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A Note on the Separation Method

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Abstract

The present paper provides explicit formulas for the estimators of the inflation parameters and of the development pattern which are used in the separation method. Although the result is essentially known, it appears that many loss reserving actuaries are not aware of it.

Consider a run-off triangle of incremental losses $\{Z_{i,k}\}_{i,k \in \{0,1,\dots,n\}, i+k \leq n}$ and assume that

$$E[Z_{i,k}] = \nu_i \lambda_{i+k} \vartheta_k$$

holds for all $i, k \in \{0, 1, \dots, n\}$ such that $i + k \leq n$, where

- $\nu_0, \nu_1, \dots, \nu_n$ are known *volume measures* of the accident years,
- $\lambda_0, \lambda_1, \dots, \lambda_n$ are unknown *inflation parameters* of the calendar years, and
- $\vartheta_0, \vartheta_1, \dots, \vartheta_n$ form an unknown *development pattern* for the development years (such that $\sum_{l=0}^n \vartheta_l = 1$).

The primary purpose of the separation method is to estimate the inflation parameters $\lambda_0, \lambda_1, \dots, \lambda_n$ from the run-off triangle.

As pointed out by in Radtke and Schmidt [2004], the parameters $\lambda_0, \lambda_1, \dots, \lambda_n$ (and $\vartheta_0, \vartheta_1, \dots, \vartheta_n$) may be estimated by the marginal-sum method, applied to the normalized incremental losses $X_{i,k} := Z_{i,k}/\nu_i$ which, in addition, are rearranged with respect to calendar years and development years instead of accident years and development years. In principle, estimation by the marginal-sum method proceeds by iteration.

In the present paper, we give explicit formulas for the marginal-sum estimators of $\lambda_0, \lambda_1, \dots, \lambda_n$ (and $\vartheta_0, \vartheta_1, \dots, \vartheta_n$). To this end, we interchange the roles of accident years and calendar years and consider the normalized incremental losses

$$Y_{i,k} := \frac{Z_{n-i-k,k}}{\nu_{n-i-k}}$$

and the parameters

$$\beta_i := \lambda_{n-i}$$

Then we have

$$E[Y_{i,k}] = \beta_i \vartheta_k$$

such that the run-off triangle of normalized incremental losses $\{Y_{i,k}\}_{i,k \in \{0,1,\dots,n\}, i+k \leq n}$ satisfies the usual assumption of the marginal-sum method.

Letting

$$T_{i,k} := \sum_{l=0}^k Y_{i,l} = \sum_{l=0}^k \frac{Z_{n-i-l,l}}{\nu_{n-i-l}}$$

we obtain

$$\beta_i = E[T_{i,n}]$$

and hence

$$\vartheta_k = \frac{E[Y_{i,k}]}{E[T_{i,n}]}$$

such that $\beta_0, \beta_1, \dots, \beta_n$ are the expected ultimate losses of the run-off triangle $\{T_{i,k}\}_{i,k \in \{0,1,\dots,n\}}$ and the parameters $\vartheta_0, \vartheta_1, \dots, \vartheta_n$ form a development pattern for incremental quotas also for the run-off triangles $\{Y_{i,k}\}_{i,k \in \{0,1,\dots,n\}, i+k \leq n}$ and $\{T_{i,k}\}_{i,k \in \{0,1,\dots,n\}, i+k \leq n}$.

It is well-known, that the marginal-sum estimators of the parameters $\beta_0, \beta_1, \dots, \beta_n$ are identical with their chain-ladder estimators, and these are given by

$$\widehat{\beta}_i := T_{i,n-i} \prod_{l=n-i+1}^n \widehat{\psi}_l = \left(\sum_{l=0}^{n-i} \frac{Z_{n-i-l,l}}{\nu_{n-i-l}} \right) \prod_{l=n-i+1}^n \widehat{\psi}_l$$

with

$$\widehat{\psi}_k := \frac{\sum_{j=0}^{n-k} T_{j,k}}{\sum_{j=0}^{n-k} T_{j,k-1}} = \frac{\sum_{j=0}^{n-k} \sum_{l=0}^k \frac{Z_{n-j-l,l}}{\nu_{n-j-l}}}{\sum_{j=0}^{n-k} \sum_{l=0}^{k-1} \frac{Z_{n-j-l,l}}{\nu_{n-j-l}}}$$

Then the marginal-sum estimators of the parameters $\lambda_0, \lambda_1, \dots, \lambda_n$ and $\vartheta_0, \vartheta_1, \dots, \vartheta_n$ are given by

$$\widehat{\lambda}_i := \widehat{\beta}_{n-i}$$

and

$$\widehat{\vartheta}_k := \begin{cases} \prod_{l=1}^n \frac{1}{\widehat{\psi}_l} & \text{if } k = 0 \\ \left(1 - \frac{1}{\widehat{\psi}_k}\right) \prod_{l=k+1}^n \frac{1}{\widehat{\psi}_l} & \text{else} \end{cases}$$

They are thus given in closed form, but in actuarial practice one will simply apply the chain-ladder method to the run-off triangle $\{T_{i,k}\}_{i,k \in \{0,1,\dots,n\}, i+k \leq n}$, as noticed already in a publication by Swiss Re [2000].

Once the marginal-sum estimators of the inflation parameters of the observable losses are known, they can be used in at least two ways to predict future losses:

- If it is assumed that there are further inflation parameters $\lambda_{n+1}, \dots, \lambda_{2n}$ such that the identity $E[Z_{i,k}] = \nu_i \lambda_{i+k} \vartheta_k$ is also valid for $i+k \geq n+1$, then one may use a curve fitting method to first estimate the remaining parameters $\lambda_{n+1}, \dots, \lambda_{2n}$ from the estimators of $\lambda_0, \lambda_1, \dots, \lambda_n$ and then predict the future incremental losses taking into account future inflation, as described in Radtke and Schmidt [2004]. Of course, the type of the curve to be fitted may considerably affect the result.
- As another possibility, one could use the estimators of $\lambda_0, \lambda_1, \dots, \lambda_n$ only to adjust the incremental losses for inflation: Putting

$$\tilde{Z}_{i,k} := \hat{\lambda}_n \frac{Z_{i,k}}{\hat{\lambda}_{i+k}}$$

for $i+k \leq n$ yields a cleaned-up run-off triangle scaled to the level of calendar year n , and now any of the common methods of loss reserving based on run-off triangles might be used to predict the future losses.¹ Of course, any particular choice of a method of loss reserving would affect the result as well, but the pros and cons of these actuarial methods are quite well understood.

The second approach has the advantage that it does not involve any assumption at all on the future development of inflation.

References

Radtke, M., and Schmidt, K. D. [2004]: *Separationsverfahren. In: Handbuch zur Schadenreservierung.* Karlsruhe: Verlag Versicherungswirtschaft.

Swiss Re [2000]: *Late claims reserves in reinsurance.* Zürich: Swiss Re.

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¹Note that scaling the volume measures by a common factor $c > 0$ would not affect the cleaned-up run-off triangle.