Interactive Evolution of a Bicontinuous Structure

FLORIAN STENGER AND **AXEL VOIGT**

ABSTRACT

The authors describe an installation that was shown at the exhibition *The Best of All Possible Worlds* at Technische Sammlungen Dresden in 2016. The installation provided an interactive experience of the evolution of a complex bicontinuous structure of two immiscible fluids. The evolution is driven by the surface tension of the interface of the two fluids, which results in a continuous reduction of the interface area. The process is mathematically described by a partial differential equation, which is numerically solved. In each time step, the structure, visualized by the fluid-fluid interface, is rendered and shown on an elastic display. According to the deformation of the display, the corresponding time frame is projected. By pushing against the elastic display, one therefore can interact with the structure and evolve it in time in a playful and intuitive manner.

ART AND TECHNOLOGY

In this work we interpret the symbiosis of art and technology by exploring physical principles, transforming them into mathematical models, developing algorithms to numerically solve these models and using the simulation results to create art. In the present case we consider two immiscible fluids of equal volume that phase-separate and form bicontinuous morphologies that coarsen in time. Such phase-separation phenomena are found in various materials, such as polymer solutions, colloidal suspensions, protein solutions or within the cytoplasm of a cell [1]. Due to their importance in materials science and biology, these phenomena are intensively studied experimentally and theoretically. The physical driving force behind this coarsening process is the surface tension of the fluid-fluid interface. The resulting three-dimensional morphologies contain a characteristic length scale. For our intention to use this coarsening process within an art exhibit, this length scale is important, as it makes the morphologies aesthetically appealing.

An appropriate mathematical model to describe this process is the Cahn-Hilliard equation [2], which can be solved numerically using Finite Elements. Combining this model with a wavy computational domain and appropriate coloring and rendering of the computed interface leads to the impression of a highly energetic fluid. Projecting the result onto a large elastic display offers the possibility for viewer interaction. Pushing against the display leads to an evolution in time and the coarsening of the structure, allowing one to explore the four-dimensional object in space-time. This is achieved by projecting the precomputed results for different time steps according to the measured deformation of the display. The resulting installation was shown at the 2016 exhibition *The Best of All Possible Worlds* at Technische Sammlungen Dresden (see Fig. 1).

Below we describe the technology used and the implementation in detail; we demonstrate the result, a playful and intuitive art exhibit, and draw conclusions.

TECHNOLOGY AND IMPLEMENTATION

The installation combines advanced display technology, computer graphics and state-of-the-art scientific computing on high-performance computers.

Elastic Display

Elastic displays are a new field in human-computer interaction. The ability to deform the display offers an additional and intuitive interaction through haptic feedback. Similar to pressure-sensitive touch screens, like the 3D Touch technology used by Apple from the iPhone 6S onwards, one can access additional functionality by varying the pressure on the display. We here use an elastic display called FlexiWall [3]. The setup is shown in Fig. 2, with a projector behind the screen and a depth sensor tracking the displacement of the projection screen at the center position.

For the depth sensor, Microsoft's Kinect device is used, calibrated to measure displacement of the elastic display in the center. This value is used to select the time frame to be projected onto the display. A linear relation between deformation and advancement in time is used. As the coarsen-

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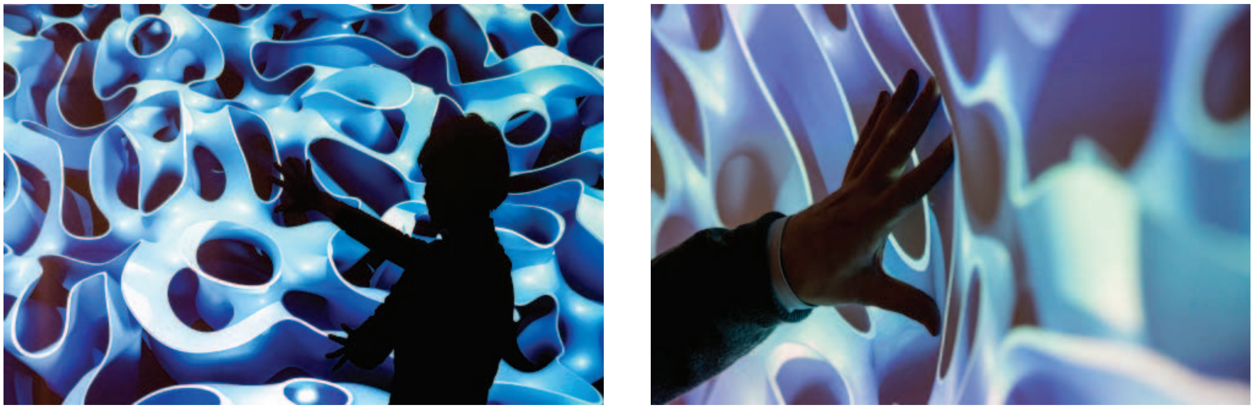

Fig. 1. Interaction with an elastic display to evolve a computational solution showing the rendered interface of a bicontinuous structure, shown at the exhibition *The Best of All Possible Worlds* at Technische Sammlungen Dresden in 2016. (© Axel Voigt. Photos: Baldauf&Baldauf.)

