Naturalness, Dark Matter and E_6 Inspired SUSY Models

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Based on:

P. Athron, D. Harries, and A. G. Williams, Phys. Rev. **D91**, 115024 [arXiv:1503.08929]

P. Athron, D. Harries, R. Nevzorov, and A. G. Williams, Phys. Lett. **B760**, 19 (2016) [arXiv:1512.07040]

P. Athron, D. Harries, R. Nevzorov, and A. G. Williams, JHEP **12**, 128 (2016) [arXiv:1610.03374]

P. Athron, C. Balázs, B. Farmer, A. Fowlie, D. Harries, and D. Kim, arXiv:1709.07895

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Australian Government
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Special Research Centre for the Subatomic Structure of Matter

The University of Adelaide

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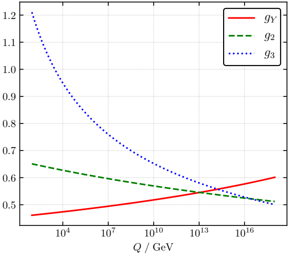
Outline

The Case for BSM Physics

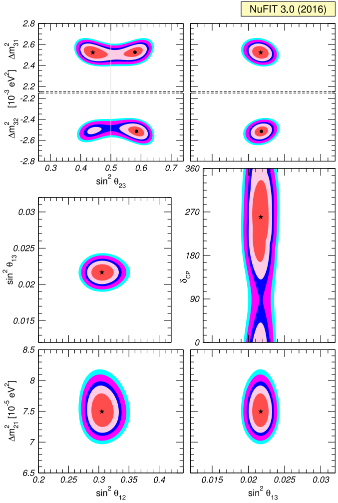
[APOD/NASA]

Dark matter?



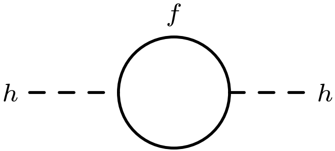
[http://flexiblesusy.hepforge.org/images.html]

Gauge unification?



[arXiv:1611.01514]

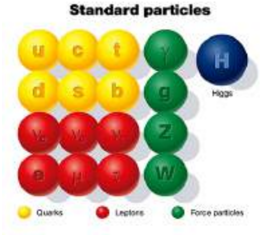
Neutrino masses?



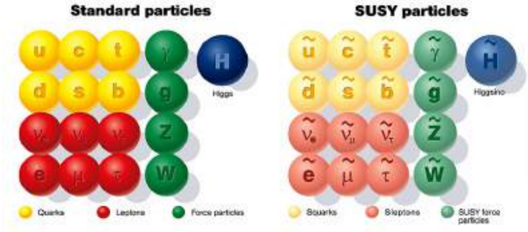
$$m_h^2 \ll \Lambda_{NP}^2?$$

i.e., the "Hierarchy Problem"?

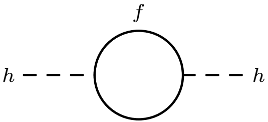
The MSSM: A Schematic View

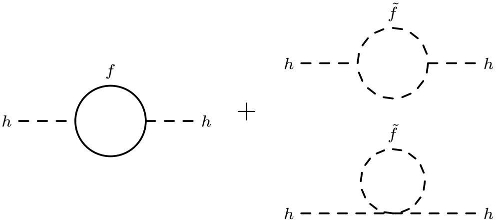


(almost)



[http://www.physics.gla.ac.uk/ppt/bsm.htm]





MSSM Field Content

- Minimal extension of the SM compatible with SUSY
- SM matter, gauge fields $\Rightarrow$ supermultiplets
- Additional Higgs doublet required
- Characterised by superpotential + soft SUSY breaking interactions
  - in general $\Rightarrow$ large number of new parameters

	$s=0$	$s=\frac{1}{2}$	$SU(3)_C$	$SU(2)_L$	$\sqrt{\frac{5}{3}}Q_i^Y$
$\hat{Q}_i$	$\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}_i$	$\begin{pmatrix}u_L\\ d_L\end{pmatrix}_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
$\hat{u}_i^c$	$\tilde{u}_{iR}^*$	u_{iR}^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
$\hat{d}_i^c$	$\tilde{d}_{iR}^*$	d_{iR}^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
$\hat{L}_i$	$\begin{pmatrix}\tilde{\nu}_L\\ \tilde{e}_L\end{pmatrix}_i$	$\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
$\hat{e}_i^c$	$\tilde{e}_{iR}^*$	e_{iR}^c	$\mathbf{1}$	$\mathbf{1}$	1
$\hat{H}_d$	$\begin{pmatrix}H_d^0\\ H_d^-\end{pmatrix}$	$\begin{pmatrix}\tilde{H}_d^0\\ \tilde{H}_d^-\end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
$\hat{H}_u$	$\begin{pmatrix}H_u^+\\ H_u^0\end{pmatrix}$	$\begin{pmatrix}\tilde{H}_u^+\\ \tilde{H}_u^0\end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$

$$W_{\rm MSSM}^{\rm RPC} = \mu(\hat{H}_d \cdot \hat{H}_u) + y_{ij}^e \hat{e}_i^c(\hat{L}_j \cdot \hat{H}_d) + y_{ij}^d \hat{d}_i^c(\hat{Q}_j \cdot \hat{H}_d) + y_{ij}^u \hat{u}_i^c(\hat{H}_u \cdot \hat{Q}_j)$$

DM in the (RPC) MSSM

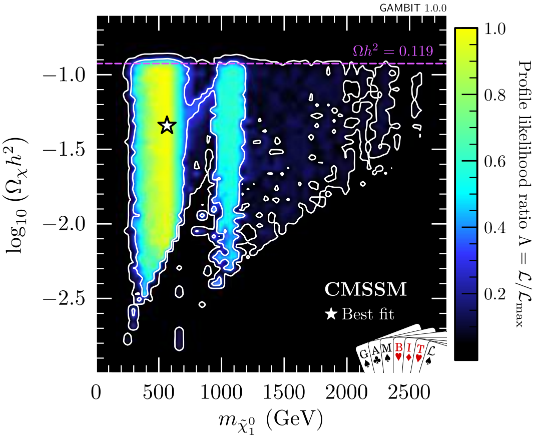
- Most general superpotential $\supset$ *B*, *L* violating interactions
- Forbid most dangerous terms? $\Rightarrow$ impose matter parity ($\equiv$ *R*-parity):

$$\boxed{Z_2^M = (-1)^{3(B-L)}}$$

- *R*-parity conservation $\Rightarrow$ electrically neutral LSP is stable, natural WIMP DM candidate
- Standard MSSM candidate is lightest $\tilde{\chi}^0$,

$$\tilde{\chi}_1^0 = N_{11}\tilde{H}_d^0 + N_{12}\tilde{H}_u^0 + N_{13}\tilde{W}_3 + N_{14}\tilde{B}$$

- Mixings N_{1i} $\Rightarrow$ different DM scenarios ("bino-like", "wino-like", ...)



[arXiv:1705.07935]

The "Little Hierarchy Problem"

Selected for a Viewpoint in *Physics*
PRL **114,** 191803 (2015) PHYSICAL REVIEW LETTERS week ending 15 MAY 2015

Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments

G. Aad *et al.**

(ATLAS Collaboration)†
(CMS Collaboration)‡
(Received 25 March 2015; published 14 May 2015)

A measurement of the Higgs boson mass is presented based on the combined data samples of the ATLAS and CMS experiments at the CERN LHC in the $H \to \gamma\gamma$ and $H \to ZZ \to 4\ell$ decay channels. The results are obtained from a simultaneous fit to the reconstructed invariant mass peaks in the two channels and for the two experiments. The measured masses from the individual channels and the two experiments are found to be consistent among themselves. The combined measured mass of the Higgs boson is $m_H = 125.09 \pm 0.21\,(\text{stat}) \pm 0.11\,(\text{syst})$ GeV.

