Differentiating accretive operators, and linearized stability for nonlinear evolution equations

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One of the particular problems in the analysis of the asymptotic behaviour of solutions to evolutionary equations is the stability of equilibrium solutions. While classical results are based on the existence of a Fréchet derivative at the equilibrium and its spectrum (stability via the linearized equation), this approach fails for an initial value problem (CP) $\dot{u}(t) + Au(t) \ni 0,\ u(0) = u_0$, governed by a nonlinear accretive operator A in a Banach space X. As a substitute for Fréchet differentiability in this context, the concept of a *resolvent derivative* $\tilde{A}$ of A at an equilibrium is introduced, and it is shown that exponential stability of the "somewhat linearized" (only resemblence to linearity of $\tilde{A}$ being $0 \in \tilde{A}0$) equation $\dot{u}(t) + \tilde{A}u(t) \ni 0$ implies local exponential stability of the equilibrium for equation (CP).
This abstract result serves to prove linearized stability results for a variety of further evolutionary equations, such as (a) initial-history (delay) problems, (b) Volterra equations, and (c) age-dependent population dynamics.
The concept also works for deriving regularity results for such problems.