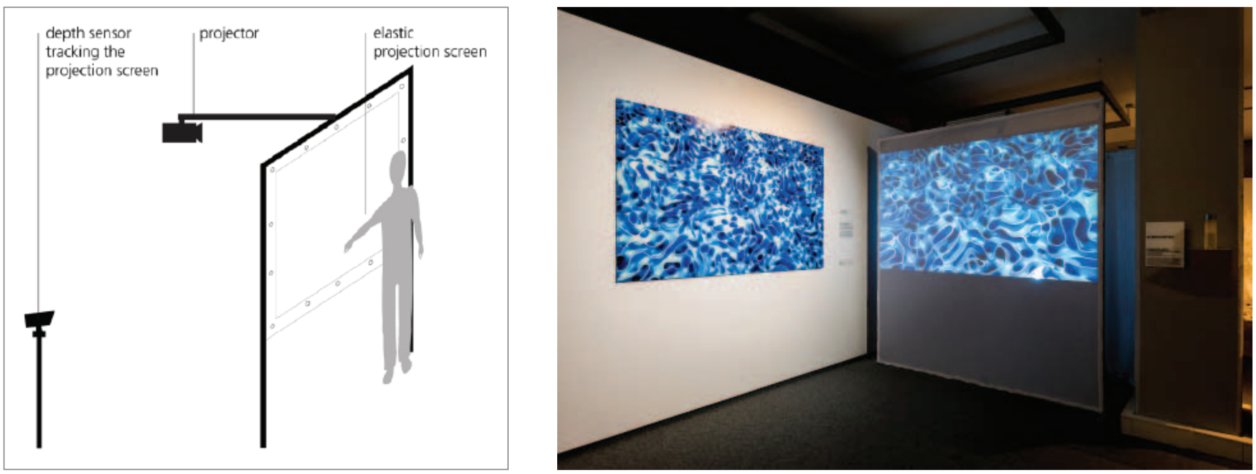

Fig. 2. (left) Core components of the system setup of the FlexiWall, adapted from Müller et al. [18]. (right) Installation at the exhibition *The Best of All Possible Worlds* at Technische Sammlungen Dresden in 2016, hiding the projector and depth sensor. (© Axel Voigt. Photo: Baldauf&Baldauf.)

ing does not evolve linearly but according to a power law and becomes slower and slower in time, the linear relation reflects the haptic feedback of the display. One has to push harder and harder to further evolve the structure in time or even to reach the maximal computed time frame. This thus ensures real-time visitor interaction with the structure. The position of the interaction with the display is not considered; the whole structure evolves according to the measured deformation in the center.

Computational Modeling

The mathematical model used is the Cahn-Hilliard equation [4]. It considers an order parameter that distinguishes between the two fluid phases. The considered equations read

$$u_t = -\nabla \cdot j, \quad j = -e\nabla\mu, \quad e\mu = -e^2\nabla^2 u + f'(u), \qquad (1)$$

with order parameter u, material flux j, chemical potential μ, quartic free energy $f(u) = 0.25(1 - u^2)^2$ and the length scale e related to the diffuse interface width. The two fluid phases are characterized by $u \approx 1$ and $u \approx -1$ and the fluid-fluid interface by $u = 0$. At the boundaries we consider zero-flux conditions $\partial_n u = \partial_n \mu = 0$, with outer normal n, and as initial condition we specify $u = 0 + \xi$, with white noise ξ, ensuring the equal volume fraction of the two phases. To allow maximal flexibility concerning the computational domain and avoid meshing of complicated domains, an implicit diffuse interface approximation [5] is used. The domain is embedded into a larger, simpler domain, and the original domain is approximated by a phase field variable $\varphi = 0.5\,(1\text{- }\tanh(r / (8^{0.5}\eta)))$, with signed distance function r, measuring the distance to the interface with $r > 0$ within the original domain and a small parameter η. We thus obtain $\varphi \approx 1$ within the original domain and $\varphi \approx 0$ outside. The equations to solve read

$$\varphi u_t = -\nabla\cdot(\varphi j), \quad j = -e\nabla\mu, \quad e\mu = -e^2\nabla\cdot(\varphi\nabla u) + \varphi f'(u). \qquad (2)$$