DOI: 10.1103/PhysRevLett.114.191803 PACS numbers: 14.80.Bn, 13.85.Qk

MSSM tree-level prediction:

$$m_{h_1}^2 \leq m_Z^2 \cos^2 2\beta \lesssim (91\ \text{GeV})^2$$

The "Little Hierarchy Problem"

- $m_{h_1} \approx 125$ GeV $\Rightarrow$ large higher order corrections

$$m_{h_1}^2 \approx m_Z^2 \cos^2 2\beta \left(1 - \frac{3}{8\pi^2}\frac{m_t^2}{v^2}\ln\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{M_t^2}\right) + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\ln\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{M_t^2} + \dots$$

- $\Rightarrow$ also large corrections to prediction for m_Z at SUSY scale M_S:

$$\frac{m_Z^2}{2} = -\mu^2 + \frac{\overbrace{m_{H_d}^2 - m_{H_u}^2}^{\text{RGE effects}} \tan^2\beta}{\tan^2\beta - 1} + \delta_{1-\text{loop}},$$

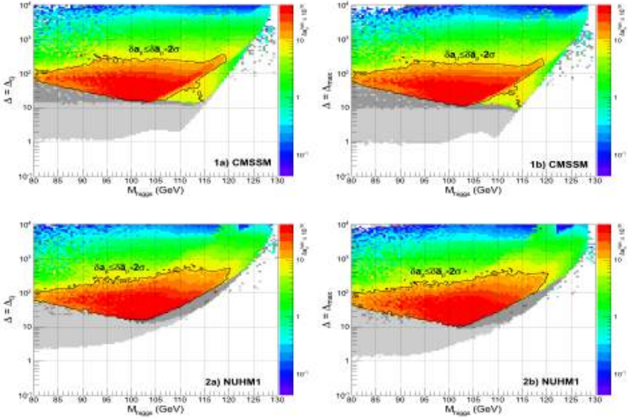
$$\delta_{1-\text{loop}} = \frac{3}{8\pi^2}\frac{m_t^2}{v^2 \cos 2\beta}\left[m_{\tilde{t}_1}^2\left(\ln\frac{m_{\tilde{t}_1}^2}{M_S^2} - 1\right) + m_{\tilde{t}_2}^2\left(\ln\frac{m_{\tilde{t}_2}^2}{M_S^2} - 1\right)\right] + \dots$$

- $\Rightarrow$ naturalness problem?

- μ-problem: $m_Z^2/2 = -\mu^2 + \dots \Rightarrow \mu \sim$ soft parameters?

The "Little Hierarchy Problem"

- E.g., constrained models ⇒ large *sensitivities* Δ



[arXiv:1203.0569]

Non-minimal SUSY Models

- Motivated by MSSM shortcomings such as LHP, μ-problem
  - also, e.g., non-zero ν masses, baryogenesis, ...
- Extend MSSM matter content and/or gauge symmetries
- Simplest example: NMSSM = MSSM + SM singlet superfield,

$$\boxed{W^{Z_3}_{\text{NMSSM}} = W^{RPC}_{\text{MSSM}}|_{\mu=0} + \lambda \hat{S}\hat{H}_d \cdot \hat{H}_u + \frac{1}{3}\kappa \hat{S}^3}$$

$$m^2_{h_1} \leq m^2_Z \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta$$

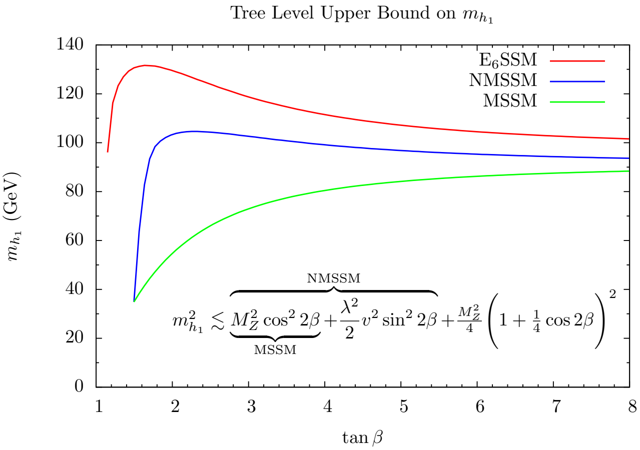
$$\langle S \rangle \neq 0 \Rightarrow \mu_{\text{eff.}} = \lambda \langle S \rangle$$

- Not without complications, may suffer from additional problems
  - e.g., domain walls in the NMSSM

$U(1)$ Extensions of the MSSM

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$$

- Well-motivated extensions of MSSM, NMSSM
- $W_{\text{USSM}} \supset \lambda \hat{S} \hat{H}_d \cdot \hat{H}_u$, $Q'_{\hat{S}} \neq 0 \Rightarrow$ $\langle S \rangle = s/\sqrt{2}$ also breaks $U(1)'$, generating massive Z'
- Consistent model requires anomaly cancellation
  - either family non-universal $U(1)'$ charges or extra matter
- Additional states $\Rightarrow$ exciting phenomenology



E_6 Inspired Models

- Lead to $U(1)$ extended models at low-energies:

$$
\begin{aligned}
E_6 &\longrightarrow SO(10) \times U(1)_\psi \\
&\longrightarrow SU(5) \times U(1)_\psi \times U(1)_\chi \\
&\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\psi \times U(1)_\chi \\
&\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'
\end{aligned}
$$

- Resulting charges $Q' = Q_\chi \cos\theta_{E_6} + Q_\psi \sin\theta_{E_6}$, e.g., class of models

$$
\begin{array}{ll}
U(1)_N: & Q_N = Q(\theta_{E_6} = \arctan\sqrt{15}) \; (\equiv \mathrm{E_6SSM}) \\
U(1)_\psi: & Q_\psi = Q(\theta_{E_6} = \pi/2) \\
U(1)_\eta: & Q_\eta = -Q(\theta_{E_6} = \pi - \arctan\sqrt{5/3}) \\
U(1)_I: & Q_I = -Q(\theta_{E_6} = \arctan\sqrt{3/5})
\end{array}
$$

- Matter content fills complete **27** representations (ensures anomaly cancellation)
  - $\Rightarrow$ additional exotic states

The E_6SSM

- $\tan\theta_{E_6} = \sqrt{15} \Rightarrow U(1)_N$ under which right-handed neutrinos are uncharged
  - allows ν masses via see-saw and successful baryogenesis [1]
- Extra $\hat{L}_4$, $\hat{\overline{L}}_4$ from incomplete $\mathbf{27}'$, $\overline{\mathbf{27}}'$ for gauge unification
- Low-energy matter content from **27**-plet:

$$(\hat{Q}_i,\ \hat{u}^c_i,\ \hat{d}^c_i,\ \hat{L}_i,\ \hat{e}^c_i) + (\hat{D}_i,\ \hat{\overline{D}}_i) + (\hat{S}_i) + (\hat{H}^u_i) + (\hat{H}^d_i)$$

- Higgs doublets $\hat{H}^d_3$, $\hat{H}^u_3$ and one singlet $\hat{S}_3$ get VEVs ($\Rightarrow$ EWSB and break $U(1)_N$)

	$SU(3)_C$	$SU(2)_L$	$\sqrt{\frac{5}{3}}Q^Y_i$	$\sqrt{40}Q^N_i$
$\hat{Q}_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	1
$\hat{u}^c_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	1
$\hat{d}^c_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	2
$\hat{L}_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	2
$\hat{e}^c_i$	$\mathbf{1}$	$\mathbf{1}$	1	1
$\hat{S}_i$	$\mathbf{1}$	$\mathbf{1}$	0	5
$\hat{H}^u_i$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	-2
$\hat{H}^d_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-3
$\hat{D}$	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$	-2
$\hat{\overline{D}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	-3
$\hat{L}_4$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	2
$\hat{\overline{L}}_4$	$\mathbf{1}$	$\overline{\mathbf{2}}$	$\frac{1}{2}$	-2

$$W_{E_6SSM} \approx y_\tau \hat{L}_3 \cdot \hat{H}^d_3 \hat{e}^c_3 + y_b \hat{Q}_3 \cdot \hat{H}^d_3 \hat{d}^c_3 + y_t \hat{H}^u_3 \cdot \hat{Q}_3 \hat{u}^c_3 + \lambda_i \hat{S}_3 \hat{H}^d_i \cdot \hat{H}^u_i + \kappa_i \hat{S}_3 \hat{D}_i \hat{\overline{D}}_i + \mu_L \hat{L}_4 \cdot \hat{\overline{L}}_4$$

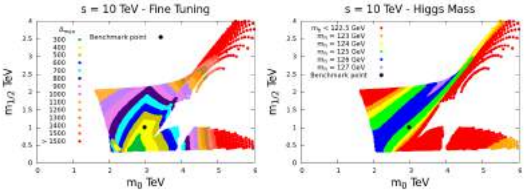
[1] S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006) (hep-ph/0510419)

The Impact of D-terms

$$m_{h_1}^2 \leq m_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta + \frac{m_Z^2}{4}\left(1 + \frac{1}{4}\cos 2\beta\right)^2$$

$$c\,\frac{m_Z^2}{2} \approx -\mu_{\text{eff.}}^2 + \frac{\bar{m}_{H_d}^2 - \bar{m}_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} + d\,\frac{m_{Z'}^2}{2}$$

- Extra D-terms $\Rightarrow$ reduced radiative corrections
- E.g., CE_6SSM shows reduced sensitivity compared to CMSSM



[arXiv:1302.5291]

- Large $m_{Z'}$ $\Rightarrow$ large contribution to m_Z
- $m_{Z'} \gg m_Z$ $\Rightarrow$ fine-tuning again?
- Coefficient $d = d(\tan\beta; \theta_{E_6})$ $\Rightarrow$ impact of $m_{Z'}$ depends on θ_{E_6}
  - $c = c(\tan\beta; \theta_{E_6}) = O(1)$

For $m_{Z'}$ large, does cost outweigh the benefit?

