Resurgence of beta spectrometry in a metrological context

Outlines

- Context and current situation
- The BetaShape code
- High precision study of atomic effects
- Outlook

The BetaShape program is available at http://www.nucleide.org/logiciels.htm

TU Dresden 2017 | X. Mougeot

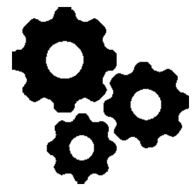
Oriented research

Fundamental research

Nuclear physics

Particle physics

Radiotoxicology



Laboratoire National Henri Becquerel

Ionizing radiation metrology

Radiochemistry

Bq, Gy and Sv units

Activity standards ~ 0.1%

Atomic and nuclear data

Applied research

Industries

Nuclear medicine

Nuclear energy

Instrumentation

Importance of beta decays

Scientific research

- Nuclear astrophysics (r-process)
- Standard Model (CKM matrix unitarity, weak magnetism)
- Beyond Standard Model (Fierz interference, sterile neutrino)
- Neutrino physics (reactor monitoring, non-proliferation)
- New detectors ($BrLa_3$)

Ionizing radiation metrology

Activity measurements by Liquid Scintillation Counting

Better knowledge of the beta spectra → **better uncertainties**

IAEA
International Atomic Energy Agency

Atomic and nuclear data

- NNDC (Brookhaven),

  → **ENSDF** nuclear decay data
- **DDEP** (International collaboration)

  Decay Data Evaluation Project
  Atomic and nuclear decay data recommended by the BIPM

Medical uses

Micro-dosimetry, internal radiotherapy

Nuclear fuel cycle

Decay heat, nuclear waste

Beta spectrum measurement is old-fashioned

Parent	Mode	E_0	Experimental shape factor	Ref.	Range	$\% E_0$	$\bar{E}_{sf}$	$\bar{E}_{calc}$
Second forbidden unique								
^{10}Be	β^-	556	$q^4+(10/3)q^2p^2+p^4$	[79]	100–500	71.9	252.33	252.00
^{22}Na	β^+	1821.02	$q^4+(10/3)\lambda_2 q^2p^2+\lambda_3 p^4$	[80]	660–1660	54.9	835.83	833.00
^{60}Co	β^-	1490.56	$q^4+(10/3)\lambda_2 q^2p^2+\lambda_3 p^4$	[81]	1350–1450	6.7	624.50	623.47
^{138}La	β^-	258	$1+407.71W-50.695/W$ $-583.794W^2+246.279W^3$	[63]	2.5–255	97.9	90.48	95.55
Third forbidden unique								
^{40}K	β^-	1311.07	$1.05q^6+6.3q^4p^2$ $+6.25q^2p^4+0.95p^6$	[82]	100–1100	76.3	583.98	583.27

X. Mougeot, Phys. Rev. C 91, 055504 (2015)

Created database of **130 experimental shape factors**

- Allowed: 36
- Forbidden unique: 25 (1st), 4 (2nd), 1 (3rd)
- Forbidden non-unique: 53 (1st), 9 (2nd), 1 (3rd), 1 (4th)

→ Few measurements below 50 keV

→ Very few high order forbidden transitions

→ 11 published shape factors since 1976!

New precise measurements are needed to test the theoretical predictions

Current situation in nuclear databases

If no experimental data → Theoretical estimates

The LogFT program is widely used in nuclear data evaluations

- Handles β and ε transitions
- Provides mean energies of β spectra, log *ft* values, β^+ and ε probabilities
- Propagates uncertainties from input parameters
- Reads and writes ENSDF files (*Evaluated Nuclear Structure Data File*)

However

- Too simple analytical models → lack of accuracy
- Forbiddenness limitation (allowed, first- and second- forbidden unique)
- Users now require β spectra and correlated ν spectra

Constraints for a new code

- Fast calculations
- Ease-of-use for both evaluators and users
- Should read and write ENSDF files

Beta spectrometry at LNHB

Purpose

Evaluation of the **shapes** of beta spectra

Calculations

Short half-lives, multiple beta decays, etc.

BetaShape
+ improvements

X. Mougeot

Measurements

Low energies

≲ 700 keV

Metallic magnetic calorimeters

M. Loidl, M. Rodrigues
C. Le Bret (PhD → 2012)

Medium energies

15 keV – 3 MeV

Si PIPS, Si(Li)

X. Mougeot
C. Bisch (PhD → 2014)
A. Singh (PhD → 2020)

The BetaShape code

Behrens and Bühring formalism

Similarly we obtain for the space components

$$\langle p|\,\mathbf{V}+\mathbf{A}\,|n\rangle = \mathrm{i}u_p^{+}\underbrace{\gamma_4\gamma_\mu}_{\begin{pmatrix}0 & \mathrm{i}\boldsymbol{\sigma}\\ \mathrm{i}\boldsymbol{\sigma} & 0\end{pmatrix}}(1+\underbrace{\lambda\gamma_5}_{\lambda\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}})u_n = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}$$

$$\times\left\{\left(\frac{\boldsymbol{\sigma}\mathbf{p}}{W_p+M_p}\chi_p^{m'}\right)^{+}\boldsymbol{\sigma}\chi_n^m+(\chi_p^{m'})^{+}\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}}{W_n+M_n}\chi_n^m-\lambda(\chi_p^{m'})^{+}\boldsymbol{\sigma}\chi_n^m\right.$$

$$\left.-\lambda\left[\left(\frac{\boldsymbol{\sigma}\mathbf{p}}{W_p+M_p}\chi_p^{m'}\right)^{+}\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}}{W_n+M_n}\chi_n^m\right]\right\}. \quad (6.38)$$

This equals to

$$\langle p|\,\mathbf{V}+\mathbf{A}\,|n\rangle = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}\left\{(\chi_p^{m'})^{+}\underbrace{\frac{\boldsymbol{\sigma}\mathbf{p}_p}{W_p+M_p}\boldsymbol{\sigma}}_{\frac{\mathbf{p}_p+\mathrm{i}(\boldsymbol{\sigma}\times\mathbf{p}_p)}{W_p+M_p}}\chi_n^m\right.$$

$$+(\chi_p^{m'})^{+}\underbrace{\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}_n}{W_n+M_n}}_{\frac{\mathbf{p}_n-\mathrm{i}(\boldsymbol{\sigma}\times\mathbf{p}_n)}{W_n+M_n}}\chi_n^m-\lambda(\chi_p^{m'})^{+}\boldsymbol{\sigma}\chi_n^m$$

$$\left.-\lambda\left[(\chi_p^{m'})^{+}\underbrace{\frac{\boldsymbol{\sigma}\mathbf{p}_p}{W_p+M_p}\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}_n}{W_n+M_n}}_{\frac{-(\mathbf{p}_p\mathbf{p}_n)\boldsymbol{\sigma}+(\boldsymbol{\sigma}\mathbf{p}_p)\mathbf{p}_n+\mathbf{p}_p(\boldsymbol{\sigma}\mathbf{p}_n)-\mathrm{i}(\mathbf{p}_p\times\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}}\chi_n^m\right]\right\}. \quad (6.39)$$

Finally we obtain for the space components

$$\langle p|\,\mathbf{V}(0)+\mathbf{A}(0)\,|n\rangle = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}$$

$$\times\left\{\left[\frac{\mathbf{p}_p}{W_p+M_p}+\frac{\mathbf{p}_n}{W_n+M_n}\right](\chi_p^{m'})^{+}\chi_n^m+(\chi_p^{m'})^{+}\right.$$

$$\times\left[\frac{\mathrm{i}(\boldsymbol{\sigma}\times\mathbf{p}_p)}{W_p+M_p}-\frac{\mathrm{i}(\boldsymbol{\sigma}\times\mathbf{p}_n)}{W_n+M_n}\right]\chi_n^m-\lambda(\chi_p^{m'})^{+}\boldsymbol{\sigma}\chi_n^m$$

$$+\lambda\frac{\mathbf{p}_p\mathbf{p}_n}{(W_p+M_p)(W_n+M_n)}\{(\chi_p^{m'})^{+}\boldsymbol{\sigma}\chi_n^m\}+\lambda\frac{\mathrm{i}(\mathbf{p}_p\times\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}$$

$$\left.\times(\chi_p^{m'})^{+}\chi_n^m-\lambda\left[(\chi_p^{m'})^{+}\frac{(\boldsymbol{\sigma}\mathbf{p}_p)\mathbf{p}_n+\mathbf{p}_p(\boldsymbol{\sigma}\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}\chi_n^m\right]\right\}. \quad (6.40)$$

$$-\frac{\mathrm{i}}{2M_A}F_M(q^2)(\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma}-F_S(q^2)q_0+\frac{1}{4(2M_A)^2}F_S(q^2)q_0(\mathbf{P}^2-\mathbf{q}^2)$$

$$\left.-\frac{\mathrm{i}}{2}\frac{1}{(2M_A)^2}F_S(q^2)q_0(\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma}\right\}\chi^{M_i} \quad (9.15)$$

$$\langle\phi_f(p_f)|\,A_0(0)\,|\phi_i(p_i)\rangle = N(\chi^{M_f})^{+}\left\{-\frac{1}{2M_A}F_A(q^2)(\mathbf{P}\boldsymbol{\sigma})\right.$$

$$-\frac{q_0}{2M_A}F_P(q^2)(\mathbf{q}\boldsymbol{\sigma})-F_T(q^2)(\mathbf{q}\boldsymbol{\sigma})+\frac{1}{4}\frac{1}{(2M_A)^2}F_T(q^2)$$

$$\left.\times[(\mathbf{P}\mathbf{q})(\boldsymbol{\sigma}\mathbf{P}+\boldsymbol{\sigma}\mathbf{q})-(\boldsymbol{\sigma}\mathbf{q})(\mathbf{P}^2-\mathbf{q}^2)]\right\}\chi^{M_i} \quad (9.16)$$

$$\langle\phi_f(p_f)|\,\mathbf{V}(0)\,|\phi_i(p_i)\rangle = N(\chi^{M_f})^{+}\left\{\frac{1}{2M_A}F_V(q^2)\mathbf{P}+\frac{\mathrm{i}}{2M_A}F_V(q^2)(\boldsymbol{\sigma}\times\mathbf{q})\right.$$

$$+\mathrm{i}F_M(q^2)(\boldsymbol{\sigma}\times\mathbf{q})-\frac{1}{2M_A}F_M(q^2)q_0\mathbf{q}-\frac{\mathrm{i}}{4M_A}F_M(q^2)q_0(\boldsymbol{\sigma}\times\mathbf{P})$$

$$-F_S(q^2)\mathbf{q}+\frac{1}{4(2M_A)^2}F_S(q^2)\mathbf{q}(\mathbf{P}^2-\mathbf{q}^2)-\frac{\mathrm{i}}{2(2M_A)^2}F_S(q^2)\mathbf{q}$$

$$\times((\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma})-\frac{\mathrm{i}}{2(2M_A)^2}F_M(q^2)\mathbf{P}((\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma})-\frac{\mathrm{i}}{4(2M_A)^2}$$

$$\left.\times F_M(q^2)(\mathbf{P}^2+\mathbf{q}^2)(\boldsymbol{\sigma}\times\mathbf{q})+\frac{\mathrm{i}}{2(2M_A)^2}F_M(q^2)(\mathbf{P}\mathbf{q})(\boldsymbol{\sigma}\times\mathbf{P})\right\}\chi^{M_i} \quad (9.17)$$

