The Higgs mass and infrared catastrophes

[MDG with variously F. Staub, K. Nickel, J. Braathen and P. Slavich, 1411.0675, 1411.4665, 1503.03098, 1511.01904, 1604.05335, 1609.06977, 1706.05372]

Mark D. Goodsell

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

cnrs

UPMC SORBONNE UNIVERSITÉS

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Overview

- Higher-order corrections to the Higgs mass: what is it used for?
- State of the art for generic theories
- The Goldstone Boson Catastrophe ...
- ... how to avoid it ...
- ... how we can exploit the solution!
- Implementation in the public code `SARAH`

The Higgs mass as a precision electroweak observable

Consider the current experimental accuracy of the Higgs mass measurement:

$$\text{ATLAS} + \text{CMS}\ (\text{Moriond 2015}): \quad m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11\,(\text{syst.})$$

The uncertainty is tiny, of the order 0.2%!
This is smaller than e.g. the 0.5% uncertainty on the top mass and on the strong gauge coupling; and compare to

1. $\sim 0.2\%$ uncertainty on M_W
2. 0.002% uncertainty on M_Z!
3. ... but $\sim 1\%$ uncertainty on $\sin^2\theta_W$

These three are known as “electroweak precision observables”: the Higgs mass deserves to now be classed among them.

SM recap

The scalar part of the Standard Model Higgs potential can be written

$$V_{SM} \supset \mu^2|H|^2 + \lambda|H|^4$$

The Higgs obtains a vev and we write

$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h) + \frac{i}{\sqrt{2}}G^0 \end{pmatrix}$$

$$\to V_{SM}^{(0)} \supset hv(\mu^2 + \lambda v^2) + \frac{1}{2}(\mu^2 + \lambda v^2)[(G^0)^2 + 2|G^+|^2] + \frac{1}{2}(\mu^2 + 3\lambda v^2)h^2$$
$$+ v\lambda(h^3 + h(G^0)^2 + 2h|G^+|^2) + \frac{\lambda}{4}h^4 + \ldots$$

We then need the potential to be at its minimum:

$$\left.\frac{\partial V}{\partial h}\right|_{h=0} = 0 = \mu^2 + \lambda v^2 + \frac{1}{v}\left.\frac{\partial \Delta V}{\partial h}\right|_{h=0}$$

This means that the Higgs mass parameter in the Lagrangian is given by

$$\mu^2 + 3\lambda v^2 = 2\lambda v^2 - \frac{1}{v}\left.\frac{\partial \Delta V}{\partial h}\right|_{h=0}.$$

The full loop corrected Higgs mass is then found by solving the on-shell condition

$$m_h^2 = 2\lambda v^2 - \frac{1}{v}\left.\frac{\partial \Delta V}{\partial h}\right|_{h=0} + \Pi_{hh}(m_h^2).$$

SM recap 2

We do not measure μ or λ directly:

- We measure v through the Fermi constant $G_F = \frac{1}{\sqrt{2}v^2}$
- The Higgs mass then gives us λ through

$$\lambda = \frac{m_h^2}{2v^2} + \text{quantum corrections}$$

- μ^2 is obtained from the minimum condition

$$\mu^2 = -\lambda v^2 - \left.\frac{\partial \Delta V}{\partial h}\right|_{h=0}$$

It is an important test of the Standard Model to measure λ directly, but:

- It seems measuring <u>four-higgs</u> couplings is well beyond the reach of the LHC
- Much theoretial work has investigated testing the $\lambda v h^3$ term, which may be possible to detect with enough luminosity.

What do we need λ for?

State-of-the-art computation includes all important two-loop effects (from [Butazzo *et al*, 1307.3536]):

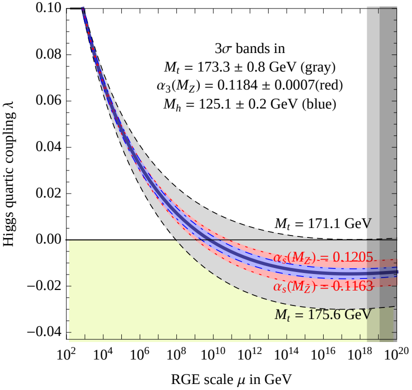
$$\lambda(\mu = m_t) = 0.12604 + 0.00206\left(\frac{m_h}{\text{GeV}} - 125.15\right) - 0.00004\left(\frac{m_t}{\text{GeV}} - 173.34\right) \pm 0.00030$$

There are now even some three-loop contributions known.