Model Dependent Contributions

- Radiative contributions, e.g.,

$$m_{H_u}^2(M_S) \sim m_{H_u}^2(M_X) + \frac{3y_t^2}{8\pi^2}[m_{H_u}^2(M_X) + m_{Q_3}^2(M_X) + A_t^2(M_X)] \ln \frac{M_S}{M_X} + \dots$$

  depend on
  - soft SUSY breaking mechanism ($\Leftrightarrow$ values of $m_{H_u}^2(M_X)$, $m_{Q_3}^2(M_X)$, etc.)
  - scale of SUSY breaking, M_X
- I.e., conclusions strongly depend on assumptions about (unknown) high-energy boundary conditions
- Contrast with *tree-level* $m_{Z'}$ effect
- Study models at low energies $\Rightarrow$ conservative picture of tuning
- Alternative: measure fine-tuning purely in terms of low-energy parameters?

Measuring Fine-tuning?

What constitutes "fine-tuning"?
Opinions differ.

- Large cancellations?

$$\Delta_{EW} = \max_i \frac{2|C_i|}{m_Z^2}, \quad C_1 = -\mu^2,\ C_2 = \frac{m_{H_d}^2}{\tan^2\beta - 1}, \ldots$$

- Extreme sensitivities?

$$\Delta_{BG} = \max_i \left| \frac{\partial \ln m_Z^2}{\partial \ln p_i} \right|, \quad p_i \in \{\text{fundamental parameters}\}$$

- High or low energy contributions (Δ_{EW} or Δ_{HS})? Definition of parameters p_i? Just m_Z?
- Model fine-tuned $\Leftrightarrow \Delta_{EW/HS/BG/\ldots} >$?
- Compare tuning between models?

A Bayesian Approach

- Rigorous framework automatically captures intuition about "naturalness"
- Bayes' theorem applied to model M, parameters $\boldsymbol{x}$, "observables" $\boldsymbol{O}$:

$$p(\boldsymbol{x}|\text{data}, M) = \frac{p(\text{data}|\boldsymbol{x}, M)p(\boldsymbol{x}|M)}{p(\text{data}|M)}, \quad p(M|\text{data}) = \frac{p(\text{data}|M)p(M)}{p(\text{data})}$$

$$p(\text{data}|M) = \int d^n x_i \; p(\text{data}|\boldsymbol{x}, M)p(\boldsymbol{x}|M), \quad p_{\text{eff.}}(x_j, \ldots) = \int dO_i \; p(\boldsymbol{O}, \boldsymbol{x}'|M, O_i^{\text{exp.}})$$

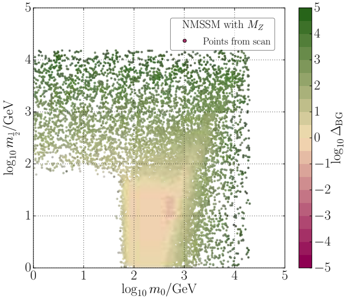
- Reparameterise in favour of $\boldsymbol{O}$, e.g.,

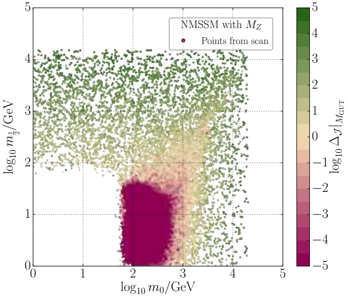
$$p(\boldsymbol{x}|M) = \mathcal{J} p(\boldsymbol{O}, \boldsymbol{x}'|M)$$

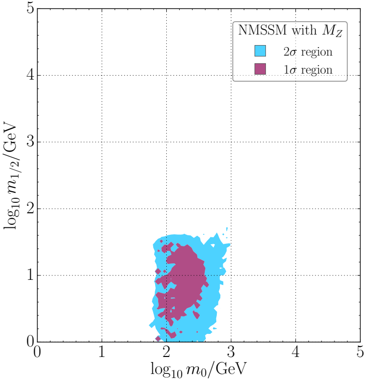
$\Rightarrow$ evidence and effective priors suppressed by Jacobian $\Delta_J \sim \mathcal{J}$,

$$\Delta_J = \left| \det \frac{\partial \ln O_i}{\partial \ln x_j} \right|$$

Example: Bayesian Naturalness in the NMSSM



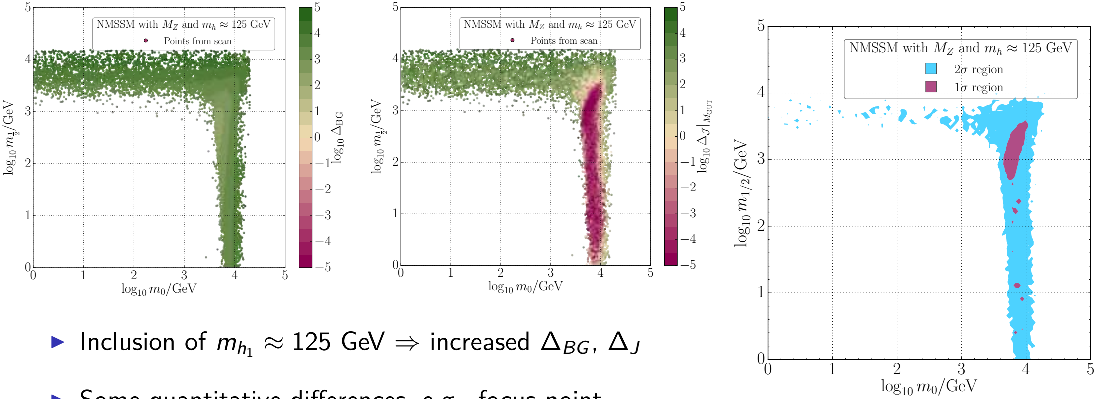




[arXiv:1709.07895]

- Δ_{BG}, Δ_J qualitatively similar
- High posterior density $\Leftrightarrow$ low fine-tuning
- Before $m_{h_1} \approx 125$ GeV, weak-scale soft parameters preferred

Example: Bayesian Naturalness in the NMSSM



- Inclusion of $m_{h_1} \approx 125$ GeV $\Rightarrow$ increased Δ_{BG}, Δ_J
- Some quantitative differences, e.g., focus point
- Bayesian analysis consistent with intuitive tuning measures

[arXiv:1709.07895]

Fine Tuning in the E_6SSM

- Use Δ_{BG} for simplicity and comparison with previous results
  - Expect similar conclusions to full Bayesian approach
- Fundamental model parameters $\Leftrightarrow$ boundary values at M_X
  - e.g. $m_{H_u}^2(M_X)$, $A_t(M_X)$, $A_\lambda(M_X)$, $M_3(M_X)$, $\ldots$
- Minimise impact of high-energy assumptions $\Rightarrow$ choose low $M_X = 20$ TeV
- Account for remaining impact with approximate solutions ($t = \ln \frac{M_S}{M_X}$, $\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4}$)

$$p(M_S) \approx p(M_X) + \frac{t}{(4\pi)^2}\left(\beta_p^{(1)}(M_X) + \frac{1}{(4\pi)^2}\beta_p^{(2)}(M_X)\right) + \frac{t^2}{2(4\pi)^4}\sum_{\{q\}} \beta_q^{(1)}(M_X) \left.\frac{\partial \beta_p^{(1)}}{\partial q}\right|_{M_X}$$

- Dependence of Δ_{BG} on $m_{Z'}$?

Numerical Methods

- Consider multiple θ_{E_6} values (role of d coefficient?)
- "Phenomenological" boundary conditions at $M_X \Rightarrow$ compute tunings for

$$p = \{\lambda, A_\lambda, m_{H_d}^2, m_{H_u}^2, m_S^2, m_{Q_3}^2, m_{u_3^c}^2, A_t, M_1, M_2, M_3, M_1'\}$$

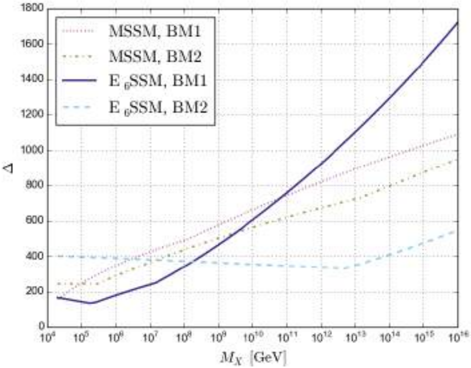
- 2-loop RGEs generated by SARAH + FlexibleSUSY
- For MSSM used modified SOFTSUSY 3.3.10
  - Assume pMSSM boundary conditions with $\mu, B \in [-1, 1]$ TeV
- Neglect kinetic mixing
  - small in models considered
  - not negligible in general (e.g. $U(1)_{B-L}$)

$2 \leq \tan\beta \leq 50$ for $U(1)_N$, $\tan\beta = 10$ for all others
$-3 \leq \lambda \leq 3$
$-10\ \mathrm{TeV} \leq A_\lambda \leq 10\ \mathrm{TeV}$
$200\ \mathrm{GeV} \leq m_{Q_3} \leq 2000\ \mathrm{GeV}$
$200\ \mathrm{GeV} \leq m_{u_3} \leq 2000\ \mathrm{GeV}$
$-10\ \mathrm{TeV} \leq A_t \leq 10\ \mathrm{TeV}$
$M_2 =$ $100\ \mathrm{GeV}, 1050\ \mathrm{GeV}, 2000\ \mathrm{GeV}$

