



Why is there more matter than antimatter? The status after 50 years

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50 years of baryogenesis

VIOLATION OF CP INVARIANCE, C ASYMMETRY, AND BARYON ASYMMETRY OF THE UNIVERSE



A. D. Sakharov

Submitted 23 September 1966

ZhETF Pis'ma 5, No. 1, 32-35, 1 January 1967

The theory of the expanding Universe, which presupposes a superdense initial state of matter, apparently excludes the possibility of macroscopic separation of matter from antimatter; it must therefore be assumed that there are no antimatter bodies in nature, i.e., the Universe is asymmetrical with respect to the number of particles and antiparticles (C asymmetry). In particular, the absence of antibaryons and the proposed absence of baryonic neutrinos implies a non-zero baryon charge (baryonic asymmetry). We wish to point out a possible explanation of C asymmetry in the hot model of the expanding Universe (see [1])

Baryogenesis: The process of creating a matter-antimatter asymmetry in the early Universe from symmetric initial conditions. We refer to baryons because protons and neutrons (which are baryons) account for most of the mass in *visible* matter.

The question of baryogenesis can in principle be answered by particle physics/quantum field theory & cosmology because these can satisfy the *Sakharov conditions*:

- (1) baryon number B violation
 - (2) charge C and charge-parity CP violation
 - (3) deviation from thermodynamic equilibrium
- } explain in this talk

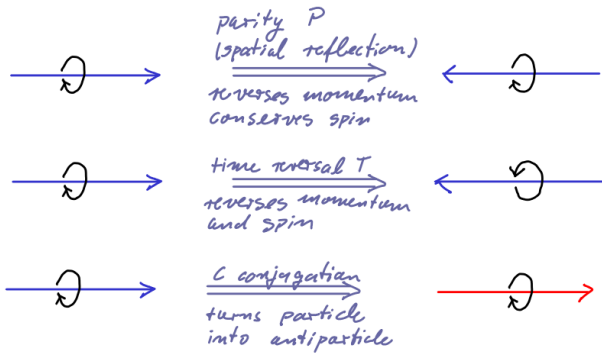
Outline

1. Introduction: The baryon asymmetry of the Universe (BAU)
2. The Sakharov conditions in the standard leptogenesis example
 - C & CP violation
 - Non-equilibrium
 - B violation
3. Leptogenesis with GeV-scale right-handed neutrinos
4. Non-equilibrium field theory methods
5. Conclusions

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Discrete symmetries



product of momentum & spin: helicity

If we can assign to all quantities in a theory definite transformation properties such as (pseudo)scalar, (pseudo)vector, charge, (axial) current, ... when applying one or more of these reflections, we call it symmetric under these transformations.

Discovery history

What was known in 1966

1928: Dirac equation	1955: CPT theorem (Pauli)	1964: CP (Cronin & Fitch)
1932: positron	1956: parity violation (Wu)	1964: CMB (Penzias & Wilson)

Challenge addressed by Sakharov:

- CPT invariance (T : time reversal) implies that particles and antiparticles have the same mass & decay rates.
- Given this toolbox (violation of discrete symmetries & cosmology), can the the baryon asymmetry of the Universe (BAU) be generated dynamically and what are the necessary conditions?
→ Sakharov conditions

Challenge/opportunity left to Physics:

- Identify the concrete mechanism realized in Nature using particle physics (including beyond the Standard Model), quantum field theory, statistical physics and cosmology

Pioneering applications of particle physics to cosmology

- Big bang nucleosynthesis (BBN) (Alpher, Bethe, Gamow, 1948)
- Cosmic Microwave Background (CMB) (Alpher, Herman, 1948)

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Ralph A. Alpher (1921–2007)

- PhD (1948)
- General Electric research lab (1955)
- American Academy of Arts and Sciences (1986)
- Distinguished research professor, Union College; Director, Dudley Observatory (1987)
- National Medal of Science (2005)



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theory,

Chall

Pione

■ Big Bang nucleosynthesis (BBN) (Alpher, Gamow, Bethe, 1948)

■ Cosmic Microwave Background (CMB) (Alpher, Herman, 1948)

Estimating the BAU in 1966*

We assume that the Universe is neutral with respect to the conserved charges (lepton, electric, and combined), but C-asymmetrical during the given instant of its development (the positive lepton charge is concentrated in the electrons and the negative lepton charge in the excess of antineutrinos over the neutrinos; the positive electric charge is concentrated in

the protons and the negative in the electrons; the positive combined charge is concentrated in the baryons, and the negative in the excess of μ -neutrinos over μ -antineutrinos).

We are unable at present to estimate theoretically the magnitude of the C asymmetry, which apparently (for the neutrino) amounts to about $[(\bar{\nu} - \nu)/(\bar{\nu} + \nu)] \sim 10^{-8} - 10^{-10}$.

$$\text{Hubble rate: } H = 100h \frac{\text{km}}{\text{s Mpc}} = h \times 2.13 \times 10^{-42} \text{ GeV}$$

$$\Rightarrow \text{energy density: } \rho = \frac{3}{8\pi} H^2 m_{\text{Pl}}^2 = h^2 \times 8.1 \times 10^{-47} \text{ GeV}^4$$

$$\Rightarrow \text{baryon number density: } n_B = \rho/m_p \approx 8.1 \times 10^{-47} \text{ GeV}^3$$

$$\text{Temperature: } T \approx 2.725 \text{ K} \approx 2.37 \times 10^{-13} \text{ GeV}$$

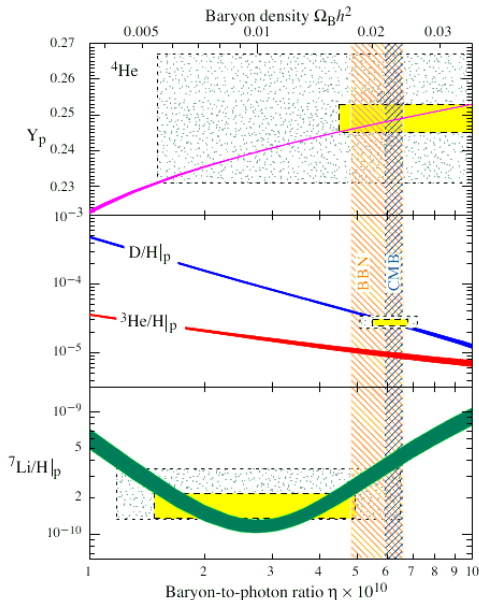
$$\Rightarrow \text{photon number density: } n_\gamma \approx 0.24 \times T^3$$

$$\Rightarrow \frac{n_B}{n_\gamma} \approx 10^{-8} \text{ (assuming } h = 0.7)$$

Estimate a bit too high – misses dark matter, dark energy.

*Alpher, Bethe, Gamow predict the CMB temperature based on the observed ^2H , ^3He and ^4He abundances given the Hubble rate and BAU. Modern reasoning in BBN is opposite: infer BAU from observed light element abundances and CMB temperature.

BAU measurements 50 years later – BBN



$$\eta_B = \frac{n_B}{n_\gamma} = (5.1-6.5) \times 10^{-10}$$

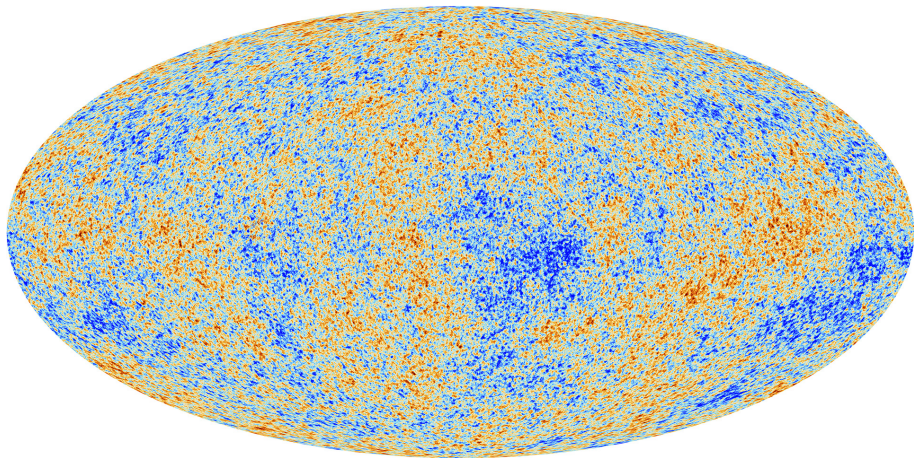
@ 95% c.l.

Baryon-to-photon ratio
 $\eta_B = \frac{n_B}{n_\gamma}$ is conserved after
electrons and positrons annihilate

At high (relativistic) temperatures
 $n_\gamma \sim$ number density of particles

One excess particle for $\sim 10^{10}$
particle-antiparticle pairs \rightarrow In
that sense, the asymmetry is tiny.

BAU measurements 50 years later – CMB

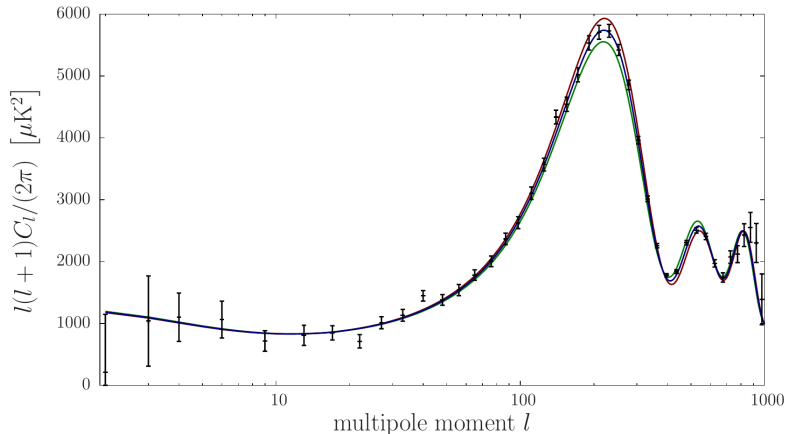


$\mathcal{O}(10^{-5})$ fluctuations on top of 2.7K radiation (image from ESA Planck).

Snapshot of the Universe at CMB formation, 380,000 y after the Bang.

BAU measurements 50 years later – CMB

...in angular momentum space \rightarrow baryon acoustic oscillations



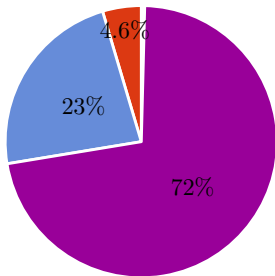
— η_B — $\eta_B + 10\%$ — $\eta_B - 10\%$ ($\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$)

First peak (roughly) corresponds to sound horizon at the time of CMB formation.

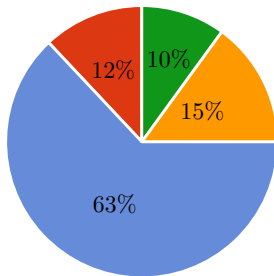
$$\eta_B = \frac{n_B}{n_\gamma} = (6.15 \pm 0.15) \times 10^{-10}$$

The cosmic pie

2017



CMB formation



dark matter

relic abundance
identity
DM genesis
asymmetry

neutrinos

relic abundance
identity ν mass
neutrino production
lepton asymmetry

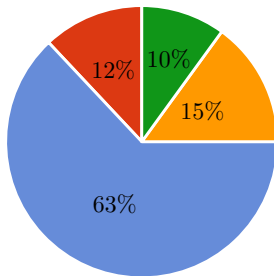
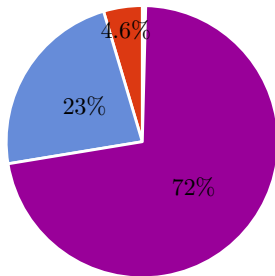
baryons

relic abundance
identity
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The cosmic pie

2017

CMB formation



dark matter

relic abundance ✓

identity ?

DM genesis ?

asymmetry ?

neutrinos

relic abundance ✓

identity ✓ ν mass ?

neutrino production ✓

lepton asymmetry ?

baryons

relic abundance ✓

identity ✓

baryogenesis ?

asymmetry ✓

Resolving the baryogenesis puzzle does not amount to identifying a new form of matter. However, the mechanism necessarily links CP violation – a hallmark of particle physics/quantum field theory – with statistical physics & cosmology.

(Overview adapted from A. Ritz)

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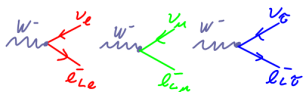
Neutrino masses & mixings

Mass parameters for *active* neutrinos from oscillation experiments

$$\Delta m_{21}^2 = 7.50 \times 10^{-5} \text{eV}^2, \Delta m_{31}^2 = 2.457 \times 10^{-3} \text{eV}^2 \text{ (NH)}, \sum_i m_i < 0.23 \text{eV}$$

[Gonzalez-Garcia, Maltoni, Schwetz (2014)], upper bound from [Planck(2015)] © 95% c.l.

Weak interaction eigenstates



Mass eigenstates are combinations of weak eigenstates:

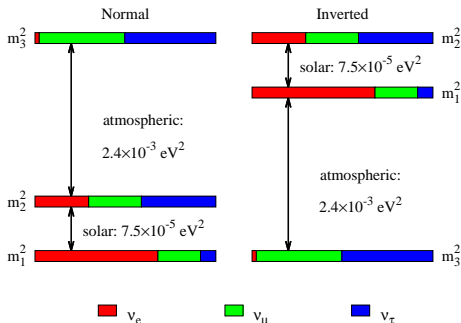
$$|\nu_n\rangle = \sum_a U_{ai}^* |\nu_a\rangle, \quad a = e, \mu, \tau, \quad n = 1, 2, 3$$

$$\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} = U^\dagger m U^*$$

for Majorana ν s, for Dirac ν s like CKM.

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

Why is the mass scale of neutrinos (several meV) much below that for other SM particles, e.g. electron (511 keV), or top quark (173 GeV)?

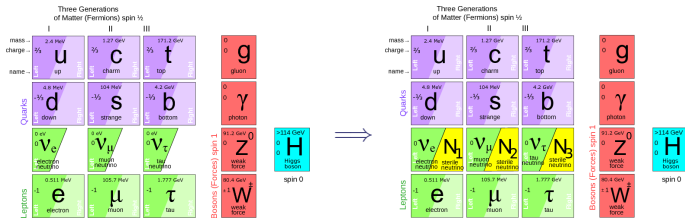


Active neutrino mass differences & mixing
[from JUNO physics paper (2015)]

Seesaw mechanism (type I)

In the Standard Model (SM), there are only left-handed (negative helicity) neutrinos
 → Add right-handed neutrinos (RHNs)

[Figure: Gninenko, Gorbunov, Shaposhnikov (2013)]



Assume here for simplicity one left-handed ν and one RHN with Majorana mass M .