$$\langle\phi_f(p_f)|\,\mathbf{A}(0)\,|\phi_i(p_i)\rangle = N(\chi^{M_f})^{+}\left\{-F_A(q^2)\boldsymbol{\sigma}+\frac{1}{2(2M_A)^2}\right.$$

$$\times F_A(q^2)\mathbf{P}^2\boldsymbol{\sigma}-\frac{1}{4(2M_A)^2}F_A(q^2)(\mathbf{P}^2+\mathbf{q}^2)\boldsymbol{\sigma}-\frac{i}{2(2M_A)^2}$$

$$\times F_A(q^2)(\mathbf{P}\times\mathbf{q})-\frac{1}{2(2M_A)^2}F_A(q^2)[(\boldsymbol{\sigma}\mathbf{P})\mathbf{P}-(\boldsymbol{\sigma}\mathbf{q})\mathbf{q}]$$

$$+\frac{1}{2M_A}F_T(q^2)[(\mathbf{P}\mathbf{p})\boldsymbol{\sigma}-\mathbf{q}(\boldsymbol{\sigma}\mathbf{P})]-F_T(q^2)q_0\boldsymbol{\sigma}$$

$$+\frac{1}{2(2M_A)^2}F_T(q^2)q_0\mathbf{P}^2\boldsymbol{\sigma}-\frac{1}{4(2M_A)^2}F_T(q^2)q_0(\mathbf{P}^2+\mathbf{q}^2)\boldsymbol{\sigma}$$

$$-\frac{\mathrm{i}}{2(2M_A)^2}F_T(q^2)q_0(\mathbf{P}\times\mathbf{q})-\frac{1}{2(2M_A)^2}F_T(q^2)q_0[(\boldsymbol{\sigma}\mathbf{P})\mathbf{P}$$

$$\left.-(\boldsymbol{\sigma}\mathbf{q})\mathbf{q}]-\frac{1}{2M_A}F_P(q^2)(\boldsymbol{\sigma}\mathbf{q})\mathbf{q}\right\}\chi^{M_i}. \quad (9.18)$$

$$\left.+\sqrt{\tfrac{2}{3}}\left(\int rI'(r)\beta\gamma_5T_{121}\right)\right\}$$

$$\mp\frac{f_P}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{D}\mathfrak{N}^{(0)}_{110}(1,1,1,1) \quad (14.101)$$

$${}^{A}F^{(0)}_{121}=\mp\lambda\,{}^{A}\mathfrak{M}^{(0)}_{121}-\frac{f_T}{R}\left[\frac{5}{\sqrt{3}}\,{}^{C}\mathfrak{N}^{(0)}_{111}-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{A}\mathfrak{M}^{(0)}_{121}\right]\mp\frac{f_P}{R}5\sqrt{\tfrac{2}{3}}\,{}^{D}\mathfrak{N}^{(0)}_{110} \quad (14.102)$$

$${}^{A}F^{(0)}_{121}(1,1,1,1)=\mp\lambda\,{}^{A}\mathfrak{M}^{(0)}_{121}(1,1,1,1)$$

$$-\frac{f_T}{R}\left\{\sqrt{\tfrac{1}{3}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta T_{111}\right)\right.$$

$$\left.-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{A}\mathfrak{M}^{(0)}_{121}(1,1,1,1)\right\}$$

$$\mp\frac{f_P}{R}\sqrt{\tfrac{2}{3}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta\gamma_5T_{110}\right) \quad (14.103)$$

$${}^{V}F^{(0)}_{211}=-{}^{V}\mathfrak{M}^{(0)}_{211}-\frac{f_M}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{C}\mathfrak{N}^{(0)}_{211} \quad (14.104)$$

$${}^{V}F^{(0)}_{220}={}^{V}\mathfrak{M}^{(0)}_{220}+\frac{f_M}{R}\sqrt{(10)}\,{}^{C}\mathfrak{N}^{(0)}_{211}\pm\frac{f_S}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{V}\mathfrak{M}^{(0)}_{220} \quad (14.105)$$

$${}^{V}F^{(0)}_{220}(1,1,1,1)={}^{V}\mathfrak{M}^{(0)}_{220}(1,1,1,1)$$

$$+\frac{f_M}{R}\left\{\sqrt{\tfrac{2}{5}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta T_{211}\right)\right.$$

$$\left.+\sqrt{\tfrac{3}{5}}\left(\int\left(\frac{r}{R}\right)rI'(r)\beta T_{231}\right)\right\}$$

$$\pm\frac{f_S}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{V}\mathfrak{M}^{(0)}_{220}(1,1,1,1) \quad (14.106)$$

$${}^{A}F^{(0)}_{221}=\pm\lambda\,{}^{A}\mathfrak{M}^{(0)}_{221}+\frac{f_T}{R}[\sqrt{(15)}\,{}^{C}\mathfrak{N}^{(0)}_{211}-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{A}\mathfrak{M}^{(0)}_{221}] \quad (14.107)$$

$${}^{A}F^{(0)}_{221}(1,1,1,1)=\pm\lambda\,{}^{A}\mathfrak{M}^{(0)}_{221}(1,1,1,1)$$

$$+\frac{f_T}{R}\left\{\sqrt{\tfrac{3}{5}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta T_{211}\right)\right.$$

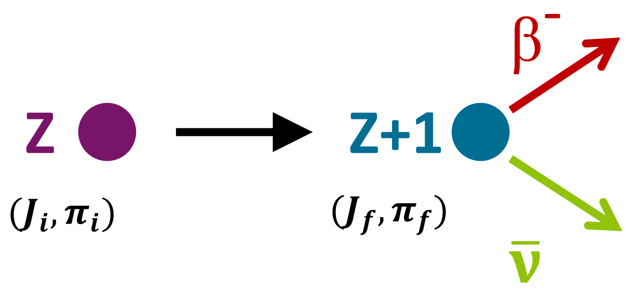
$$-\sqrt{\tfrac{2}{5}}\left(\int\left(\frac{r}{R}\right)rI'(r)\beta T_{231}\right)$$

$$\left.-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^{A}\mathfrak{M}^{(0)}_{221}(1,1,1,1)\right\} \quad (14.108)$$

H. Behrens and W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

More than 600 pages!

Basics of beta decay



ΔJ	$\pi_i \pi_f$	Classification
$0, 1$	1	Allowed
$0, 1$	-1	1^{st} fnu
> 1	$(-1)^{\lvert\Delta J\rvert}$	$\lvert\Delta J\rvert^{th}$ fnu
> 1	$(-1)^{\lvert\Delta J\rvert-1}$	$(\lvert\Delta J\rvert - 1)^{th}$ fu

$\Delta J = \lvert J_f - J_i \rvert$

fnu: forbidden non-unique

fu: forbidden unique

Electroweak interaction

$$\left.\begin{array}{l} M_{W+,W-,Z0} \sim 80 \text{ GeV} \\ E_{\max}(\beta) \lesssim 50 \text{ MeV} \end{array}\right\}$$

Fermi: **4 particles interacting at one vertex**

Free neutron decay

$$H_\beta = \frac{G_\beta}{\sqrt{2}} \left[\overline{\psi}_p \gamma_\mu (1 + \lambda\gamma_5) \psi_n \times \overline{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu + \text{h.c.} \right]$$

Hadron current

Lepton current

Neutrino

$\boldsymbol{m_{\overline{\nu}} \sim 0}$ → β spectrum very poorly modified, in the endpoint region

Nucleus

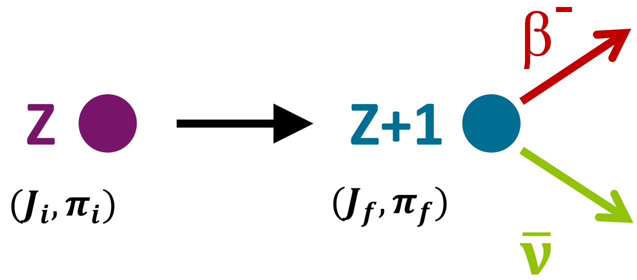
- **Point charge**, **spherical symmetry** → no deformation of the nucleus
- $M_{\text{nucleus}} \sim \infty \rightarrow \boldsymbol{E_{\text{recoil}} \sim 0}$

Physics modelling in BetaShape

Coulomb part (Fermi function)

Beta spectrum $\frac{\mathrm{d}N}{\mathrm{d}W} \propto \boxed{p\ W\ q^2}\ \boxed{F_0 L_0}\ \boxed{C(W)}$

Phase space — Shape factor



Nuclear current can be **factored out** for **allowed** and **forbidden unique** transitions

$$C(W) = (2L-1)! \sum_{k=1}^{L} \lambda_k \frac{p^{2(k-1)}\ q^{2(L-k)}}{(2k-1)![2(L-k)+1]!}$$

$L = 1$ if $\Delta J = 0$
$L = \Delta J$ otherwise

$$F_0 L_0 = \frac{\alpha_{-1}^2 + \alpha_1^2}{2p^2} \qquad \lambda_k = \frac{\alpha_{-k}^2 + \alpha_k^2}{\alpha_{-1}^2 + \alpha_1^2} = 1?$$

→ Solving the Dirac equation for the leptons is sufficient with these assumptions

Forbidden **non-unique** transitions calculated according to the ***ξ* approximation**

if $2\xi = \alpha Z / R \gg E_{\max}$
1st fnu → allowed
applied to 2nd, 3rd, etc.

Assumptions → Corrections

- Analytical screening corrections
- Radiative corrections

Propagation of uncertainty on $E_{\max}$

X. Mougeot, Phys. Rev. C 91, 055504 (2015)

Relativistic electron wave functions

$$\Psi(\vec{r}) = \begin{pmatrix} S_\kappa f_\kappa(r)\chi^\mu_{-\kappa} \\ g_\kappa(r)\chi^\mu_\kappa \end{pmatrix}$$

Radial component

Spin-angular functions
→ spherical harmonics expansion

Electron wave function
→ spherical symmetry

$$\begin{cases} \dfrac{\mathrm{d}f_\kappa}{\mathrm{d}r} = \dfrac{(\kappa-1)}{r} f_\kappa - [W - 1 - V(r)]g_\kappa \\ \dfrac{\mathrm{d}g_\kappa}{\mathrm{d}r} = [W + 1 - V(r)]f_\kappa - \dfrac{(\kappa+1)}{r} g_\kappa \end{cases}$$

Dirac equation
→ coupled differential equations

Analytical solutions
(approximate)

M.E. Rose, *Relativistic Electron Theory*, Wiley and Sons (1961)

nucleus = point charge + very approximate correction for its spatial extension

} LogFT treatment

Power series expansion
(exact solutions)

$$\begin{Bmatrix} f(r) \\ g(r) \end{Bmatrix} = \frac{(pr)^{k-1}}{(2k-1)!!} \sum_{n=0}^{\infty} \begin{Bmatrix} a_n \\ b_n \end{Bmatrix} r^n$$

nucleus = uniformly charged sphere
→ fast computation of the solutions

H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

} BetaShape treatment

Analytical screening corrections

Rose

M.E. Rose, Phys. Rev. 49, 727 (1936)

Thomas-Fermi $V_0(Z, \beta^{\pm})$

$\Rightarrow W \rightarrow W' = W \pm V_0$ in all quantities except in neutrino energy

→ **non-physical discontinuity** for β^- spectrum

→ **identical for all transitions**

N.B. Gove and M.J. Martin, Nucl. Data Tables 10, 205 (1971)