As shown in Li et al. [6], this converges for $\eta \to 0$ to the original problem with the appropriate boundary conditions. Eq. (2) is solved numerically using a finite element discretization in space and a semi-implicit finite difference discretization in time [7,8]. The simulation tool AMDiS [9,10] is used for the implementation, which allows for efficient parallel computations. Figure 3 shows (left) the phase field variable φ, (middle) the adaptively refined mesh with a high resolution only within the original domain $\varphi \approx 1$ and (right) a snapshot of the solution u, which is visualized through the interface $u = 0$, displayed only within the region $\varphi \approx 1$.

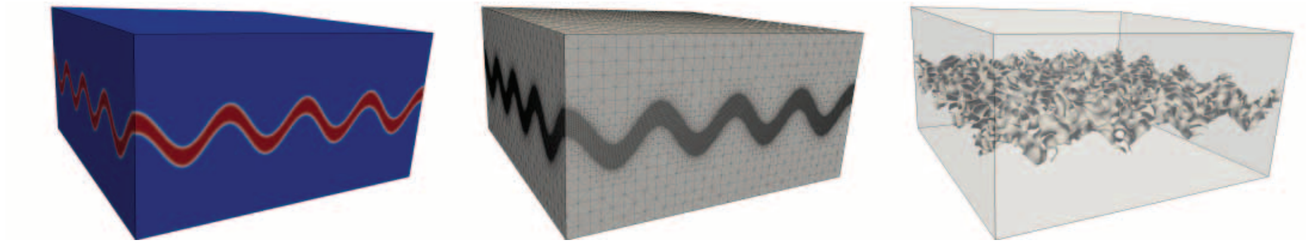

Fig. 3. (left) Phase field function φ with values 1 (red) and 0 (blue); (middle) adaptively refined mesh with fine resolution within the region $\varphi \approx 1$; (right) solution $u = 0$ within the region $\varphi \approx 1$. The interface is displayed with a finite thickness. The visualization is done by ParaView [19]. (© Axel Voigt)

The mesh contains 62.5 million tetrahedral elements, and approximately 300 time steps are computed. The simulations are done at the high-performance computer JURECA at the Jülich Supercomputing Center.

Postprocessing

In order to extract and render the interface, various post-processing steps are required. For each time step ParaView [11] is used to clip the mesh at $\varphi = 0.5$ to extract the original domain. The resulting mesh is composed of both tetrahedra and hexahedra.

The next step extracts the fluid-fluid interface, which is obtained by clipping the mesh at two different level-sets u. We use $u = -0.1$ and $u = 0.1$ to obtain a thin layer around the interface $u = 0$. From the resulting volume mesh, still consisting of tetrahedra and hexahedra, the filters in ParaView extract the surface and triangulate it. The resulting surface mesh contains about 12.5 million elements and has very poor quality. Substantial improvement is achieved by the meshing tool MeshConv [12], which decimates the number of triangles to about one million and improves the quality of the mesh without loss of accuracy. The software MeshConv is also used to convert the resulting surface mesh into a format that the rendering software POV-Ray [13] can use. Rendering is then achieved with the same settings for each time step, and the resulting 3600-×-2400-pixel images are stored on a standard PC.

RESULTS

These elements—the rendered images in each time step and the elastic display with projector and depth sensor—form the interactive installation. Color Plate D gives an impression of the playful and intuitive experience of exploring the bicontinuous structure. Pushing against the display evolves the structure in time.

CONCLUSION

The described installation provides a playful and intuitive experience of the physical principles of phase separation in fluidic systems. This is achieved by an interactive art exhibit using advanced technologies, including mathematical modeling using a Cahn-Hilliard equation and an implicit diffuse interface approximation [14], scientific computing using adaptive finite element methods and parallel computations on high-performance computers [15], intensive preprocessing and rendering, and advanced display technologies using an elastic display called FlexiWall [16]. All the technology is invisible to the visitor, who only sees the projection of a static image and is asked to push against it. The deformation of the elastic display causes the structure to evolve in time, showing the coarsening process with various details resulting from topological changes of the interface.

The installation was shown at the exhibition *The Best of All Possible Worlds* at Technische Sammlungen Dresden in 2016. To provide this experience to a broader audience, a version with standard display technology has been developed and made available for Linux, Windows and macOS on the IMAGINARY platform for open mathematics [17]. The deformation of the display is thereby emulated by the time of interaction. We here also consider the position of interaction. A local blending of the images at different time frames, chosen according to the position, allows for a local evolution in time. A version for mobile devices is also available on Google Play Store under the name "spacetime."