Yukawa coupling $Y \bar{\nu} \phi^0 N \xrightarrow{\langle \phi^0 \rangle = v = 174 \text{ GeV}}$ Dirac mass $m_D = Yv$ ($H \equiv \phi$ Higgs field) →

$$\text{Mixing mass matrix: } \frac{1}{2} (\bar{\nu} \bar{N}^c) \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu^c \\ N \end{pmatrix} \text{ where } M \gg m_D \rightarrow$$

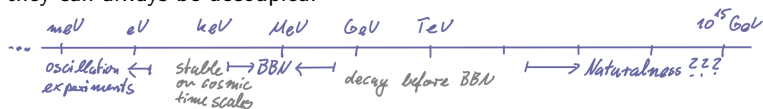
$$\text{Eigenvalues: } \left\{ \frac{1}{2} \left(M \mp \sqrt{M^2 + 4m_D^2} \right) \right\} \approx \{M, m_D^2/M\} = \{M, Y^2 v^2/M\} \rightarrow m = Y^2 v^2/M$$

$$\text{Eigenvectors: } \begin{pmatrix} \nu^{\text{light}} \\ \nu^{\text{heavy } c} \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu \\ N^c \end{pmatrix} \text{ where } \theta = m_D/M + \mathcal{O} \left(\frac{m_D^3}{M^3} \right)$$

[P. Minkowski (1977); Gell-Mann, Ramond, Slansky (1979)]

Mass scale of RHNs

- For $Y = \mathcal{O}(1)$ (as for τ lepton, t and b quarks), light neutrino masses point to superheavy scale 10^{14} – 10^{16} GeV. However, smaller Y and lighter RHNs are not excluded.
- Leaving leptogenesis aside, RHNs are allowed throughout the mass range because they can always be decoupled.



- Barring strong decoupling, BBN yields strong constraints for masses $\lesssim 100$ MeV, oscillation experiments for masses \lesssim eV.
- Naturalness: In absence of SUSY or other cancellation mechanism, the RHNs will contribute to the Higgs mass. In order to avoid destabilization, require

$$\Delta m_\phi^2 = \sum_i \frac{[Y^\dagger Y]_{ii}}{4\pi^2} M_i^2 \log \frac{M_i}{\mu} \sim \frac{m M^3}{4\pi^2 v^2} \log \frac{M}{\mu} \lesssim m_\phi^2 \rightarrow M_N \lesssim 10^7 \text{ GeV}$$

[Vissani (1997)]



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- Weak interactions distinguish between left- and right-chiral fermions (chirality \Leftrightarrow helicity in the relativistic limit. Left/right chiral fermions have negative/positive helicity). They couple to left-handed fermions and right-handed antifermions. Hence they violate C and P “maximally”. However, they still conserve the combination CP .
- CP can be violated by complex Yukawa couplings or mass terms, provided the phase cannot be removed by phase redefinitions. CP violation is a genuine quantum effect (see next slides).
- Need C and CP violation for left-handed fermions not to cancel asymmetry in right-handed fermions.

CP -violating interactions

Noether current of a complex scalar field: $j^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) = -i(\phi \partial^\mu \phi^* - \phi^* \partial^\mu \phi)$

For scalar fields, C conjugation is realized through complex conjugation of the coupling constants, similarly CP for fermions (a little more work to show this).

CP violation & quantum interference

Squared amplitude for some CP violating process:

$$\left| a_1 \cdots a_m |\mathcal{A}_a| e^{i\varphi_a} + b_1 \cdots b_n |\mathcal{A}_b| e^{i\varphi_b} \right|_{\text{x-term}}^2 \supset (a_1 \cdots a_m b_1^* \cdots b_n^* e^{i(\varphi_a - \varphi_b)} + \text{c.c.}) |\mathcal{A}_a \mathcal{A}_b|$$

a_i, b_i : coupling constants

$\arg(a_1 \cdots a_m b_1^* \cdots b_n^*)$: "weak" phase,

CP odd

$\mathcal{A}_{a,b}$: amplitudes stripped of coupling constants

$\varphi_{a,b}$: ("strong") phases of $\mathcal{A}_{a,b}$, CP even

Rate for CP conjugate process:

$$\left| a_1^* \cdots a_m^* |\mathcal{A}_a| e^{i\varphi_a} + b_1^* \cdots b_n^* |\mathcal{A}_b| e^{i\varphi_b} \right|_{\text{x-term}}^2 \supset (a_1^* \cdots a_m^* b_1 \cdots b_n e^{i(\varphi_a - \varphi_b)} + \text{c.c.}) |\mathcal{A}_a \mathcal{A}_b|$$

Difference: $(a_1 \cdots a_m b_1^* \cdots b_n^* - a_1^* \cdots a_m^* b_1 \cdots b_n)(e^{i(\varphi_a - \varphi_b)} - e^{-i(\varphi_a - \varphi_b)}) |\mathcal{A}_a \mathcal{A}_b|$

$$= 4 \text{Im}[e^{i(\varphi_a - \varphi_b)}] \text{Im}[a_1^* \cdots a_m^* b_1 \cdots b_n] |\mathcal{A}_a \mathcal{A}_b|$$

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Difference: $(a_1 \cdots a_m b_1^* \cdots b_n^* - a_1^* \cdots a_m^* b_1 \cdots b_n) (e^{i(\varphi_a - \varphi_b)} - e^{-i(\varphi_a - \varphi_b)}) |\mathcal{A}_a \mathcal{A}_b|$

$$= 4 \text{Im}[e^{i(\varphi_a - \varphi_b)}] \text{Im}[a_1^* \cdots a_m^* b_1 \cdots b_n] |\mathcal{A}_a \mathcal{A}_b|$$

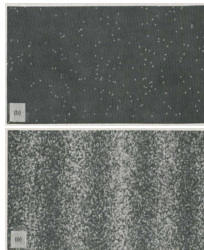
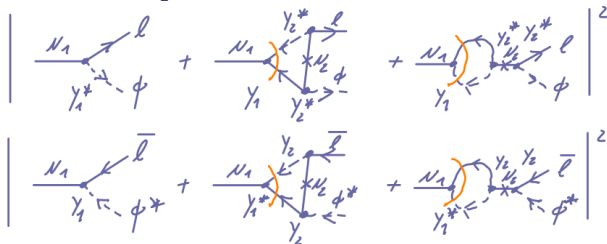
$\text{Im}[e^{i\varphi_{a,b}}]$ comes from *coherent superposition of different quantum states*. For calculable problems, these often correspond

- to **on-shell cuts** in a Feynman diagram (typical view on “standard” leptogenesis with ultraheavy RHNs)
- or to flavour **mixing and oscillations** (typical view on leptogenesis with GeV-scale RHNs).

There are parametric regimes where both pictures overlap.

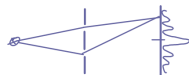
CP violation in leptogenesis

$$\mathcal{L}_{\text{SM}} \rightarrow \mathcal{L}_{\text{SM}} + \frac{1}{2} \bar{N}_i^c (i\cancel{\partial} - M_{ij}) N_j - Y_{ia}^* \bar{l}_a \phi^\dagger N_i - Y_{ia} \bar{N}_i \phi l_a; \quad a = e, \mu, \tau; \quad i = 1, 2, \dots$$



[Tomonura et al. (1989)]

The creation matter-antimatter asymmetry a quantum effect. Like the electron having passed through both slits, each lepton we find in the Universe went through a history where it has always been a lepton as well as one where it initially was an antilepton.

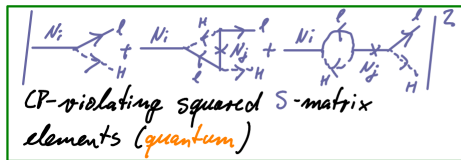


“Wave function” & “vertex” contributions:

$$\varepsilon_{N_i}^{\text{wf}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_i M_j}{M_i^2 - M_j^2} \frac{\text{Im}[(Y Y^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}} \quad (\text{decay asymmetry: } \varepsilon = \frac{\Gamma_{N_i \rightarrow lH} - \Gamma_{N_i \rightarrow \bar{l}H^*}}{\Gamma_{N_i \rightarrow lH} + \Gamma_{N_i \rightarrow \bar{l}H^*}})$$

$$\varepsilon_{N_i}^{\text{vertex}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_j}{M_i} \left[1 - \left(1 + \frac{M_j^2}{M_i^2} \right) \log \left(1 + \frac{M_j^2}{M_i^2} \right) \right] \frac{\text{Im}[(Y Y^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}} \quad [\text{Fukugita, Yanagida (1986); Covi, Roulet, Vissani (1996)}]$$

Standard approach to "standard" leptogenesis



$$L[f] = \mathcal{L}[f]$$

Boltzmann equation (classical)

lepton asymmetry

Momentum integration $n(\mathbf{x}, t) = \int \frac{d^3 p}{(2\pi)^3} f(\mathbf{x}, t, \mathbf{p})$ - Boltzmann eqs. \rightarrow fluid eqs.

$$\nabla_\mu j_X^\mu = \partial_t n_X - \nabla \cdot \mathbf{j}_X + 3H n_X = \mathcal{C}_X$$

$$\mathcal{C}_X = \frac{1}{2E_X} \int \prod_i \frac{d^3 p_i}{(2\pi)^3 2E_i} \delta^4(p_X + p_{A1} + \dots - p_{B1} - \dots)$$

$$\times \left\{ (1 \pm f_X)(1 \pm f_{A1}) \dots f_{B1} \dots |\mathcal{M}_{B_1 B_2 \dots \rightarrow X A_1 A_2 \dots}|^2 - f_X f_{A1} \dots (1 \pm f_{B1}) \dots |\mathcal{M}_{X A_1 A_2 \dots \rightarrow B_1 B_2 \dots}|^2 \right\}$$

n_X : number density

\mathbf{j}_X : current density

j^μ : four-current

∇_μ : cov. derivative

H : Hubble rate

\mathcal{C}_X : collision term

f_i : distribution fn.




Heuristic substitution of quantum field theoretical reaction rates into the collision term \mathcal{C} of classical Boltzmann equations: \rightarrow Not a derivation from first principles

Real intermediate state (RIS) problem

- Interference of tree & loop amplitudes $\rightarrow CP$ violation.

$$\left| \begin{array}{c} N_1 \text{---} \phi \\ \diagdown \\ \ell \\ \diagup \\ N_1 \end{array} + \begin{array}{c} \phi \\ | \\ N_1 \text{---} \phi \\ \diagdown \\ \ell \\ \diagup \\ N_2 \end{array} + \begin{array}{c} \phi \\ | \\ N_1 \text{---} \phi \\ \diagdown \\ \ell \\ \diagup \\ N_2 \end{array} \right|^2 \quad (*)$$

- CP violating contributions (“strong phase”) from discontinuities \rightarrow loop momenta where **cut** particles are on shell.

- Is  an extra process or is it already accounted for by  and  ?

- Including (*) only $\rightarrow CP$ asymmetry is already generated in equilibrium. $\Downarrow CPT \Downarrow$ theorem.
(CPT invariance requires to break T thermodynamically in order to make CP effective)

(Inverse) decays & CP asymmetry

- Consider the squared matrix elements, ε being the decay asymmetry.

$$\begin{array}{cc}
 \begin{array}{c} \text{---} \swarrow \\ \text{---} \searrow \\ \text{---} \end{array} & |\mathcal{M}_{N \rightarrow l\phi}|^2 \sim 1 + \varepsilon & \begin{array}{c} \text{---} \swarrow \\ \text{---} \searrow \\ \text{---} \end{array} & |\mathcal{M}_{N \rightarrow \bar{l}\phi^*}|^2 \sim 1 - \varepsilon \\
 \begin{array}{c} \text{---} \swarrow \\ \text{---} \searrow \\ \text{---} \end{array} & \xrightarrow{\underline{CPT}} & \begin{array}{c} \text{---} \swarrow \\ \text{---} \searrow \\ \text{---} \end{array} & \xrightarrow{\underline{CPT}} & \begin{array}{c} \text{---} \swarrow \\ \text{---} \searrow \\ \text{---} \end{array} & |\mathcal{M}_{\bar{l}\phi^* \rightarrow N}|^2 \sim 1 + \varepsilon & \begin{array}{c} \text{---} \swarrow \\ \text{---} \searrow \\ \text{---} \end{array} & \xrightarrow{\underline{CPT}} & \begin{array}{c} \text{---} \swarrow \\ \text{---} \searrow \\ \text{---} \end{array} & |\mathcal{M}_{l\phi \rightarrow N}|^2 \sim 1 - \varepsilon
 \end{array}$$

- Naive multiplication[†] suggests that an asymmetry is generated already in equilibrium: $\Gamma_{\bar{l}\phi^* \rightarrow l\phi} \sim 1 + 2\varepsilon$, $\Gamma_{l\phi \rightarrow \bar{l}\phi^*} \sim 1 - 2\varepsilon$

- Ad hoc fix: Subtract real intermediate states (RIS) from [Kolb, Wolfram (1980)].



Fix of the approach by *a posteriori* imposing CPT theorem

Better Way Out

- Compute the real time (time dependent perturbation theory), non-equilibrium (statistical physics) evolution of the quantum field theory states of interest.

[†] Do not try this at home: The unstable N are not asymptotic states of a unitary S matrix \rightarrow conflict with the CPT theorem.

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Number densities in the expanding Universe

Need to track the evolution of n_N (RHN number density) and $n_L = n_\ell - n_{\bar{\ell}}$ (lepton charge density).