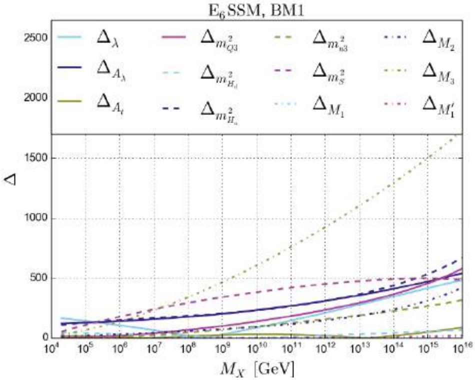
- Other soft scalar masses $= 5$ TeV, $M_1 = M_1' = 300$ GeV, $M_3 = 2$ TeV

High-scale Model Dependence

- Δ_{BG} highly model-dependent
  - E.g., E_6SSM BM1 (unconstrained) vs. BM2 ($\sim$ CE_6SSM)

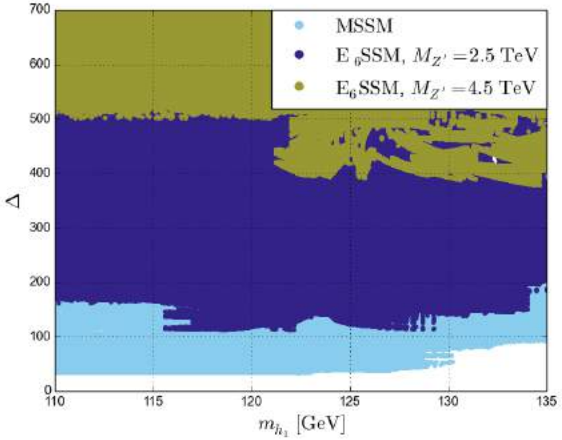


[arXiv:1503.08929]



- Large M_X $\Rightarrow$ RG contributions (e.g., M_3 in E_6SSM BM1) are dominant
- High-scale models: impact of $m_{Z'}$ limits not easily seen

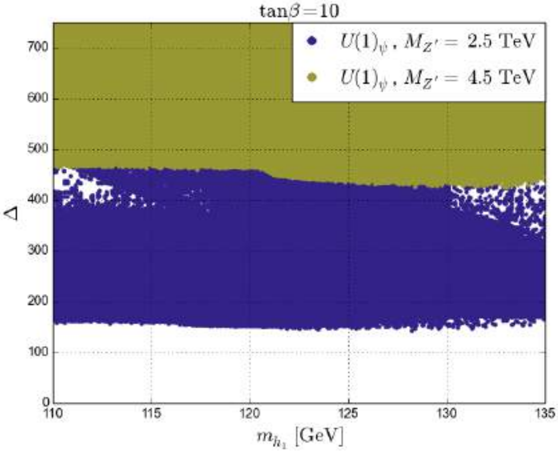
$U(1)_N$: $Q_N = Q(\theta_{E_6} = \arctan\sqrt{15})$, $d(\tan\beta = 10) \approx 0.40$



[arXiv:1503.08929]

- Low $M_X \Rightarrow \tilde{t}$ tuning reduced
- $m_{Z'}$ sets lower limit on Δ_{BG}
  - $\Rightarrow$ conservative limit on naturalness in E$_6$SSM
- $m_{Z'} \approx 2.5$ TeV (old limit!) $\Rightarrow$ minimum tuning already exceeds MSSM
  - $\approx$ equivalent to MSSM with $\geq$ 700 GeV $\tilde{\chi}^{\pm}$ ($\Rightarrow$ lower bound on μ)
- Warning: only 1-loop t, $\tilde{t}$ + leading 2-loop Higgs mass calculation $\Rightarrow$ large Higgs mass uncertainty

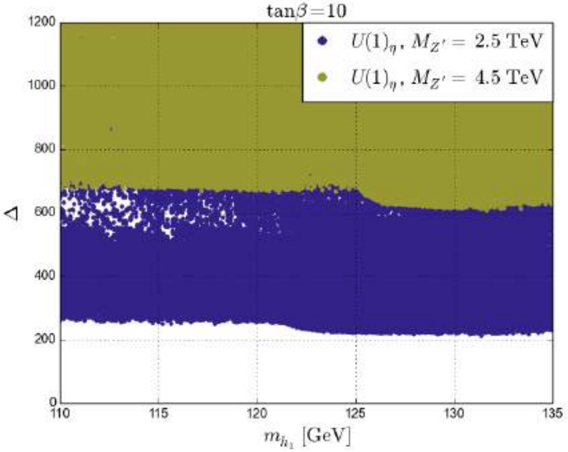
$U(1)_\psi$: $Q_\psi = Q(\theta_{E_6} = \pi/2)$, $d(\tan\beta = 10) \approx 0.50$



[arXiv:1503.08929]

- Minimum Δ_{BG} depends on θ_{E_6}
  - $\Rightarrow$ size of tuning depends on low-energy $U(1)$
- E.g., $U(1)_\psi$, $d \approx 0.5$ compared to ≈ 0.4 in E_6SSM
- $\Rightarrow$ lower bound for given $m_{Z'}$ increased
- $d \neq 0 \Rightarrow m_{Z'}$ still sets minimal sensitivity

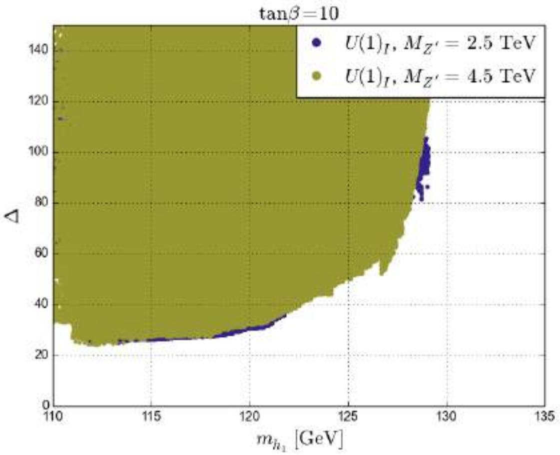
$U(1)_\eta$: $Q_\eta = -Q(\theta_{E_6} = \pi - \arctan\sqrt{5/3})$, $d(\tan\beta = 10) \approx 0.81$



[arXiv:1503.08929]

- Largest D-term contribution, $d \approx 0.81$
- $\Rightarrow$ largest tuning for given $m_{Z'}$
  - Minimum Δ_{BG} substantially larger than in $U(1)_N$, $U(1)_\psi$
- Current $m_{Z'}$ limits $\Rightarrow$ models with large D-terms already "moderately" tuned
- Severity of tuning depends strongly on θ_{E_6} (i.e. charges)

$U(1)_I$: $Q_I = -Q(\theta_{E_6} = \arctan\sqrt{3/5})$, $d(\tan\beta = 10) \approx -0.01$



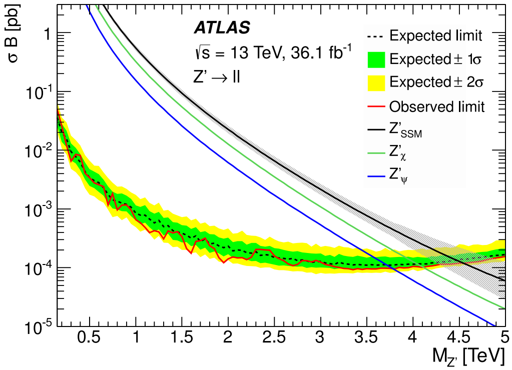
[arXiv:1503.08929]

- Example of model with suppressed D-terms ($Q_{H_u} = 0$)
- $\Rightarrow m_{Z'}$ sensitivity much reduced
- Trade-off: suppresses D-term contribution to m_{h_1}:

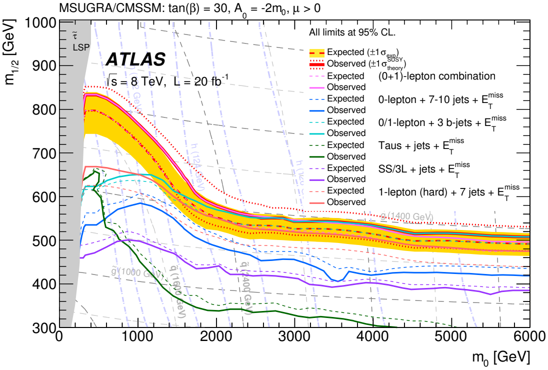
$$m_{h_1}^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{\lambda^2}{2} v^2 \sin^2 2\beta + g_1'^2 v^2 (Q_{H_d} \cos^2\beta + \underbrace{Q_{H_u} \sin^2\beta}_{=0})^2$$

- For current $m_{Z'}$ limits, tuning is MSSM-like

E_6SSM: $M_{Z'} \sim M_S$



[arXiv:1707.02424]



