SDFEM for singularly perturbed problem with two parameters in two dimensions

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Abstract

We consider a streamline-diffusion finite element method (SDFEM) for the following singularly perturbed eliptic problem

$$\begin{aligned} -\varepsilon_1 \Delta u + \varepsilon_2 b(x,y) u_x + c(x,y) u = f(x,y) \quad \text{in} \quad \Omega = (0,1) \times (0,1), \\ u = 0 \quad \text{on} \quad \partial\Omega, \end{aligned} \tag{1}$$

with $b(x,y) \geq b_0 > 0$, $c(x,y) \geq c_0 > 0$, $(x,y) \in \Omega$, where b, c and f are sufficiently smooth functions, b_0, c_0 are constants, ε_1, ε_2 are small perturbation parameters and f satisfies the compatibility conditions $f(0,0) = f(0,1) = f(1,0) = f(1,1) = 0$. Also, we assume $c(x,y) - \frac{\varepsilon_2}{2}\text{div}b(x,y) \geq \gamma > 0$, $(x,y) \in \Omega$, for some constant γ. The problem (1) is characterized by exponential layers at $x = 0$ and $x = 1$, parabolic layers at $y = 0$ and $y = 1$ and corner layers at four corners of Ω. The width of exponential layers depends on the relation between ε_1 and ε_2. For $\varepsilon_2 = 0$ the problem (1) is a reaction-diffusion problem as opposed to $\varepsilon_2 = 1$ when it becomes a convection-diffusion problem.

We will analyse the superconvergence property of the SDFEM of problem (1) and give optimal parameter choices for maximal stability in the induces streamline diffusion norm.

Keywords: singularly perturbed problem, two small parameters, SDFEM, stabilization parameter.

References

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[1]Supported by the German Academic Exchange Service (DAAD) with a scholarship in the programme Research Stays for University Academics and Scientists, 2017.

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