Necessary ingredients to the fluid equation:

- Decay rate $N \rightarrow \ell\phi$: $\Gamma \times n_N$ where $\Gamma = |Y|^2 M / (8\pi)$
- Rate for inverse decays $\ell\phi \rightarrow N$: $\Gamma \times n_N^{\text{eq}}$ where n_N^{eq} is the equilibrium Fermi-Dirac distribution
- Asymmetry production in n_L : $\varepsilon \times \Gamma \times (n_N - n_N^{\text{eq}})$
(Need RIS method or better first principle derivation to get this term straight.)
- Washout of n_L : $\Gamma \times \mathcal{O}(1)$ due to different normalization
- Dilution from expansion of the Universe:

Friedmann eq. $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3m_{\text{Pl}}^2}$, H : Hubble rate, a : scale factor

w/o interactions: $n(t) = n_0 a^{-3}(t) \Leftrightarrow \frac{dn(t)}{dt} = -3n_0 a^{-4} \dot{a} = -3Hn(t)$

Non-equilibrium and neutrinos

Out-of-equilibrium dynamics

Simplest Meaningful Network of Fluid Equations Describing Leptogenesis*:

$$\frac{dn_{N_i}}{dt} + 3Hn_{N_i} = -\Gamma_i(n_{N_i} - n_{N_i}^{\text{eq}})$$

$$\Gamma_i = \frac{|Y_i|^2}{8\pi} M_i$$

$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon\Gamma_i(n_{N_i} - n_{N_i}^{\text{eq}}) - Wn_L$$

$$W = \frac{\Gamma_i}{4} \left(\frac{M}{T}\right)^{\frac{3}{2}} e^{-\frac{M}{T}}$$

n_{N_i} : number density of N_i ; n_L : lepton charge density; W : washout rate;
 $\varepsilon = (\Gamma_{N_i \rightarrow \ell H} - \Gamma_{N_i \rightarrow \bar{\ell} H^*}) / (\Gamma_{N_i \rightarrow \ell H} + \Gamma_{N_i \rightarrow \bar{\ell} H^*})$: decay asymmetry; T : temperature

→ Best compromise between large L violating rate (1st S. condition) and and large deviation from equilibrium (3rd S. condition):

$\Gamma \sim H$ for $T \sim M$ (i.e. at freezeout, when the RHNs become Maxwell suppressed).

$$\rightarrow Y^2 M / (8\pi) \sim T^2 / m_{\text{Pl}} \sim M^2 / m_{\text{Pl}} \stackrel{m \sim \frac{Y^2 v^2}{M}}{\Rightarrow} m \sim 8\pi v^2 / m_{\text{Pl}} \sim 0.1 \text{ meV}$$

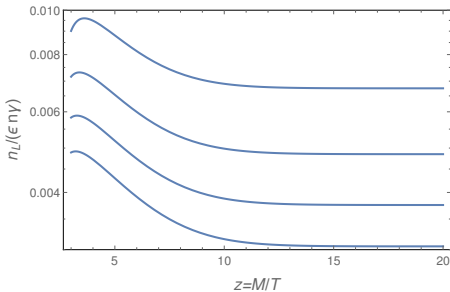
Light Neutrino Mass Scale points to Role of RHNs in Baryogenesis

- Out-of-equilibrium property of the N independent of the mass scale.
- Tendency of being somewhat close to equilibrium, i.e. $\Gamma \gg H$ around $T \sim M \rightarrow$ strong washout. However, \exists a lot of parametric freedom.

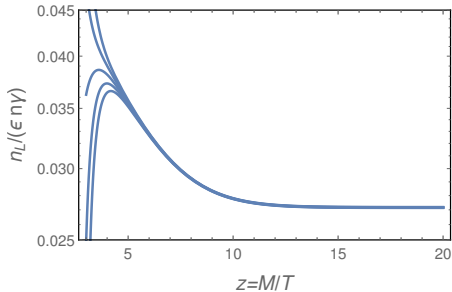
* Assume $\|M\| \gg M_W$ for now, get back to case $\|M\| \sim \text{GeV}$ later

Evolution of lepton asymmetry for strong washout

Varying washout strength



Varying initial conditions



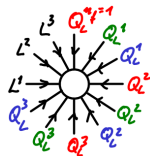
→ Strong washout leads to independence of initial conditions.

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How leptogenesis makes baryons

- Baryon number B and lepton number L are conserved by the SM Lagrangian at the *classical* level.
- L (and fermion number) is violated by Majorana masses (beyond the SM addition).
- At quantum level – chiral anomaly: $\partial^\mu j_\mu^{B+L} \neq 0$
- Instanton/sphaleron configurations: $\Delta(B + L) = 6n$ where $n \in \mathbb{Z}$ ($\Delta(B - L) = 0$ is still conserved)
- Sphaleron processes are fast above the electroweak scale $T_{EW} \approx 140\text{GeV} \rightarrow$ lepton asymmetry implies baryon asymmetry in chemical equilibrium $\mu_B + \mu_L = 0$ (for full answer, must consider all reactions & conservation laws)
- Below the electroweak temperature, sphaleron processes are exponentially suppressed, and B is frozen in.



Sakharov conditions in leptogenesis

(C asymmetry). In particular, the absence of antibaryons and the proposed absence of baryonic neutrinos implies a non-zero baryon charge (baryonic asymmetry). We wish to point out a possible explanation of C asymmetry in the hot model of the expanding Universe (see [1]) by making use of effects of CP invariance violation (see [2]). To explain baryon asymmetry, we propose in addition an approximate character for the baryon conservation law.

We assume that the baryon and lepton conservation laws are not absolute and should be energy dependent with $n \sim 10^{10} \text{ cm}^{-3}$ and $\epsilon \sim 10^{-10} \text{ erg/cm}^3$.

M. A. Markov (see [5]) proposed that during the early stages there existed particles with maximum mass on the order of one gravitational unit ($M_0 = 2 \times 10^{-5} \text{ g}$ in ordinary units), and called them maximons. The presence of such particles leads unavoidably to strong violation of thermodynamic equilibrium. We can visualize that neutral spinless maximons (or

2

1

3

- 1 B violated, in particular through interplay of L -violating Majorana-mass and $B + L$ -violating sphaleron
- 2 CP violated through RH neutrino Yukawa couplings and Majorana masses, C through chiral nature of interactions
- 3 Deviation from equilibrium because weakly coupled RHNs do not adapt quickly to equilibrium distribution, intriguing connection with light neutrino masses

Situation in the Standard Model (SM)

Check Sakharov conditions:

- 1 B violated due to anomalous $B + L$ violation via sphaleron processes.
- 2 CP violation in CKM matrix, C and P violation in weak interactions.
- 3 Deviation from equilibrium due to expansion of the Universe.

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However, conditions for baryogenesis not met *quantitatively*:

- CP rephasing invariant normalised to electroweak scale is tiny:

$$\text{Im} \left[\det[m_u m_u^\dagger, m_d m_d^\dagger] \right] \approx -2J m_t^4 m_b^4 m_c^2 m_s^2, \quad J \approx 3 \times 10^{-5},$$

$$2J \frac{m_t^4 m_b^4 m_c^2 m_s^2}{T^{12}} \approx 3 \times 10^{-19} \text{ for } T = 100 \text{ GeV}.$$

(CP violation is non-perturbatively enhanced in neutral meson systems and therefore observable. The perturbative suppression applies also to e.g. electric dipole moments.)

- Deviation from equilibrium $H/\Gamma \sim (T^2/m_{\text{Pl}})/(g^4 T) = g^{-4} T/m_{\text{Pl}}$ with $g = \mathcal{O}(1)$ is tiny unless T is very high ($m_{\text{Pl}} = 1.2 \times 10^{19} \text{ GeV}$).

Loophole: For $m_H < 70 \text{ GeV}$, first order phase transition [Kajantie, Laine, Rummukainen, Shaposhnikov (1996)]. Ruled out by discovery $m_H = 125 \text{ GeV}$. Still possible beyond the SM when light bosons couple to the Higgs field.

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Baryogenesis requires physics beyond the Standard Model.

Baryogenesis puzzle in new physics scenarios

- Leptogenesis (this talk)
- Electroweak baryogenesis – requires extension of the SM to provide strong first order phase transition, extra CP violation (permanent electric dipole moments?) → perhaps the best prospects for testability
- Decay of scalar condensates or Q-balls, e.g. from SUSY flat directions (Affleck-Dine)
- Connection with asymmetric dark matter
- Other paradigms less connected to specific models

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Hard to solve because

- we do not know whether we have all pieces,
- we do not know whether the pieces we have are part of the puzzle,
- some of the pieces probably not even exist.



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Main concerns raised about standard leptogenesis

Testability

- Unless there is resonant enhancement, leptogenesis in the strong washout regime requires $M \gtrsim 10^{10}$ GeV because $\varepsilon \sim Y^2$ [Davidson, Ibarra (2002)].
- In this mass range, RHNs will remain hypothetical for a long time. NB that $0\nu\beta\beta$ decay is only sensitive to the Majorana masses of the light neutrinos.

Field theoretical and cosmological issues

- In absence of SUSY, destabilization of the Higgs mass for $M \gtrsim 10^7$ GeV
- In presence of SUSY, overproductions of relic gravitinos (that either would lead to overabundant dark matter, if stable, or spoil BBN, if unstable) for $T \gtrsim M \gtrsim 10^{10}$ GeV.

Alternative: leptogenesis from GeV-scale RHNs

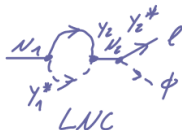
- Also based on type-I seesaw, but with lighter RHNs
- Makes use of interplay of lepton flavour effects and freezeout of B at the electroweak scale [Akhmedov, Rubakov, Smirnov (1998); Shaposhnikov, Asaka (2005)]

The flavour loophole

Lepton-number violating



Lepton-number conserving



Consider the LNC but lepton-flavour violating source.

$$\rightarrow \text{Initially, } \sum_{a=e,\mu,\tau} \left(\frac{B}{3} - L_a \right) = 0$$

Partial, flavour-dependent washout (inverse decays to RHNs) $\rightarrow \sum_{a=e,\mu,\tau} \left(\frac{B}{3} - L_a \right) \neq 0$

Full washout would lead to the erasure of all asymmetries, $B/3 - L_a \rightarrow 0$

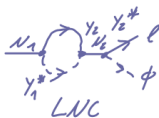
Effectively realize partial washout of B for $0.1\text{GeV} \gtrsim M_{1,2} \lesssim T_{\text{EW}}$, such that B number is frozen in (due to sphaleron suppression) before washout is complete.

Enhanced asymmetries



- LNV Majorana mass insertions necessary

$$\rightarrow \epsilon \propto \frac{M_1 M_2}{M_1^2 - M_2^2}$$



- No LNV Majorana mass insertions
- Gauge interactions at finite T enhance phase space



$$\rightarrow \epsilon \propto \frac{T^2}{M_1^2 - M_2^2}$$

- Off diagonal correlations build up through flavour oscillations among the RHNs. These first occur at the temperature $T_{\text{osc}} \sim (|M_i^2 - M_j^2| m_{\text{Pl}})^{1/3}$, i.e. for $M_{1,2} \sim \text{GeV} \rightarrow T_{\text{osc}} \sim 10^5 \text{GeV}$.
 \rightarrow Large enhancement of asymmetry opens up possibility of leptogenesis from GeV-scale RHNs without strong parametric tuning [Drewes, BG (2012)].
- NB for $M_{1,2} \gg T_{\text{EW}}$, these early asymmetries typically experience strong washout.*

*For exceptions, see [BG (2014)].

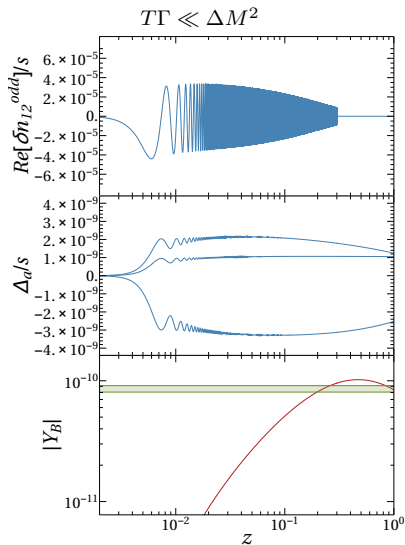
Coherent superpositions & oscillations

- RHNs (momentum \mathbf{k} , helicity h) produced through a lepton ℓ_a are a coherent superposition of mass eigenstates $|N_a\rangle = \sum_i Y_{ia}|N_i\rangle$
- Not a mass eigenstate and consequently, no energy eigenstate of Hamiltonian $H \approx M^2/(2k^0)$ for relativistic RHNs
- Matrix-valued generalized distribution function $f_{Nhij}(\mathbf{k}) \leftrightarrow$ Density matrix $|N_a\rangle\langle N_a|$
- $\dot{f}_{Nh} = -\frac{i}{2k^0}[M^2, f_{Nh}] \rightarrow i[M^2, f_{Nh}]_{ij} = i(M_i^2 - M_j^2)f_{Nhij} \rightarrow$ oscillating phase in off-diagonal correlations of RHNs
- These are CP -even phases that lead to an asymmetric production rate of doublet leptons.
- In the limit of fast oscillations, this corresponds to a resummation of the “standard” wave-function contribution.



Leptogenesis from oscillation – dynamics

Generation of the BAU

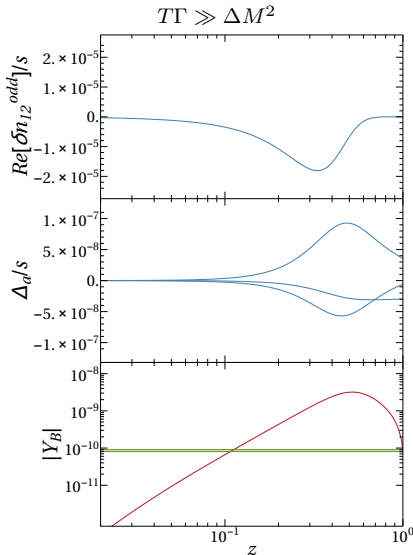
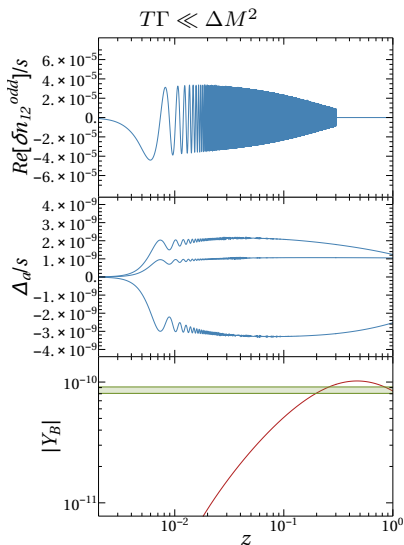


- RHNs perform first oscillation at $T_{\text{osc}} \sim (|M_i^2 - M_j^2| m_{\text{Pl}})^{1/3}$.
- Off-diagonal correlations lead to CP violating source for lepton flavour asymmetries (*purely flavoured*).
- Contributions from subsequent oscillations average out.
- Transfer of asymmetries into helicity asymmetries of RHNs leads to $Y_L \neq 0$. Large active-sterile mixing possible if one active flavour is more weakly washed out than the other two.
- Asymmetry frozen in at T_{EW} , where sphalerons are quenched by the developing Higgs vev.
- $\delta n = \int d^3k / (2\pi)^3 \delta f_N \rightarrow$ **momentum averaging**

$z = T_{\text{EW}}/T$; Δ_a : lepton asymmetry in flavour $a = e, \mu, \tau$; δn : number density of RHNs;
 Y_B : entropy-normalized baryon asymmetry; s : entropy density. [Drewes, BG, Gueter, Klarić (2016)]

Leptogenesis from oscillation – dynamics

Oscillatory vs overdamped regime



$z = T_{EW}/T$; Δ_a : lepton asymmetry in flavour $a = e, \mu, \tau$; δn : number density of RHNs;
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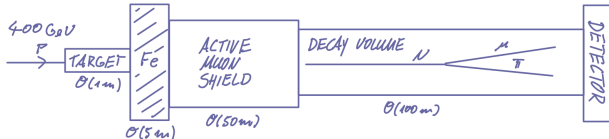
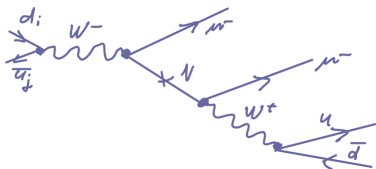
Direct & indirect searches for GeV-scale RHNs