[arXiv:1507.05525]

E_6SSM: Discrete Symmetries

- Most general superpotential

$$W \supset g^D_{ijk} \hat{D}_i \hat{Q}_j \cdot \hat{Q}_k + \tilde{g}^E_{ijk} \hat{e}^c_i \hat{D}_j \hat{u}^c_k + y^U_{ijk} \hat{u}^c_i \hat{H}_{uj} \cdot \hat{Q}_k + y^D_{ijk} \hat{d}^c_i \hat{Q}_j \cdot \hat{H}_{dk} + \ldots$$

$\Rightarrow$ unacceptable B, L violation and large FCNCs

- Impose
  - either exact Z_2^L or exact Z_2^B to forbid B, L violating couplings
  - approximate Z_2^H to suppress FCNCs
- Resulting Higgs, singlet couplings

$$\lambda_{ijk} \hat{S}_i \hat{H}_{dj} \cdot \hat{H}_{uk} \to \lambda \hat{S} \hat{H}_{d3} \cdot \hat{H}_{u3} + \lambda_{\alpha\beta} \hat{S} \hat{H}_{d\alpha} \cdot \hat{H}_{u\beta} + \tilde{f}_{\alpha\beta} \hat{S}_\alpha \hat{H}_{d\beta} \cdot \hat{H}_{u3} + f_{\alpha\beta} \hat{S}_\alpha \hat{H}_{d3} \cdot \hat{H}_{u\beta}$$

$\Rightarrow$ LSP, NSLP is a (ruled out) "inert" neutralino

- Simplest viable models impose *another* exact Z_2^S
- None of these Z_2 symmetries commute with E_6

The SE$_6$SSM

- E_6 inspired model arising from 5D or 6D orbifold GUT [2]
- Complete **27**-plets supplemented by components of extra **27**$'$-, $\overline{\mathbf{27}}'$-plets
- Stabilise Higgs potential $\Rightarrow$ pure singlet $\hat{\phi}$
- $U(1)_\psi \times U(1)_\chi \to U(1)_N \times Z_2^M$ at intermediate scale $\Rightarrow$ automatically conserved Z_2^M
- Dangerous B, L violating operators, FCNCs forbidden by single exact $\tilde{Z}_2^H$

$$\begin{aligned} W_{\mathrm{SE_6SSM}} = {} & \lambda \hat{S}(\hat{H}_d \cdot \hat{H}_u) - \sigma \hat{\phi} \hat{S} \hat{\overline{S}} + \frac{\kappa}{3} \hat{\phi}^3 + \frac{\mu}{2} \hat{\phi}^2 + \Lambda_F \hat{\phi} + \lambda_{\alpha\beta} \hat{S} (\hat{H}^d_\alpha \cdot \hat{H}^u_\beta) \\ & + \kappa_{ij} \hat{S} \hat{D}_i \hat{\overline{D}}_j + \tilde{f}_{i\alpha} \hat{S}_i (\hat{H}_u \cdot \hat{H}^d_\alpha) + f_{i\alpha} \hat{S}_i (\hat{H}^u_\alpha \cdot \hat{H}_d) + g^D_{ij} (\hat{Q}_i \cdot \hat{L}_4) \hat{\overline{D}}_j \\ & + h^E_{i\alpha} \hat{e}^c_i (\hat{H}^d_\alpha \cdot \hat{L}_4) + \mu_L (\hat{L}_4 \cdot \hat{\overline{L}}_4) + \tilde{\sigma} \hat{\phi} (\hat{L}_4 \cdot \hat{\overline{L}}_4) + W_{\mathrm{MSSM}}(\mu = 0) \end{aligned}$$

[2] R. Nevzorov, Phys. Rev. **D** 87, 015029 (2013) (arXiv:1205.5967)

EWSB in the SE_6SSM

- At physical minimum,

$$\langle H_d^0 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle H_u^0 \rangle = \frac{v_2}{\sqrt{2}}, \quad \langle S \rangle = \frac{s_1}{\sqrt{2}}, \quad \langle \overline{S} \rangle = \frac{s_s}{\sqrt{2}}, \quad \langle \phi \rangle = \frac{\varphi}{\sqrt{2}}$$

- S, $\overline{S}$ develop along nearly D-flat direction $\langle S \rangle = \langle \overline{S} \rangle$ with

$$\langle S \rangle \approx \langle \overline{S} \rangle \sim \frac{M_S}{\sigma}$$

- Small $\sigma \Rightarrow M_{Z'}^2 \sim g_1'^2 Q_S^2 (s_1^2 + s_2^2)$ far heavier than M_S
- $\therefore$ can have $M_{Z'}$ far above limits while keeping sparticles (relatively) light
- D-term contribution to EW scale also suppressed:

$$c \frac{M_Z^2}{2} \approx -\frac{\lambda^2 s_1^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} + d \frac{g_1'^2 Q_S^2 (s_1^2 - s_2^2)}{2}$$

Dark Matter Candidates?

- Conserved Z_2^M, $\tilde{Z}_2^H \Rightarrow$ *two* (distinct) DM candidates
- "Exotics" $\equiv Z_2^E$ odd states, where $\tilde{Z}_2^H = Z_2^M \times Z_2^E$
- Z_2^E conserved $\Rightarrow$ lightest exotic is stable
- Limits from exotic Higgs decays, DM direct detection $\Rightarrow$ inert singlinos $\tilde{S}_i$ form subdominant hot DM, $m_{\tilde{S}_i} \ll 1$ eV
- $M_{Z'} \gg M_S \gg M_Z \Rightarrow$ singlet dominated $\tilde{\chi}^0$'s decouple
- $\Rightarrow$ account for Ωh^2 with MSSM-like $\tilde{\chi}_1^0$?

	$\tilde{Z}_2^H$	Z_2^M	Z_2^E
$\hat{Q}_i, \hat{u}_i^c, \hat{d}_i^c, \hat{L}_i, \hat{e}_i^c, \hat{N}_i^c$	$-$	$-$	$+$
$\hat{H}_\alpha^u, \hat{H}_\alpha^d, \hat{S}_i, \hat{D}_i, \hat{\overline{D}}_i$	$-$	$+$	$-$
$\hat{H}_u, \hat{H}_d$	$+$	$+$	$+$
$\hat{S}, \hat{\overline{S}}$	$+$	$+$	$+$
$\hat{L}_4, \hat{\overline{L}}_4$	$+$	$-$	$-$

Enlarged (8×8) $\tilde{\chi}^0$ sector:

$$M_{\tilde{\chi}^0} = \begin{pmatrix} A & C^T \\ C & B \end{pmatrix}$$

The CSE$_6$SSM

- General model is complicated
  - $O(200)$ new parameters (assuming no new sources of CP-violation)
  - Many masses and mixings
- Consider constrained model (CSE$_6$SSM) inspired by gravity mediated SUSY breaking
- Universal soft breaking parameters: $M_{1/2}$, A_0, B_0, m_0
- Interested in mechanism decoupling Z' from EWSB conditions $\Rightarrow$ can have large $s = \sqrt{s_1^2 + s_2^2}$
- Higgsino mass set by $\mu_{\text{eff}} = \lambda s_1/\sqrt{2} \Rightarrow$ acceptable LSP mass ($\lesssim$ TeV) for small λ
- $\Rightarrow$ other exotic couplings must be small, otherwise exotic states are tachyonic

Parameter Space Scans

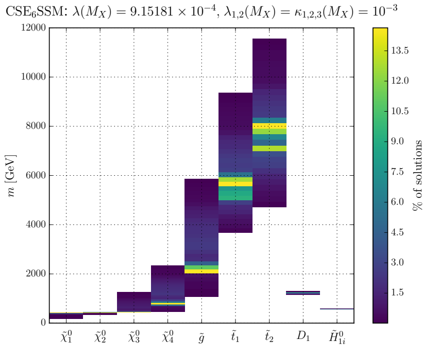
- Focus on heavy Z', $s = 650$ TeV, choose fixed μ_{eff}
- Achieve using semi-analytic solutions for soft parameters:

$$M_i(Q) = p_i(Q)M_{1/2} + q_i(Q)A_0\,, \quad A_i(Q) = e_i(Q)A_0 + f_i(Q)M_{1/2}\,,$$
$$m_i^2(Q) = a_i(Q)m_0^2 + b_i(Q)M_{1/2}^2 + c_i(Q)A_0M_{1/2} + d_i(Q)A_0^2\,, \ldots$$

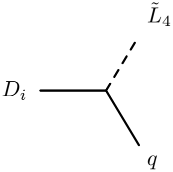
- Fix m_0 from EWSB, $m_0^2 \sim -\frac{b_{H_u}}{a_{H_u}}M_{1/2}^2 - \ldots$
- Implemented in FlexibleSUSY for full 1-loop masses and 2-loop RGEs
  - Resulting "semi-analytic solver" forms part of FlexibleSUSY 2.0 [3], along with many other updates.
- Require $\Omega h^2 \leq 0.1187$ (micrOMEGAs) and $m_{h_1} = 125.09 \pm 3$ GeV
- Compare with CMSSM for $|\mu| \sim 400$ GeV and $|\mu| \sim 1$ TeV