This is vital for stability analysis: check for running of λ through solving RGEs up to high scales

$$\frac{d\lambda}{d\log\mu} = \frac{2}{16\pi^2}\left[\lambda(12\lambda + 6y_t^2) - 3y_t^4 + \ldots\right]$$

i.e. to determine the fate of the Standard Model!



How well do we know λ?

We can solve the equation

$$m_h^2 = 2\lambda v^2 - \frac{\partial \Delta V}{\partial h}\bigg|_{h=0} + \Pi_{hh}(m_h^2)$$

for λ once we know m_h, e.g. by iterating. This gives e.g.

Loop order	Butazzo et al (on-shell)	SARAH	SMH (Landau gauge)
Tree level	0.12917	0.12786	0.12786
One loop	0.12774	0.12647	0.12580
Two loops	0.12604	0.12619	0.12541

These use three different schemes, and although the calculation in SARAH is missing some subdominant corrections, we see the differences are small.
But these differences were indeed important for the stability analysis – since the RGEs are with respect to $\log\mu$, changes in λ lead to exponential shifts of μ.

Even more important for BSM

It is even more important for BSM to predict scale of new physics. For example, SUSY predicts:

$$\lambda = \frac{1}{8}(g_Y^2 + g_2^2)\cos^2 2\beta + \text{quantum corrections.}$$

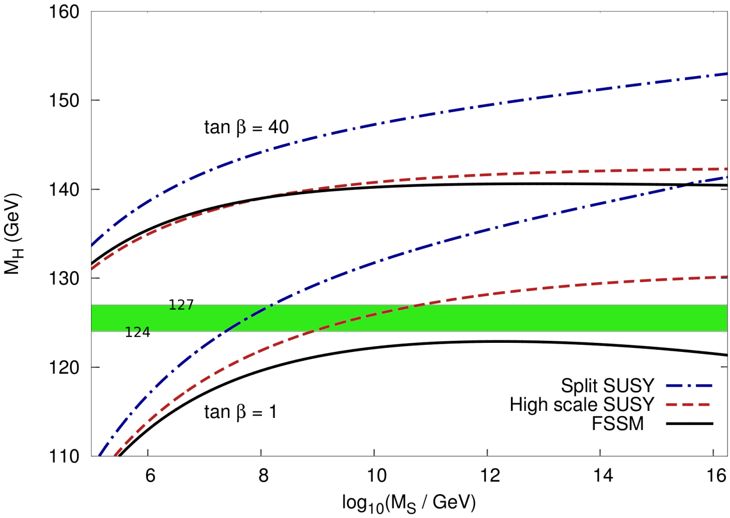
This comes from two Higgs doublets H_u, H_d which couple to up and down quarks; only one remains light, and we define

$$H_u^0 = \sin\beta \frac{1}{\sqrt{2}}(v + h) + \ldots, \qquad H_d^0 = \cos\beta \frac{1}{\sqrt{2}}(v + h) + \ldots$$

For heavy SUSY, we can neglet v, and β becomes the mixing angle between mass eigenstates.
By extracting λ and running it up to high scales, we can use this to predict the scale of the strongly-coupled SUSY particles!
In particular, we find that there can be a maximum scale of new physics when λ runs to zero ...

BSM examples

E.g. three different models taken from [Benakli, Darmé, MDG, Slavich, 1312.5220]



Large corrections to λ

On the other hand, we can have models where the loop corrections to λ are huge! E.g. consider a real singlet scalar with a $\mathbb{Z}_2$ symmetry extending the Standard Model:

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} M_S^2 S^2 - \frac{\kappa_2}{2} |H|^2 S^2 - \frac{1}{4} S^4$$

We find (using `SARAH`) for $\lambda_S = 0.1$, $\kappa_2 = 3$, $M_S = 500$ GeV (perfectly allowed by unitarity etc) that

λ Loop order	$Q = m_t$	$Q = M_S$
Tree level	0.12914	0.12919
One loop	0.09634	0.09856
Two loops	0.09490	0.09541

We see that perturbativity is well under control, but the SM Higgs quartic is rather different!

