Affine Processes - Topics for the oral Exam

Not relevant for exam are subsection 3.3 on exponential moments & the (unnumbered) subsection of Chapter 5 on Doob's h-transform. Lemmas and Propositions are not asked, but you should be able to explain the content of all theorems and be able to sketch the proof (if it was covered in the lecture). I will also not ask about the proofs of the Kolmogorov extension theorem and the Filipovic-Teichmann theorem on finite dimensional realizations of Heath-Jarrow-Morton models.

Chapter 1: Affine Diffusions

* Ornstein-Uhlenbeck process, Feller Diffusion (Definition and Properties).
* Thm 1.1.: OU-process and Feller diffusion are affine processes; Riccati equations
* Existence and uniqueness of strong solution for the Feller SDE;
* Thm 1.2: Behavior of Feller diffusion at the boundary, meaning of the Feller condition, McKean's argument for inaccessibility of the zero.
* Properties of the Squared Bessel process, scale function of BESQ;
* Multivariate Affine Diffusions: Definition, Riccati equations,
* Invariance of half-spaces (inward pointing drift & parallel diffusion) & admissibility conditions
* Thm 1.11: Existence of affine diffusion processes under admissibility conditions & the affine transform formula

Chapter 2: Applications

* Forward rate agreement (FRA), short rate and forward rate.
* Thm 2.1.: Equations for bond prices and forward rates in affine short rate models.
* Heath-Jarrow-Morton model on Hilbert space and existence of finite dimensional realizations.
* Pricing of Calls and Puts with Fourier methods. Thm 2.2: Option Pricing in Affine Stochastic Volatility models (sketch)
* The Heston model as an affine stochastic volatility model.

Chapter 3: General Theory of Affine Processes

* Markov processes, Kolmogorov Extension Theorem; Feller processes and the infinitesimal generator;
* Levy processes and semi-martingale characteristics.
* Definition of affine processes on convex cones; regularity property; Levy-Khintchine form of functions F,R (with proof).
* Thm 3.6: Generator of a regular affine process on a convex cone;
* Thm 3.8: Semi-martingale characteristics of affine process with sketch of proof;

Chapter 4: More Applications: Implied Volatility in the Heston model

* The implied volatility surface
* Large deviations and the Gärtner-Ellis theorem (Thm 4.1)
* Application of Gärtner-Ellis theorem to the Heston model (Thm 4.4, sketch only).
* Conclusion on the shape of the volatility surface in the Heston model & SVI-parametrization.

Chapter 5: Matrix-Valued Affine Processes

* Basics on Matrix-Valued stochastic processes.
* The Wishart process: Definition and SDE characterization (= Thm 5.6).
* Bru's theorem (=Thm 5.8) and SDEs for the Eigenvalues of the Wishart process + sketch of proof.
* Thm 5.9. on first Eigenvalue collision and first singularity time of the Wishart process.
* Wishart process as a matrix-valued affine process + its Laplace transform.