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Comment on "Degenerate mobilities in phase field models are insufficient to capture surface diffusion" [Appl. Phys. Lett. 107, 081603 (2015)]

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In a recent paper, Lee *et al.*[1] argue about Cahn-Hilliard equations as approximations for motion by surface diffusion. The considered equations read

$$u_t = -\nabla \cdot \mathbf{j}, \quad \mathbf{j} = -\epsilon M(u)\nabla\mu, \tag{1}$$

$$\epsilon\mu = -\epsilon^2\nabla^2 u + f'(u), \tag{2}$$

with order parameter u, quartic free energy $f(u) = \frac{1}{4}(1-u^2)^2$, mobility function $M(u) = 1-u^2$, and the length scale ϵ related to the diffuse interface width. In leading order, these equations formally converge if $\epsilon \to 0$ to motion by surface diffusion with an additional bulk diffusion term, which had been ignored in several previous studies. This additional term can alter the long time behaviour, as shown in Refs. 1 and 2 and is thus important to consider. However, several points in Ref. 1 are at least misleading. This includes the general statement of the title, which is already contradicted by the authors, who admit that a double obstacle or logarithmic free energy instead of the used quartic free energy $f(u)$ gives rise to motion by surface diffusion without the additional bulk diffusion term.[3] But even if the authors use a terminology where the term *phase field models* is only associated with equations based on a quartic free energy, their conclusion is wrong. It has already been pointed out by Gugenberger *et al.*,[4] who showed using matched asymptotic analysis, that the model in Ref. 5 converges also to motion by surface diffusion without the additional bulk diffusion term. With the introduced notation and scaling, the model in Ref. 5 reads

$$u_t = -\nabla \cdot \mathbf{j}, \quad \mathbf{j} = -\epsilon M(u)\nabla\mu, \tag{3}$$

$$\epsilon g(u)\mu = -\epsilon^2\nabla^2 u + f'(u), \tag{4}$$

which is obtained by defining $u = 2\phi - 1$, rescaling time by $1/\epsilon^2$, considering an isotropic free energy and neglecting the viscous term. There are two differences: The mobility function is a higher order polynomial with $M(u) \approx f(u)$ and the presence of a stabilizing function $g(u)$, which is also defined such that $g(u) \approx f(u)$. Such a stabilizing function is commonly used in classical phase field models for solidification.[6] Both modifications alter the analysis in Ref. 1. The higher order polynomial in $M(u)$ already suppresses the presence of a normal flux in leading order and as shown in Ref. 4 the combination with the stabilizing function ensures also at next-to-leading order proper convergence results.

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