Classic BSM perspective on the Higgs mass

For many years the standard example has been the MSSM for $\sim$ TeV-scale SUSY:

- Quartic predicted to be determined entirely by gauge couplings at tree level – in large M_H limit have

$$\lambda = \frac{1}{8}(g_Y^2 + g_2^2)\cos^2 2\beta = \frac{M_Z^2}{2v^2}\cos^2 2\beta$$

- Hence $\rightarrow m_h(\text{tree}) \leqslant M_Z$
- $\delta m_h^2(\text{loops}) \geqslant (125\text{GeV})^2 - (M_Z)^2 \geqslant (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$
- Can have $\delta m_h(\text{two loops}) \lesssim 10$ GeV
  $\rightarrow \delta m_h^2(\text{two loops}) \sim 15\% m_h^2!$
- While at three-loop order, have $\delta m_h \sim$ few hundred MeV,
  $\rightarrow \delta m_h^2(\text{three loops}) \lesssim 1\% m_h^2$

Much work has led to: full one-loop calculation, two loops full diagrammatic calculation for $\alpha_s \alpha_t$ only; effective potential approximation and gaugeless limit for (Yukawa coupling)^4 diagrams, and three-loop $\alpha_s^2 \alpha_t$.

Fixed order vs. EFT

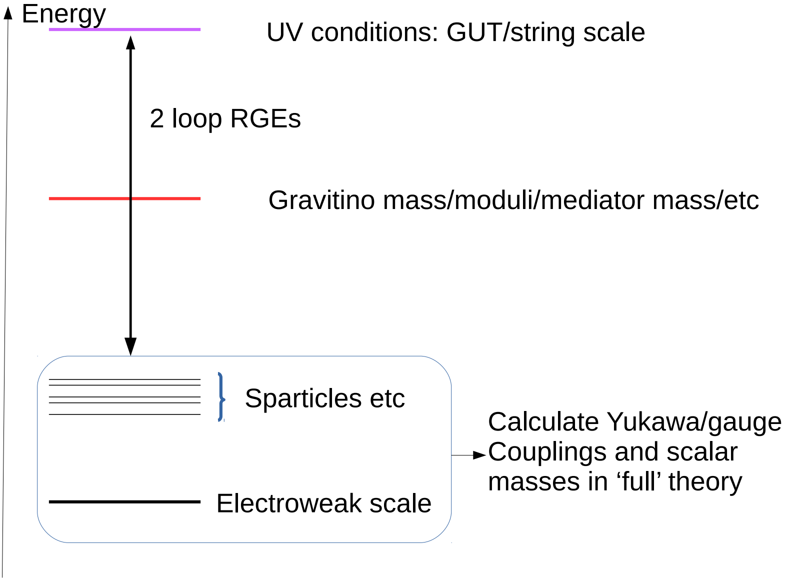
So I have alluded to the two approaches to the Higgs mass calculation:

1. Traditional "fixed-order:" include all of the states in the theory and calculate the Higgs mass at e.g. M_{SUSY}. This is still appropriate if there are some light states or Higgs mixing with other scalars.

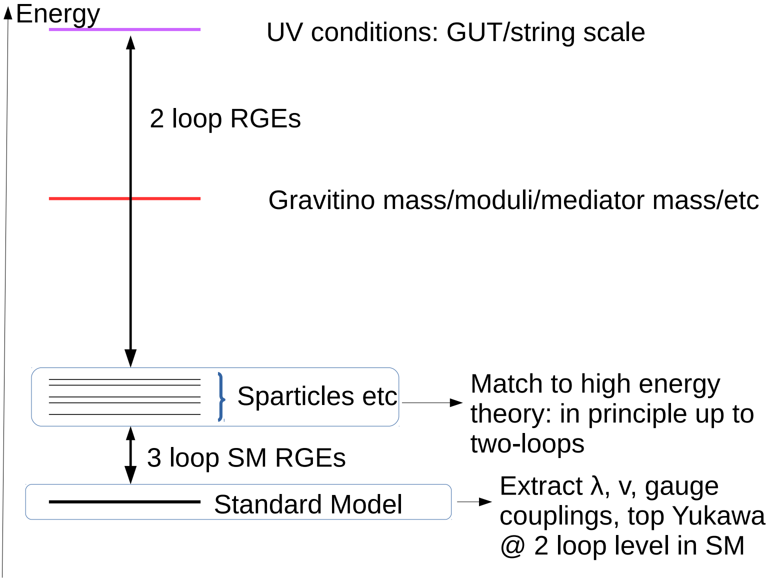
2. Effective Field Theory approach: assume that the Standard Model is valid up to some matching scale. This way large logarithms are automatically resummed through RGEs (3 or more loops in the Standard Model, 2 otherwise) and therefore <u>much</u> more accurate when new physics states are heavy.

