Workshop on Optimization

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Abstracts

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Contents

Evolution of Random Complexes from LEGO(R) Bricks in a Washing Machine

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People use their washing machines to clean LEGO bricks. Typically, during the washing process, also some of the bricks join together randomly, forming complexes. I realized the phenomenon and became curiously obsessed: more than 100 washing runs gave a lot of interesting insights. See for instance some of them via this link list:

http://www.althofer.de/brick-washing-links.html

In particular, I have become a great fan of Helen Hansma's primordial sandwich model for the origins of terrestrial life, because an analogy to her setting (organic molecules between thin mica sheets) gave also best complex building in LEGO washing sessions.

Besteuerte Matrixspiele: Auswirkungen auf den Erwarteten Transfer

Marlis Bärthel
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In Spielsituationen im Casino, an Wettbörsen, auf dem Finanzmarkt o.ä. könnte sich die Einführung von Steuern auf den monetären Umsatz auswirken. Mithilfe eines spieltheoretischen Ansatzes fragen wir: Wie verändert sich der Erwartete Transfer eines Matrixspiels, wenn der Gewinner eine prozentuale Steuer abgeben muss? Die Analyse dieses elementaren Modells zeigt deutlich: Steuern sind kein Allheilmittel zur Eindämmung aggressiven Spielverhaltens, denn Steuern können zu erhöhten erwarteten Umsätzen führen.

Der Vortrag gibt eine Einführung in das betrachtete Modell und einen Überblick über einige überraschende Phänomene bei besteuerten Matrixspielen.

Upper Bounds for Heuristic Approaches to the Strip Packing Problem

Torsten Buchwald and Guntram Scheithauer
Technische Universität Dresden, Faculty of Mathematics and Natural Sciences

We present an implementation of the FFDH heuristic for the three- dimensional case, which is used to construct a new algorithm with absolute per- formance ratio of at most 5. We also show, that this algorithm has absolute perfor- mance ratio of at most 4.25 for the z-oriented three-dimensional SPP. Furthermore, we prove that the absolute performance ratio for the z-oriented three-dimensional SPP is at most 4, if the container has squared base area. Moreover, we show that the absolute performance ratio of this algorithm is at most 3.25 for items and container with squared base area. Based on this algorithm, we prove a general upper bound for the optimal height, which depends on the continuous lower bound and the maximum height lower bound, and show that the combination of both lower bounds also has an absolute worst case performance ratio of at most 5 for the classic three-dimensional SPP. We also show that the layer-relaxation has a worst case performance ratio of at most 4.25 for the z-oriented three-dimensional SPP.

Solution of bilevel optimization problems

Stephan Dempe
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Consider a bilevel optimization problem

$$\min_{x,y}\{F(x,y) : G(x) \leq 0,\ (x,y) \in \text{gph } \Psi\}, \tag{1}$$

where $(x,y) \in \text{gph } \Psi$ iff $y \in \Psi(x)$ and

$$\Psi(x) := \operatorname*{Argmin}_y \{f(x,y) : g(x,y) \leq 0\}.$$

Here all the functions are assumed to be sufficiently smooth. Moreover, $y \mapsto f(x,y),\ y \mapsto g_i(x,y)$ are convex functions.

One way to solve problem (1) is to replace the lower level problem by its Karush-Kuhn-Tucker conditions provided some regularity condition is satisfied:

$$\min_{x,y,u}\{F(x,y) : G(x) \leq 0,\ \nabla_y L(x,y,u) = 0,\ u \geq 0,\ g(x,y) \leq 0,\ u^\top g(x,y) = 0)\}, \tag{2}$$

where $L(x,y,u) = f(x,y) + u^\top g(x,y)$. There are a number of difficulties related:

1. Both problems are equivalent only if a regularity condition is satisfied and if global optimal solutions are computed, see [2].

2. Regularity conditions are not generically satisfied at optimal solutions of the lower level problem.

Because of the second point, the use of the Fritz-John conditions is suggested in the paper [1]:

$$\begin{aligned}\min_{x,y,u}\{F(x,y) : G(x) \leq 0,\ \nabla_y L_0(x,y,u) = 0,\ u_0, u \geq 0,\ g(x,y) \leq 0,& \\ u^\top g(x,y) = 0),\ \sum_{i=0}^{p} u_i = 1\},&\end{aligned} \tag{3}$$

where $L(x,y,u) = u_0 f(x,y) + \sum_{i=1}^{p} u_i g_i(x,y)$.

