04 s	0.05 s	0.09 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	0.37 s	0.11 s	0.03 s	0.03 s	0.55 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	93.3%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	86.7%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	73.3%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	40.0%	91.7%	65.0%	100.0%	5.6%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	33.3%	83.3%	55.0%	93.8%	0.0%

Tr1b

Dimensions: $N = 6$, $n_x = 60$, $m_g = 228$, $\boldsymbol{n = 516}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	14	3	19	18	3
# runs with $\varepsilon^* \leq 10^{-8}$	18	19	20	20	3
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	143.50	151.67	42.42	70.06	180.00
Evaluations of F					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	760.29	738.00	52.11	94.94	986.67
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	698.06	1188.74	54.10	140.75	986.67
Evaluations of G					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	143.50	151.67	42.42	70.06	170.67
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	135.89	253.11	42.25	81.05	170.67
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	8.64 s	7.65 s	2.22 s	3.86 s	6.98 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	8.02 s	11.57 s	2.20 s	4.28 s	6.98 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	3.57 s	2.57 s	1.17 s	2.22 s	2.41 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	2.44 s	2.33 s	0.17 s	0.30 s	3.15 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	85.7%	100.0%	89.5%	94.4%	66.7%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	71.4%	66.7%	68.4%	77.8%	33.3%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	35.7%	0.0%	21.1%	44.4%	0.0%

Tr1c

Dimensions: $N = 7$, $n_x = 80$, $m_g = 304$, $\boldsymbol{n = 688}$

	Alg. 1	**Alg. 2**	**Alg. 3**	**Alg. 4**	**Alg. 5**
# runs terminated at a solution	14	1	20	20	1
# runs with $\varepsilon^* \leq 10^{-8}$	19	14	20	20	1
$\varnothing$ number of iterations per run (for runs terminated at a solution only)	143.36	162.00	80.60	82.25	115.00
Evaluations of $\boldsymbol{F}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	654.07	880.00	108.60	108.40	692.00
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	637.58	2045.14	108.60	108.40	692.00
Evaluations of $\boldsymbol{G}$					
$\varnothing$ number of evaluations per run (for runs terminated at a solution only)	143.36	162.00	80.60	82.25	113.00
$\varnothing$ number of evaluations per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	138.37	367.36	80.60	82.25	113.00
CPU time					
$\varnothing$ CPU time per run (for runs terminated at a solution only)	11.05 s	12.79 s	8.73 s	6.68 s	4.88 s
$\varnothing$ CPU time per run (for all runs terminated with $\varepsilon^* \leq 10^{-8}$)	10.65 s	23.91 s	8.73 s	6.68 s	4.88 s
$\varnothing$ CPU time for subproblems per run (for runs term. at a solution only)	5.29 s	5.59 s	6.33 s	4.38 s	0.83 s
$\varnothing$ CPU time for line search per run (for runs terminated at a solution only)	2.72 s	3.58 s	0.45 s	0.45 s	2.90 s
Local Convergence Rate (for runs terminated at a solution)					
last step size = 1 (glob. LPN) or last step is LPN (hybrid)	100.0%	100.0%	100.0%	100.0%	100.0%
last 2 step sizes = 1 (glob. LPN) or last 2 steps are LPN (hybrid)	92.9%	100.0%	100.0%	100.0%	100.0%
last 3 step sizes = 1 (glob. LPN) or last 3 steps are LPN (hybrid)	85.7%	100.0%	100.0%	100.0%	100.0%
last improvement of $\|F\|_\infty$ is $\geq 10^2$	85.7%	100.0%	80.0%	80.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^3$	57.1%	100.0%	65.0%	55.0%	0.0%
last improvement of $\|F\|_\infty$ is $\geq 10^4$	14.3%	100.0%	25.0%	40.0%	0.0%

References

[1] A. DREVES, F. FACCHINEI, A. FISCHER, AND M. HERRICH, *A new error bound result for Generalized Nash Equilibrium Problems and its algorithmic application*, Comput. Optim. Appl., 59 (2014), pp. 63–84.

[2] A. DREVES, F. FACCHINEI, C. KANZOW, AND S. SAGRATELLA, *On the solution of the KKT conditions of generalized Nash equilibrium problems*, SIAM J. Optim., 21 (2011), pp. 1082–1108.

[3] A. FISCHER, M. HERRICH, A. F. IZMAILOV, M. V. SOLODOV, *A globally convergent LP-Newton method*, available from `http://w3.impa.br/~optim/solodov.html`