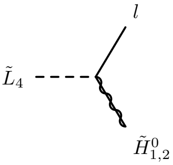
[3] P. Athron, M. Bach, D. Harries, T. Kwasnitza, J.-h. Park, D. Stöckinger, A. Voigt, and J. Ziebell, arXiv:1710.03760

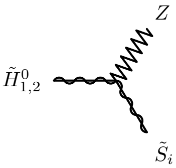
Sparticle Mass Spectrum

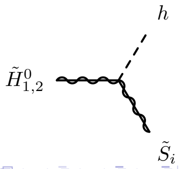


- EWSB conditions $\Rightarrow m_0 > M_{1/2}, A_0$
- $\therefore$ MSSM sfermions out of reach of run II
- Light exotic fermions can be observable
  - Exotic leptoquarks D_i:
    $p\,p \to t\,\bar{t}\,\tau^+\,\tau^- + E_T^{miss} + X$, $p\,p \to b\,\bar{b} + E_T^{miss} + X$
  - Charged, neutral inert Higgsinos:
    $p\,p \to W\,W/Z\,Z/W\,Z + E_T^{miss} + X$

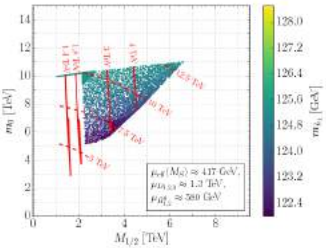


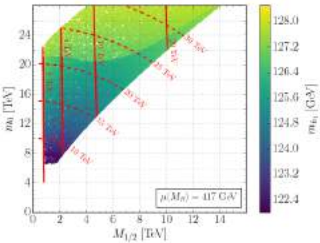


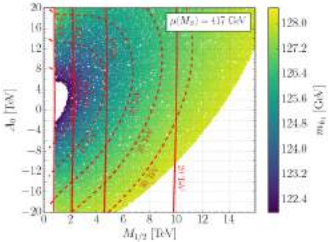


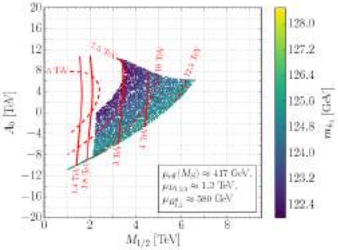


$\mu_{(\text{eff.})} \approx 400$ GeV: Parameter Space Restrictions







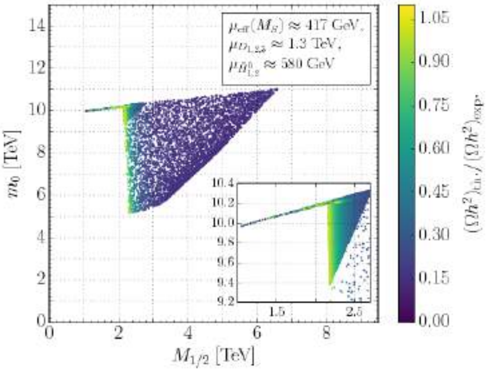


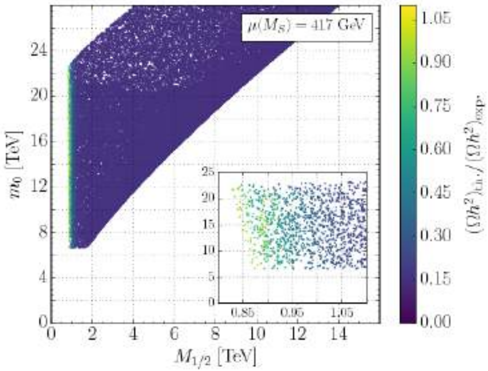
[arXiv:1610.03374]

- Successful EWSB + $m_{h_1} \approx 125$ GeV $\Rightarrow$ large $m_0 > M_{1/2}, A_0$
- $m_{h_1} \approx 125$ GeV important constraint on range of variation of $M_{1/2}$, A_0
  - Additional constraints in CSE$_6$SSM from tachyonic CP-even and CP-odd Higgs states
- Sfermions heavy, but gluino and EW-inos can be observable

$\mu_{(\text{eff.})} \approx 400$ GeV: Relic Density

[arXiv:1610.03374]

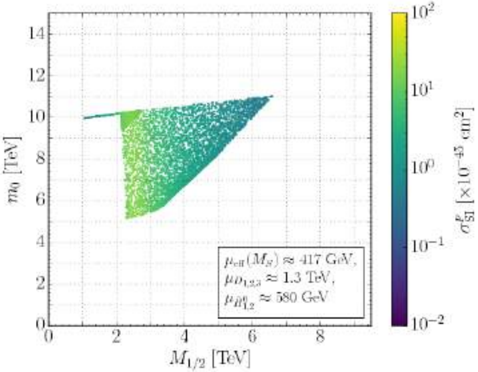




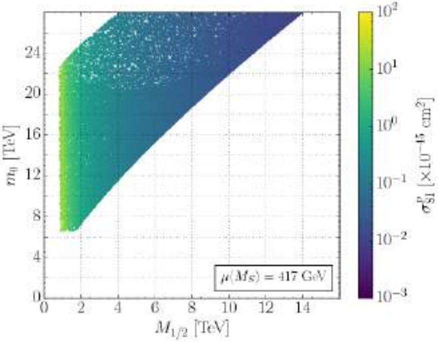
- $\Omega h^2 \approx 0.1187 \Rightarrow$ "well-tempered" bino-Higgsino $\tilde{\chi}^0$ ($\mu_{\text{eff.}} \sim M_1$)
- Pair annihilations $\tilde{\chi}^0_1 \tilde{\chi}^0_1 \to \bar{f}\, f$

- MSSM-like nature of neutralino sector clear – almost identical behaviour
- Existence of A-funnel at $\tan\beta = 10$ notable difference in CSE$_6$SSM

$\mu_{(\text{eff.})} \approx 400$ GeV: Direct Detection Cross Section



- Well-tempered $\tilde{\chi}_1^0 \Rightarrow$ large mixing, σ_{SI} exceeds, e.g., 90% LUX limits
- $M_1 \gg \mu_{\text{eff.}}$ both mixing and number density suppressed

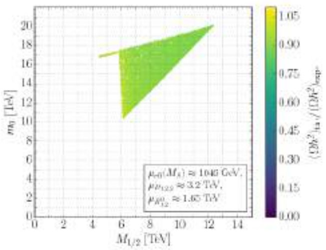


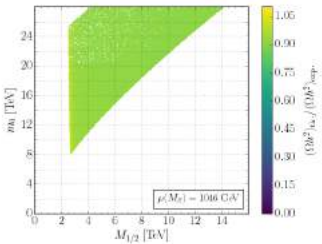
[arXiv:1610.03374]

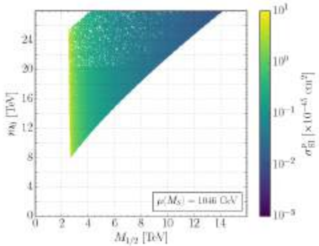
- σ_{SI} set by $g_{h_1\chi_1\chi_1}$ (t-channel h exchange):

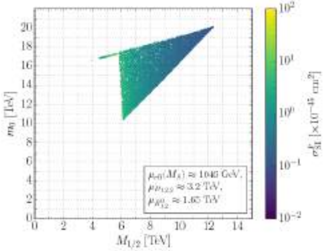
$$g_{h_1\chi_1\chi_1} \approx \frac{1}{2}\left(\sqrt{\frac{3}{5}}g_1 N_{14} - g_2 N_{13}\right)\left[N_{11}(U_h)_{11} - N_{12}(U_h)_{12}\right]$$

$\mu_{(\text{eff.})} \approx 1$ TeV: Pure Higgsino DM Candidate







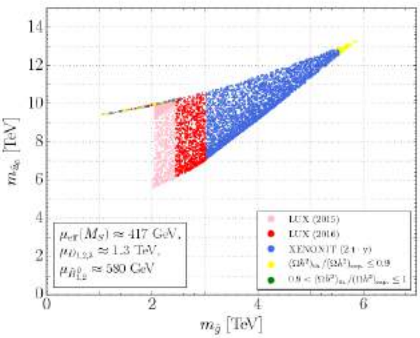


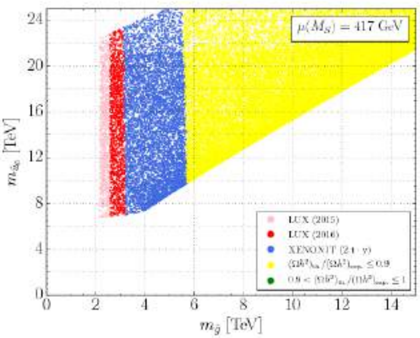
- Similar parameter space constraints due to m_{h_1}, tachyonic states
- Suppress $\tilde{B}$ fraction $\Rightarrow$ large $M_{1/2}$, $m_{\tilde{g}} \gtrsim 4$ TeV, $m_{\tilde{q}} \gtrsim 10$ TeV
- Exotics not forced to be heavy
- σ_{SI} (mostly) acceptably small