Bühring

W. Bühring, Nucl. Phys. A 430, 1 (1984)

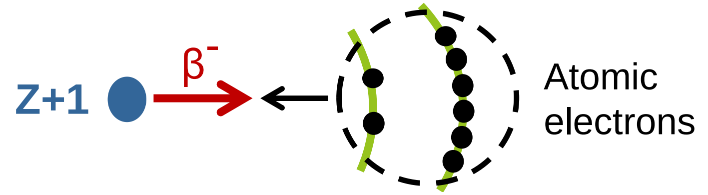
All quantities depend on the normalization of the electron wave functions

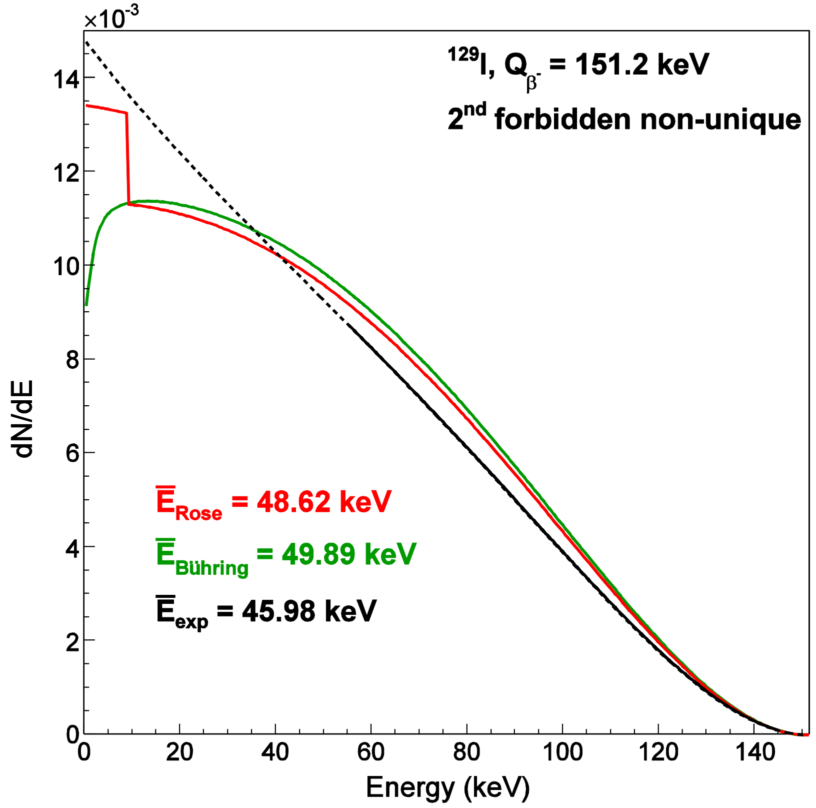
⇒ **Analytical solutions** and **leading order** at the nucleus + **asymptotic** solutions

Hulthén screened potentials → **Salvat's preferred**

F. Salvat *et al.*, Phys. Rev. A 36, 467 (1987)

→ **acting on Fermi function and λ_k parameters, thus different according to the forbiddenness**





- **More refined** + **no breakdown** at low energy
- Rose's correction can be deduced

Radiative corrections

Electrons $\rightarrow \quad \times[1+\delta_R(W,Z)]$

$$\delta_R(W,Z) = \delta_1(W) + \delta_2(Z) + \delta_3(Z) + \delta_4(Z)$$

$$\delta_1(W) = \frac{\alpha}{2\pi} g(W,q)$$

$$g(W,q) = 3\ln\left(\frac{m_p}{m_e}\right) - \frac{3}{4} + \frac{4}{\beta} L\left(\frac{2\beta}{1+\beta}\right) + 4\left(\frac{\tanh^{-1}\beta}{\beta} - 1\right)\left[\frac{q}{3W} - \frac{3}{2} + \ln(2q)\right] + \frac{\tanh^{-1}\beta}{\beta}\left[(1+\beta^2)\frac{q^2}{3W^2} - 4\tanh^{-1}\beta\right]$$

$$\delta_2(Z) = 1.1\,|Z|\alpha^2 \frac{m_p}{m_e}$$

$$\delta_3(Z) = \frac{Z^2\alpha^3}{\pi}\left(3\ln 2 - \frac{3}{2} + \frac{\pi^2}{3}\right)\frac{m_p}{m_e}$$

$$\delta_4(Z) = \frac{|Z|\alpha^3}{2\pi}\frac{m_p}{m_e}$$

A. Sirlin, Phys. Rev. 164, 1767 (1967)
W. Jaus, Phys. Lett. 40, 616 (1972)

Virtual photons, internal bremsstrahlung.

Only **outer** radiative corrections influence the energy dependence of the β spectrum.

Analytical solutions from QED for **allowed** transitions.

Neutrinos $\rightarrow \quad \times[1+\delta_\nu(q)]$

$$\delta_\nu(q) = \frac{\alpha}{2\pi} h(W)$$

$$h(W) = 3\ln\left(\frac{m_p}{m_e}\right) + \frac{23}{4} + \frac{8}{\beta} L\left(\frac{2\beta}{1+\beta}\right) + 8\left(\frac{\tanh^{-1}\beta}{\beta} - 1\right)\ln(2W\beta) + 4\frac{\tanh^{-1}\beta}{\beta}\left(\frac{7+3\beta^2}{8} - 2\tanh^{-1}\beta\right)$$

A. Sirlin, Phys. Rev. D 84, 014021 (2011)

$$\beta = p/W$$

Spence function $L(x) = \int_0^x \frac{\ln(1-t)}{t} dt$

Calculated quantities in BetaShape

- **Experimental shape factors** (database of 130 elements)
- **Mean energy** $\bar{E} = \int_0^{E_0} E \cdot N(E)\mathrm{d}E / \int_0^{E_0} N(E)\mathrm{d}E$
- **Log *ft* value**

☑ $f_{\beta^-} = \int_1^{W_0} N(W)\mathrm{d}W$

☒ $f_{\varepsilon/\beta^+} = f_\varepsilon + f_{\beta^+}$

Partial half-life: $t_i = T_{1/2}/I_\beta \quad \rightarrow \log ft$

provided that $f_{\beta^+} \neq 0$
and $I_{\beta^+} \neq 0$
$\rightarrow \log ft = \log\left(\frac{f_{\beta^+}}{I_{\beta^+}} T_{1/2}\right) + \log\left(\frac{1+f_\varepsilon/f_{\beta^+}}{1+I_\varepsilon/I_{\beta^+}}\right)$

However

$$\frac{I_\varepsilon}{I_{\beta^+}} = \frac{\lambda_\varepsilon}{\lambda_{\beta^+}} = \frac{K_{\mathrm{nuc}} \sum_x n_x C_x f_x}{K_{\mathrm{nuc}} \int_1^{W_0} N(W)\mathrm{d}W} \approx \frac{f_\varepsilon}{f_{\beta^+}}$$

C_x: lepton dynamics

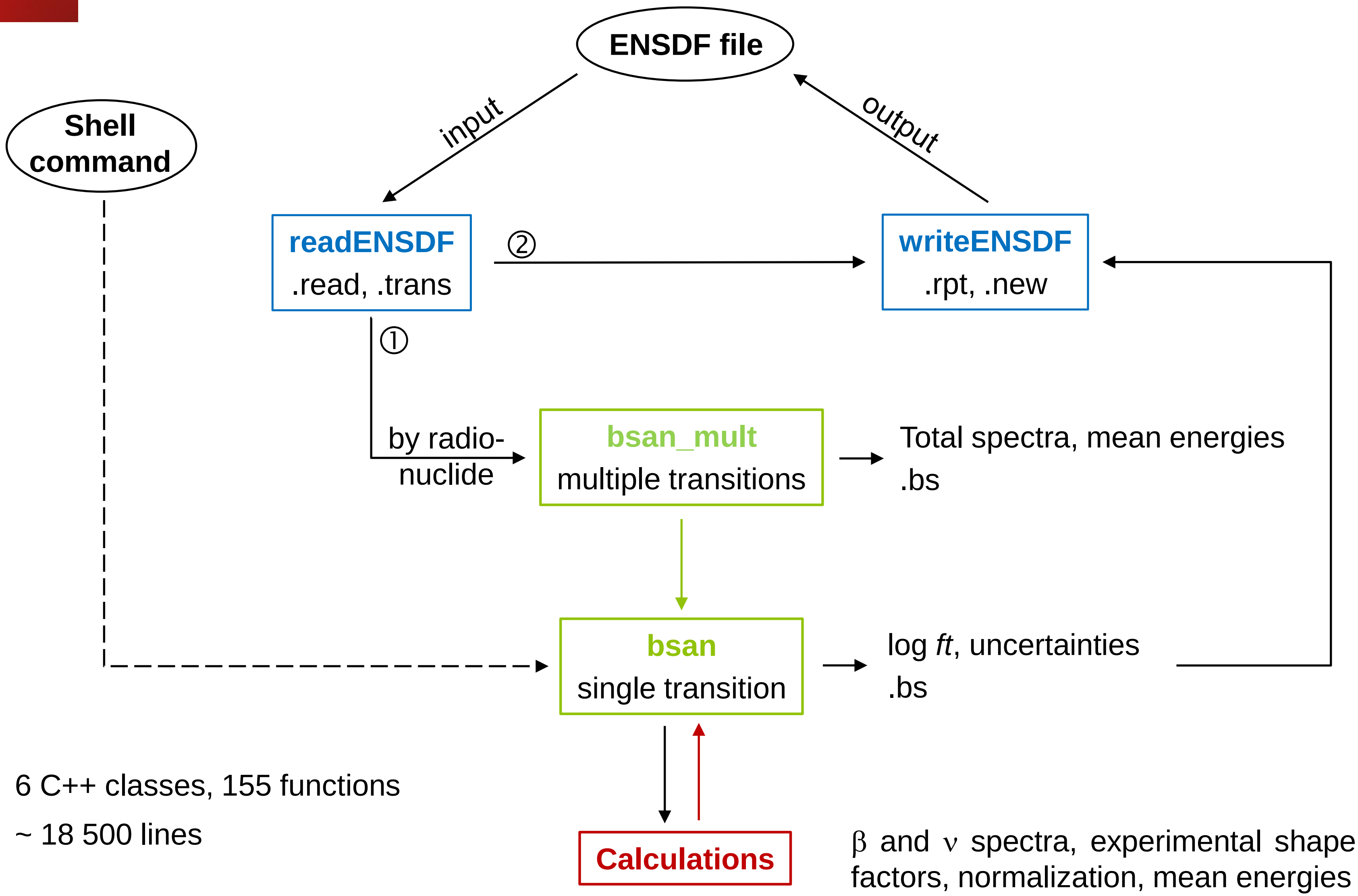
K_{nuc}: nuclear structure (allowed, forbidden unique)

n_x: relative occupation number of the orbital, not accounted for in the LogFT program

For allowed and forbidden unique electron capture transitions, one has

$$\rightarrow \log ft \approx \log\left(\frac{f_{\beta^+}}{I_{\beta^+}} T_{1/2}\right)$$

Structure of the BetaShape code



- 6 C++ classes, 155 functions
- ~ 18 500 lines

Output file

.bs

single transition

Transition parameters and options for calculation

Experimental shape factor

Mean energies, log *ft* values, analysis parameters

β and ν spectra

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-------------------------------------------------------------------------------------------

BetaShape
Analytical version: 1.0 (10/06/2016)
Author: X. Mougeot (xavier.mougeot@cea.fr)
CEA, LIST, Laboratoire National Henri Becquerel (LNHB), Gif-sur-Yvette F-91191, France
Please cite: X. Mougeot, Physical Review C 91, 055504; Erratum Phys. Rev. C 92, 059902 (2015)

-------------------------------------------------------------------------------------------

Parent nucleus: 18-Ar-41 [7/2*] g.s.  -->  Daughter nucleus: 19-K-41 [3/2+] g.s.
Calculation of the 1st forbidden unique transition from the beta - decay of Ar-41

Bühring's screening correction is considered.