Direct searches:

- Mixing between active neutrinos and RHNs:

$$U_{ai} \approx \theta_{ai} = v Y_{ai}^\dagger / M_i$$

- GeV-scale RHNs can be produced in heavy meson (D , B) decays in B factories or beam dump experiments.
- Sensitivity of B factories is limited because RHNs can decay outside of the detector.
- SHiP – Search for Hidden Particles: Proposed beam dump facility @ CERN:

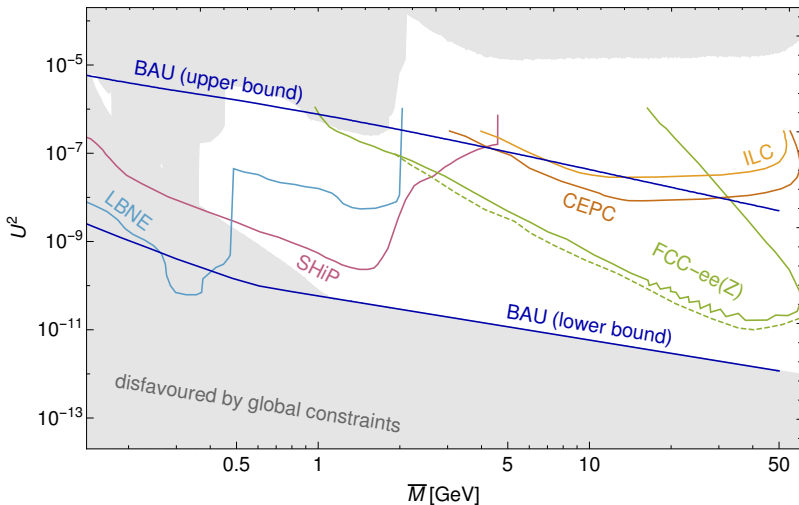


Indirect signals:

- Rate for $0\nu\beta\beta$ decay in type I seesaw mechanism can be enhanced for GeV-scale RHNs compared to other mass regions.
- Lepton universality in meson decays.
- Charged lepton flavour violation.
- Direct searches yield however most stringent bounds in mass/mixing plane. [Drewes, BG (2015)]

Experimental prospects

Normal hierarchy

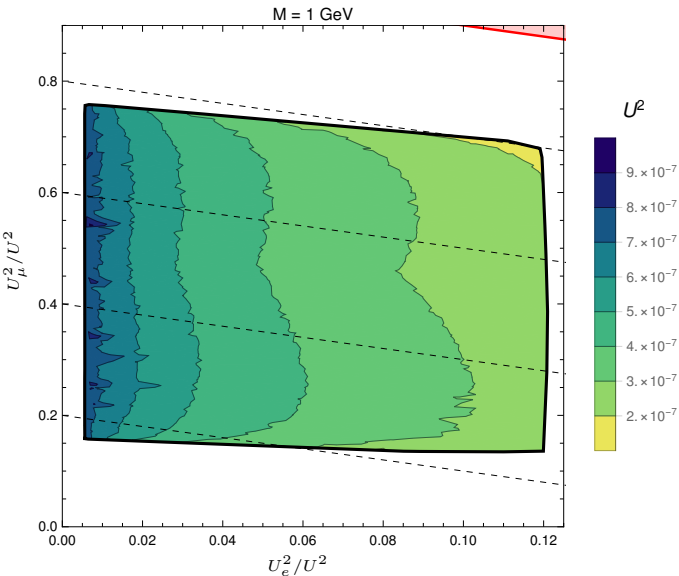


Active-sterile mixing: $U_{ai} \approx \theta_{ai} = vY_{ai}^\dagger/M_i$; $U^2 = \text{tr}\theta^\dagger\theta$; $n_N = 2$; $\bar{M} = \frac{M_1 + M_2}{2}$

[Drewes, BG, Gueter, Klarić (2016)]

GeV-scale leptogenesis

Normal hierarchy, $n_N = 2$



- Maximal $U^2 = \sum_a U_a^2$ for viable leptogenesis shows particular flavour patterns
- Larger U^2 require smaller U_e^2/U^2 in order for the asymmetry stored in ℓ_e to survive washout prior to sphaleron freezeout

[Drewes, BG, Gueter, Klarić (2016)]

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Kinetic & fluid equations

- Set of equations describing a non-equilibrium system of a large number of particles
- Should be derivable from first principles
 - classical: Liouville equations (with interaction potentials)
 - quantum: Schwinger-Dyson equations in Closed-Time-Path (CTP) formalism
- Truncations/approximations
 - Reduction to *irreversible kinetic equations* in the form of *Boltzmann equations* (or variants thereof taking e.g. account of flavour coherence, quantum statistics)
 - Further reduction down to *fluid equations* (in terms of number densities and bulk flows instead of particle distributions)
 - Allows for analytical/numerical treatment

Liouville/Schwinger-Dyson equations
(microscopic kinetic equations)

Boltzmann-like kinetic equation for distribution functions

fluid equations for number densities/bulk flows

Closed-Time-Path Approach

- In-in generating functional (in contrast to “in-out” for S matrix elements): [Schwinger (1961); Keldysh (1965); Calzetta & Hu (1988)]

$$\begin{aligned} Z[J_+, J_-] &= \int \mathcal{D}\phi(\tau) \mathcal{D}\phi_{\text{in}}^- \mathcal{D}\phi_{\text{in}}^+ \langle \phi_{\text{in}}^- | \phi(\tau) \rangle \langle \phi(\tau) | \phi_{\text{in}}^+ \rangle \langle \phi_{\text{in}}^- | \rho | \phi_{\text{in}}^+ \rangle \\ &= \int \mathcal{D}\phi^- \mathcal{D}\phi^+ e^{i \int d^4x \{ \mathcal{L}(\phi^+) - \mathcal{L}(\phi^-) + J_+ \phi^+ - J_- \phi^- \}} \end{aligned}$$

- The Closed Time Path:



Aim: Calculate $\langle \text{in} | \mathcal{O}(t) | \text{in} \rangle$

- Path-ordered Green functions:

$$i\Delta^{ab}(u, v) = -\frac{\delta^2}{\delta J_a(u) \delta J_b(v)} \log Z[J_+, J_-] \Big|_{J_{\pm}=0} = i \langle \mathcal{C}[\phi^a(u) \phi^b(v)] \rangle$$

$$\text{e.g. } j^\mu(x) = \text{tr}[\gamma^\mu \langle \mathcal{C}[\psi^-(x_1) \bar{\psi}^+(x_2)] \rangle]_{x_1=x_2=x}$$

Wigner Transformation of Two-Point Functions (Green Function or Self Energy)

$$A(k, x) = \int d^4r e^{ik \cdot r} A(x + r/2, x - r/2) \rightarrow \sim \text{distribution function}$$

x : average coordinate – macroscopic evolution

$r \rightarrow k$: relative coordinate – microscopic (quantum)

Path Ordered Green Functions @ Tree Level

- Four propagators (two of which are linearly independent):

$$\left. \begin{aligned} i\Delta^<(u, v) &= i\Delta^{+-}(u, v) = \langle \phi(v)\phi(u) \rangle \\ i\Delta^>(u, v) &= i\Delta^{-+}(u, v) = \langle \phi(u)\phi(v) \rangle \end{aligned} \right\} \text{Wightman functions}$$
$$i\Delta^T(u, v) = i\Delta^{++}(u, v) = \langle T[\phi(u)\phi(v)] \rangle \quad \text{Feynman propagator}$$
$$i\Delta^{\bar{T}}(u, v) = i\Delta^{--}(u, v) = \langle \bar{T}[\phi(u)\phi(v)] \rangle \quad \text{Dyson propagator}$$

- Perturbation theory can be formulated in terms of tree-level Wigner-space propagators:

$$i\Delta^<(p, t) = 2\pi\delta(p^2 + m^2) [\vartheta(p^0)f(\mathbf{p}, t) + \vartheta(-p^0)(1 + \bar{f}(-\mathbf{p}, t))]$$

$$i\Delta^>(p, t) = 2\pi\delta(p^2 + m^2) [\vartheta(p^0)(1 + f(\mathbf{p}, t)) + \vartheta(-p^0)\bar{f}(-\mathbf{p}, t)]$$

$$i\Delta^T(p, t) = \frac{i}{p^2 - m^2 + i\epsilon} + 2\pi\delta(p^2 + m^2) [\vartheta(p^0)f(\mathbf{p}, t) + \vartheta(-p^0)\bar{f}(-\mathbf{p}, t)]$$

$$i\Delta^{\bar{T}}(p, t) = \frac{-i}{p^2 - m^2 - i\epsilon} + 2\pi\delta(p^2 + m^2) [\vartheta(p^0)f(\mathbf{p}, t) + \vartheta(-p^0)\bar{f}(-\mathbf{p}, t)]$$

- $f(\mathbf{p}, t)$, $\bar{f}(\mathbf{p}, t)$: Particle and antiparticle distribution functions. Carry flavour indices (relevant for leptogenesis: sterile & active flavour).

Leptogenesis in the CTP Approach

■ Schwinger-Dyson equations relevant for leptogenesis:

[Buchmüller, Fredenhagen (2000); De Simone, Riotto (2007); Garny, Hohenegger, Kartatvtsev, Lindner (2009-); Beneke, BG, Herranen, Schwaller (2010-); Anisimov, Buchmüller, Drewes, Mendizabal (2010-)]

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{tr} \left[\text{---} \overset{N}{\text{---}} \text{---} + \text{---} \text{---} \right]_{\substack{>< \\ <>}} + \dots$$

$= \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{P}^0 \text{---}] + \dots$

$= \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{P}^0 \text{---}] + \dots$

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■ Truncation that yields leading asymmetry in non-degenerate regime

$$M_i \Gamma_i, M_j \Gamma_j \ll |M_i^2 - M_j^2|:$$

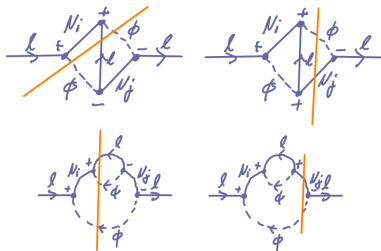
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$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[\text{---} \overset{N}{\text{---}} \text{---} \right]_{\substack{>< \\ <>}}$$

- Non-minimal truncations → e.g. systematic inclusion of thermal corrections.

Unitarity Restored (without RIS)

- CTP approach readily yields *inclusive* rates for the creation of the charge asymmetry.
- No need to separately remove unwanted/unphysical contribution *a posteriori*.



- Loop insertions in propagator for N must be resummed unless (typical $|\mathbf{p}| \sim T$)

$$|M_i^2 - M_j^2|/\sqrt{\mathbf{p}^2 + M_i^2} \gg \Gamma_i \sim \begin{cases} Y^2/(16\pi)M_i & \text{for } M_i \gg T \\ Y^2 g^2 \log g T & \text{for } M_i \ll T \end{cases} .$$

Leptogenesis from Oscillations/Resonant Limit

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{Y}^0] = \int \frac{dk^0}{2\pi} \text{tr} \left[\text{diagram 1} + \text{diagram 2} \right] \gg \ll + \dots$$

$= \int d_e(\vec{k}^0) - \int \bar{d}_e(\vec{k}^0) + \dots$

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} [\mathcal{Y}^0] = \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[\text{diagram 3} \right] \gg \ll + \dots$$

$= \int d_\nu(\vec{k}^0)$

Evolution for Matrix-Valued RHN Distributions δf_{Nh} (i.e. deviation from equilibrium form f_N^{eq})

$$\delta f'_{Nh} + \frac{a^2(\eta)}{2k^0} i[M^2, \delta f_{Nh}] + f_N^{\text{eq}'} = -2 \left\{ \text{Re}[Y^* Y^t] \frac{k \cdot \hat{\Sigma}_N^A}{k^0} - i h \text{Im}[Y^* Y^t] \frac{\tilde{k} \cdot \hat{\Sigma}_N^A}{k^0}, \delta f_{Nh} \right\}$$

$\hat{\Sigma}_N^A$: spectral (cut part) self energy, $a(\eta), \eta$: scale factor and conformal time, $\iota \equiv d/d\eta$,

h : helicity, $\tilde{k} = (|\mathbf{k}|, |k^0| \mathbf{k}/|\mathbf{k}|)$

- $i[M^2, \delta f_{Nh}]_{ij} = i(M_i^2 - M_j^2) \delta f_{Nhij}$ for diagonal $M^2 \rightarrow$ RHN “flavour” oscillations
- Off-diagonal entries of δf_{Nhij} correspond to interference between the different N_i that give rise to CP violation (“strong phases”)
- Note: If $\delta f'_{Nh}$ and off diagonal elements of the collision term can be neglected, recover result from “standard resonant leptogenesis” \rightarrow next slide
- Evolution equations are well behaved for $\Delta M^2 \rightarrow 0$. Solutions δf_{Nh} enter into the resummed RHN propagators. [BG, Herranen (2010); Iso, Shimada (2014); BG, Gautier, Klaric (2014)]

Regulator for Resonant Leptogenesis in Strong Washout

Wave-function contribution:

$$\epsilon_{Ni}^{\text{wf}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_i M_j (M_i^2 + M_j^2)}{(M_i^2 - M_j^2)^2 + R} \frac{\text{Im}[(YY^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}}$$

- In the degenerate limit, this will dominate over the vertex contribution.
- Proposed forms for regulator R ($\bar{M} = (M_i + M_j)/2$):

- $R = \frac{\bar{M}^4}{64\pi^2} (YY^\dagger)_{jj}^2 = \bar{M}^2 \Gamma_j^2$ [Pilaftsis (1997); Pilaftsis, Underwood (2003)]
- $R = \frac{\bar{M}^4}{64\pi^2} ([YY^\dagger]_{ii} - [YY^\dagger]_{jj})^2$ [Anisimov, Broncano, Plümacher (2005)]
- $R = \frac{\bar{M}^4}{64\pi^2} ([YY^\dagger]_{ii} + [YY^\dagger]_{jj})^2$ [Garny, Hohenegger, Kartavtsev (2011)]
- $R = \frac{M^4}{64\pi^2} \frac{([YY^\dagger]_{11} + [YY^\dagger]_{22})^2}{[YY^\dagger]_{11}[YY^\dagger]_{22}} ((\text{Im}[YY^\dagger]_{12})^2 + \det YY^\dagger)$

Obtained by algebraic solution to oscillation equation, neglecting $\delta f'_N$

[BG, Gautier, Klaric (2014); Iso, Shimada (2014)]

Resonant Leptogenesis in the Strong Washout Regime

Quasistatic Approximation

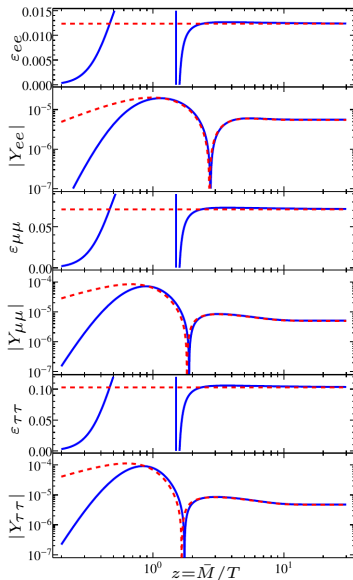
- Neglect derivative term provided the eigenvalues (that originate from the mass *and* the damping terms) in the kinetic equation for are larger than the Hubble rate $H \rightarrow$ obtain linear system of equations for δf_{Nh} . [BG, Gautier, Klaric (2014);

Iso, Shimada (2014)].