In the talk the relations between problems (2) and (3) will be discussed especially with respect to their use for solving problem (1).

[1] G.B. Allende, G. Still: Solving bilevel programs with the KKT-approach. Mathematical Programming **138** (2013) 309–332
[2] S. Dempe, J. Dutta: Is bilevel programming a special case of a mathematical program with complementarity constraints? Mathematical Programming **131** (2012) 37–48

A Bundle Algorithm for Solving Bilevel Optimization Problems

Susanne Franke and Stephan Dempe
Technische Universität Bergakademie Freiberg, Faculty of Mathematics and Computer Science

Our basis is the optimistic bilevel programming problem

$$\begin{array}{ll} \min\limits_{x,y} & F(x,y) \\ \text{s.t.} & \begin{cases} G(x) \le 0 \\ y \in \Psi(x) := \underset{z}{\operatorname{Argmin}} \{f(x,z) : \; g(x,z) \le 0\} \end{cases} \end{array} \tag{4}$$

with the point-to-set mapping Ψ as the solution set mapping of the lower level problem which depends on the upper level variable x.

We use the optimal value reformulation

$$\begin{array}{ll} \min\limits_{x,y} & F(x,y) \\ \text{s.t.} & \begin{cases} G(x) \le 0 \\ g(x,y) \le 0 \\ f(x,y) \le \varphi(x) \end{cases} \end{array} \tag{5}$$

of the problem (4) with $\varphi(x) := \min\limits_{y} \{f(x,y) : \; g(x,y) \le 0\}$ being the lower level optimal value function. We assume that all functions are Lipschitz continuous.

Using the concept of partial calmness (which was first introduced in [1]) allows us to formulate suitable constraint qualifications for (5) and move the constraint $f(x,y) - \varphi(x) \le 0$ which is not given explicitly as a penalty term to the objective function.

Kiwiel [2] proposed an inexact bundle method for solving convex optimization problems. There, the bundle is a set consisting of trial points, the respective objective function values and one subgradient for every point. At every newly calculated trial point, it is evaluated whether the point provides a sufficient improvement of the objective function value or not. Depending on this decision, the bundle is updated such that a sequence of trial points converging to the optimal solution of the problem is constructed. In the talk, the algorithm is extended to the nonconvex case such that it can be applied to the bilevel programming problem. Therefore, ideas from [3] are used.

[1] F.H. Clarke: Optimization and Nonsmooth Analysis. SIAM, 1987
[2] K.C. Kiwiel: A proximal bundle method with approximate subgradient linearizations. SIAM Journal on Optimization **16** (2006) 1007–1023
[3] H. Schramm, J. Zowe: A version of the bundle idea for minimizing a nonsmooth function: conceptual idea, convergence analysis, numerical results. SIAM Journal on Optimization **2** (1992) 121–152

New Inequalities for One-dimensional Relaxations of the Two-dimensional Rectangular Strip Packing Problem

Isabel Friedow and Guntram Scheithauer
Technische Universität Dresden, Faculty of Mathematics and Natural Sciences

We investigate a heuristic for the two-dimensional rectangular strip packing problem that constructs a feasible two-dimensional packing by placing one-dimensional cutting patterns obtained by solving the horizontal one-dimensional bar relaxation. To represent a solution of the strip packing problem, a solution of a horizontal bar relaxation has to satisfy, among others, the vertical contiguous condition. That means that there must exist such an ordering of cutting patterns that all items representing one rectangle are located in consecutive patterns. To strengthen the one-dimensional horizontal bar relaxation with respect to that vertical contiguity we formulate new inequalities.

Software for Solving Multiobjective Location Problems

Christian Günther and Marcus Hillmann
Martin-Luther-Universität Halle-Wittenberg, Faculty of Natural Sciences II

The aim of this talk is to present a tool (based on MATLAB) for solving multiobjective location problems. The classical single-facility multiobjective location problem consists in minimizing the distances between a new facility (one point in the plane) and given facilities (finite points in the plane) simultaneously, where simultaneous minimization is understood in the sense of multiobjective optimization. In our talk we present mathematical model formulations of different scalar and multiobjective location problems. Moreover, we present decomposition methods for solving non-convex extended multiobjective location problems. At the end of our talk we give an overview for expected future development steps of our software.