End-point energy: 2491.60 (40) keV     Energy step: 8 keV     Intensity: 0.00784 (19)

An experimental shape factor has been found: (q^2 + l_2*p^2)
Energy range of the measurement: 1330 - 2420 keV
From [1961KA19] G.R. Kartashov, N.A. Burgov, A.V. Davydov,  Izvest. Akad. Nauk SSSR, Ser. Fiz. 25, 189 (1961) or Columbia Tech. Transl. 25, 184 (1962)

Input mean energy: 1076.60 (20) keV
Mean energy from the calculated spectrum: 1072.92 (19) keV
Mean energy from the experimental shape factor: 1076.05 (19) keV
Mean energy from the calculated spectrum if lk=1: 1076.0 (33) keV

Input log ft value: 9.72
Log ft value from the calculated spectrum: log ft 9.735 (11)   with components: log f 3.81177 (42) and log partial T1/2 5.924 (11)
Log ft value from the experimental shape factor: log ft 9.728 (11)   with component: log f 3.80453 (42)
Log ft value from the calculated spectrum if lk=1: log ft 9.740 (12)   with component: log f 3.816 (5)

Agreement of the experimental and calculated spectra in [1330,2420] keV: 99.98 %
Corresponding disagreement: 1.75e-02 %
Variation of the mean energies: -2.91e-01 %

E(keV)     dN/dE calc.    unc.          dN/dE exp.     unc.          dN/dE lk=1     unc.
     0     1.37491e-06    4.56604e-10   1.36282e-06    4.54479e-10   1.35951e-06    1.58534e-08
     8     1.41489e-06    4.67613e-10   1.40958e-06    4.67776e-10   1.39901e-06    1.63497e-08
    16     1.46578e-06    4.82021e-10   1.46404e-06    4.83425e-10   1.44933e-06    1.69282e-08
    24     1.52758e-06    4.99830e-10   1.52621e-06    5.01425e-10   1.51049e-06    1.75890e-08
               ⋮                            ⋮                            ⋮
  2472     6.09655e-09    2.46104e-10   6.32780e-09    2.54503e-10   6.12687e-09    2.71767e-10
  2480     2.15057e-09    1.49183e-10   2.24064e-09    1.54856e-10   2.16118e-09    1.54915e-10
  2488     2.07631e-10    4.86465e-11   2.18157e-10    5.09102e-11   2.08648e-10    4.76298e-11
 2491.6    0.00000e+00    0.00000e+00   0.00000e+00    0.00000e+00   0.00000e+00    0.00000e+00


-------------------------
Antineutrino spectrum
-------------------------

Mean energy from the calculated spectrum: 1416.48 (29) keV
Mean energy from the experimental shape factor: 1416.24 (29) keV
Mean energy from the calculated spectrum if lk=1: 1413.4 (34) keV

E(keV)     dN/dE calc.    unc.          dN/dE exp.     unc.          dN/dE lk=1     unc.
     0     0.00000e+00    0.00000e+00   0.00000e+00    0.00000e+00   0.00000e+00    0.00000e+00
     8     1.06916e-09    5.37713e-13   1.07023e-09    5.37166e-13   1.07440e-09    5.77388e-12
    16     4.23052e-09    2.12127e-12   4.23474e-09    2.11910e-12   4.25140e-09    2.30004e-11
    24     9.41583e-09    4.70705e-12   9.42521e-09    4.70220e-12   9.46266e-09    5.15326e-11
               ⋮                            ⋮                            ⋮
```

Output file

.bs

single transition

Transition parameters and options for calculation

```
-------------------------------------------------------------------------------------------

BetaShape
Analytical version: 1.0 (10/06/2016)
Author: X. Mougeot (xavier.mougeot@cea.fr)
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Please cite: X. Mougeot, Physical Review C 91, 055504; Erratum Phys. Rev. C 92, 059902 (2015)

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```
      8     1.41489e-06    4.67613e-10    1.40958e-06    4.67776e-10    1.39901e-06    1.63497e-08
     16     1.46578e-06    4.82021e-10    1.46404e-06    4.83425e-10    1.44933e-06    1.69282e-08
     24     1.52758e-06    4.99830e-10    1.52621e-06    5.01425e-10    1.51049e-06    1.75890e-08
                ⋮                              ⋮                              ⋮
   2472     6.09655e-09    2.46104e-10    6.32780e-09    2.54503e-10    6.12687e-09    2.71767e-10
   2480     2.15057e-09    1.49183e-10    2.24064e-09    1.54856e-10    2.16118e-09    1.54915e-10
   2488     2.07631e-10    4.86465e-11    2.18157e-10    5.09102e-11    2.08648e-10    4.76298e-11
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      0     0.00000e+00    0.00000e+00    0.00000e+00    0.00000e+00    0.00000e+00    0.00000e+00
      8     1.06916e-09    5.37713e-13    1.07023e-09    5.37166e-13    1.07440e-09    5.77388e-12
     16     4.23052e-09    2.12127e-12    4.23474e-09    2.11910e-12    4.25140e-09    2.30004e-11
     24     9.41583e-09    4.70705e-12    9.42521e-09    4.70220e-12    9.46266e-09    5.15326e-11
                ⋮                              ⋮                              ⋮
```

Output file

.bs

single transition

```
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```

Experimental shape factor

An experimental shape factor has been found: (q^2 + l_2*p^2)
Energy range of the measurement: 1330 - 2420 keV
From [1961KA19] G.R. Kartashov, N.A. Burgov, A.V. Davydov, Izvest. Akad. Nauk SSSR, Ser. Fiz. 25, 189 (1961)


```
Log ft value from the calculated spectrum if lk=1: log ft 9.740 (12)   with component: log f 3.816 (5)

Agreement of the experimental and calculated spectra in [1330,2420] keV: 99.98 %
Corresponding disagreement: 1.75e-02 %
Variation of the mean energies: -2.91e-01 %
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	⋮		⋮		⋮	
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2491.6	0.00000e+00	0.00000e+00	0.00000e+00	0.00000e+00	0.00000e+00	0.00000e+00

```
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```

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	⋮		⋮		⋮	

Output file

.bs

single transition

```
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Analytical version: 1.0 (10/06/2016)
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Log ft value from the experimental shape factor: log ft 9.728 (11)   with component: log f 3.80453 (42)
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Agreement of the experimental and calculated spectra in [1330,2420] keV: 99.98 %
Corresponding disagreement: 1.75e-02 %
Variation of the mean energies: -2.91e-01 %
```

Mean energies, log *ft* values, analysis parameters


```
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Mean energy from the calculated spectrum: 1072.92 (19) keV
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```
16   4.23052e-09   2.12127e-12   4.23474e-09   2.11910e-12   4.25140e-09   2.30004e-11
24   9.41583e-09   4.70705e-12   9.42521e-09   4.70220e-12   9.46266e-09   5.15326e-11
  ⋮                          ⋮                          ⋮
```

Output file

.bs

single transition

```
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BetaShape
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```

E (keV)	dN/dE calc.	unc.	dN/dE exp.	unc.	dN/dE 1k=1	unc.
0	1.37491e-06	4.56604e-10	1.36282e-06	4.54479e-10	1.35951e-06	1.58534e-08
8	1.41489e-06	4.67613e-10	1.40958e-06	4.67776e-10	1.39901e-06	1.63497e-08
16	1.46578e-06	4.82021e-10	1.46404e-06	4.83425e-10	1.44933e-06	1.69282e-08
24	1.52758e-06	4.99830e-10	1.52621e-06	5.01425e-10	1.51049e-06	1.75890e-08
32	1.59207e-06	5.18314e-10	1.59014e-06	5.19795e-10	1.57433e-06	1.82615e-08

```
Mean energy from the calculated spectrum: 1072.92 (19) keV
Mean energy from the experimental shape factor: 1076.05 (19) keV
```

β and ν spectra

```
-------------------------
Antineutrino spectrum
-------------------------

Mean energy from the calculated spectrum: 1416.48 (29) keV
Mean energy from the experimental shape factor: 1416.24 (29) keV
Mean energy from the calculated spectrum if 1k=1: 1413.4 (34) keV
```

E (keV)	dN/dE calc.	unc.	dN/dE exp.	unc.	dN/dE 1k=1	unc.
0	0.00000e+00	0.00000e+00	0.00000e+00	0.00000e+00	0.00000e+00	0.00000e+00
8	1.06916e-09	5.37713e-13	1.07023e-09	5.37166e-13	1.07440e-09	5.77388e-12
16	4.23052e-09	2.12127e-12	4.23474e-09	2.11910e-12	4.25140e-09	2.30004e-11
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```
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Mean energy from the calculated spectrum if 1k=1: 1413.4 (34) keV
```
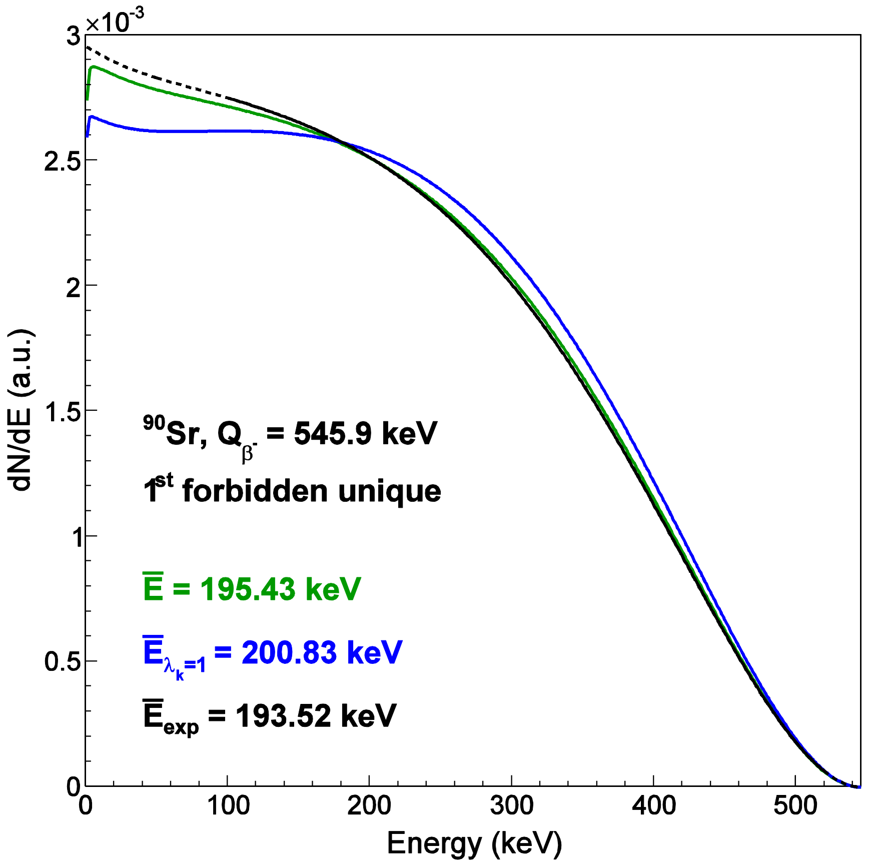
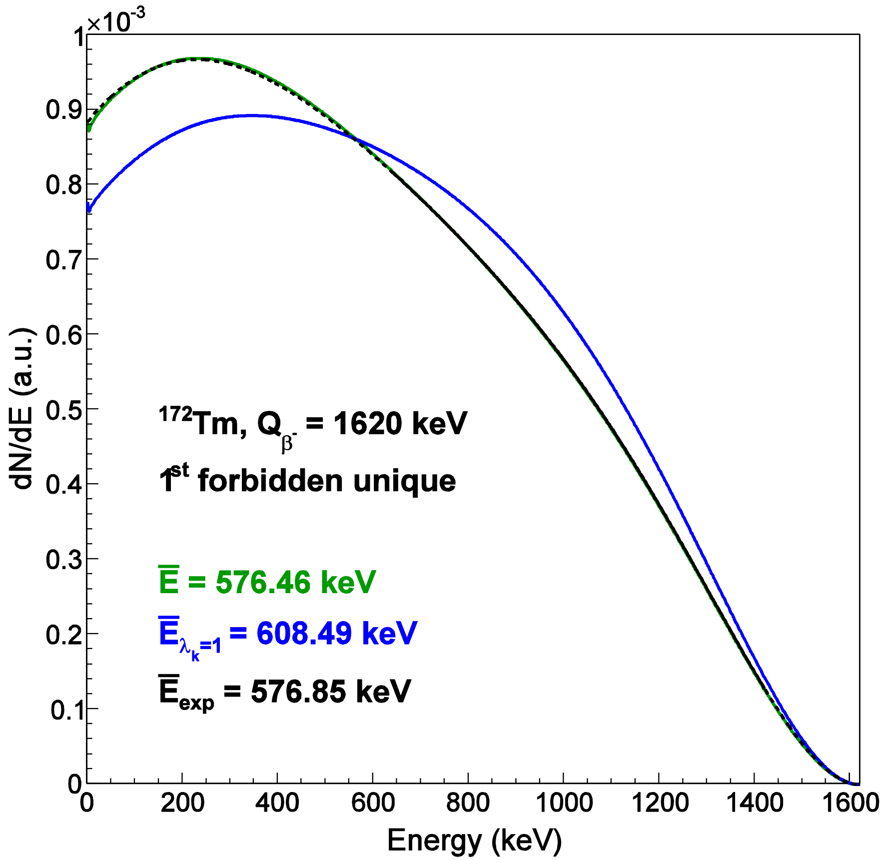