The precision available for both is almost identical now, but there are technicalities still to resolve for matching the EFT calculation for a general theory to the SM.

Conventional approach



EFT approach



State of the art for generic models

A summary of what can be done for the generic case:

	Conventional approach	SM	EFT matching
λ/Scalar masses	'Gaugeless EP' 2-loop	'Full' 2-loop/ partial 3 loop	'Gaugeless EP*' 2-loop
Gauge couplings	1-loop	2-loop	1-loop
Yukawas	1-loop	2-loop	1-loop
v	1-loop	2-loop	1-loop
RGEs	2-loop	3 or 4 loops	N/A

- Clearly the extraction of parameters is very important, e.g. in SM $m_h^2 = 2\lambda v^2$ so two-loop extraction of v is technically necessary.
- I will elaborate more in the following on what 'Gaugeless' and 'Gaugeless EP*' mean ...
- But corrections to λ or equivalently computing the scalar masses are the most important – and the subject of this talk!

Extracting λ in the EFT

There is a standard way to calculate threshold corrections to the Higgs quartic:

- Identify combination of scalars in high energy theory (HET) that corresponds to the Higgs: $H = R_{ij}\phi_j$
- Calculate $V_{\text{eff}}(|H|^2)$ in HE theory
- Find $\frac{\partial^4 V_{\text{eff}}}{\partial |H|^4}$.
- Main problem is that generic expressions are painful and not known beyond one loop. There are even subtleties at one loop.

Extracting λ in the EFT: alternative approach

The alternative method, (well known here – see e.g. [Athron, Stöckinger et al 1609.00371, 1710.03760]) – implemented in `FlexibleSUSY` and later `SARAH` is:

- Calculate $M_h^2(m_h^2)$ (i.e. pole mass) in SM and in HET at the matching scale M:

$$M^2_{h,SM}(p^2) = 2\lambda v^2 + \Delta M^2_{h,SM}(p^2)$$

- Set them equal:

$$\rightarrow \lambda = \frac{1}{2v^2}\left[M^2_{h,HET}(m_h^2) - \Delta M^2_{h,SM}(m_h^2)\right]$$

Here we only compute two-point diagrams $\rightarrow$ computationally much easier.

- Hence a code (`SARAH`) that can compute the Higgs mass at two loops via the conventional method can also calculate the λ thresholds ...
- However: there are subtleties involving subleading logs $\rightarrow$ for general theories the results available are not genuinely two-loop, and break down for large scales $\rightarrow$ work in progress.

Calculation of the Higgs mass

The Higgs mass is corrected order by order through two effects:

1. Self energy corrections
$$m^2_{\text{pole}} = m^2_0 + \Pi(m^2_{\text{pole}})$$
2. Shifts to the minimum conditions: we define the potential in terms of real scalars with vevs v to be

$$\begin{aligned} V(v) =& V^{\text{tree}} + \Delta V \equiv \frac{1}{2} m^2_{\text{run}} v^2 + V^{\text{tree}}_{\lambda} + \Delta V \\ \to 0 =& m^2_{\text{run}} v + \left(\frac{\partial V^{\text{tree}}_{\lambda}}{\partial v} + \frac{\partial \Delta V}{\partial v} \right) \end{aligned}$$

If we take v as fixed to all orders (which is convenient since couplings depend on v) we must shift m^2_{run} so that

$$\to m^2_0 = \frac{\partial^2 V^{\text{tree}}}{\partial v^2} = m^2_{\text{run}} + \frac{\partial^2 V^{\text{tree}}_{\lambda}}{\partial v^2} = \underbrace{\left(\frac{\partial^2 V^{\text{tree}}_{\lambda}}{\partial v^2} - \frac{1}{v} \frac{\partial V^{\text{tree}}_{\lambda}}{\partial v} \right)}_{\text{tree-level mass}} - \frac{1}{v} \frac{\partial \Delta V}{\partial v}.$$

So we need the tadpole diagrams as well as self-energies to calculate the mass; note that if we took the masses fixed instead of the vevs we would still have a shift in the mass due to a shift in v (c.f. Higgs tree-level mass of $2\lambda v^2$).

