TECHNISCHE UNIVERSITÄT DRESDEN

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Sommersemester 2025

Dresdner Mathematisches Seminar

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Heat kernel estimates for the fractional Laplacian and related non-local operators

The fractional Laplacian $?^{?/2}$ in a d-dimensional Euclidean space is a non-local integral operator with the singular kernel of the form $|x-y|^{-d-?}$, $0<?<2$. As in the case of the standard Laplacian ?, the fundamental solution of the corresponding heat equation is called the *heat kernel*. The heat kernel has a probabilistic interpretation: it represents the transition densities $p(t,x,y)$ from point x to point y in time t for the corresponding strong Markov process – the isotropic ?-stable process.

Unlike the Laplacian, where the heat kernel is given by an explicit formula (the Gaussian kernel), the closed-form expression for the heat kernel of the fractional Laplacian remains unknown. Therefore, the best one can hope for are sharp two-sided estimates.

I will begin by introducing the basic properties of the fractional Laplacian, including its connection to the heat semigroup, the associated Dirichlet form, and the fundamental heat kernel estimates. A key aspect of these estimates is how they arise naturally from exact scaling properties.

Next, I will discuss various formulations of the fractional Laplacian on open subsets of Euclidean space, such as the *reflected*, *restricted*, and *censored* fractional Laplacians. For each case, I will explain the corresponding Markov process and show the relevant heat kernel estimates. All these operators share the feature that their singular kernels are either equal or comparable to $|x-y|^{-d-?}$, – the so-called *uniformly elliptic case*. The analysis relies on a combination of functional-analytic and probabilistic techniques.

I will conclude with some very recent results for scenarios where the singular kernel may vanish at the boundary. I will also motivate the study of such degenerate cases. Surprisingly, the resulting heat kernel estimates exhibit qualitatively different behavior, highlighting new phenomena in this setting.

Mittwoch, 16.07.2025, 17:00 Uhr – Willers-Bau, Raum A 124
Leitung: Prof. Dr. René Schilling

Vor dem Vortrag findet **ab 16:30 Uhr** ein gemeinsames **Kaffee-/ Teetrinken** vor dem Vortragsraum WIL A 124 statt.

Bereich Mathematik und Naturwissenschaften
Fakultät Mathematik