SDFEM for singularly perturbed problem with two parameters in two dimensions

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Abstract

We consider a streamline-diffusion finite element method (SDFEM) for the following singularly perturbed elliptic problem

\[-\varepsilon_1 \Delta u + \varepsilon_2 b(x, y)u_x + c(x, y)u = f(x, y) \quad \text{in} \quad \Omega = (0, 1) \times (0, 1),
\]

\[u = 0 \quad \text{on} \quad \partial \Omega,\]

with \(b(x, y) \geq b_0 > 0, c(x, y) \geq c_0 > 0, (x, y) \in \Omega\), where \(b, c\) and \(f\) are sufficiently smooth functions, \(b_0, c_0\) are constants, \(\varepsilon_1, \varepsilon_2\) are small perturbation parameters and \(f\) satisfies the compatibility conditions \(f(0, 0) = f(0, 1) = f(1, 0) = f(1, 1) = 0\). Also, we assume \(c(x, y) - \varepsilon_2 \frac{\nabla b(x, y)}{2} \geq \gamma > 0, (x, y) \in \Omega\), for some constant \(\gamma\). The problem (1) is characterized by exponential layers at \(x = 0\) and \(x = 1\), parabolic layers at \(y = 0\) and \(y = 1\) and corner layers at four corners of \(\Omega\). The width of exponential layers depends on the relation between \(\varepsilon_1\) and \(\varepsilon_2\). For \(\varepsilon_2 = 0\) the problem (1) is a reaction-diffusion problem as opposed to \(\varepsilon_2 = 1\) when it becomes a convection-diffusion problem.

We will analyse the superconvergence property of the SDFEM of problem (1) and give optimal parameter choices for maximal stability in the induces streamline diffusion norm.

Keywords: singularly perturbed problem, two small parameters, SDFEM, stabilization parameter.

References


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