Newton-Typ-Methoden für nichtglatte restringierte Gleichungssysteme und Anwendungen

Markus Herrich
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Der Vortrag befasst sich mit lokalen Konvergenzeigenschaften von Newton-Typ-Verfahren zur Lösung von restringierten Gleichungssystemen $F(z) = 0$ bei $z \in \Omega$. Im ersten Teil des Vortrags wird ein neues lokales Konvergenzresultat für das restringierte Levenberg-Marquardt-Verfahren präsentiert. Dazu werden Voraussetzungen vorgestellt, unter denen dieses Verfahren lokal quadratisch gegen eine Lösung des Ausgangsproblems konvergiert. Diese Voraussetzungen implizieren weder die lokale Eindeutigkeit von Lösungen noch Differenzierbarkeit von F.

Im zweiten Teil des Vortrags erfolgt eine Diskussion der Konvergenzvoraussetzungen für den Fall, dass F eine stückweise stetig differenzierbare Funktion ist. Es wird sich herausstellen, dass alle Konvergenzvoraussetzungen gelten, falls eine gewisse Menge lokaler Fehlerschrankenbedingungen erfüllt ist und außerdem diejenigen Nullstellen der Auswahlfunktionen, die keine Nullstellen von F sind, nicht zur Menge Ω gehören.

Am Ende diskutieren wir die neuen Bedingungen für restringierte Gleichungssysteme, die durch Umformulierung von KKT-Systemen verallgemeinerter Nash-Gleichgewichtsprobleme entstehen. Insbesondere wird eine Vollrangbedingung vorgestellt, unter der das restringierte Levenberg-Marquardt-Verfahren lokal quadratisch konvergiert.

Spaltengenerierung zur Lösung des Schichtplanungsproblems von Zugbegleitern

Kirsten Hoffmann
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Steigender Kostendruck zwingt Verkehrsunternehmen zum effizienten Einsatz ihres Personals mit dem Ziel der Kostensenkung und Produktivitätserhöhung. Speziell im Schienenpersonennahverkehr sind Zugbegleiter und Triebfahrzeugführer von zentraler Bedeutung. Aufgrund der fortschreitenden Automatisierung der Fahrzeuge verschwinden zunehmend betriebliche Aufgaben von den Schichtplänen der Zugbegleiter. Infolgedessen geht in Deutschland der Trend weg von einer ständigen Begleitung aller Zugleistungen hin zu einer partiellen Abdeckung, deren Größe anhand einer Prüfquote gemessen wird. Dieser Vortrag beschäftigt sich mit den Auswirkungen dieser neuen Entwicklungen und stellt ein Modell für die Schichtplanung von Zugbegleitern unter Berücksichtigung von Prüfquoten, rechtlichen und tariflichen Bestimmungen vor. Aufgrund der enormen Problemgröße mit mehreren Millionen möglichen Schichten wird eine Spaltengenerierungstechnik verbunden mit einem genetischen Algorithmus zur Lösung dieses Modells entwickelt.

Optimale Diskrete Empfangsstrahlformung

Johannes Israel
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Strahlformung (*engl. beamforming*) bezeichnet eine Signalverarbeitungstechnik für Antennenarrays zur Anpassung der Richtcharakteristik. Bei der Empfangsstrahlformung werden mit Amplituden- und Phasenstellgliedern die empfangenen Signale an den Antennenelementen individuell gewichtet. In der Literatur werden zahlreiche Verfahren für eine optimale Einstellung der Beamforming-Gewichte (Amplituden und Phasen) für verschiedene Szenarien beschrieben, wobei die Variablen für gewöhnlich als stetig vorausgesetzt werden. In praktischen Anwendungen ist hingegen eine Implementierung mit diskreten Amplituden- und Phasenstellgliedern oftmals einfacher und kostengünstiger zu realisieren. Das Signal-zu-Interferenz-und-Rausch-Verhältnis (SINR) ist die übliche (nichtkonkave) Zielfunktion für die Empfangsstrahlformung.