The effective potential approach

Now we turn to calculating two-loop corrections to the Higgs mass/quartic.
One significant simplification to calculations is to take $p^2 = 0$; this is then equivalent to taking

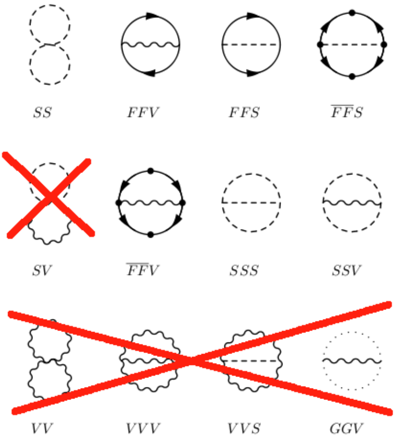
$$\Pi(0) = \frac{\partial^2 \Delta V}{\partial v^2}.$$

Hence the "effective potential limit." When the scale of new physics is above the electroweak scale this is a good approximation, and is better than might be expected even for the Standard Model.

The gaugeless limit

The 'gaugeless limit' is a popular simlification in both SM and BSM:

- Set $g_Y = g_2 = 0$ in two-loop calculation (and any other couplings of broken gauge groups in BSM models) – but keep the important g_3!



- Justified by smallness of α: even if g_2 is not very small, $\alpha_2 \equiv \alpha/s_W^2 \simeq 0.03$, c.f. $\alpha_t \simeq 0.08$, $\alpha_s \simeq 0.12$
- ... and also by lack of large logs involving weak bosons. The approximation works very well – typical correction to the Higgs mass of $\mathcal{O}(10-100)$ MeV.
- On the other hand, it dramatically simplifies calculations.
- Has a special place in the MSSM because $\lambda \propto g_Y^2 + g_2^2$ at tree level $\rightarrow$ kills Higgs self-couplings in the loops.

Generic calculations

- Some contributions of the effective potential are known for the Standard Model up to three and four loop order ...
- Otherwise it is only known in Landau gauge up to two loops. [S. Martin, 01] gave the expression in dimensional regularisation ($\overline{\text{DR}}$ and $\overline{\text{MS}}$) for generic theories.
- [Martin, '03] gave the two-loop scalar self-energies up to $\mathcal{O}(g^2)$ in gauge couplings (don't need g^4 in the gaugeless limit).
- In [MDG, Nickel, Staub 1503.03098] we calculated the tadpoles, and substantial simplifications for massless gauge fields.
- We have implemented in `SARAH` a diagrammatic calculation for self-energies and tadpoles in a "generalised effective potential and gaugeless limit."

The Goldstone Boson Catastrophe

But there is a technical barrier for any theory other than the gaugeless limit of the MSSM: the Goldstone Boson Catastrophe. Note that this includes the Standard Model where it was studied by [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14]!

- Consider for simplicity the Abelian Goldstone Model of one complex scalar $\Phi = \frac{1}{\sqrt{2}}(v + h + iG)$ and tree-level potential

$$V = \mu^2|\Phi|^2 + \lambda|\Phi|^4.$$

- This is a nice prototype for the Standard Model in Landau gauge – but a subtle difference is that the Goldstone boson is physical!
- At tree level, the tadpole equation gives $\mu^2 + \lambda v^2 = 0$, and the masses are $m_G^2 = \mu^2 + \lambda v^2, M_h^2 = \mu^2 + 3\lambda v^2$.
- Recall the potential from earlier:

$$V^{(0)} \supset hv(\mu^2 + \lambda v^2) + \frac{1}{2}(\mu^2 + \lambda v^2)G^2 + \frac{1}{2}(\mu^2 + 3\lambda v^2)h^2 + ...$$

- But we use $m_G^2 \equiv \mu^2 + \lambda v^2$ to calculate loops, and once we include loop corrections we have

$$0 = \mu^2 + \lambda v^2 + \frac{1}{v}\frac{\partial \Delta V}{\partial v}$$

- ... hence $m_G^2 = \mathcal{O}(1 - \text{loop})$ and is of indefinite sign!