⋮ ⋮ ⋮

$\lambda_k = 1$ approximation





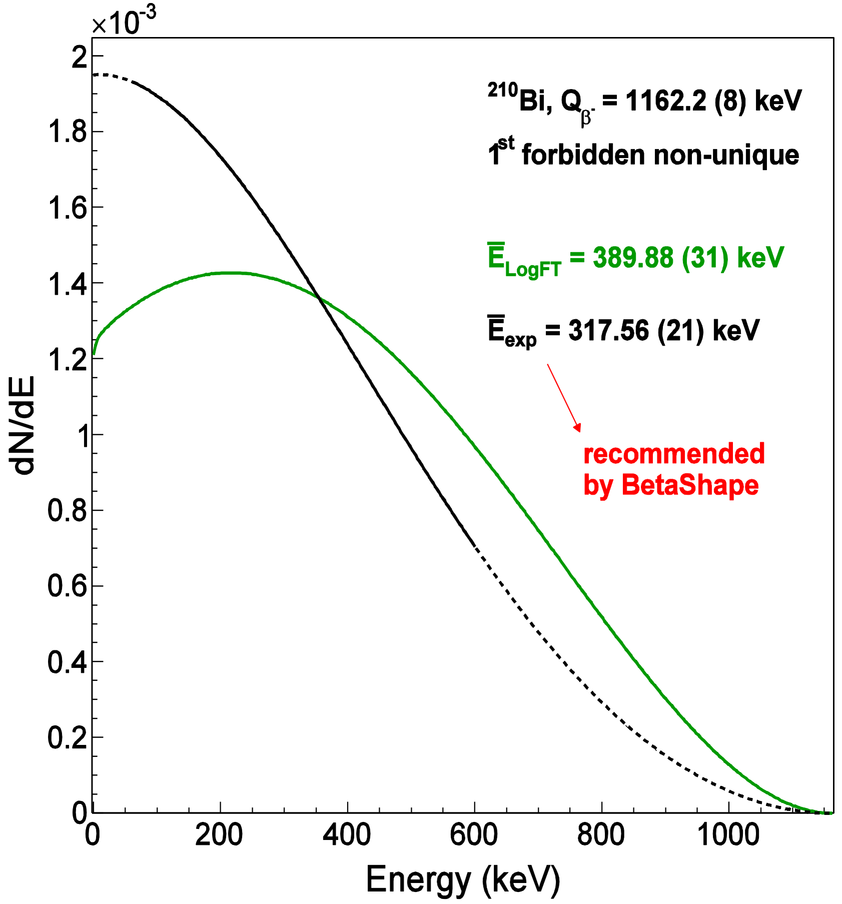
Complete $(1 - R^2) = 0.06$ % and $|\Delta\bar{E}| = 0.99$ %

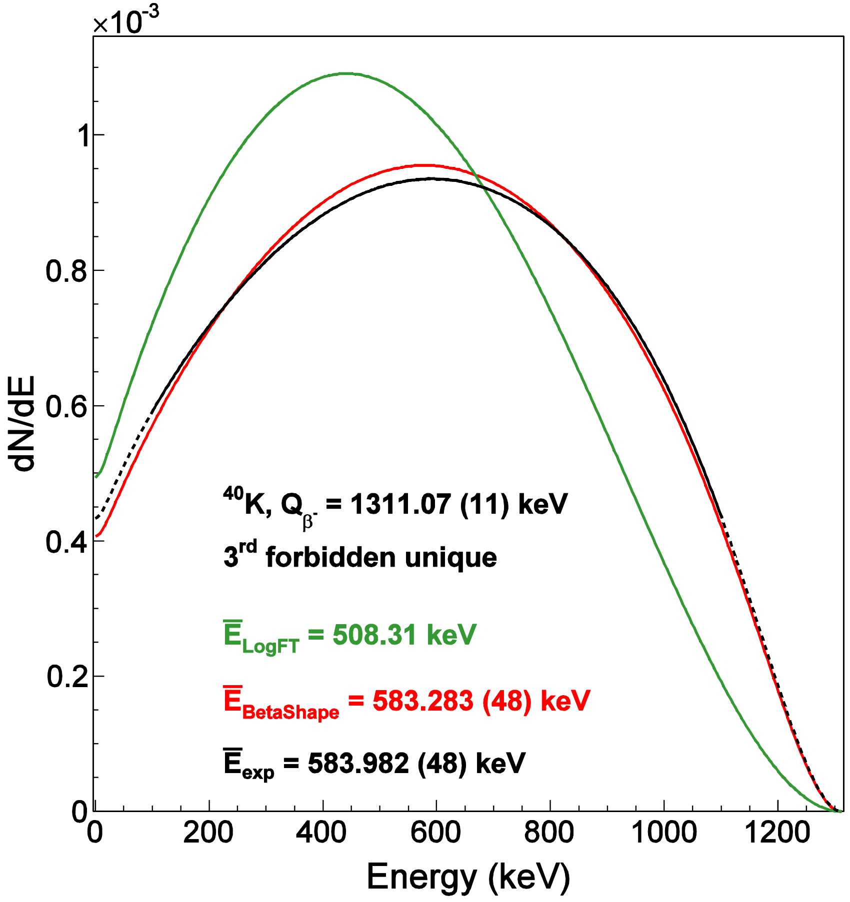
$\lambda_k = 1$ $(1 - R^2) = 0.93$ % and $|\Delta\bar{E}| = 3.8$ %

Complete $(1 - R^2) = 0.003$ % and $|\Delta\bar{E}| = 0.17$ %

$\lambda_k = 1$ $(1 - R^2) = 5.9$ % and $|\Delta\bar{E}| = 5.5$ %

Examples of improved calculations



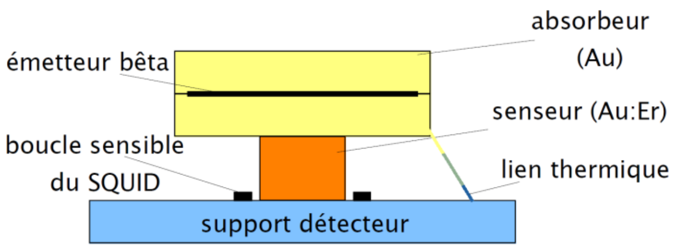


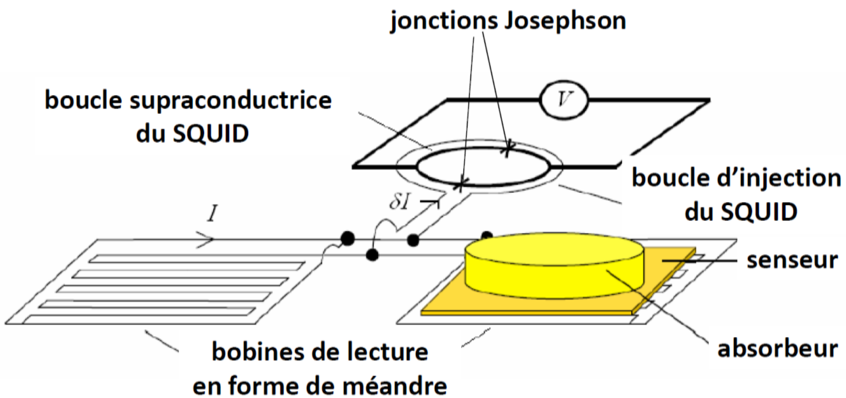
These two transitions are calculated as allowed by the LogFT program.

