Hans-Görg Roos
Martin Stynes
Lutz Tobiska

Robust Numerical Methods for Singularly Perturbed Differential Equations

SPIN Springer's internal project number, if known

Convection-Diffusion-Reaction and Flow Problems

May 27, 2008

Springer
Berlin Heidelberg New York
Hong Kong London
Milan Paris Tokyo

Preface

The analysis of singular perturbed differential equations began early in the twentieth century, when approximate solutions were constructed from asymptotic expansions. (Preliminary attempts appear in the nineteenth century – see [vD94].) This technique has flourished since the mid-1960s and its principal ideas and methods are described in several textbooks; nevertheless, asymptotic expansions may be impossible to construct or may fail to simplify the given problem and then *numerical approximations* are often the only option.

The systematic study of numerical methods for singular perturbation problems started somewhat later – in the 1970s. From this time onwards the research frontier has steadily expanded, but the exposition of new developments in the analysis of these numerical methods has not received its due attention. The first textbook that concentrated on this analysis was [DMS80], which collected various results for ordinary differential equations. But after 1980 no further textbook appeared until 1996, when *three* books were published: Miller et al. [MOS96], which specializes in upwind finite difference methods on Shishkin meshes, Morton's book [Mor96], which is a general introduction to numerical methods for convection-diffusion problems with an emphasis on the cell-vertex finite volume method, and [RST96], the first edition of the present book. Nevertheless many methods and techniques that are important today, especially for partial differential equations, were developed after 1996. To give some examples, layer-adapted special meshes are frequently used, new stabilization techniques (such as discontinuous Galerkin methods and local subspace projections) are prominent, and there is a growing interest in the use of adaptive methods. Consequently contemporary researchers must comb the literature to gain an overview of current developments in this active area. In this second edition we retain the exposition of basic material that underpinned the first edition while extending its coverage to significant new numerical methods for singularly perturbed differential equations.

Our purposes in writing this introductory book are twofold. First, we present a structured and comprehensive account of current ideas in the numerical analysis of singularly perturbed differential equations. Second, this important area has many open problems and we hope that our book will stimulate their investigation. Our choice of topics is inevitably personal and reflects our own main interests.

We have learned a great deal about singularly perturbed problems from other researchers. We thank those colleagues who helped and influenced us; these include V.B. Andreev, A.E. Berger, P.A. Farrell, A. Felgenhauer, E.C. Gartland, Ch. Großmann, A.F. Hegarty, R.B. Kellogg, N. Kopteva, G. Lube, N. Madden, G. Matthies, J.J.H. Miller, K.W. Morton, F. Schieweck, G.I. Shishkin, E. Süli, and R. Vulanović; in particular Herbert Goering and Eugene O'Riordan guided our initial steps in the area. Our research colleague T. Linß deserves additional thanks for providing many of the figures in this book.

Our work was supported by the Deutsche Forschungsgemeinschaft and by the Boole Centre for Research in Informatics at the National University of Ireland, Cork. We are grateful to them, to the Mathematisches Forschungsinstitut in Oberwolfach for its hospitality, and to Springer-Verlag for its cooperation.

Contents

Part II Parabolic Initial-Boundary Value Problems in One Space Dimension

Part IV The Incompressible Navier-Stokes Equations