- Regulator:

$$R = \frac{\bar{M}^4}{64\pi^2} \frac{([YY^\dagger]_{11} + [YY^\dagger]_{22})^2}{[YY^\dagger]_{11}[YY^\dagger]_{22}} \times ((\text{Im}[YY^\dagger]_{12})^2 + \det YY^\dagger)$$

- Applies in strong washout regime, *i.e.* a large portion of parameter space.



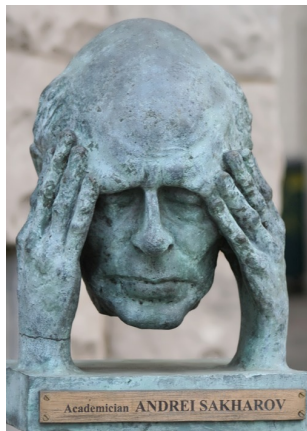
blue: full result; red: result using effective decay asymmetry ε ; $Y_{aa} = n_{La a}/s$. Here, $\bar{M}\Gamma \gg \Delta M^2$.

Table of Contents

1. Introduction: The baryon asymmetry of the Universe (BAU)
2. The Sakharov conditions in the standard leptogenesis example
 - C & CP violation
 - Non-equilibrium
 - B violation
3. Leptogenesis with GeV-scale right-handed neutrinos
4. Non-equilibrium field theory methods
5. Conclusions

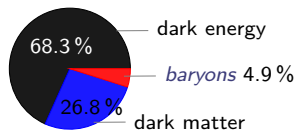
Conclusions

- “Standard” leptogenesis is perhaps the simplest realistic realization of Sakharov’s conditions and gains credibility from the observed neutrino mass scale (due to non-equilibrium condition) but it is hard to impossible to test.
- Leptogenesis with GeV-scale RHNs is a slightly more involved mechanism that relies however on essentially the same ingredients (type-I seesaw). Motivations: testability, no need of introducing new mass scales above the Standard Model.
- Leptogenesis is a good setting for methodical developments at the interface of particle & statistical physics that are applicable to the many other scenarios of baryogenesis.
- After 50 years: Baryogenesis is one of the chief motivations for considering physics beyond the SM, one of the greatest mysteries of Science.



Dynamical Generation of the BAU: Sakharov Conditions

- Most of the mass in visible matter is made up of protons & neutrons, *i.e.* *baryons*, almost no anti-baryons.
- Baryon-to-photon ratio $\eta_B = (6.16 \pm 0.15) \times 10^{-10}$ [Planck, 68% confidence level], *i.e.* in the early Universe, there was one extra quark per ten billion quark-antiquark pairs.
- Creating the baryon (B) asymmetry of the Universe (BAU) from symmetric initial conditions (*baryogenesis*) requires [Sakharov (1968)]
 - 1 B violation,
 - 2 charge (C) and charge-parity (CP) violation,
 - 3 departure from equilibrium.



[Planck (2013)]

Remark on 3rd Condition

- CPT -invariance theorem is the QFT generalization of the time reversal invariance well-known from classical mechanics and electrodynamics.
- Need to break time-reversal (T) invariance to allow for CP asymmetry.
- Realize this thermodynamically (2nd law, *non-equilibrium* \rightarrow irreversible processes).
- Will look at 2nd & 3rd condition more closely on example of leptogenesis.

Kinetic Equations in Early Universe Cosmology

Inferring the Asymmetry from Observations

- Baryon-to-photon ratio $\eta_B = (6.16 \pm 0.15) \times 10^{-10}$ can be inferred from observations
 - light elements produced during Big Bang Nucleosynthesis (BBN),
 - anisotropies of the cosmic microwave background (CMB), in particular baryon acoustic oscillations (BAOs).
- Theoretical predictions from *Boltzmann equations* that model network of nuclear reactions or scatterings of photons, electrons and nuclei/ons.

$$\nabla_\mu j_X^\mu = \partial_t n_X - \nabla \cdot \mathbf{j}_X + 3Hn_X = \mathcal{C}_X$$

$$\mathcal{C}_X = \frac{1}{2E_X} \int \prod_i \frac{d^3 p_i}{(2\pi^3) 2E_i} \delta^4(p_X + p_{A_1} + \dots - p_{B_1} - \dots) \\ \times \left\{ (1 \pm f_X)(1 \pm f_{A_1}) \dots f_{B_1} \dots |\mathcal{M}_{B_1 B_2 \dots \rightarrow X A_1 A_2 \dots}|^2 \right. \\ \left. - f_X f_{A_1} \dots (1 \pm f_{B_1}) \dots |\mathcal{M}_{X A_1 A_2 \dots \rightarrow B_1 B_2 \dots}|^2 \right\}$$

n_X : number density

\mathbf{j}_X : current density

j^μ : four-current

∇_μ : cov. derivative

H : Hubble rate

\mathcal{C}_X : collision term

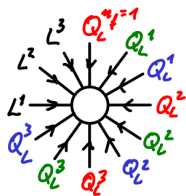
f_i : distribution fn.

- Agreement between BBN & CMB values for BAU huge success
→ aim to repeat this e.g. for baryogenesis or dark matter abundance

Situation in the Standard Model (SM)

Check Sakharov conditions:

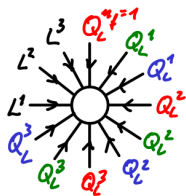
- 1 B violated due to anomalous $B + L$ violation via sphaleron processes.
- 2 CP violation in CKM matrix, C and P violation in weak interactions.
- 3 Deviation from equilibrium due to expansion of the Universe.



Situation in the Standard Model (SM)

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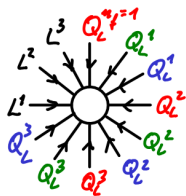
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However, conditions for baryogenesis not met *quantitatively*:

- CP rephasing invariant normalised to electroweak scale is tiny:

$$\text{Im} \left[\det[m_u m_u^\dagger, m_d m_d^\dagger] \right] \approx -2J m_t^4 m_b^4 m_c^2 m_s^2, \quad J \approx 3 \times 10^{-5},$$

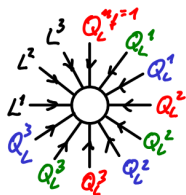
$$2J \frac{m_t^4 m_b^4 m_c^2 m_s^2}{T^{12}} \approx 3 \times 10^{-19} \text{ for } T = 100 \text{ GeV}.$$

- Deviation from equilibrium $H/\Gamma \sim (T^2/m_{\text{Pl}})/(g^4 T) = g^{-4} T/m_{\text{Pl}}$ is tiny unless T is very high ($m_{\text{Pl}} = 1.2 \times 10^{19} \text{ GeV}$).

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Baryogenesis requires physics beyond the Standard Model.

Baryogenesis Puzzle in New Physics Scenarios

- Leptogenesis (this talk)
- Electroweak baryogenesis – requires extension of the SM to provide strong first order phase transition, extra CP violation \rightarrow perhaps the best prospects for testability
- Decay of scalar condensates or Q-balls, *e.g.* from SUSY flat directions (Affleck-Dine)
- Connection with asymmetric dark matter
- Other paradigms less connected to specific models

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Hard to solve because

- we do not know whether we have all pieces,
- we do not know whether the pieces we have are part of the puzzle,
- some of the pieces probably not even exist.



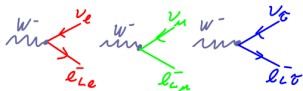
Non-Equilibrium and Neutrinos

Neutrinos Masses & Mixings

Mass Parameters for *Active* Neutrinos from Oscillation Experiments

$$\Delta m_{21}^2 = 7.50 \times 10^{-5} \text{eV}^2, \Delta m_{31}^2 = 2.457 \times 10^{-3} \text{eV}^2 \text{ (NH)}, \sum_i m_i < 0.23 \text{eV}$$

[Gonzalez-Garcia, Maltoni, Schwetz (2014)], upper bound from [Planck(2015)] @ 95% c.l.



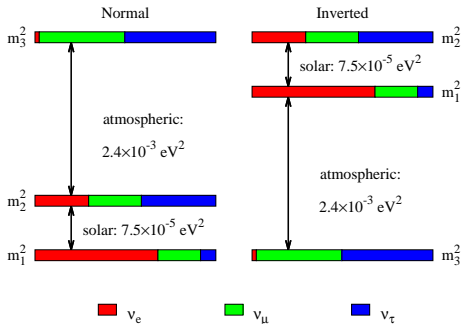
$$|\nu_n\rangle = \sum_a U_{ai}^* |\nu_a\rangle,$$

$$a = e, \mu, \tau; n = 1, 2, 3$$

$$\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} = U^\dagger m U^*$$

for Majorana neutrinos, for Dirac neutrinos just like CKM mechanism.

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.



- Why is the mass scale of neutrinos (several meV) much below that for other SM particles, e.g. electron (511 keV), or top quark (173 GeV)?

Non-Equilibrium and Neutrinos

Seesaw Mechanism

- Type I seesaw: Introduce n_N hypothetical right-handed neutrinos N (RHNs) with Majorana mass matrix M (can take it to be diagonal), couple these to Standard Model lepton $\ell = (\nu, e_L)$ and Higgs doublets ϕ , $\langle |\phi| \rangle = v = 174 \text{ GeV}$:

$$\mathcal{L}_{\text{SM}} \rightarrow \mathcal{L}_{\text{SM}} + \frac{1}{2} \bar{N}_i^c (i \not{\partial} - M_{ij}) N_j - Y_{ia}^* \bar{\ell}_a \phi^\dagger N_i - Y_{ia} \bar{N}_i \phi \ell_a; \quad a = e, \mu, \tau$$

- $(3 + n_N) \times (3 + n_N)$ matrix: $\frac{1}{2} (\bar{\nu} \quad \bar{N}^c) \underbrace{\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}}_{=: \mathcal{M}} \begin{pmatrix} \nu^c \\ N \end{pmatrix}, \quad m_D = Y^\dagger v$

- Assume $\|M\| \gg \|m\| \rightarrow$ block-diagonalization

$$\begin{pmatrix} m & 0 \\ 0 & M \end{pmatrix} = \tilde{U} \mathcal{M} \tilde{U}^\dagger, \quad \tilde{U} = \begin{pmatrix} 1 - \theta \theta^\dagger & \theta \\ -\theta^\dagger & 1 - \theta^\dagger \theta \end{pmatrix} + \mathcal{O}(\theta^3), \quad \theta = Y^\dagger v M^{-1}$$

\rightarrow Majorana mass matrix for *light* neutrinos: $m = v^2 Y^\dagger M^{-1} Y^*$, diagonalised by U , for heavy neutrinos $M_N = M + \frac{1}{2} (\theta^\dagger \theta M + M \theta^T \theta^*) + \mathcal{O}(\theta^3)$, diagonalised by U_N

- $\nu^{\text{light}} = U^\dagger ((\mathbb{1} - \frac{1}{2} \theta \theta^\dagger) \nu - \theta N^c)$, $\nu^{\text{heavy}} = U_N^\dagger ((\mathbb{1} - \frac{1}{2} \theta^T \theta^*) N - \theta^T \nu^c)$
- RHNs are their only antiparticles \rightarrow can decay either $N_i \rightarrow \ell \phi$ or $N_i \rightarrow \bar{\ell} \phi^\dagger$ and therefore violate lepton number L .

This talk

Comprehensive overview of leptogenesis in the type I seesaw mechanism throughout the parameter space, in particular for all mass scales M .

Non-Equilibrium Dynamics and Neutrinos

Out-of-Equilibrium Dynamics

Simplest Meaningful Network of Kinetic Equations Describing Leptogenesis:

$$\frac{dn_{N_i}}{dt} + 3Hn_{N_i} = \Gamma_i(n_{N_i} - n_{N_i}^{\text{eq}})$$

$$\Gamma_i = \frac{|Y_i|^2}{8\pi} M_i$$

$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon \Gamma_i(n_{N_i} - n_{N_i}^{\text{eq}}) - Wn_L$$

$$W = \frac{\Gamma_i}{4} \left(\frac{M}{T}\right)^{\frac{3}{2}} e^{-\frac{M}{T}}$$

n_{N_i} : number density of N_i ; n_L : lepton charge density; W : washout rate;
 $\varepsilon = (\Gamma_{N_i \rightarrow \ell H} - \Gamma_{N_i \rightarrow \bar{\ell} H^*}) / (\Gamma_{N_i \rightarrow \ell H} + \Gamma_{N_i \rightarrow \bar{\ell} H^*})$: decay asymmetry; T : temperature

→ Best compromise between large L violating rate[†] (1st S. condition) and large deviation from equilibrium (3rd S. condition):

$\Gamma \sim H$ for $T \sim M$ (i.e. at freezeout, when the RHNs become Maxwell suppressed).

$$\rightarrow Y^2 M / (8\pi) \sim T^2 / m_{\text{Pl}} \sim M^2 / m_{\text{Pl}} \stackrel{m \sim \frac{Y^2 v^2}{M}}{\Rightarrow} m \sim 8\pi v^2 / m_{\text{Pl}} \sim 0.1 \text{ meV}$$

Light Neutrino Mass Scale points to Role of RHNs in Baryogenesis

- Out-of-equilibrium property of the N independent of the mass scale.
- Tendency of being somewhat close to equilibrium, i.e. $\Gamma \gg H$ around $T \sim M \rightarrow$ strong washout. However, \exists a lot of parametric freedom.

[†]Asymmetry in lepton sector transferred to baryons by the sphaleron.