One loop

At one loop, this is benign enough:

- For tadpoles proportional to hGG coupling

$$T \sim \lambda v \int \frac{d^d q}{q^2 - m_G^2} \propto m_G^2 (\overline{\log}\, m_G^2 - 1)$$

- For masses, the self-energy diagrams give

$$\Pi \sim \lambda^2 v^2 \int \frac{d^d q}{(q^2 - m_G^2)((q+p)^2 - m_G^2} \propto (\overline{\log}\, p^2 - 2)$$

- So we see that we need to include momentum at one loop for this model (or the Standard Model in Landau gauge)

Beyond one loop

At two loops we find that the tadpole equations give (with $A(x) \equiv x(\log x/Q^2 - 1)$)

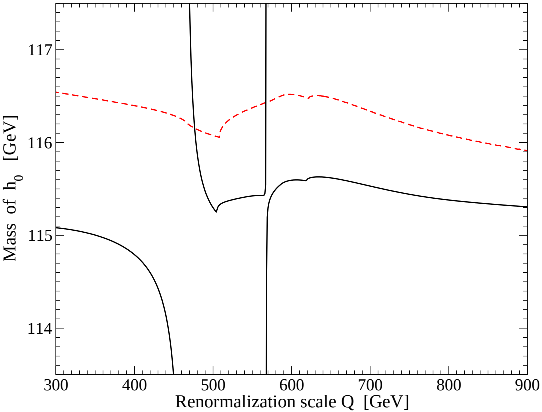
$$0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log \frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{M_h^2} A(M_h^2)\right] + \overbrace{\ldots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}}$$

The problem then extends to two-loop self energies, and becomes even worse for three-loop tadpoles etc.

GB Catastrophe in the MSSM

The problem was identified early on when trying to use the effective potential approach on the full MSSM potential – From S. Martin [hep-ph/0211366]:



Solid line: including EW effects, dashed line without

This shows both the GB catastrophe near $Q = 568$ GeV and the ‘Higgs boson catastrophe’ near 463 GeV.

The special case of the MSSM

So what happened after 2002?

- Martin's calculation was in any case not publically available, nor were there closed-form expressions.
- Instead, until recently almost all spectrum generators for the MSSM (`SPheno`, `SoftSUSY`, `FeynHiggs`) used routines from P. Slavich for $\alpha_s \alpha_t$ and (Yukawa^4) corrections – performed in the gaugeless limit at two loops.
- But: in the MSSM the quartic coupling is given at tree-level by the gauge couplings:

$$\mathcal{L} \supset -\frac{g_Y^2 + g_2^2}{8} (|H_u|^2 - |H_d|^2)^2 \underset{g_Y, g_2 \to 0}{\longrightarrow} 0.$$

- This means that the Goldstone boson <u>does not couple</u> to the Higgs, so the dangerous terms are absent!
- For a long time the problem remained neglected.

Ways out

When we implemented the two-loop calculation in `SARAH`, we had to confront the problem. What to do?

- Calculations in the Standard Model have used Feynman gauge. In general this is much more complicated, the generic expressions are not available – and it anyway does not actually completely solve the problem!
- Otherwise we could simply ignore the phases introduced in the potential and try to find a renormalisation scale Q where $m_G^2 > 0$. But the masses will have a huge sensitivity to the IR effects so we can no longer trust the calculation, because of the way that spectrum generators implement the corrections.
- Indeed, the problem is more serious, because all loop computations were performed using the tree-level masses, which would give <u>exactly</u> massless Goldstones in Landau gauge $\rightarrow$ everywhere divergent tadpoles and self-energies.

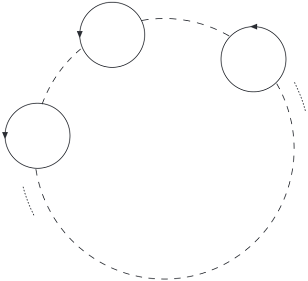
Resummation

A solution for the Standard Model was proposed in [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14]:

The daisy diagram contributes the most singular term at any fixed loop order; it has the most soft Goldstone propagators – each term looks like

$$\int d^4 q \frac{(\Pi_{GG}(q^2))^n}{(q^2 - m_G^2)^n} \sim (\Pi_{GG}(0))^n \frac{\partial^n f(m_G^2)}{\partial (m_G^2)^n}$$

$$f(m_G^2) = -\frac{i}{2} C \int d^d q \log(-q^2 + m_G^2)$$



- $f(x) \equiv \frac{1}{4} x^2 (\overline{\log} x - \frac{3}{2})$.
- But if we sum it to all orders, then we will just find $f(m_G^2 + \Pi_{GG}(0))$

Now, thanks to Goldstone's theorem, we know that the Goldstone boson mass must vanish on-shell, so

$$\mu^2 + \lambda v^2 + \Pi_{GG}(0) = 0 = -\frac{1}{v} \frac{\partial \Delta V}{\partial v} + \Pi_{GG}(0)$$

and since $f(0) = 0$, $f(m_G^2 + \Pi_{GG}(0))$ is finite and has a vanishing first derivative.