[arXiv:1610.03374]

Current and Future Limits

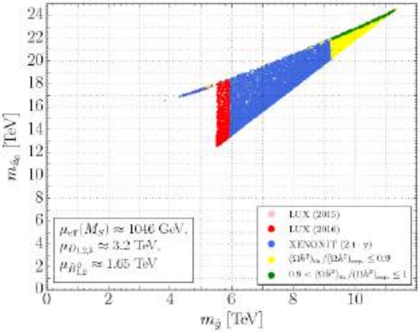
[arXiv:1610.03374]





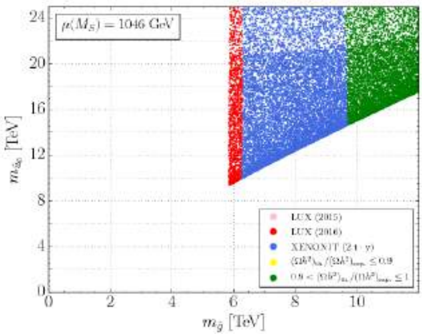
- LUX $\Rightarrow$ already stringent limits on highly mixed scenarios
- XENON1T expected to cover many of the remaining solutions
- SD limits (LUX, IceCube) can also be relevant
- *A*-funnel solutions can survive, but in reach of LHC run II $\Rightarrow$ complementarity of searches

Current and Future Limits



- Essentially no collider limits for $\mu_{\text{eff.}} \approx 1$ TeV (except possibly exotics)
- LUX now excludes mixed $\tilde{\chi}_1^0$ even when $m_{\tilde{\chi}_1^0} \approx 1$ TeV

- Non-mixed scenarios expected to be discoverable at XENON1T
- Larger $M_{1/2}$ and heavy exotics $\Rightarrow$ only accessible at, e.g., LZ



[arXiv:1610.03374]

Summary

- E_6 inspired models can address, e.g., little hierarchy problem and μ-problem of MSSM
- Additional D-terms $\Rightarrow$ also potential source of fine-tuning
- Bayesian analysis of BSM models automatically incorporates intuitions about naturalness
- Traditional measures, e.g., Δ_{BG}, agree qualitatively with Bayesian methods
- Limits on $m_{Z'}$ $\Rightarrow$ conservative limits on Δ_{BG}
  - Not dependent on assumptions about high-energy model
- Depends strongly on $U(1)'$, e.g., $U(1)_N$ compared to $U(1)_I$

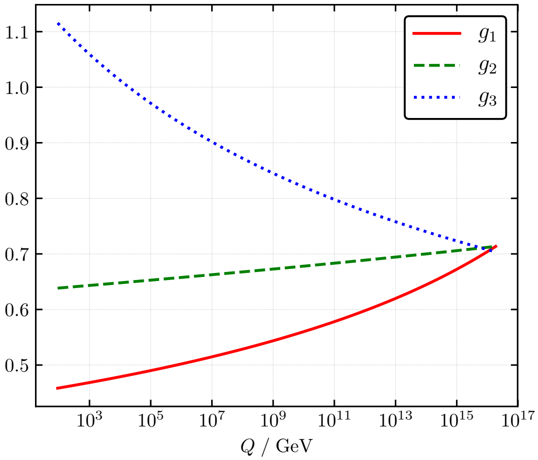
Summary

- Simplest phenomenologically viable variants, e.g. E_6SSM, face other problems: multiple discrete symmetries, $m_{Z'} \sim M_S$ ...
- SE_6SSM is a well-motivated extension with an exact custodial symmetry
- DM relic density can be fitted by MSSM-like $\tilde{\chi}_1^0$
- Direct detection searches $\Rightarrow$ stringent limits on allowed mixing
  - Mixed states fully accounting for Ωh^2 excluded
- XENON1T expected to probe much of remainder space, complementary probe to LHC searches
- E_6 exotics can be light $\Rightarrow$ important possible discovery channel

Thank you for listening!

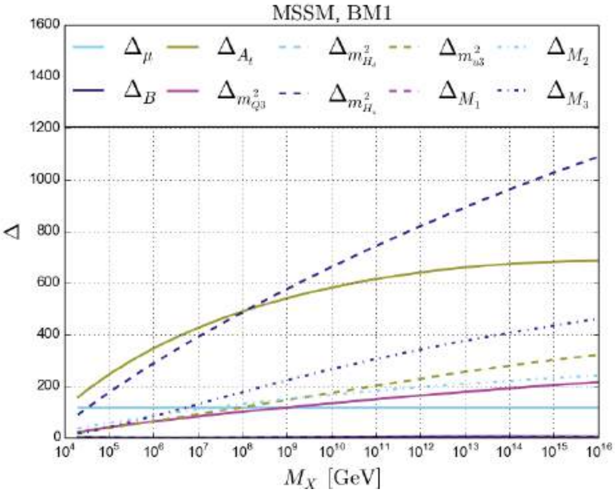
Additional Slides

CMSSM Gauge Coupling Running



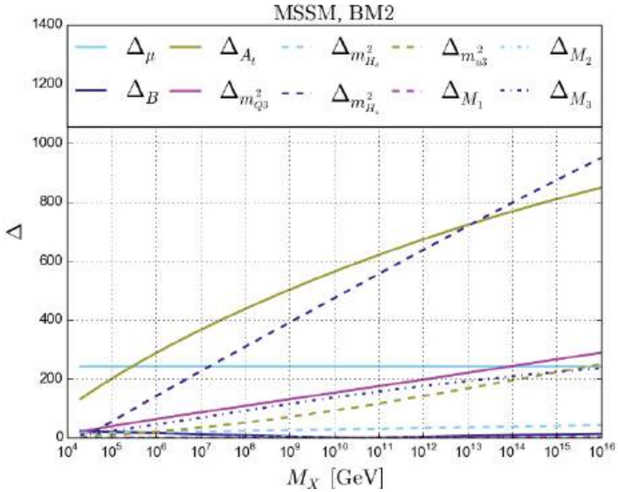
[http://flexiblesusy.hepforge.org/images.html]

MSSM BM1: Tuning Breakdown



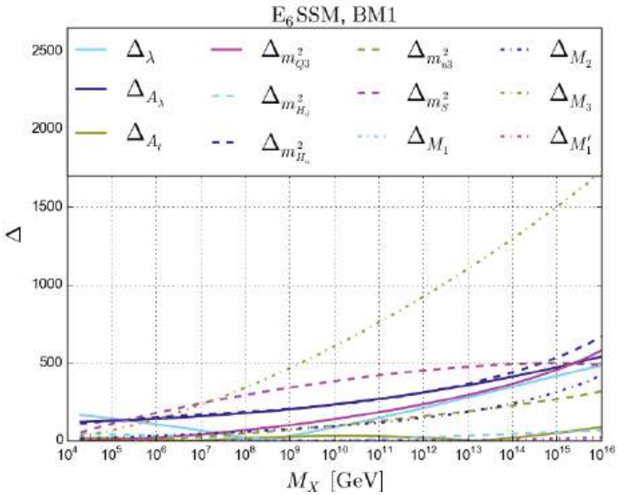
[arXiv:1503.08929]

MSSM BM2: Tuning Breakdown



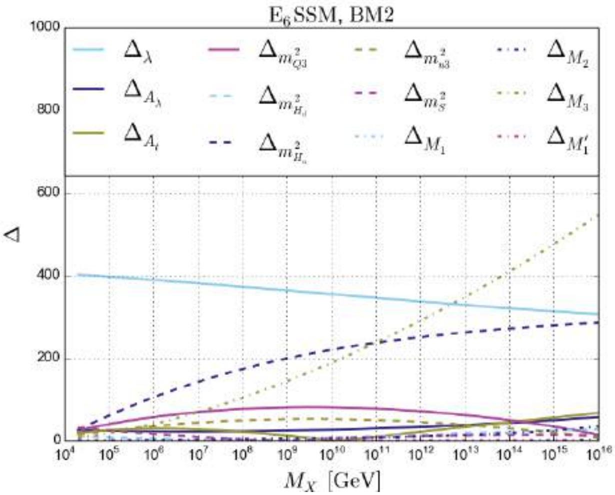
[arXiv:1503.08929]

E_6SSM BM1: Tuning Breakdown



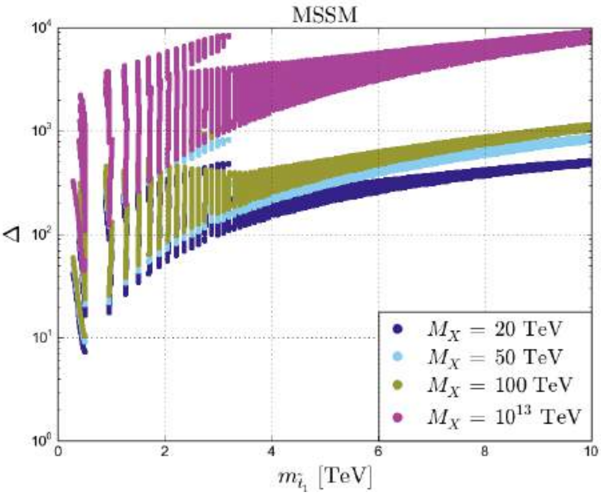
[arXiv:1503.08929]

