Summary: Lecture on Random Graphs and Networks

Chapter 0&1 (Introduction & Graph Theory Basics): Examples of real-world networks, Typical features (sparsity, small diameter, universal laws), Basic mathematical terminology, Adjacency matrix, Degree distribution.

Chapter 2 (Structural Importance & Perron-Frobenius Theory): Measuring centrality/structural importance of vertices. Dynamic model of 'importance recursion', Non-negative, stochastic, primitive and irreducible matrices, Simplicity of Eigenvectors and Jordan decomposition. Perron's theorem and sketch of proof. Perron-Frobenius theorem and comparison to Perron's theorem. Corollary on convergence of 'importance recursion'. Eigenvector centrality and PageRank-Algorithm.

Chapter 3 (Concentration Inequalities): Markov- and Chernoff-Bernstein-inequalities with sketch of proof. Azuma's inequality, McDiarmid's inequality with sketch of proof.

Chapter 4 (Erdös-Renyi-Model and its degree distribution): Definitions of ER(n,p) and ER(n,k) models. Expected degree distribution (general + approximation in the sparse regime). Actual degree distribution and concentration for large n, including sketch of proof. Empiricial considerations

Chapter 5 (Emergence of the Giant Component): What is the giant component? Main result on existence/non-existence of giant component. Exploration process and sketch of proof: Recursion for explored but unsaturated vertices, recursions for upper + lower bounds. Relation to skip-free random walks.

Chapter 6: Does not exist :-)

Chapter 7 (The preferential attachment model and power-law distributions): Scale-free property, Pareto distribution, log-log-plots, heavy tailed distributions, Power-law tails.
Def. of PA(p) model, power-law-property of expected degree distribution with sketch of proof. Concentration of actual degree distribution for large n.

Chapter 8 (Spectral graph analysis): Random walks on graphs, equilibrium distributions. Spectral characterization of bipartite graphs. Definition of graph Laplacian, relation to random walk matrix. Rayleigh quotients, variational characterization of Eigenvalues & Intuition. Conductance and Cheeger's inequality with sketch of proof.