Atomic effects

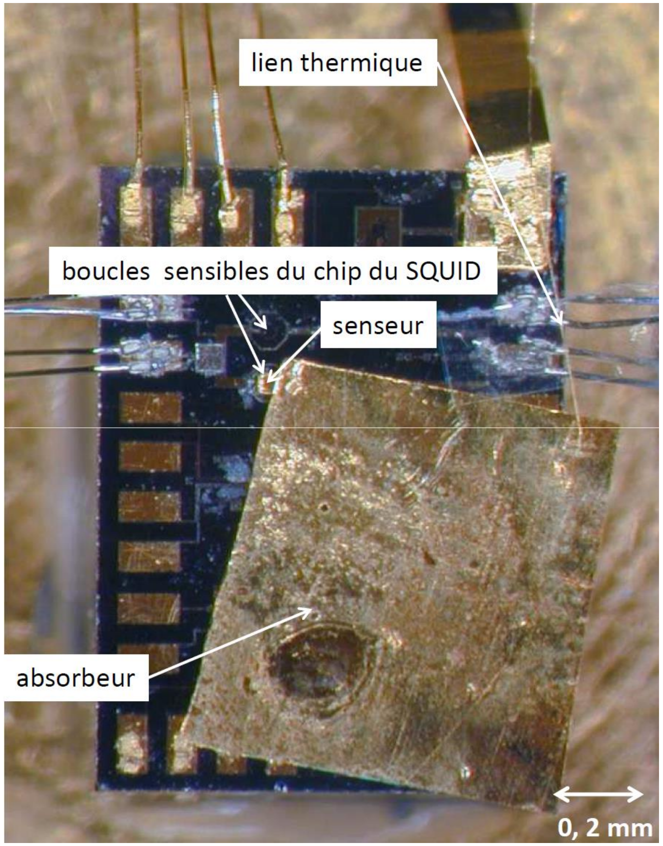
Metallic magnetic calorimetry

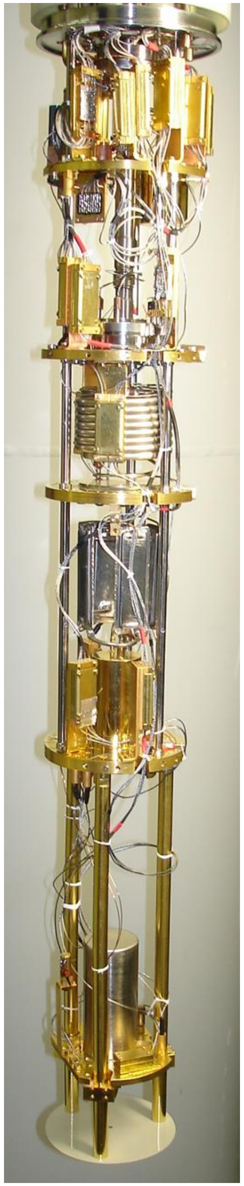
Direct magnetic coupling





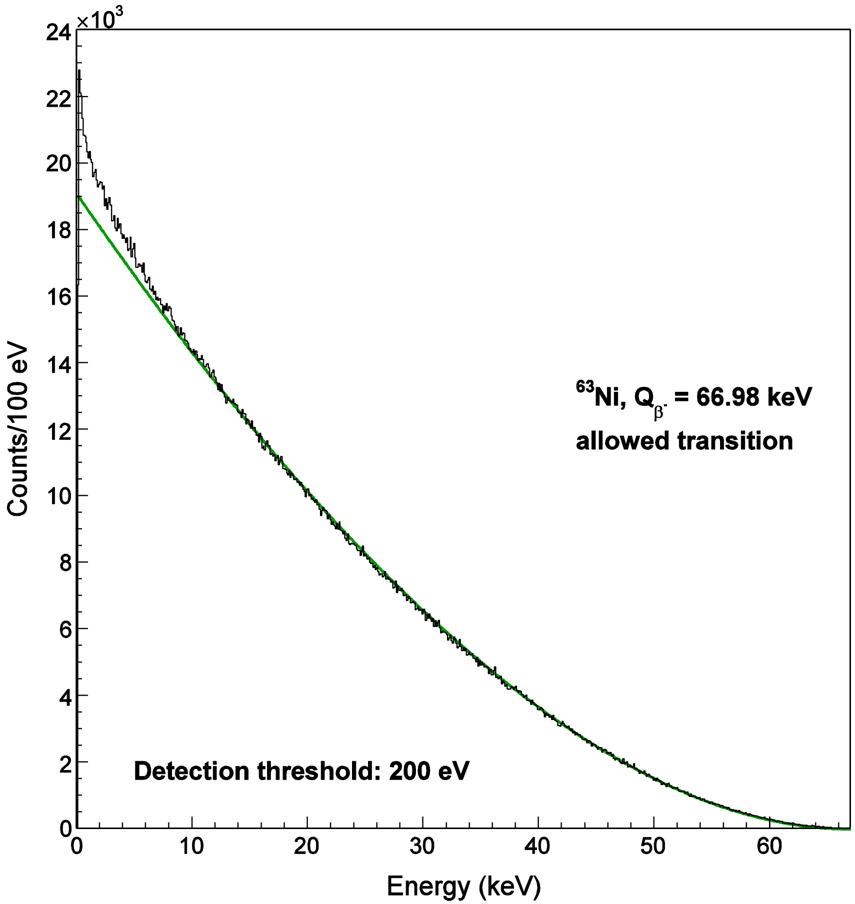
Indirect magnetic coupling

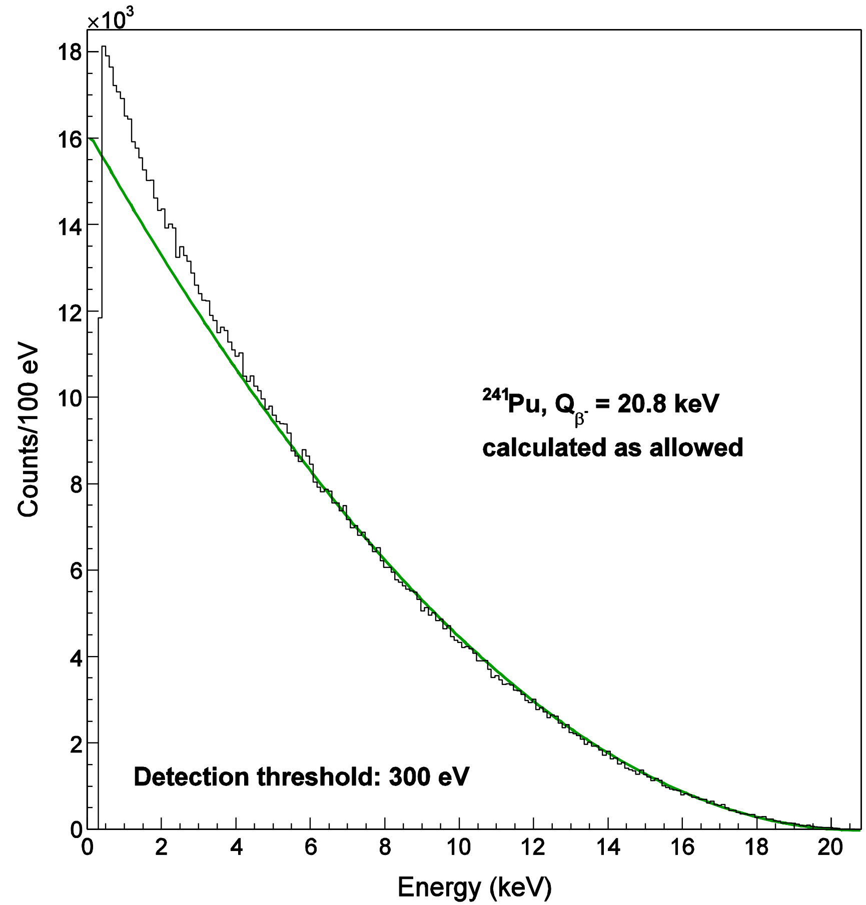




System cooled down to 10 mK

^{63}Ni and ^{241}Pu beta spectra



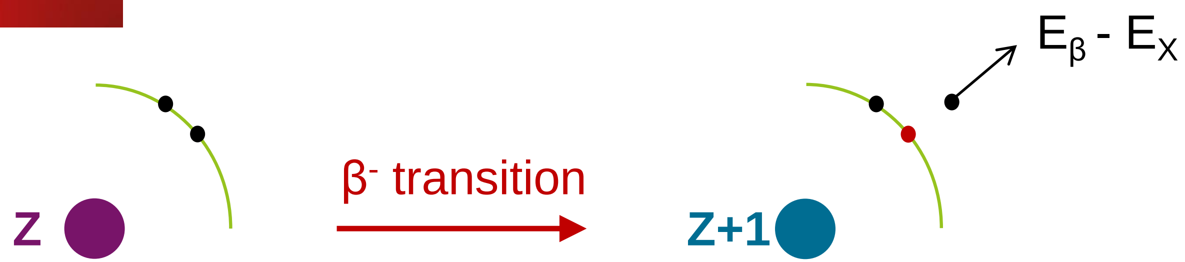


Classical beta calculations fail to reproduce these "simple" spectra

1st forbidden non-unique transition calculated as **allowed**

$2\xi = \alpha Z/R \gg E_0$ = 20.8 keV ≪ 19.8 MeV

The atomic exchange effect



N.C. Pyper, M.R. Harston, Proc. Roy. Soc. Lond. A 420, 277 (1988)

X. Mougeot *et al.*, Phys. Rev. A 86, 042506 (2012)

First work using analytical wave functions

Atomic exchange effect

→ Indistinguishable from the direct decay to a final continuum state

→ Depends on the **overlap of the continuum and bound electron wave functions**

→ **Allowed transitions**: only the ***ns* orbitals** are reachable

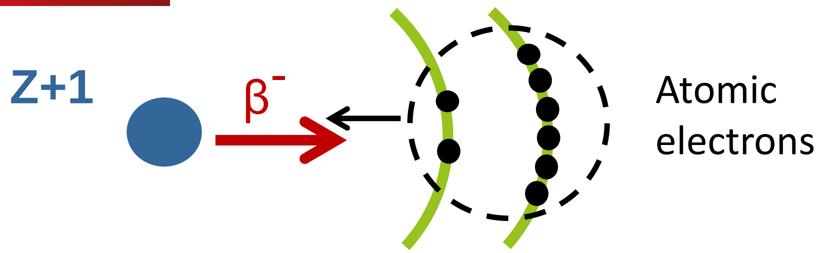
Spectrum correction factor $$\left[1 + \eta_{ex}^{T}(E)\right]$$

Total exchange factor $$\eta_{ex}^{T}(E) = \sum_{n} \eta_{ex}^{ns}(E) + \sum_{\substack{m,n \\ (m \neq n)}} \mu_m \mu_n$$

Subshell contribution $$\eta_{ex}^{ns}(E) = f\left(\mu_n^2 - 2\mu_n\right)$$

with $$\mu_n = \langle Es' | ns \rangle \frac{g_{n,\kappa}^{b}(R)}{g_{\kappa}^{c}(R)}, \quad f = \frac{g_{\kappa}^{c}(R)^2}{g_{\kappa}^{c}(R)^2 + f_{\kappa}^{c}(R)^2}$$

A new screening correction



H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

Screening

Generally corrected for using a constant Thomas-Fermi potential, which creates a **non physical discontinuity** in the spectrum.

Evaluating the **wave functions at the nuclear surface** cannot provide a good result because of the **weakness of the screened potentials** in this region.