E_6SSM BM2: Tuning Breakdown



[arXiv:1503.08929]

Low $M_X \Rightarrow$ Low $\tilde{t}$ Tuning



[arXiv:1503.08929]

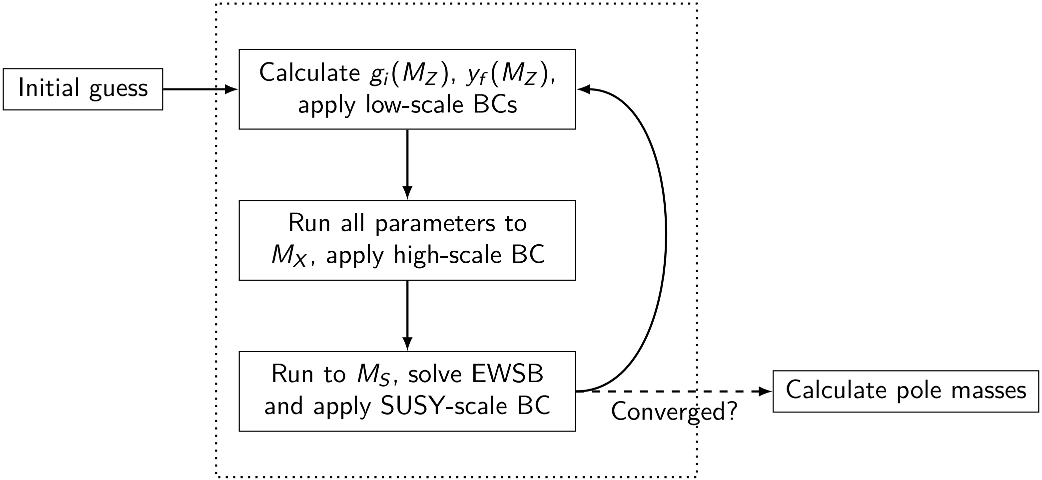
Semi-analytic Solutions

- Useful technique for studying constrained models (e.g. CE_6SSM)
- RGEs for SUSY parameters independent of soft SUSY breaking parameters (up to effects of threshold corrections)
- RGEs for SUSY breaking parameters are system of linear ODEs with variable coefficients
- Combine with boundary conditions $\Rightarrow$ semi-analytic solutions,

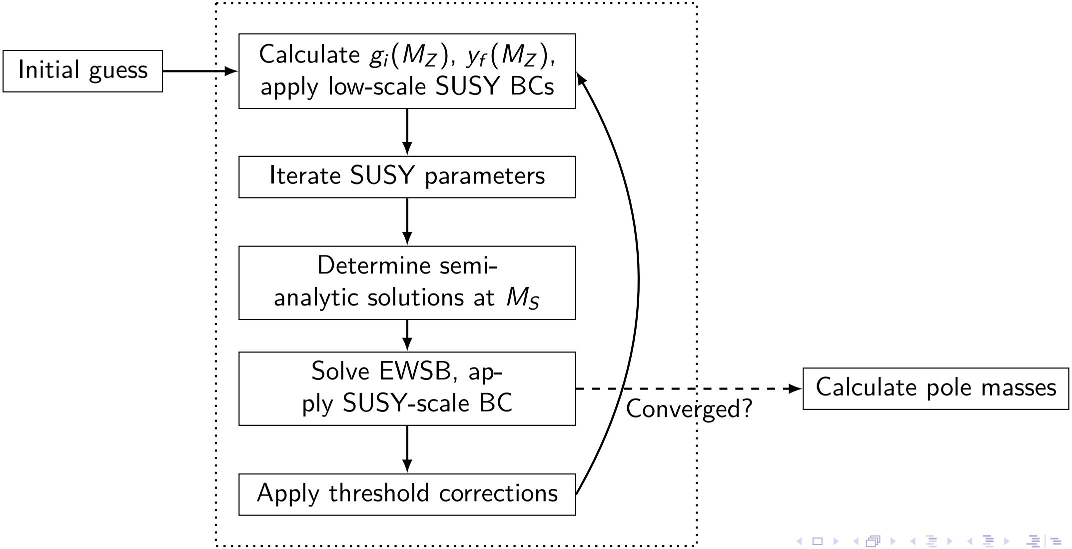
$$M_i(Q) = p_i(Q)M_{1/2} + q_i(Q)A_0\,, \quad A_i(Q) = e_i(Q)A_0 + f_i(Q)M_{1/2}\,,$$
$$m_i^2(Q) = a_i(Q)m_0^2 + b_i(Q)M_{1/2}^2 + c_i(Q)A_0M_{1/2} + d_i(Q)A_0^2\,, \ldots$$

- Coefficients depend only on SUSY parameters, calculated numerically
- Relate SUSY scale parameters to high-scale inputs, e.g. trade m_0 for μ_{eff}
  - Contrast with traditional approach in CMSSM: μ output for given m_0
  - N.B. all results here are for fixed μ

Two-scale Algorithm

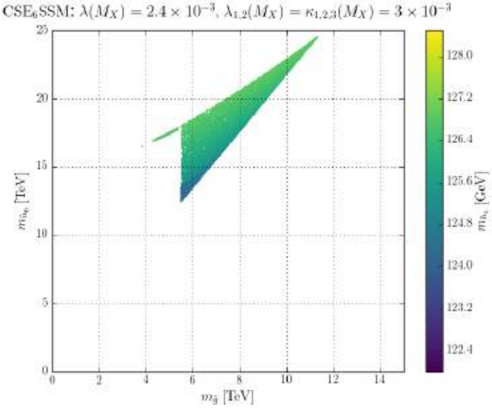


Semi-analytic Algorithm



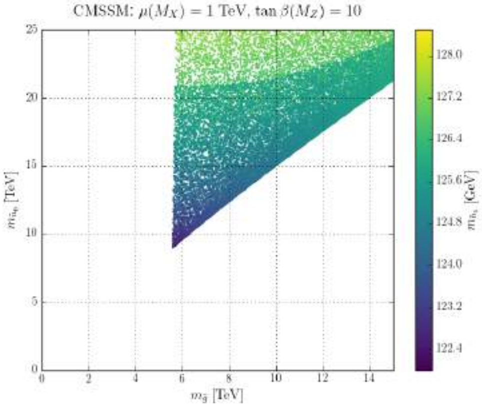
CP-even Higgs Mass in the CSE_6SSM

- Split spectrum $\Rightarrow$ large logarithms contribute
- Exotic contributions small, mainly $\tilde{t}$'s
- Use EFT calculation of m_{h_1} (SUSYHD)
  - cross-checked with prototype FlexibleEFTHiggs



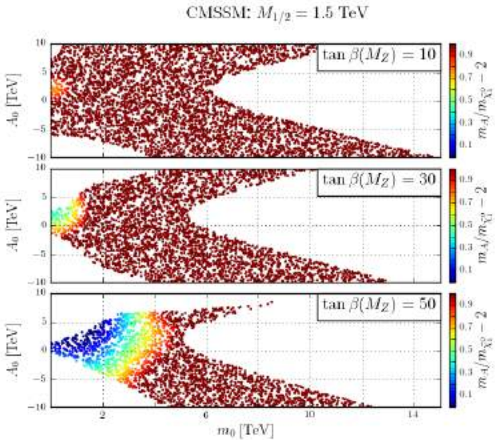
[arXiv:1610.03374]

$m_{h_1} \lesssim 130$ GeV
(i.e. bounded from above, as usual)



A-funnel in the CSE$_6$SSM

- Solutions at lower $M_{1/2}$ due to $m_{A_1} \sim 2m_{\tilde{\chi}_1^0}$
- CMSSM: A-funnel requires $\tan\beta \gtrsim 40$
- CSE$_6$SSM: tune A_0 for given $\tan\beta$, $M_{1/2}$ to that $m_{A_1} \to 0$, keeping $m_{\tilde{\chi}_1^0} \sim$ fixed
- Lightest state A_1 mixture of singlets for $s \gg M_S \gg v$ ($\tan\delta \approx \frac{s_1 s_2}{\varphi\sqrt{s_1^2+s_2^2}}$):



$$m_{A_1}^2 \approx \cos^2\delta\left(-2B\mu - 3\frac{\kappa A_\kappa}{\sqrt{2}}\varphi - \sqrt{2}\xi\frac{\Lambda}{\varphi} + \frac{9}{2}\sigma\kappa s_1 s_2 + 2\sqrt{2}\frac{\sigma\mu s_1 s_2}{\varphi} + \frac{\sigma s_1 s_2 \Lambda}{\varphi^2}\right)$$