Boundary observation and control for fractional heat and wave equations

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Abstract

We will start the talk by introducing the general notion of controllability of evolution equations and its connections to the observation of the associated dual system.

Secondly, we will introduce the fractional Laplace operator $(-\Delta)^s$ $(0 < s < 1)$ and its basic properties.

Next, we will establish boundary observability and control for the fractional heat equation. Our approach introduces a novel synthesis of techniques from fractional partial differential equations and control theory, combining several key ingredients in an original and effective manner as follows:

- *Boundary observability for low-frequency solutions of the fractional wave equation.* We begin by analyzing the associated fractional wave equation. Using a fractional analogue of Pohozaev's identity, we establish a partial boundary observability result for the low-frequency solutions. The corresponding observability time horizon increases with the eigenmode frequency, reflecting the inherently slower propagation speed of the fractional waves.
- *Transmutation to the parabolic setting.* Using transmutation techniques, we transfer the observability results from the wave setting to the parabolic one. This yields a frequency-dependent observability inequality for the fractional heat equation, which - via duality - enables control of its low-frequency components.
- *Frequency-wise iteration.* Leveraging the dissipative nature of the fractional heat equation, we develop an iterative procedure to successively control the entire frequency spectrum of solutions. The condition $s \in (1/2, 1)$ is crucial in this analysis, as it guarantees sufficient decay of high-frequency components, enabling the convergence of the iteration.
- *Duality.* By a duality argument, we derive boundary observability from the boundary controllability of the fractional heat equation. Remarkably, this type of boundary observability result is entirely new in the multi-dimensional setting and appears to be out of reach for existing methods.

The talk will be accessible to a large audience avoiding any technical difficulties.

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