Im Vortrag wird das diskrete SINR-Maximierungsproblem eingeführt und dessen exakte Lösung mittels Branch-and-Bound vorgestellt. Dabei wird insbesondere die effiziente Bestimmung oberer Schranken für die Teilprobleme im Branch-and-Bound-Verfahren diskutiert. Mit Hilfe der exakten Lösung kann auch die Güte zuvor entwickelter approximativer Ansätze besser beurteilt werden.

Der Vortrag basiert auf gemeinsamen Arbeiten mit Andreas Fischer, Eduard Jorswieck, John Martinovic und Marat Mesyagutov.

On Mathematical Programs with Cardinality Constraints

Christian Kanzow
Julius-Maximilians-Universität Würzburg, Faculty of Mathematics and Computer Science

This talks considers optimization problems with cardinality constraints. It is shown that this problem can be reformulated as a mixed integer (binary) program whose standard relaxation turns out to have the same solutions as the original cardinality constrained problem. We therefore elaborate further on this relaxation, discuss the relation between the local minima and consider suitable first-order optimality conditions. Based on this relaxation, we further propose an algorithm, discuss its convergence properties and present some corresponding numerical results. It should be noted that our relaxation of the mathematical program with cardinality constraints has a structure which looks almost like an MPCC (mathematical program with complementarity constraints). In fact, it is also possible to derive a complete MPCC-reformulation of the cardinality-constrained program and to apply the well-known MPCC-machinery. However, it turns out that a direct treatment of the problem yields stronger results than what would be obtained by the MPCC-approach.

This talk is based on joint work with Oleg Burdakov, Michael Cervinka, and Alexandra Schwartz.

Nichtglatte Optimierung – Stetige äußere Subdifferentiale, Methode und Konvergenz

Martin Knossalla
Friedrich-Alexander-Universität Erlangen-Nürnberg, Naturwissenschaftliche Fakultät

Es wird eine neue Strategie zur Lösung von Optimierungsproblemen mit lokal Lipschitz-stetiger Zielfunktion $f : \mathbb{R}^n \to \mathbb{R}$ vorgestellt. Als Grundlage werden sogenannte stetige äußere Subdifferentiale $G^f : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ eingeführt. Es wird ein Abstiegsverfahren basierend auf diesen stetigen Subdifferentialen entwickelt und dessen globale Konvergenz bewiesen. Für die Klasse der Optimalwertfunktionen der Gestalt $\varphi(x) := \inf\{\vartheta(x, y) \,|\, y \in \Omega(x)\}$ mit reellwertigen Funktionen $\vartheta : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ und mengenwertigen Abbildungen $\Omega : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$ wird ein stetiges äußeres Subdifferential G^φ konstruiert. Eine Anwendung zur Optimierung einer Optimalwertfunktion aus den Wirtschaftswissenschaften schließt diesen Vortrag ab.

Variable Ordering Structures in Set Optimization

Elisabeth Köbis
Friedrich-Alexander-Universität Erlangen-Nürnberg, Faculty of Science

We introduce a variable upper set less order relation and consider set optimization problems equipped with this variable ordering structure. Following a set approach, we provide characterizations for optimal solutions. Furthermore, we consider sections of feasible elements of set optimization problems.

Pure State Constrained Bilevel Optimal Control Problems with Finite-dimensional Lower Level

Patrick Mehlitz
Technische Universität Bergakademie Freiberg, Faculty of Mathematics and Computer Science

In this talk we are going to present results from our recent paper [1] where the authors derived non-degenerated necessary Pontryagin-type optimality conditions for optimal control problems of the following type:

$$\begin{aligned} F(x(T),y) &\to \min_{x,u,y} && \\ \dot{x}(t)-\phi(t,x(t),u(t)) &= 0_n && \text{a.e. } t\in[0,T] \\ x(0)-x_0 &= 0_n && \\ G(t,x(t)) &\leq 0_p && \text{for all } t\in[0,T] \\ u(t) &\in \mathcal{U} && \text{a.e. } t\in[0,T] \\ y &\in \Psi(x(T)). && \end{aligned} \tag{6}$$