Resummation 2

Both papers agree that we should use instead use the resummed potential

$$\hat{V}_{\rm eff} \equiv V_{\rm eff} + \frac{1}{16\pi^2}\left[f(m_G^2 + \Delta) - \sum_{n=0}^{l-1} \frac{\Delta^n}{n!} \left(\frac{\partial}{\partial m_G^2}\right)^n f(m_G^2)\right].$$

The two potentials only differ by terms of order $l+1$. The two papers then differ in how to define Δ (it is not quite $\Pi_{GG}(0)$):

- [Martin, '14] proposed to expand the potential at two loops as a series in m_G^2, and use this to define Δ_1, using $A(x) = 2f'(x) = x(\overline{\log}\, x - 1)$:

  $$V^{(2)} \equiv V^{(2)}|_{m_G^2=0} + \frac{1}{2}\Delta_1 A(m_G^2) + \frac{1}{2}\Omega m_G^2 + \mathcal{O}(m_G^4).$$

  ... so the first derivative is also free of divergences:

  $$\frac{\partial \hat{V}_{\rm eff}}{\partial v} = \frac{\partial V^{(2)}|_{m_G^2=0}}{\partial v} + \frac{1}{2}\Omega \frac{\partial m_G^2}{\partial v} + \mathcal{O}(3-\text{loop})$$

- [Elias-Miro, Espinosa, Konstandin, '14] proposed to use $\Delta_1 \equiv \Pi_g(0)$, defined in terms of the self energy excluding "soft" Goldstones.

Generalising

If we want to apply this to general theories, however, we have two problems:

1. Identifying the Goldstone boson(s) among the scalars: in general the fields can mix!
2. Taking derivatives of the potential as a function of masses and couplings generally means taking derivatives of mixing matrices.

[Martin, Kumar '16] applied this to the MSSM with CP conservation, where they could use 2×2 matrices and do all the derivatives explicitly.

We can do better by taking all of the derivatives implicitly.

We can do better still by adopting a different solution.

On-shell scheme

We saw that we can cure the IR divergences by resumming the Goldstone boson propagators, so that the effective mass in the loop functions became $m_G^2 + \Delta = 0$. But we can do this more directly by just putting the Goldstone boson on shell:

$$(m_G^2)^{\text{run.}} \equiv (m_G^2)^{\text{OS}} - \Pi_{GG}((m_G^2)^{\text{OS}})$$

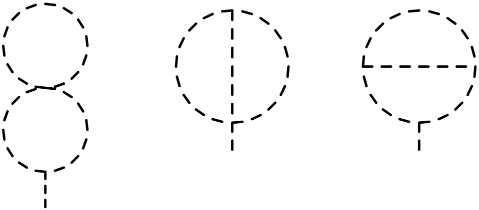
We can do this directly in the tadpole equations – and also the self-energies! So then there should be no need to take derivatives of couplings ... exactly what we want. For example, applying the above shift to the one loop tadpole gives a two-loop correction:

$$\frac{\partial V}{\partial v} \supset \frac{\lambda v}{16\pi^2} A(m_G^2) = \frac{\lambda v}{16\pi^2}\left[\underbrace{A((m_G^2)^{\text{OS}}}_{\to 0} - \underbrace{\Pi_{GG}((m_G^2)^{\text{OS}}) \log \frac{(m_G^2)^{\text{OS}}}{Q^2}}_{\text{cancels divergent part}} + \underbrace{\ldots}_{3-\text{loop}}\right]$$

We also see that $\Pi_{GG}((m_G^2)^{\text{OS}}) = \Pi_g(0)$ (at least at this loop order) automatically!

