**Sums of i.i.d. random variables with exponential weights**

It is well known that a random walk $S_n = \sum_{k=1}^{n} X_k$, with $(X_k)$ i.i.d. having finite expecation diverges almost surely to $\infty$ if and only $E(X_1) > 0$, while for $E(X_1) = 0$ it oscillates. Less well known is the study of random walks when $E|X_1| = \infty$. In 1973, Erickson [1] obtained an integral criterion characterising when the corresponding random walk diverges to $\infty$, $-\infty$, or when it oscillates. In this talk we are interested in the divergence behaviour of $W_n = \sum_{k=1}^{n} c^k X_k$, where $(X_k)$ is i.i.d. and $0 < c < 1$. It is well known that this sum converges almost surely if and only if $E \log^+ |X_1| < \infty$, but we are interested in the divergence behaviour when $E \log^+ |X_1| = \infty$. We give sufficient analytic conditions for $W_n$ to exhibit an oscillating behaviour (i.e. $\infty = \liminf_{n\to\infty} W_n < \limsup_{n\to\infty} W_n = \infty$ almost surely), as well as a sufficient condition for the almost sure limit to exist in the sense that $\lim_{n\to\infty} W_n = \infty$. The talk is based on joint work with A. Lindner and R. Maller.

[1] K. Bruce Erickson. “The strong law of large numbers when the mean is undefined”. In: Transactions of the American Mathematical Society 185 (1973), pp. 371–381.