Therein, $\Psi\colon \mathbb{R}^n \to 2^{\mathbb{R}^k}$ denotes the solution-set-mapping of the fully convex parametric optimization problem

$$\min_y \big\{ f(x(T),y) \,|\, g(x(T),y) \leq 0_q \big\} \tag{7}$$

and the functions $F, f\colon \mathbb{R}^n\times\mathbb{R}^k \to \mathbb{R}$, $\phi\colon [0,T]\times\mathbb{R}^n\times\mathbb{R}^m \to \mathbb{R}^n$, $G\colon [0,T]\times\mathbb{R}^n \to \mathbb{R}^p$, and $g\colon \mathbb{R}^n\times\mathbb{R}^k \to \mathbb{R}^q$ are assumed to be continuously differentiable while the initial state x_0 and the final time $T>0$ are fixed. Furthermore, the set $\mathcal{U}\subseteq\mathbb{R}^m$ is assumed to be non-empty and Borel-measurable. We claim the state function x to be an element of the Sobolev space $W^n_{1,1}[0,T]$ while the control u is chosen from $L^m_1[0,T]$. The lower level variable y comes from $\mathbb{R}^k$.

A typical example where a problem of type (6) arises is the continuous version of the natural-gas-cash-out-problem (cf. [2]). It is used to model the trading and transportation of gas in Mexico respecting the special structure of the corresponding market.

The main difficulties of problem (6) comprise its bilevel structure and the appearence of pure state constraints in the upper level problem, i.e. constraints which cannot be influenced by the choice of the control directly (cf. [3]). We start by transforming the bilevel programming problem (6) into a single-level optimal control problem with non-smooth terminal conditions using the optimal-value-function of the lower level problem (7). It will be demonstrated that any maximum principle derived from this surrogate problem is likely to be a degenerated optimality condition (i.e. it holds at any feasible point of (6)). Consequently, we apply an adaptation of the famous partial calmness condition, introduced by Ye and Zhu in [4], to derive a second single-level surrogate problem whose objective is non-smooth. Claiming some standard constraint qualifications allows to derive a non-degenerated maximum principle for this problem and hence, for (6). We conclude our presentation by means of an illustrative example.

[1] F. Benita, S. Dempe, P. Mehlitz: Bilevel optimal control problems with pure state constraints and finite-dimensional lower level, Preprint 2015-01, Fakultät für Mathematik und Informatik, Technische Universität Bergakademie Freiberg, 2015

[2] V.V. Kalashnikov, R.Z. Ríos-Mercado: A natural gas cash-out problem: A bilevel programming framework and a penalty function method. Optimization and Engineering **7** (2006) 403–420

[3] R. Vinter: Optimal Control. Birkhäuser, Boston, 2010

[4] J.J. Ye, D.L. Zhu: Optimality conditions for bilevel programming problems. Optimization **33** (1995) 9–27

Chance Constraint Models for Multi-Failures in the Design of Communication Networks

Sebastian Richter
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For a given network topology we suppose that the data stream is disrupted by failure of components of the network or exterior forces leading to failures, which both can occur with a certain probability. For each node pair of the network a routing subgraph has to be determined such that the overall loss of data due to events that lead to failures is small with high probability. We present a cutting plane and a robust approach which yield reasonably good solutions assuming that the failures are caused by at most two events.

Stochastic Bilevel Programming Problem

Rizo Saboiev
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Bilevel programming problem act as optimization problem whose constraint region is determined implicitly by another mathematical programming problem [1]. An ordered hierarchy structure between two decision makers appear, when the decision makers have the conflict objectives. The decision maker at the upper level (leader) optimize his/her objective function first, after the possible reaction of the decision maker at the lower level (the follower) considered. Hereupon, the follower selects his/her decision under the given decision of the leader.

Stochastic programming with bilevel programming problem has been investigated in [2],[3]. But in general these both problems studied together very little. The bilevel knapsack problem, which was first considered by Dempe and Richter [4], recently investigated with stochastic right-hand sides in paper of Ozaltin et al. [5]. Kalashnikov et al. [6] presented a bilevel multi-stage stochastic programming problem for the energy modeling, to formulate the natural gas cash-out problem. Material on single-level stochastic programming can be found in Chapter 27 of [7]. Bilevel stochastic linear programming problems with quantile criterion has been investigated in [8].