→ Implementation of a **new screening correction** which:

- avoids complete calculation of lepton and nuclear matrix elements
- is available only for allowed transitions up-to-now

$$C_{sc} = 1 - \frac{\Delta R_{unsc}}{\Delta R_{sc}} \cdot \left(1 - \frac{f_{sc}}{f_{unsc}}\right)$$

exchange formalism → f factor

$$f^{-1} = \frac{\overline{g_{-1}^2}}{\overline{g_{-1}^2} + \overline{f_{-1}^2}}$$

mean value → spatial extension

$$\overline{g_\kappa^2} = \frac{1}{\Delta R} \int_{\Delta R} g_\kappa^2(r)dr$$

Coulomb potential for electron bound wave functions

Point charge — **Exchange potential**

$$V_{ex}(r) = K\,\frac{3\alpha}{2}\left(\frac{3Z}{4\pi^2}\right)^{1/3}\left[\frac{1}{r}\frac{\mathrm{d}^2\phi(r)}{\mathrm{d}r^2}\right]^{1/3}$$

for atomic electrons (**fermions**)

Total potential

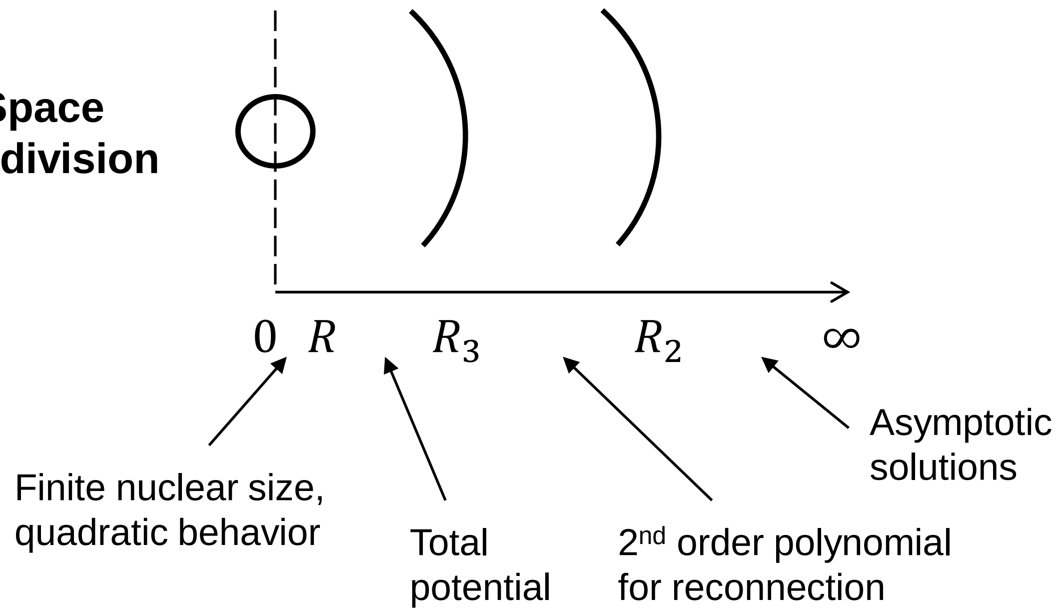
$$V(r) = -\frac{\alpha Z}{r}\phi(r) - V_{ex}(r)$$

Screened potential

$$\phi(r) = \sum_{i=1}^{N} a_i \mathrm{e}^{-\beta_i r}$$

F. Salvat *et al.*, Phys. Rev. A 36, 467 (1987)

Space subdivision



A power series expansion of the total potential is required.

Numerical procedure

- For bound states, the **orbital energy** is **not known** in advance

  → **iterative procedure**

- **Orbital energy** → Oscillation frequency of the wave functions

  → **Accuracy** of the **overlap**

⟹ **Adjustment** of $\boldsymbol{V_{ex}}$ to reach the "good" energies in

J.P. Desclaux, At. Data Nucl. Data Tab. 12, 311 (1973)

Inspection

Useful **tabulated parameters** for β spectra, electron capture, electron polarization, β-γ angular correlation, etc.

Tabulated **screened** parameters $F_0L_0^*/F_0L_0$ and λ_2^*/λ_2 **only**, but for **very few energies**

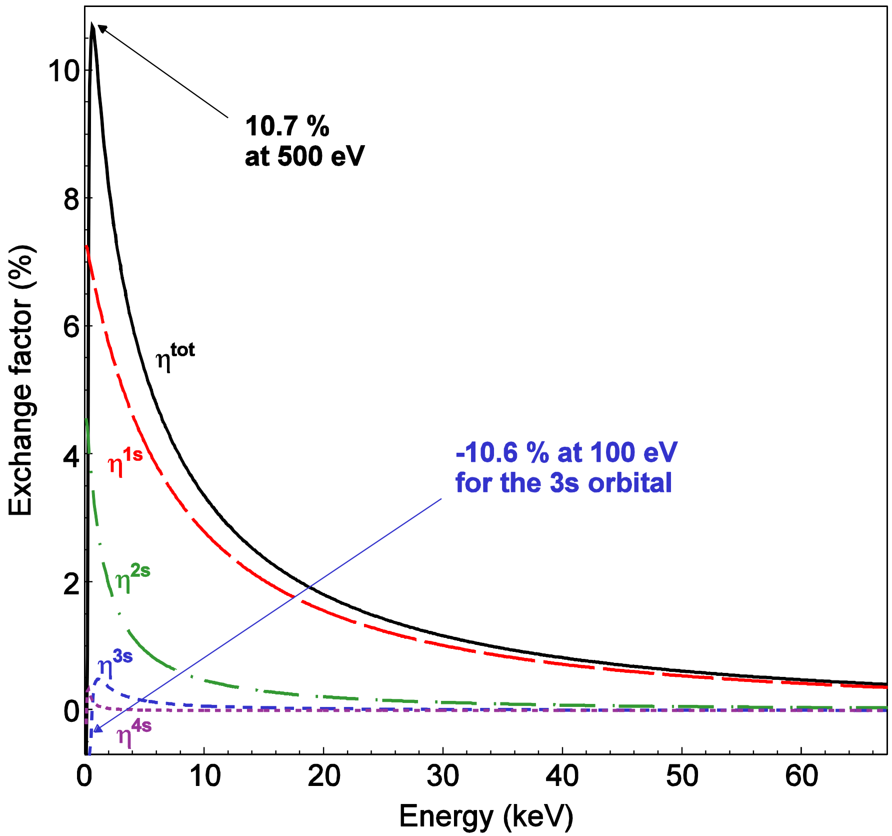
H. Behrens, J. Jänecke, Landolt-Börnstein, New Series, Group I, vol. 4, Springer Verlag, Berlin (1969)

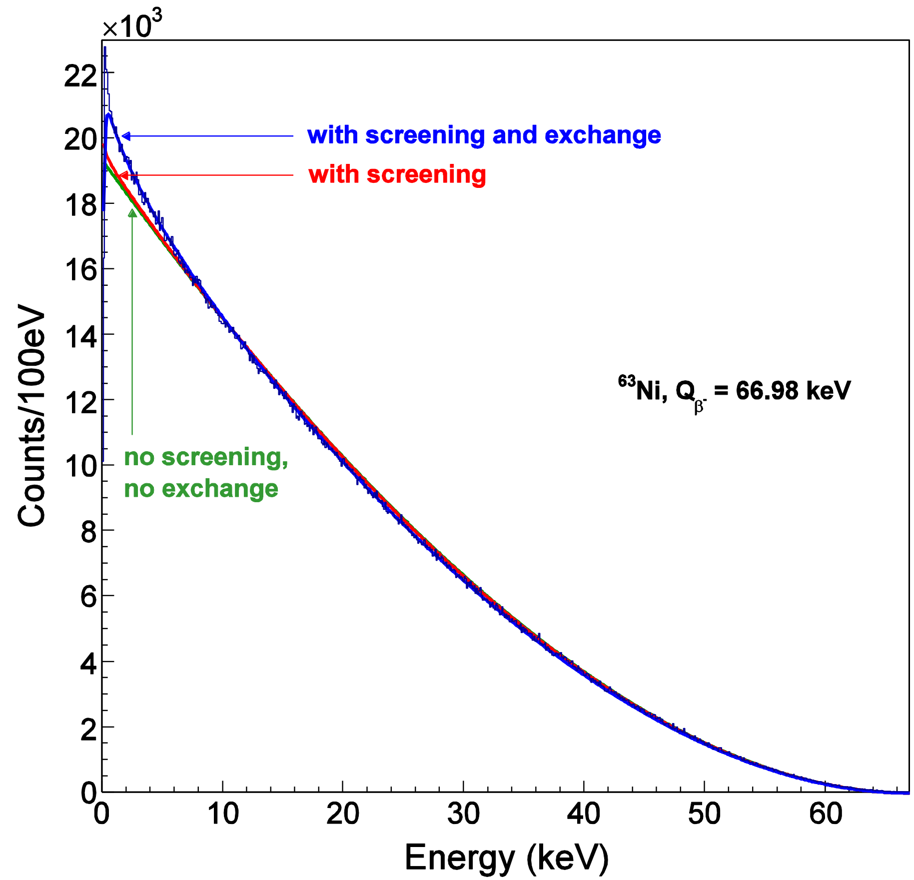
For both the continuum and bound wave functions,

Without screening: parameters perfectly reproduced

With screening: parameters in excellent agreement, despite different potentials

^{63}Ni





Analytic: $\bar{E} = 17.45$ keV

With screening: $\bar{E} = 17.40$ keV

With screening and exchange: $\bar{E} = 17.14$ keV

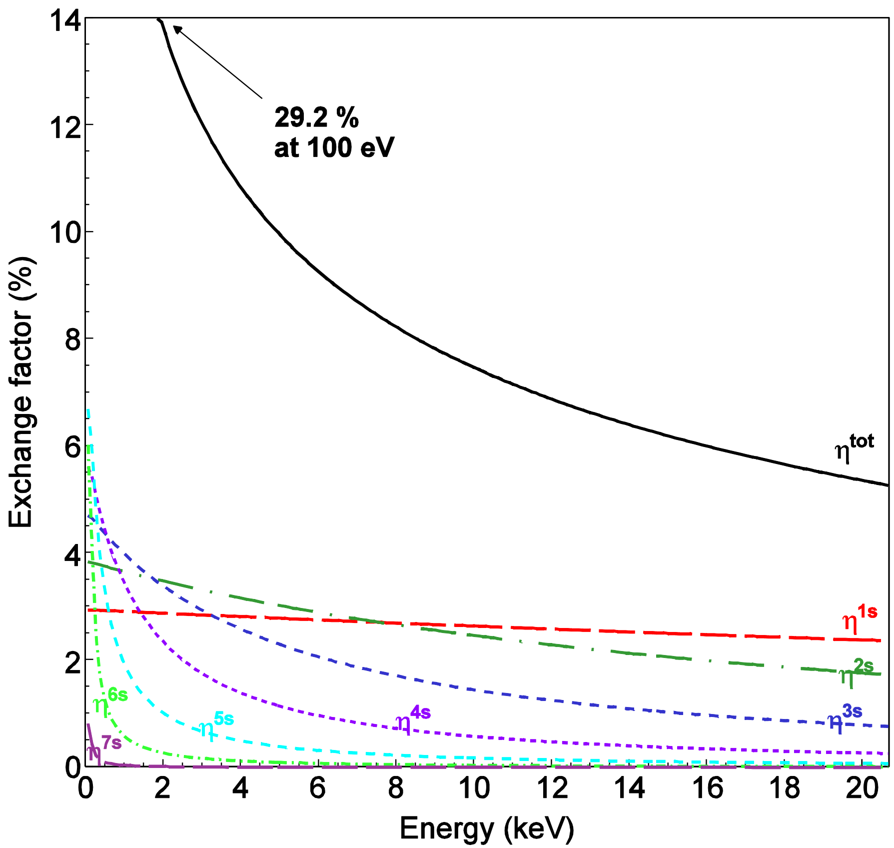
Mean energy of the spectrum decreased by **1.8 %**