CP Violation & Quantum Interference

Example *leptogenesis*: Recall $\mathcal{L} \supset - \sum_{i=1,2} (Y_i \bar{N}_i \phi \ell + Y_i^* \bar{\ell} \phi^\dagger N_i)$

Consider first $N_1 \rightarrow \ell \phi$:

$$\begin{aligned}
 & \left| \begin{array}{c} \text{Tree diagram: } N_1 \rightarrow \ell \phi \text{ via } \gamma_1^* \\ \text{Loop diagram 1: } N_1 \rightarrow \ell \phi \text{ via } \gamma_1 \text{ and } \gamma_2^* \\ \text{Loop diagram 2: } N_1 \rightarrow \ell \phi \text{ via } \gamma_1 \text{ and } \gamma_2 \end{array} \right|^2 \\
 &= \left| Y_1^* |\mathcal{A}_{\text{tree}}| + Y_1 Y_2^{*2} |\mathcal{A}_{\text{loop}}| e^{i\varphi_{\text{loop}}} \right|_{\text{x-term}}^2 \supset (Y_1^{*2} Y_2^2 e^{-i\varphi_{\text{loop}}} + Y_1^2 Y_2^{*2} e^{i\varphi_{\text{loop}}}) |\mathcal{A}_{\text{tree}} \mathcal{A}_{\text{loop}}|
 \end{aligned}$$

Rate for CP conjugate process $N_1 \rightarrow \bar{\ell} \phi^\dagger$:

$$\left| Y_1 |\mathcal{A}_{\text{tree}}| + Y_1^* Y_2^2 |\mathcal{A}_{\text{loop}}| e^{i\varphi_{\text{loop}}} \right|_{\text{x-term}}^2 \supset (Y_1^2 Y_2^{*2} e^{-i\varphi_{\text{loop}}} + Y_1^{*2} Y_2^2 e^{i\varphi_{\text{loop}}}) |\mathcal{A}_{\text{tree}} \mathcal{A}_{\text{loop}}|$$

Difference: $4\text{Im}[e^{i\varphi_{\text{loop}}}] \text{Im}[Y_1^{*2} Y_2^2] |\mathcal{A}_{\text{tree}} \mathcal{A}_{\text{loop}}|$

CP Violation & Quantum Interference

Example *leptogenesis*: Recall $\mathcal{L} \supset - \sum_{i=1,2} (Y_i \bar{N}_i \phi l + Y_i^* \bar{l} \phi^\dagger N_i)$

Consider first $N_1 \rightarrow l \phi$:

$$\left| \begin{array}{c} \text{Tree diagram: } N_1 \rightarrow l \phi \text{ via } Y_1^* \\ \text{Loop diagram: } N_1 \rightarrow l \phi \text{ via } Y_1 \text{ and } Y_2^* \text{ (with } N_2 \text{ loop)} \\ \text{Loop diagram: } N_1 \rightarrow l \phi \text{ via } Y_1 \text{ and } Y_2 \text{ (with } N_2 \text{ loop)} \end{array} \right|^2 \\
 = \left| Y_1^* |\mathcal{A}_{\text{tree}}| + Y_1 Y_2^{*2} |\mathcal{A}_{\text{loop}}| e^{i\varphi_{\text{loop}}} \right|_{\text{x-term}}^2 \supset (Y_1^{*2} Y_2^2 e^{-i\varphi_{\text{loop}}} + Y_1^2 Y_2^{*2} e^{i\varphi_{\text{loop}}}) |\mathcal{A}_{\text{tree}} \mathcal{A}_{\text{loop}}|$$

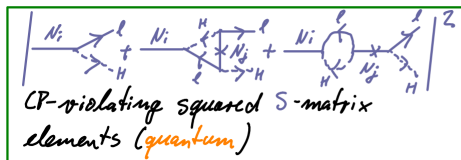
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Difference: $4\text{Im}[e^{i\varphi_{\text{loop}}}] \text{Im}[Y_1^{*2} Y_2^2] |\mathcal{A}_{\text{tree}} \mathcal{A}_{\text{loop}}|$

- $\text{Im}[e^{i\varphi_{\text{loop}}}]$ comes precisely from contributions where the anti-Higgs and the anti-lepton in the loops propagate on shell (*i.e.* they fulfill the energy-momentum relation of real particles). \rightarrow **on-shell cuts**
- *Quantum interference* between the “direct” path and the case where antiparticles are produced, then rescattered into particles.

Leptogenesis – Standard Approach



$$L[\psi] = e[\psi]$$

Boltzmann equation (classical)

decay asymmetry:

$$\epsilon_{N_i} = \frac{\Gamma_{N_i \rightarrow \ell H} - \Gamma_{N_i \rightarrow \bar{\ell} H^*}}{\Gamma_{N_i \rightarrow \ell H} + \Gamma_{N_i \rightarrow \bar{\ell} H^*}}$$

Lepton Asymmetry

[Fukugita, Yanagida (1986);
Covi, Roulet, Vissani (1996)]

“Wave Function” & “Vertex” Contributions:

$$\epsilon_{N_i}^{\text{wf}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_i M_j}{M_i^2 - M_j^2} \frac{\text{Im}[(YY^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}}$$

$$\epsilon_{N_i}^{\text{vertex}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_j}{M_i} \left[1 - \left(1 + \frac{M_j^2}{M_i^2} \right) \log \left(1 + \frac{M_j^2}{M_i^2} \right) \right] \frac{\text{Im}[(YY^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}}$$

- Resonant enhancement for $M_i \rightarrow M_j$.
- Understand situation when $|M_i^2 - M_j^2| \gg M_{i,j} \Gamma_{N_{i,j}}$ does *not* hold.

Kinetic Equations – Strong Washout, Unflavoured

Reparametrization and Nonrelativistic Approximations

Kinetic Equations

$$\frac{dn_{N1}}{dt} + 3Hn_{N1} = \Gamma_1(n_{N1} - n_{N1}^{\text{eq}})$$

$$\Gamma_1 = \frac{|Y_1|^2}{8\pi} M_1$$

$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon \Gamma_1(n_{N1} - n_{N1}^{\text{eq}}) - Wn_L$$

$$W = \frac{\Gamma_1}{4} \left(\frac{M}{T}\right)^{\frac{3}{2}} e^{-\frac{M}{T}}$$

- Normalize number densities to entropy density $s = g_* \frac{2\pi^2 T^3}{45} \propto 1/V \rightarrow$ Hubble dilution term drops. (g_* : number of relativistic dofs., $g_* = 106.75$ in the SM.)

$$Y_{N1} = n_{N1}/s, Y_L = n_L/s.$$

- Introduce dimensionless parameter $z = M_1/T \rightarrow d/dt = z/H(z=1) d/dz$.

$$H = \frac{\dot{a}}{a} = \frac{\dot{z}}{z} \Leftrightarrow dz = zH dt = \frac{1}{z} H(z=1) dt$$

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Reparametrization and Nonrelativistic Approximations

Kinetic Equations

$$\frac{dY_{N1}}{dz} = \frac{z\Gamma_1}{H(z=1)} (Y_{N1} - Y_{N1}^{\text{eq}})$$

$$\Gamma_1 = \frac{|Y_1|^2}{8\pi} M_1$$

$$\frac{dY_L}{dz} = \varepsilon \frac{z\Gamma_1}{H(z=1)} (Y_{N1} - Y_{N1}^{\text{eq}}) - \frac{zW}{H(z=1)} Y_L$$

$$W = \frac{\Gamma_1}{4} z^{\frac{3}{2}} e^{-z}$$

- Normalize number densities to entropy density $s = g_* \frac{2\pi^2 T^3}{45} \propto 1/V \rightarrow$ Hubble dilution term drops. (g_* : number of relativistic dofs., $g_* = 106.75$ in the SM.)
 $Y_{N1} = n_{N1}/s$, $Y_L = n_L/s$.

- Introduce dimensionless parameter $z = M_1/T \rightarrow d/dt = z/H(z=1) d/dz$.

- Strong washout ($\Gamma_1 \gg H$) \rightarrow freezeout $W \sim H$ occurs when $M_1 \gg T$
 \rightarrow non-relativistic approximations applicable:

$$n_{N1}^{\text{eq}} = \int \frac{d^3p}{(2\pi)^3} 2e^{-\sqrt{p^2 + M_1^2}/T} = \frac{T^3}{\pi^2} z^2 K_2(z) \approx 2^{-\frac{1}{2}} \pi^{-\frac{3}{2}} z^{\frac{3}{2}} e^{-z} T^3$$

- Can further approximate $\frac{z\Gamma_1}{H(z=1)} (Y_{N1} - Y_{N1}^{\text{eq}}) = \frac{dY_{N1}^{\text{eq}}}{dz}$.

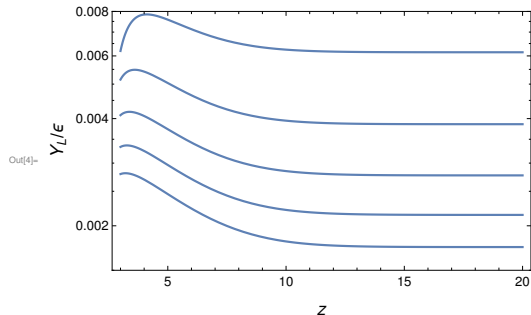
Result depends only on

- ε (trivially),
- washout strength $K := \Gamma_1/H(z=1)$ (NB $K \propto Y^2/M \propto m$ up to flavour mixing).

Solutions for the Freeze-Out Asymmetry

Varying Washout Strength

```
YNeq[z_] = 2^(-1/2) * Pi^(-3/2) * z^(3/2) * Exp[-z];  $\epsilon = 1$ ;  
S = Table[NDSolve[  
  {YL'[z] ==  $\epsilon$  * D[YNeq[z], z] - Kws / 4 * z^(5/2) * Exp[-z] * YL[z], YL[0.1] == 0},  
  YL, {z, 0.5, 100}], {Kws, {10, 15, 20, 25, 30}}];
```



Analytic Approximation:

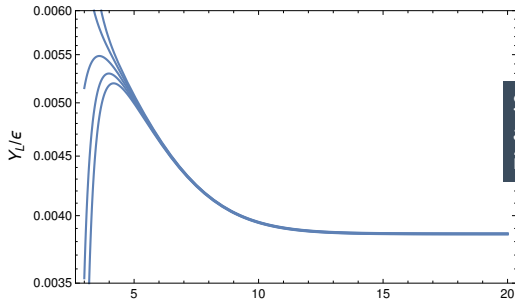
$$\eta_B \simeq 0.96 \times 10^{-2} \epsilon \kappa,$$

where $\kappa = \frac{2}{z_B(K)K}$, $z_B(K) = 1 + \frac{1}{2} \log \left(1 + \frac{\pi K^2}{1024} \left[\log \frac{3125\pi K^2}{1024} \right]^5 \right)$ [Buchmüller, Di Bari, Plümacher (2004)]

Solutions for the Freeze-Out Asymmetry

Varying Initial Conditions

```
YNeq[z_] = 2^(-1/2) * Pi^(-3/2) * z^(3/2) * Exp[-z]; e = 1;  
With[{Kws = 15},  
  S = Table[NDSolve[  
    {YL'[z] == e * D[YNeq[z], z] - Kws / 4 * z^(5/2) * Exp[-z] * YL[z], YL[0.1] == IC},  
    YL, {z, 0.5, 100}], {IC, {-1, -0.5, 0, 0.5, 1}}];  
];
```

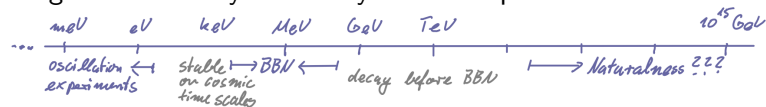


Strong washout \rightarrow freeze-out asymmetry approximately independent from initial asymmetry.

NB The evolution of the asymmetries in the crossover between relativistic and nonrelativistic regimes for $z \lesssim 1$ has not yet been studied in all details but is irrelevant for the freeze-out asymmetry in the strong washout regime.

Mass Scale of RHNs

- Unless there is resonant enhancement, leptogenesis in the strong washout regime requires $M \gtrsim 10^{10}$ GeV because $\varepsilon \sim Y^2$ [Davidson, Ibarra (2002)].
- Leaving leptogenesis aside, RHNs are allowed throughout the mass range because they can always be decoupled.



- Barring strong decoupling, BBN yields strong constraints for masses $\lesssim 100$ MeV, oscillation experiments for masses \lesssim eV.
- In absence of SUSY or other cancellation mechanism, the RHNs will contribute to the Higgs mass. In order to avoid destabilization, require
$$\Delta m_\phi^2 = \sum_i \frac{[Y^\dagger Y]_{ii}}{4\pi^2} M_i^2 \log \frac{M_i}{\mu} \sim \frac{m M^3}{4\pi^2 v^2} \log \frac{M}{\mu} \lesssim m_\phi^2 \rightarrow M_N \lesssim 10^7 \text{ GeV}$$
 [Vissani (1997)]
- In presence of SUSY, there is a slight tension of the high temperatures with gravitino production that can either lead to overclosure (*i.e.* too much dark matter, if stable) or a conflict with BBN (if unstable).

Flavoured Leptogenesis

- Neutrino Yukawa couplings: $Y_{ia}\bar{N}_i\ell_a\phi$
- When insensitive to lepton flavour, can $\ell_a \rightarrow U_{ab}\ell_b$, $Y_{ia} \rightarrow Y_{ib}U_{ba}^\dagger$ such that
$$Y = \begin{pmatrix} Y_{11} & 0 & 0 \\ Y_{21} & Y_{22} & 0 \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \rightarrow N_1 \text{ decays only produce } (U\ell)_1, \text{ which is a linear combination of } \ell_e, \ell_\mu, \ell_\tau.$$

→ Back to flavour basis – quantum correlations of charge densities:

$$\begin{pmatrix} q_{ee} & q_{e\mu} & q_{e\tau} \\ q_{\mu e} & q_{\mu\mu} & q_{\mu\tau} \\ q_{\tau e} & q_{\tau\mu} & q_{\tau\tau} \end{pmatrix}$$

- When $H \sim \frac{T^2}{m_{\text{Pl}}} \lesssim \Gamma_{\tau,\mu} \sim h_{\tau,\mu}^2 T$, i.e. $T \lesssim 10^{12} \text{GeV}$ (for τ) or $T \lesssim 10^9 \text{GeV}$ (for μ), the flavour coherence is destroyed by the SM lepton Yukawa couplings $h_{\tau,\mu}$.

→ $\begin{pmatrix} q_{ee} & 0 & 0 \\ 0 & q_{\mu\mu} & 0 \\ 0 & 0 & q_{\tau\tau} \end{pmatrix}$ → The resulting linear combination is only partly aligned with the one that is produced in the decays of N_1 . → $\mathcal{O}(1)$ suppression of washout.

[Abada, Davidson, Josse-Michaux, Losada, Riotto (2006); Nardi, Nir, Roulet, Racker (2006)]

- For initially lepton flavour violating contributions with vanishing $L = L_e + L_\mu + L_\tau$: washout of different flavours → lepton number violating asymmetry at freeze out that receives contribution from the PMNS phase δ – but from other phases as well. [Pascoli, Petcov, Riotto (2006)]
- Even for $T \gtrsim 10^{12} \text{GeV}$, the asymmetries from the decays of the heavier RHNs correspond to linear combinations of flavour that are in general misaligned with the one washed out by the lightest of the RHNs [Di Bari (2005)]. → Richer phenomenology, less predictivity.