In this paper we investigate bilevel programming problem where the lower level presents a two-stage linear stochastic programming problem. The support probabiity distributation in our case is finite. In order to solve the stated bilevel programming problem we use the tools from convex analysis to reformulate it as one-level optimization problem.

[1] S. Dempe: Foundations of Bilevel Programming. Kluwer, Dordrecht, 2002
[2] P.B. Luh, T.S. Chang, T. Ning: Pricing: Problems with a continuum of customers as stochastic Stackelberg games. Journal of Optimization Theory and Applications **55** (1987) 119–131
[3] M. Patriksson, L. Wynter: Stochastic mathematical programs with equilibrium constraints. Operations Research Letters **25** (1999) 159–167
[4] S. Dempe, K. Richter: Bilevel programming with knapsack constraints. Central European Journal of Operations Research **8** (2000) 93–107
[5] O.Y. Özaltın, O.A. Prokopyev, A.J. Schaefer: The Bilevel Knapsack Problem with Stochastic Right-Hand Sides. Operations Research Letters **38** (2010) 328–333
[6] V.V. Kalashnikov, G.A. Pérez-Valdés, A. Tomasgard, N.I. Kalashnykova: Natural gas cash-out problem: bilevel stochastic optimization approach. European Journal of Operational Research **206** (2010) 18–33
[7] A.A. Gaivoronski: Stochastic Optimization in Telecommunications. Handbook of Optimization in Telecommunications (M.G. C. Resende and P. M. Pardalos, Editors), Springer, 2006
[8] S.V. Ivanov: Bilevel stochastic linear programming problems with quantile criterion. Automation and Remote Control **75** (2014) 107–118

Optimal Embeddings of Trees in the Line

Uwe Schwerdtfeger
Technische Universität Chemnitz, Faculty of Mathematics

We consider embeddings of bipartite graphs into Euclidean space. We wish to minimise the sum of the squared norms of the embedding points subject to distance constraints, namely adjacent nodes shall be embedded at distance at least one. This problem is the rescaled dual of minimising the largest eigenvalue of a weighted Laplacian of the graph. Gring, Helmberg and Reiss studied properties of optimal solutions one of which is that every bipartite graph has a one-dimensional embedding. Optimal solutions can be computed efficiently by semidefinite programming which indicates that there should be a polynomial time algorithm which operates "directly" on the graph. The aim of this talk is to devise such an algorithm for trees. This is joint work with Israel de Souza Rocha and Christoph Helmberg.

Scalarization in Geometric and Functional Vector Optimization Revisited

Christiane Tammer, Marius Durea, and Radu Strugariu
Martin-Luther-Universität Halle-Wittenberg, Faculty of Natural Sciences II

The aim of this talk is to provide a survey on some recent results on the field of optimality conditions in vector optimization with geometric and inequality / equality constraints. Moreover, the discussion we initiate leads us to consider new situations which were not previously studied.

In the last twenty years, the literature dedicated to necessary optimality conditions for Pareto efficiency in terms of generalized differentiation had an important growth. In general, dealing with weak Pareto optimality is a problem already solved in several very general cases and certain nonlinear scalarization techniques played important roles in the early as well as in the subsequent developments. Another well-known fact is that, in many classic infinite dimensional Banach spaces, the natural ordering cone has empty topological interior, and therefore the notion of weak Pareto minimum is not operable. Nevertheless, in finite dimensional normed vector spaces, as well as in few infinite dimensional Banach spaces, the usual positive cones have non-empty interior, and in such a case, every local Pareto minimum is a local weak Pareto minimum. On the other hand, the possibility to work with weak solutions is a very important advantage from theoretical point of view, because in this case one has some powerful scalarization techniques, which enable one to get necessary optimality conditions for vectorial problems on the base of a similar issue for a scalar one. Nevertheless, the case of genuine Pareto optimality, allowing the ordering cone to have empty topological interior in Banach spaces, is a much harder issue. Roughly speaking, we can identify three possibilities, which were developed up to now in order to overcome the difficulties in this case. The first one is to not use scalarization at all and to impose some additional conditions on the ordering cone (such as dually compactness property) and on the image space (the Asplund property). The second possibility is to still use a nonlinear scalarization technique and to impose conditions on the feasible set, while the third one is to change the scalarizing method and to use some concepts of approximate efficiency and derive optimality conditions moving away from the reference point.