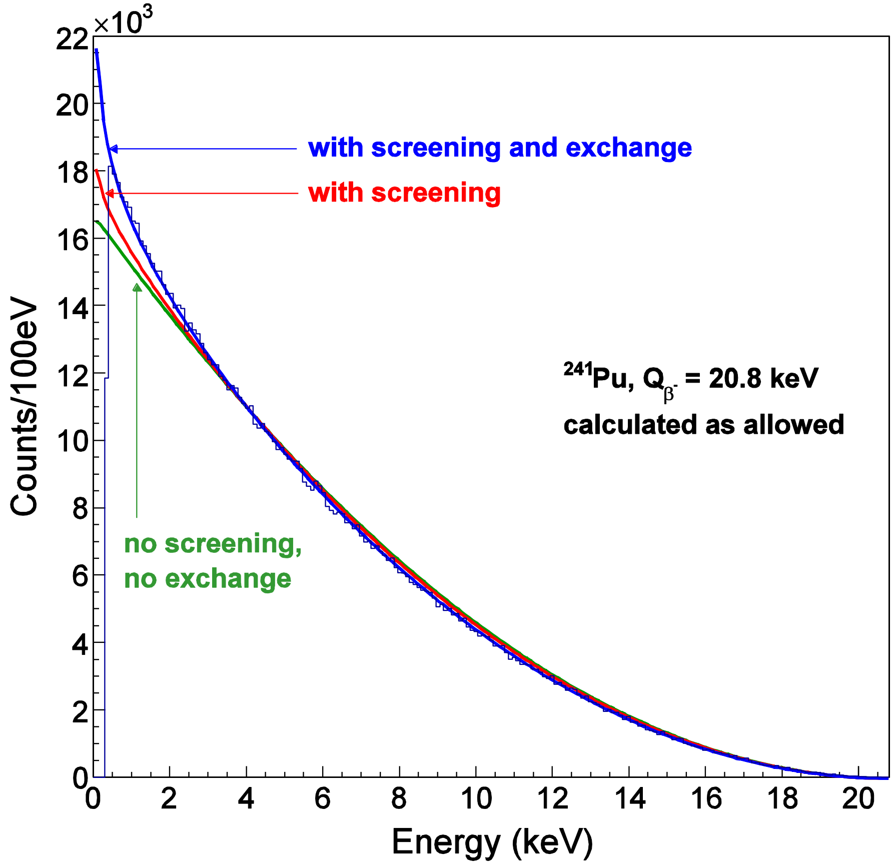
Allowed transition

Experimental spectrum

C. Le-Bret, PhD thesis, Université Paris 11 (2012)

^{241}Pu





Analytic: $\bar{E} = 5.24\ \mathrm{keV}$

With screening: $\bar{E} = 5.18\ \mathrm{keV}$

With screening and exchange: $\bar{E} = 5.03\ \mathrm{keV}$

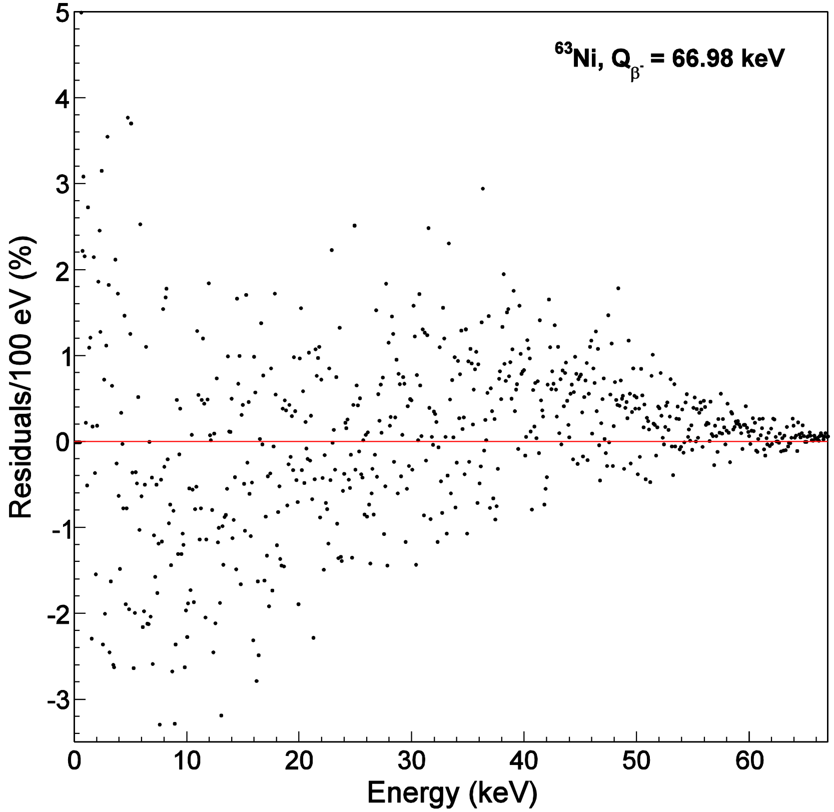
Mean energy of the spectrum decreased by **4** %

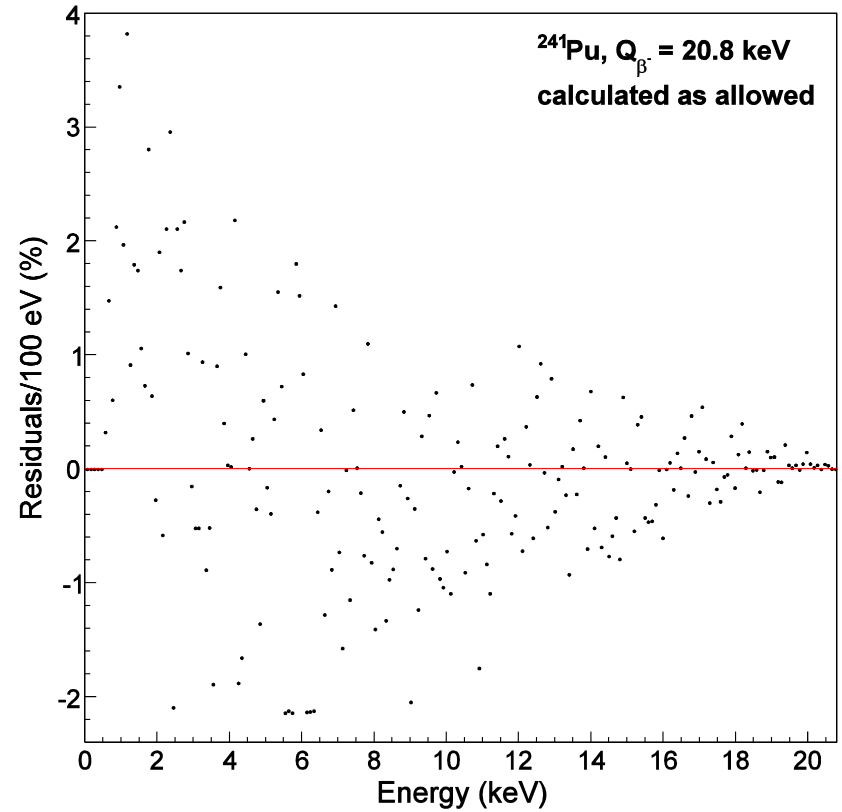
Calculated as **allowed**

Experimental spectrum

M. Loidl *et al.*, App. Radiat. Isot. 68, 1454 (2010)

Quality of the calculations





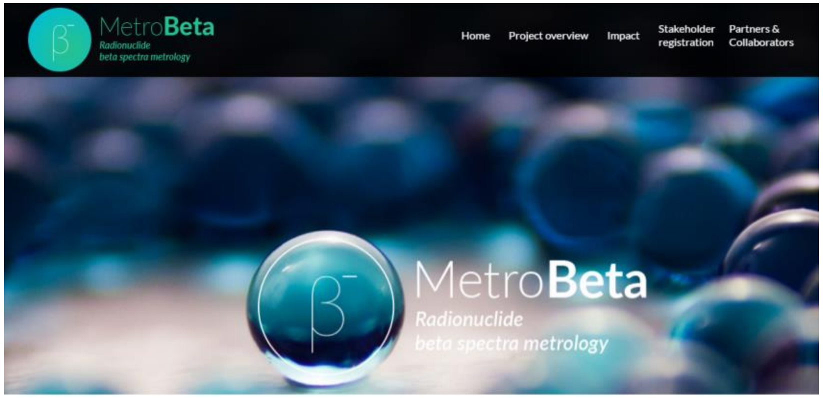
^{63}Ni		^{241}Pu
$\bar{r_i} = 0.093\ \%$	**Residuals mean**	$\bar{r_i} = 0.0019\ \%$
$(1 - R^2) = 0.028\ \%$	**Disagreement**	$(1 - R^2) = 0.040\ \%$
$\sigma_{r_i} = 1.03\ \%$	**Overall uncertainty**	$\sigma_{r_i} = 0.99\ \%$

X. Mougeot, C. Bisch, Phys. Rev. A 90, 012501 (2014)

Outlook

The EMPIR project MetroBeta (2016-2019)

http://metrobeta-empir.eu/



8 partners from 6 countries
- France (LNHB)
- Germany (PTB Berlin and Braunschweig, Heidelberg University)
- Czech Republic (CMI)
- Poland (UMCS)
- Switzerland (IRA)
- The Netherlands (Gonitec)

Work packages
- WP1: Theoretical calculations of beta spectra
- WP2: High-resolution beta spectrometry based on Metallic Magnetic Calorimeters
- WP3: Measurements of beta spectra with other methods
- WP4: Comparison and validation of measurements
- WP5: Creating Impact

LNHB is highly involved
- Coordination of the project (Mark A. Kellett)
- Coordination of WP1 (X. Mougeot)
- Scientific work scheduled in WP1, WP2, WP4, WP5

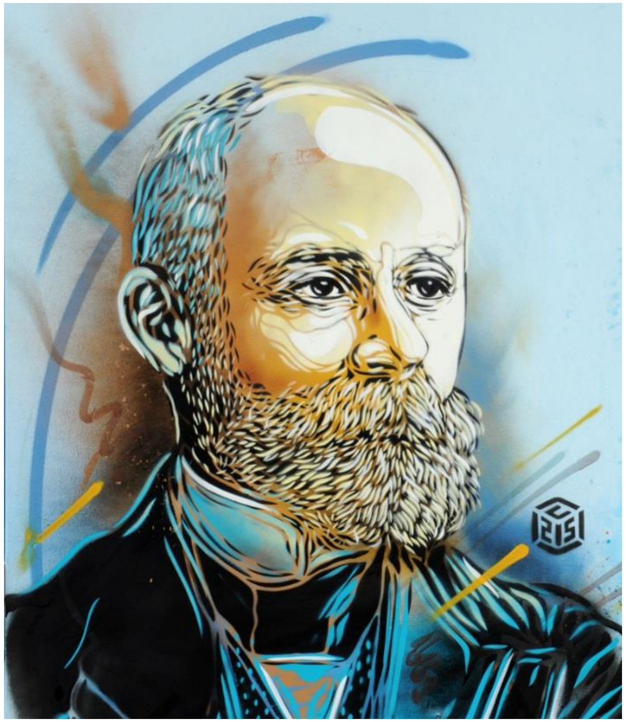
Conclusion

- **BetaShape**: ENSDF files, improved modelling, mean energies, log *ft* values, β^-/β^+ and correlated $\bar{\nu}_e/\nu_e$ spectra, multiple transitions, propagation of uncertainties.
- The BetaShape program is now the reference code for DDEP evaluations. Available at http://www.nucleide.org/logiciels.htm

- **Metallic magnetic calorimetry** has been demonstrated to have a great potential for high precision beta spectrometry.
- Exchange and screening effects have been demonstrated to have a great influence on the spectrum shape at low energy.

Unmentioned studies in progress

- Measurements with silicon detectors.
- Inclusion of the nuclear structure in the beta spectrum calculation.
- Improved modelling for electron capture transitions.



Street art from **C215** at
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Thank you for your attention

Commissariat à l'énergie atomique et aux énergies alternatives
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