Flavoured Leptogenesis

Partial Flavour (De)Coherence

- Kinetic equations for number densities & flavour correlations of (anti-)leptons δn_ℓ^\pm :

$$\frac{\partial \delta n_{\ell ab}^\pm}{\partial t} = \underbrace{\pm S_{ab}}_{\substack{CP\text{-} \\ \text{violating} \\ \text{source}}} \mp \underbrace{i \Delta \omega_{ab}^{\text{th}} \delta n_{\ell ab}^\pm}_{\substack{\text{oscillations} \\ \text{induced by} \\ \text{thermal masses}}} - \underbrace{[W, \delta n_\ell^\pm]_{ab}}_{\text{washout}} - \underbrace{\gamma^{\text{bl}} (\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-)}_{\substack{\text{flavour-blind} \\ \text{pair-creation \&} \\ \text{annihilation}}} - \underbrace{\Gamma_{ab}^{\text{fl}}}_{\substack{\text{flavour-} \\ \text{sensitive} \\ \text{damping}}}$$

$$\Gamma^{\text{fl}} \propto h_\tau^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \delta n_\ell^\pm \right] - 2h_\tau^2 \delta n_{\text{R}}^\pm \rightarrow \text{directly damps off-diagonal correlations}$$

- thermal masses like to induce flavour oscillations in opposite directions*

oscillations overdamped

like to keep these aligned

$\delta n_{\ell ab}^+$

$\delta n_{\ell ab}^-$

$W^{\text{or } Z}, B$

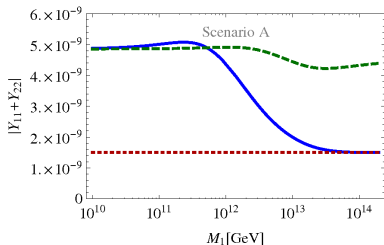
$$\Gamma^{\text{direct}} \sim (h_{aa}^2 + h_{bb}^2) T$$

$$\Gamma^{\text{overdamped}} \sim \frac{(h_{aa}^2 - h_{bb}^2)^2}{g_2^4} T$$

where $a, b = e, \mu, \tau$

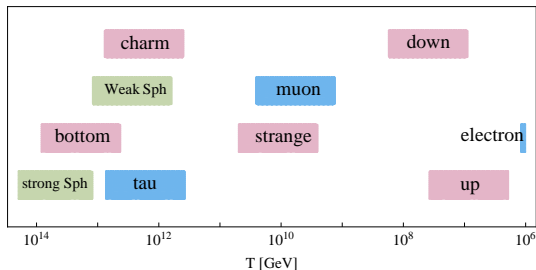
- Can interpolate between fully flavoured and unflavoured regimes.

[Beneke, BG, Herranen, Fidler, Schwaller (2010)]



Spectator Effects

- Rate for SM processes mediated by Yukawa coupling h and gauge coupling g in the relativistic limit
 $\Gamma_X \sim Tg^2h^2 \log g$, Hubble rate $H \sim T^2$.
 \rightarrow As the Universe cools, more and more SM processes come into equilibrium.



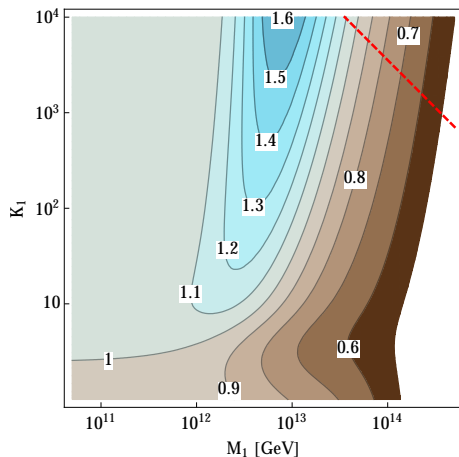
Temperature bands $T_X \leq T \leq 20T_X$ where T_X is the equilibration temperature $\Gamma_X(T_X) = H(T_X)$.

- Besides Yukawa mediated processes, there are also the chiral anomalies (strong and weak sphalerons).
- Asymmetries are transferred partly to *all* SM degrees of freedom \rightarrow suppression of washout. [Barbieri, Creminelli, Strumia, Tetradis (2000)]
- Rephrase kinetic equations in terms of conserved SM charges, $\Delta_a = B/3 - L_a$.

$$\frac{dY_{\Delta_a}}{dz} = \varepsilon_a \frac{z\Gamma_1}{H(z=1)} (Y_{N1} - Y_{N1}^{\text{eq}}) - \frac{zW}{H(z=1)} (Y_{La} + \frac{1}{2}Y_H)$$
 where $Y_{La} = A_{ab}\Delta_b$ and $Y_H = C_a\Delta_a \rightarrow$ another $\mathcal{O}(1)$ effect.
E.g. for reaction $X + Y \leftrightarrow Z$, impose $\mu_X + \mu_Y - \mu_Z = 0$. \rightarrow linear maps A_{ab}, C_a .
 This procedure holds for *fully* equilibrated spectator fields.

Partially Equilibrated Spectators

- Fully equilibrated spectators are not a realistic assumption in most regions of parameter space.
- At early times, the deviation of the RHNs from equilibrium is large \rightarrow large asymmetries present.
- These asymmetries are transferred to the spectators.
- For partially equilibrated spectators, these asymmetries are also partially protected from washout.
- Moderate enhancement effect for thermal initial conditions of the RHNs. Vanishing initial conditions (where the effect may be larger) are yet to be explored.



Freeze out asymmetry for partially equilibrated τ -Yukawa coupling h_τ divided by the limit $\tau \rightarrow \infty$. [BG, Schwaller (2014)]




Need for Going beyond the Standard Approach

Real Intermediate State (RIS) Problem

- Interference of tree & loop amplitudes $\rightarrow CP$ violation.

$$\left| \begin{array}{c} N_1 \text{---} \phi \\ \diagdown \\ \ell \\ \diagup \\ N_1 \end{array} + \begin{array}{c} \phi \\ | \\ N_1 \text{---} \phi \\ \diagdown \\ \ell \\ \diagup \\ N_2 \end{array} + \begin{array}{c} \phi \\ | \\ N_1 \text{---} \phi \\ \diagdown \\ \ell \\ \diagup \\ N_2 \end{array} \right|^2 \quad (*)$$

- CP violating contributions from discontinuities
 \rightarrow loop momenta where **cut** particles are on shell.

- Is  an extra process or is it already accounted for by  and  ?

- Including (*) only $\rightarrow CP$ asymmetry is already generated in equilibrium. \Downarrow CPT \Downarrow theorem.

Need for Going beyond the Standard Approach

(Inverse) Decays & CP Asymmetry

- Consider the squared matrix elements, ε being the decay asymmetry.

$$\begin{array}{cc}
 \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} & |\mathcal{M}_{N \rightarrow l\phi}|^2 \sim 1 + \varepsilon & \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} & |\mathcal{M}_{N \rightarrow \bar{l}\phi^*}|^2 \sim 1 - \varepsilon \\
 \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} & \xrightarrow{CPT} & \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} & \xrightarrow{CPT} \\
 \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} & |\mathcal{M}_{\bar{l}\phi^* \rightarrow N}|^2 \sim 1 + \varepsilon & \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \end{array} & |\mathcal{M}_{l\phi \rightarrow N}|^2 \sim 1 - \varepsilon
 \end{array}$$

- Naive multiplication* suggests that an asymmetry is generated already in equilibrium: $\Gamma_{\bar{l}\phi^* \rightarrow l\phi} \sim 1 + 2\varepsilon$, $\Gamma_{l\phi \rightarrow \bar{l}\phi^*} \sim 1 - 2\varepsilon$

- Ad hoc fix: Subtract real intermediate states (RIS) from [Kolb, Wolfram (1980)].



Better Way Out

- Compute the real time (time dependent perturbation theory), non-equilibrium (statistical physics) evolution of the quantum field theory states of interest.
- Since charges and currents are e.g. given by $j^\mu(x) = \langle \bar{\psi}(x) \gamma^\mu \psi(x) \rangle$, calculate in particular evolution of two-point functions.

*Do not try this at home: The unstable N are not asymptotic states of a unitary S matrix \rightarrow conflict with the CPT theorem.

Closed-Time-Path Approach

- In-in generating functional (in contrast to “in-out” for S matrix elements): [Schwinger (1961); Keldysh (1965); Calzetta & Hu (1988)]

$$\begin{aligned}
 Z[J_+, J_-] &= \int \mathcal{D}\phi(\tau) \mathcal{D}\phi_{\text{in}}^- \mathcal{D}\phi_{\text{in}}^+ \langle \phi_{\text{in}}^- | \phi(\tau) \rangle \langle \phi(\tau) | \phi_{\text{in}}^+ \rangle \langle \phi_{\text{in}}^- | \rho | \phi_{\text{in}}^+ \rangle \\
 &= \int \mathcal{D}\phi^- \mathcal{D}\phi^+ e^{i \int d^4x \{ \mathcal{L}(\phi^+) - \mathcal{L}(\phi^-) + J_+ \phi^+ - J_- \phi^- \}}
 \end{aligned}$$

- The Closed Time Path:



- Path-ordered Green functions:

$$i\Delta^{ab}(u, v) = -\frac{\delta^2}{\delta J_a(u) \delta J_b(v)} \log Z[J_+, J_-] \Big|_{J_{\pm}=0} = i \langle \mathcal{C}[\phi^a(u) \phi^b(v)] \rangle$$

$$\text{e.g. } j^\mu(x) = \text{tr}[\gamma^\mu \langle \mathcal{C}[\psi^-(x_1) \bar{\psi}^+(x_2)] \rangle]_{x_1=x_2=x}$$

Wigner Transformation of Two-Point Functions (Green Function or Self Energy)

$$A(k, x) = \int d^4r e^{ik \cdot r} A(x + r/2, x - r/2) \rightarrow \sim \text{distribution function}$$

x : average coordinate – macroscopic evolution

$r \rightarrow k$: relative coordinate – microscopic (quantum)

Schwinger-Dyson & Kadanoff-Baym Equations

Feynman Rules

- Vertices either + or -.
- Connect vertices $a = \pm$ and $b = \pm$ with $i\Delta^{ab}$.
- Factor -1 for each - vertex.

- Schwinger-Dyson equations \rightarrow

- These describe in principle the full time evolution. However, truncations, e.g. perturbation theory, are needed.

- The $\langle, \rangle \equiv +-, -+$ parts of the Schwinger-Dyson equations are the celebrated Kadanoff-Baym equation:

$$(-\partial^2 - m^2)\Delta^{\langle, \rangle} - \Pi^H \odot \Delta^{\langle, \rangle} - \Pi^{\langle, \rangle} \Delta^H = \underbrace{\frac{1}{2} (\Pi^{\rangle} \odot \Delta^{\langle} - \Pi^{\langle} \odot \Delta^{\rangle})}_{\text{collision term}}$$

- Remaining linear combination gives pole-mass equation:

$$(-\partial^2 - m^2)i\Delta^{R,A} - \Pi^{R,A} \odot i\Delta^{R,A} = i\delta^4, \quad R, A: \text{retarded, advanced,}$$

$$\Pi^H, \Delta^H = \text{Re}[\Pi^R, \Delta^R]$$

- First principle derivation of Boltzmann-like *kinetic equations*.

[Keldysh (1965); Calzetta & Hu (1988)]

$$i\Delta^{ab} = i\Delta^{(ab)} + cd i\Delta^{(c)ac} \odot \mathbb{T}^{cd} \odot \Delta^{db}$$

$$\text{====} = \text{---} + \text{---} \textcircled{\Pi} \text{---}$$

$$A(x, w) \odot B(w, y) = \int d^4w A(x, w) B(w, y)$$

Leptogenesis in the CTP Approach

■ Schwinger-Dyson equations relevant for leptogenesis:

[Beneke, BG, Herranen, Schwaller (2010)]

$$\frac{d}{dt} \underbrace{\int \frac{dk^0}{2\pi} \text{tr} [\gamma^0 \text{---}]}_{= \int d_e(\vec{k}^0) - \int d_e(\vec{k}^0)} = \int \frac{dk^0}{2\pi} \text{tr} \left[\text{---} \overset{N}{\text{---}} \overset{L}{\text{---}} + \text{---} \right]_{\begin{smallmatrix} >< \\ <> \end{smallmatrix}}$$

$$\frac{d}{dt} \underbrace{\int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} [\gamma^0 \text{---}]}_{= 4 f_\nu(\vec{k}^0)} = \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[\text{---} \overset{L}{\text{---}} \right]_{\begin{smallmatrix} >< \\ <> \end{smallmatrix}} + \dots$$

■ Truncation that yields leading asymmetry in non-degenerate regime

$M_i \Gamma_i, M_j \Gamma_j \ll |M_i^2 - M_j^2|$:

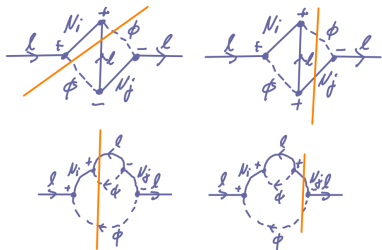
$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{tr} [\gamma^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{tr} \left[\text{---} \overset{N}{\text{---}} + \text{---} \overset{L}{\text{---}} + \text{---} \right]_{\begin{smallmatrix} >< \\ <> \end{smallmatrix}}$$

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} [\gamma^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[\text{---} \overset{L}{\text{---}} \right]_{\begin{smallmatrix} >< \\ <> \end{smallmatrix}}$$

■ Non-minimal truncations \rightarrow e.g. systematic inclusion of thermal corrections.

Unitarity Restored (without RIS)

- CTP approach readily yields *inclusive* rates for the creation of the charge asymmetry.
- No need to separately remove unwanted contribution. No hand waving explanation needed why these are unphysical.



Note: Statistical Factors now on External and Internal Lines

$$\blacksquare \text{ Asymmetry} \propto [1 - f_\ell(\mathbf{p}) + f_\phi(\mathbf{k})] \times [1 - f_\ell(\mathbf{p}') + f_\phi(\mathbf{k}')].$$

Resonant Leptogenesis

Regulator for Decay Asymmetry

Consider the *mass-degenerate* regime $M_1 \rightarrow M_2$ and $M_{1,2} \gtrsim T$ (strong washout).