We want to point out the main ideas developed around the useful properties of nonlinear scalarization functionals in the context of necessary optimality conditions for vector optimization problems; this was and still is a fruitful research area.

The Lipschitzianity of Convex Vector and Set-valued Functions

Anh Tuan Vu
Martin-Luther-Universität Halle-Wittenberg, Faculty of Natural Sciences II

It is well known that every scalar convex function is locally Lipschitz on the interior of its domain in finite dimensional spaces. The aim of this paper is to extend this result for both vector functions and set-valued mappings acting between infinite dimensional spaces with an order generated by a proper convex cone C. Under the additional assumption that the ordering cone C is w-normal, we prove that a locally C-bounded C-convex vector function is Lipschitz on the interior of its domain by two different ways. Moreover, we derive necessary conditions for Pareto minimal points of vector-valued optimization problems where the objective function is C-convex and C-bounded.

Investigating Mixed-integer Hulls Using a MIP-solver

Matthias Walter and Volker Kaibel
Otto-von-Guericke-Universität Magdeburg, Faculty of Mathematics

We present a preliminary version of a software tool which, given a system of linear inequalities and a subset of integer variables, investigates the associated mixed-integer hull. In particular, it detects all equations and some facets valid for the hull with exact arithmetic. The generation of facets can be controlled by specifying an interesting objective function. This is in contrast to usual convex-hull algorithms which produce the entire description of the hull, but run out of resources for small dimensions already. Based on the performance of the MIP solver used this software can handle much larger dimensions. The approach is related to work on target cuts by Buchheim, Liers and Oswald in 2008.

Extended Formulations: Compact Formulations vs. Limitations

Stefan Weltge, Volker Kaibel, Jon Lee, and Matthias Walter
Otto-von-Guericke-Universität Magdeburg, Faculty of Mathematics

This talk is about recent theoretical developments in the field of extended formulations. After explaining the main concept, we revisit some small size formulations associated to classical combinatorial optimization problems. In the second part of the talk, we give an idea about proving lower bounds on sizes of extended formulations. Finally, we include an alternative, simple proof of the fact that any extended formulation associated to the MAX-CUT problem must have exponential size.

Dualitätsaussagen für Erweiterte Minimax Standortprobleme

Oleg Wilfer

Technische Universität Chemnitz, Faculty of Mathematics

Es werden Erweiterte Standortprobleme diskutiert, wie sie von Drezner [2] erstmalig 1991 formuliert wurden. Dabei wird für jeden gegebenen Standort die gewichtete Summe der Entfernungen zu allen Einrichtungen plus möglicher Set-Up Kosten ermittelt und der maximale Wert dieser Summen minimiert. Anwendungsmöglichkeiten solcher Problemstellungen liegen beispielsweise bei gewissen Notfallszenarien, bei denen die Standorte der Krankenwagen nicht notwendigerweise mit den Standorten der Krankenhäuser zusammenfallen müssen. Unter dieser Annahme können die Fahrtzeiten verkürzt werden, da die verfügbaren Krankenwagen die Patienten nicht zu den ursprünglichen, sondern zu den nächstgelegenen Krankenhäusern transportieren. Trotz praktischer Bedeutung haben diese Probleme in der Literatur bislang nur wenig Beachtung erfahren. Das Ziel dieser Untersuchung ist, mit den Mitteln der Konjugierten Dualität duale Probleme zu konstruieren sowie notwendige und hinreichende Optimalitätsbedingungen zu formulieren. Unter der Verwendung dieser Optimalitätsbedingungen wird zudem eine geometrische Charakterisierung der Menge der optimalen Lösungen der entsprechenden konjugiert dualen Probleme vorgestellt. Abschließend werden einige Beispiele zur Veranschaulichung der Resultate besprochen.

[1] R.I. Bot, S.-M. Grad, G. Wanka: Duality in Vector Optimization. Springer, Berlin, 2009

[2] Z. Drezner: The Weighted Minimax Location Problem with Set-up Costs and Extensions. RAIRO-Operations Research **25** (1991) 55–64

[3] C. Michelot, F. Plastria: An Extended Multifacility Minimax Location Problem Revisited. Annals of Operations Research **111** (2002) 167–179