Acknowledgments

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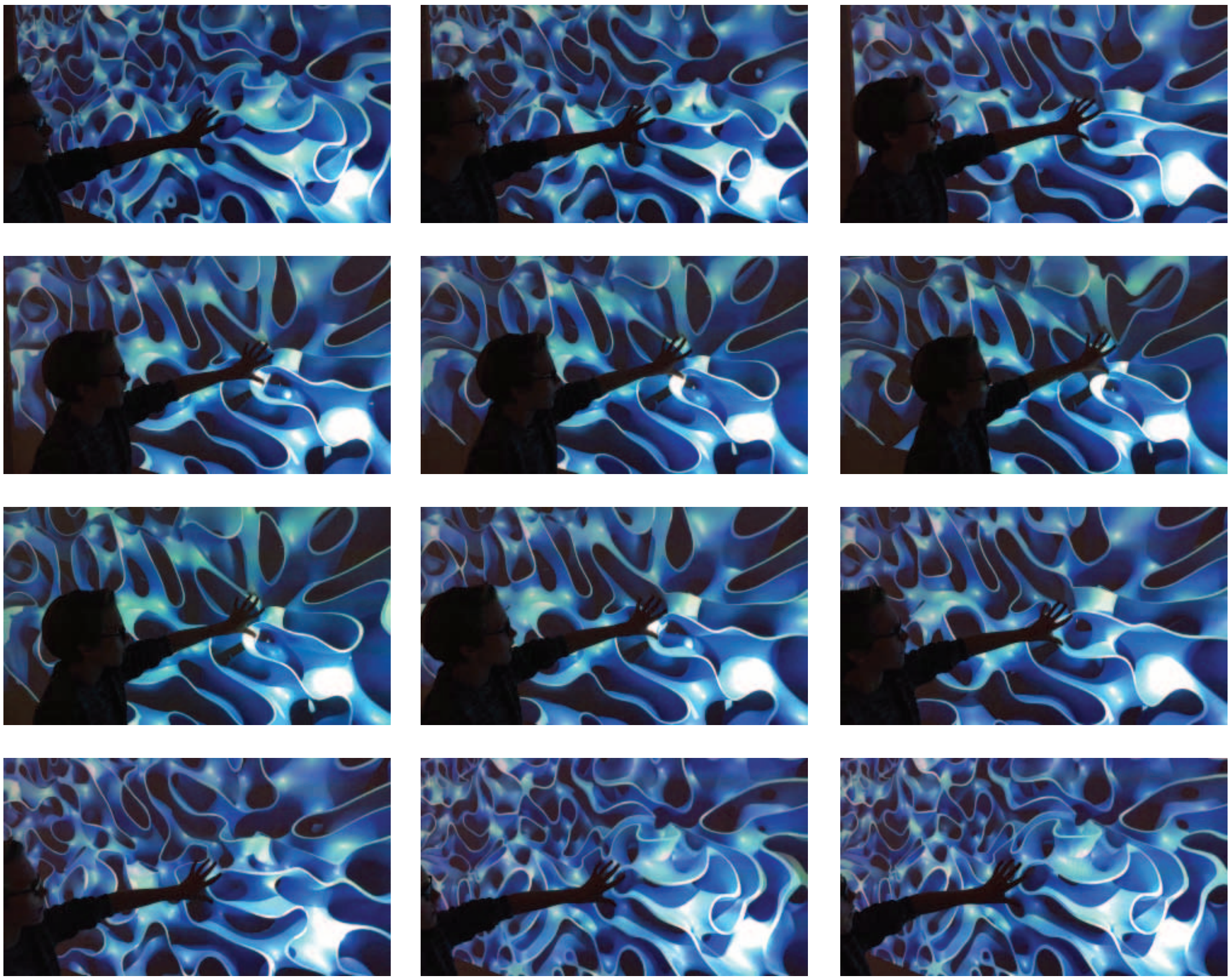
Both authors have presented their artwork at various exhibitions, including the art gallery of the 3D PrintShow in London 2012 and the Bridges exhibition in Seoul 2014.

COLOR PLATE D: **INTERACTIVE EVOLUTION OF A BICONTINUOUS STRUCTURE**



A visitor to the exhibition evolves the structure by pushing against the elastic display. This demonstrates the playful and intuitive interaction. The whole video can be found on YouTube (see Ref. [20] on page 330). (© Axel Voigt) (See article in this issue by Florian Stenger and Axel Voigt.)