Decay Asymmetry

- Wave-function contribution:

$$\varepsilon_{Ni}^{\text{wf}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_i M_j (M_i^2 + M_j^2)}{(M_i^2 - M_j^2)^2 + R_j} \frac{\text{Im}[(YY^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}}$$

- In the degenerate limit, this will dominate over the vertex contribution.
- Proposed forms for regulator R ($\bar{M} = (M_i + M_j)/2$):

- $R_j = \frac{\bar{M}^4}{64\pi^2} (YY^\dagger)_{jj}^2 = \bar{M}^2 \Gamma_j^2$ [Pilaftsis (1997); Pilaftsis, Underwood (2003)]

- $R_j = \frac{\bar{M}^4}{64\pi^2} ([YY^\dagger]_{ii} - [YY^\dagger]_{jj})^2$ [Anisimov, Broncano, Plümacher (2005)]

- $R_j = \frac{\bar{M}^4}{64\pi^2} ([YY^\dagger]_{ii} + [YY^\dagger]_{jj})^2$ [Garny, Hohenegger, Kartavtsev (2011)]

Problem: When $\bar{M}\Gamma \ll |M_i^2 - M_j^2| = \Delta M^2$ not satisfied, cannot expand the RHN propagator as

$$= N + \dots$$

Resonant Leptogenesis

Back to Schwinger-Dyson form

$$\frac{d}{dt} \underbrace{\int \frac{d\vec{k}^0}{2\pi} \text{tr} [\not{y}^0 =]}_{= \not{d}_e(\vec{k}^0) - \not{f}_e(\vec{k}^0)} = \int \frac{d\vec{k}^0}{2\pi} \text{tr} \left[\text{diagram 1} + \text{diagram 2} \right]_{><}^{<>}$$

$$\frac{d}{dt} \underbrace{\int \frac{d\vec{k}^0}{2\pi} \text{sign} k^0 \text{tr} [\not{y}^0 =]}_{= \not{d}_\nu(\vec{k}^0)} = \int \frac{d\vec{k}^0}{2\pi} \text{sign} k^0 \text{tr} \left[\text{diagram 3} \right]_{><}^{<>} + \dots$$

Evolution for Matrix-Valued RHN Distributions δf_{Nh}

$$\delta f'_{Nh} + \frac{a^2(\eta)}{2k^0} i[M^2, \delta f_{Nh}] + f_N^{\text{eq}'} = -2 \left\{ \text{Re}[Y^* Y^t] \frac{k \cdot \hat{\Sigma}_N^A}{k^0} - i h \text{Im}[Y^* Y^t] \frac{\vec{k} \cdot \hat{\Sigma}_N^A}{k^0}, \delta f_{Nh} \right\}$$

$\hat{\Sigma}_N^A$: spectral (cut part) self energy; $a(\eta), \eta$: scale factor and conformal time; $\iota \equiv d/d\eta$;
 h : helicity.

- $i[M^2, \delta f_{Nh}]_{ij} = i(M_i^2 - M_j^2) \delta f_{Nhij}$ for diagonal $M^2 \rightarrow$ RHN “flavour” oscillations
- Off-diagonal entries correspond to interference between the different N_i that give rise to CP violation.

- Evolution equations are well behaved for $\Delta M^2 \rightarrow 0$. Solutions δf_{Nh} enter into the resummed RHN propagators. [BG, Herranen (2010)]

Resonant Leptogenesis in the Strong Washout Regime

Quasistatic Approximation

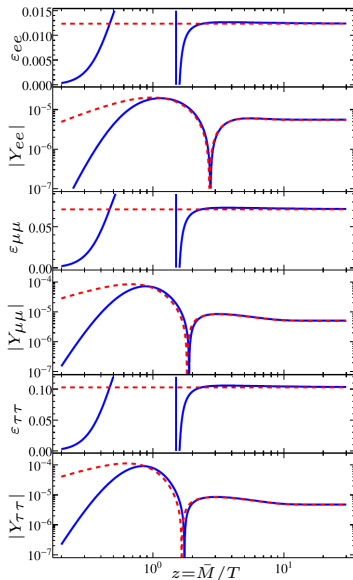
- Neglect derivative term provided the eigenvalues (that originate from the mass *and* the damping terms) in the kinetic equation for are larger than the Hubble rate $H \rightarrow$ obtain linear system of equations for δf_{Nh} . [BG, Gautier, Klaric (2014);

Iso, Shimada (2014)].

- Regulator:

$$R = \frac{\bar{M}^4}{64\pi^2} \frac{([YY^\dagger]_{11} + [YY^\dagger]_{22})^2}{[YY^\dagger]_{11}[YY^\dagger]_{22}} \times ((\text{Im}[YY^\dagger]_{12})^2 + \det YY^\dagger)$$

- Applies in strong washout regime, *i.e.* a large portion of parameter space.



blue: full result; red: result using effective decay asymmetry ε ; $Y_{aa} = n_{La a}/s$. Here, $\bar{M}\Gamma \gg \Delta M^2$.

Leptogenesis from Oscillations

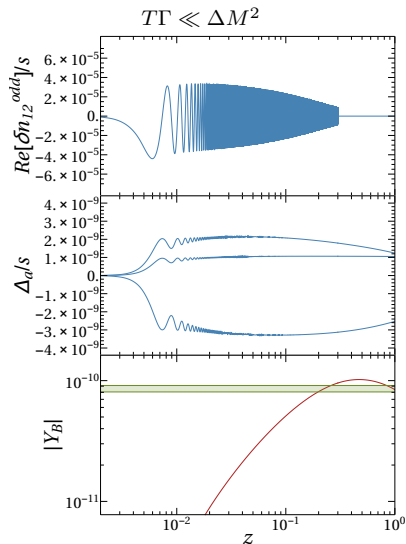
- RHNs perform first oscillation at temperature $T_{\text{osc}} \sim (|M_i^2 - M_j^2| m_{\text{Pl}})^{1/3}$
 - Off-diagonal correlations for the RHNs
 - Sizeable asymmetries generated around T_{osc} (see plots on next slide)
- Since $T_{\text{osc}} \gg M_i$, the RHNs are typically relativistic and thermal effects are of leading importance → calls for CTP techniques.
- Typically, “early asymmetries” are washed out by the time $T \sim M$. Only asymmetry produced around $T \sim M$ survives (strong washout).

Loopholes to Preserve Early Asymmetries

- GeV-scale RHNs (→ $T_{\text{osc}} \sim 10^5$ GeV) typically do not equilibrate prior to the electroweak phase transition where B settles to final value (sphaleron freezeout).
[Akhmedov, Rubakov, Smirnov (1998); Asaka, Shaposhnikov (2005); BG, Drewes (2012)].
 - Early asymmetry preserved until baryon number freezes in.
 - One individual active flavour (typically (ν_e, e_L)) is only weakly washed out, such that early asymmetries survive [BG (2014)].
-
- Computations are numerically challenging and analytic estimates rough because of oscillation scales, damping scales and levels of mass degeneracy that can be very different through parameter space.

Leptogenesis from Oscillation – Dynamics

Generation of the BAU

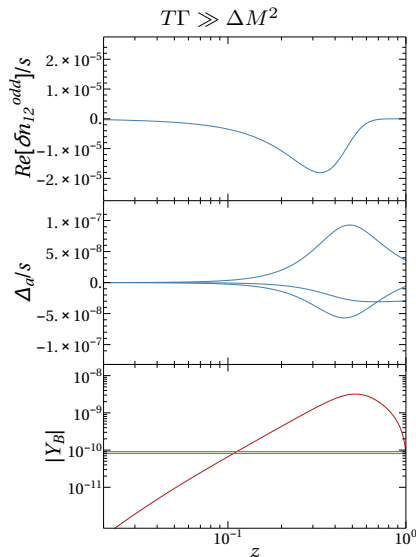
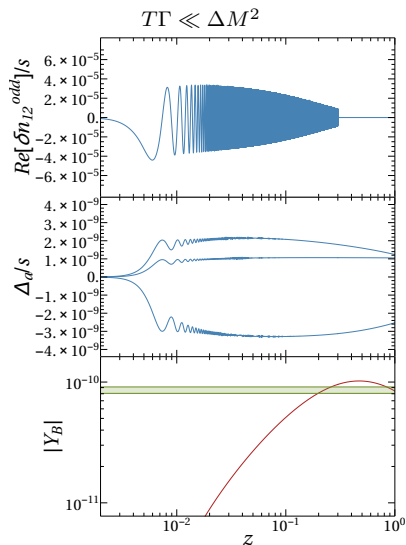


- RHNs perform first oscillation at $T_{\text{osc}} \sim (|M_i^2 - M_j^2| m_{\text{Pl}})^{1/3}$.
- Off-diagonal correlations lead to CP violating source for lepton flavour asymmetries (*purely flavoured*).
- Contributions from subsequent oscillations average out.
- Transfer of asymmetries into helicity asymmetries of RHNs leads to $Y_L \neq 0$.
- Asymmetry frozen in at T_{EW} , where sphalerons are quenched by the developing Higgs vev.

$z = T_{\text{EW}}/T$; Δ_a : lepton asymmetry in flavour $a = e, \mu, \tau$; δn : number density of RHNs; Y_B : entropy-normalized baryon asymmetry. [with Marco Drewes, Dario Gueter, Juraj Klarić (in preparation)]

Leptogenesis from Oscillation – Dynamics

Oscillatory vs Overdamped Regime



$z = T_{\text{EW}}/T$; Δ_a : lepton asymmetry in flavour $a = e, \mu, \tau$; δn : number density of RHNs;
 Y_B : entropy-normalized baryon asymmetry. [with Marco Drewes, Dario Gueter, Juraj Klarić (in preparation)]

Leptogenesis from Oscillation of GeV-scale RHNs

Asymmetries

- Source term for flavoured early asymmetries around T_{osc} :

$$S_{ab} = \sum_{\substack{c,i,j \\ i \neq j}} \frac{32i}{M_{ii}^2 - M_{jj}^2} \int \frac{d^3k}{(2\pi)^3 2\sqrt{\mathbf{k}^2 + M_{ii}^2}}$$

$$\times \left\{ (Y_{ai}^\dagger Y_{ic} Y_{cj}^\dagger Y_{jb}) \left[(M_{ii}^2 + 2\mathbf{k}^2) (\hat{\Sigma}_N^{A02} + \hat{\Sigma}_N^{Ai2}) - 4|\mathbf{k}| \sqrt{\mathbf{k}^2 + M_{ii}^2} \hat{\Sigma}_N^{A0} \hat{k}^i \hat{\Sigma}_N^{Ai} \right] \right.$$

$$\left. + (Y_{ai}^\dagger Y_{ic}^* Y_{cj}^\dagger Y_{jb}) M_{ii} M_{jj} \hat{\Sigma}_{N\mu}^A \hat{\Sigma}_N^{A\mu} \right\} \times \delta f_{N_{hii}}(\mathbf{k}).$$

■ $\hat{\Sigma}_N^A =$  [Besak, Bödeker (2012); BG, Glowna, Schwaller (2013)]

RHNs relativistic $\rightarrow \hat{\Sigma}_N^A$ dominated by thermal effects

- Lepton number violating contribution $\sim M^2/\Delta M^2$ (Majorana mass insertion) requires $\Delta M^2/M^2 \rightarrow 0$ for resonant enhancement.
- Lepton flavour violating (lepton number conserving) contribution $\sim T^2/\Delta M^2 \rightarrow$ large enhancement for $\Delta M^2 \ll T^2 \rightarrow$ no/less pronounced mass degeneracy needed.
- Leptogenesis is viable with non-degenerate (in mass) RHNs of the GeV scale [Drewes, BG (2012)] and for masses $\gtrsim 5 \times 10^3$ GeV [BG (2014)].
- Enhanced early production of RHNs is favourable for leptogenesis. This may happen when extra degrees of freedom in scenarios beyond the Standard Model can be radiated in the scattering processes at high temperatures.

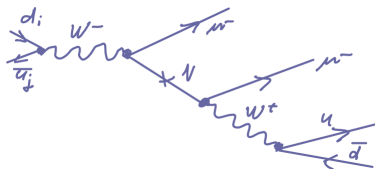
Direct & Indirect Searches for GeV-scale RHNs

Direct searches:

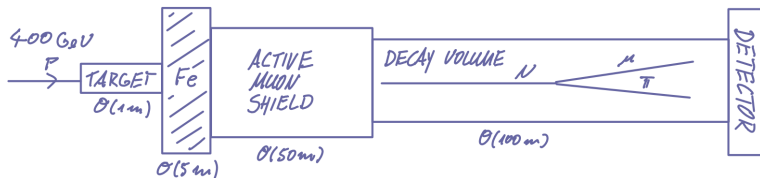
- Mixing between active neutrinos and RHNs:

$$U_{ai} \approx \theta_{ai} = v Y_{ai}^\dagger / M_i$$

- GeV-scale RHNs can be produced in heavy meson (D , B) decays in B factories or beam dump experiments.



- Sensitivity of B factories is limited because RHNs can decay outside of the detector.
- SHiP – Search for Hidden Particles: Proposed beam dump facility @ CERN:

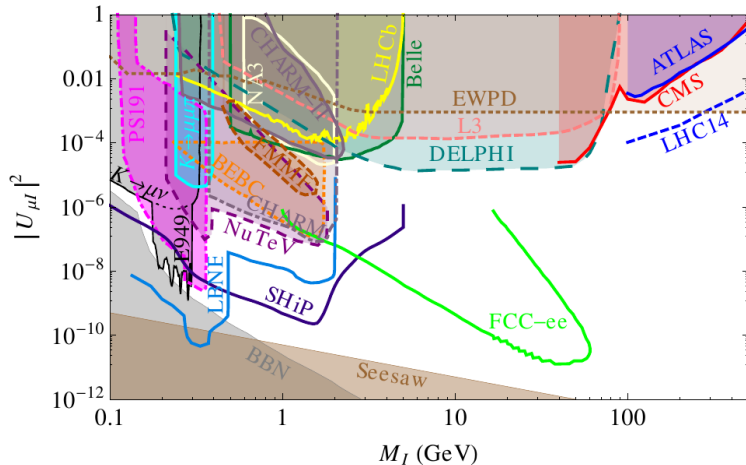


Indirect signals:

- Rate for $0\nu\beta\beta$ decay in type I seesaw mechanism can be enhanced for GeV-scale RHNs compared to other mass regions.
- Lepton universality in meson decays.
- Charged lepton flavour violation.

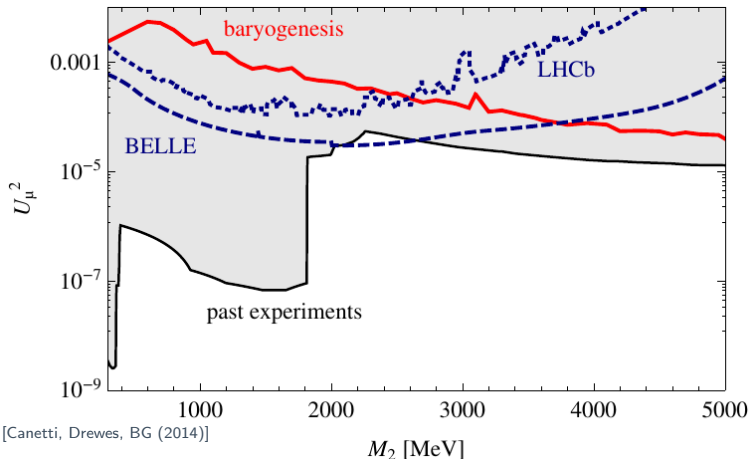
Searches for GeV-Scale RHNs and Leptogenesis

Direct search bounds:



[from SHiP physics case paper (2015)]

Searches for GeV-scale RHNs and Leptogenesis



- Goal: Find regions where leptogenesis is viable (and understand these in the high-dimensional parameter space).
- Need better numerical performance, precision & better analytical understanding. [Drewes, BG, Gueter, Klarić (in progress)]