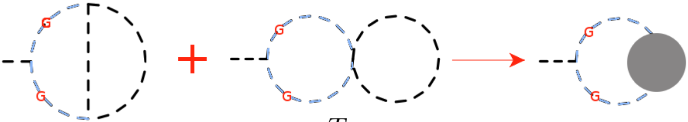
Illustration

To see why this works, let us look at the scalar-only case. There are three classes of tadpole diagrams:



T_{SS} T_{SSS} T_{SSSS}

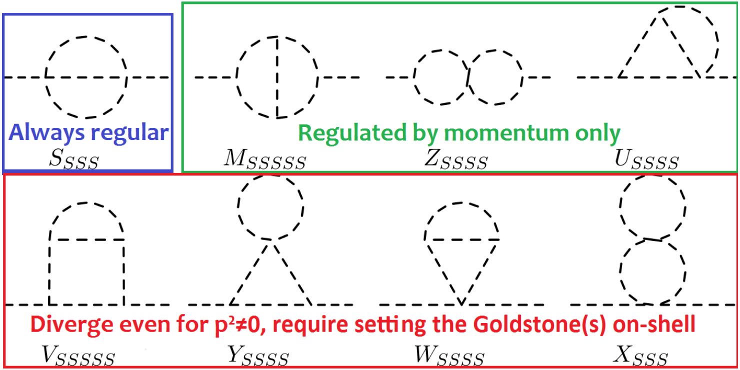
We find that the divergences only come from the T_{SS} and T_{SSSS} topologies, and they correspond to a Goldstone self-energy as a subdiagram and exactly cancel out against the on-shell shift:



T_{SSSS} T_{SS}

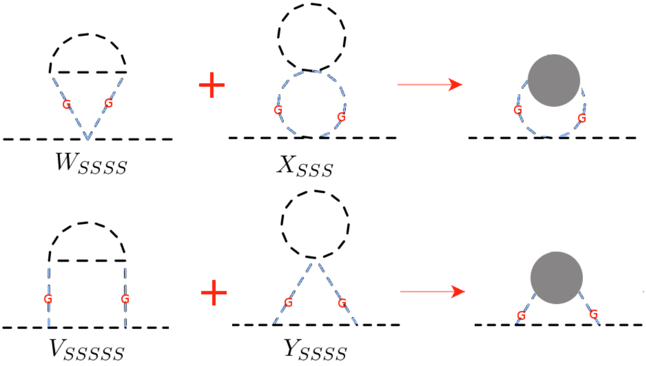
Mass diagrams

We also find that we can apply our on-shell scheme to the cancellation of divergences in self-energies! This seemed hopeless in the former approaches ... We can divide the topologies into three categories:



Mass diagram divergences

Again we find that the divergences in m_G^2 arise from Goldstone boson propagator subdiagrams:



... and once more the one-loop shifts from our on-shell scheme exactly cancel the divergences (but leave a finite momentum-dependent piece).

Generalised effective potential limit

Since we see that there are classes of diagrams that are divergent when the $p^2 \equiv s \neq 0$ and the Goldstone bosons are on-shell, the obvious response is that we cannot avoid using momentum dependence – but this is computationally expensive. Instead, we can expand the self-energies as:

$$\Pi^{(2)}_{ij}(s) = \frac{\overline{\log}(-s)}{s}\Pi^{(2)}_{-1\,l,ij} + \frac{1}{s}\Pi^{(2)}_{-1,ij} + \Pi^{(2)}_{l^2,ij}\,\overline{\log}^2(-s) + \Pi^{(2)}_{l,ij}\,\overline{\log}(-s) + \Pi^{(2)}_{0,ij} + \sum_{k=1}^{\infty}\Pi^{(2)}_{k,ij}\frac{s^k}{k!}$$

If we discard all terms $\mathcal{O}(s)$ and higher, we have a generalised effective potential approximation! We can find closed forms for the singular terms, e.g.

$$U(0,0,0,u) = (\overline{\log}\,u - 1)\,\overline{\log}(-s) - \frac{\pi^2}{6} + \frac{5}{2} - 2\,\overline{\log}\,u - \frac{1}{2}\overline{\log}^2\,u + \mathcal{O}(s).$$

This turns out to be a very good approximation, e.g. even in the Standard Model:

	SARAH/SPheno		SMH (code by Martin & Robertson)	
ξ	1	0.01	0	
2ℓ momentum dependence	partial $s = m_h^2\vert^{\text{tree}}$	partial $s = m_h^2\vert^{\text{tree}}$	full iterative	none $s = 0$
$m_h^{2\ell}$ (GeV)	125.083	125.134	125.176	125.121

Alternative perspective

Another way to solve the problem is to instead expand the masses used in the loop functions perturbatively. Originally, this appears as the need to solve the tadpole equations consistently, because both sides depend on μ:

$$\begin{aligned}\mu^2 =& -\lambda v^2 - \frac{1}{v}\frac{\partial \Delta V(\mu)}{\partial v} \\ =& (\mu^2)^{\text{tree}} - \frac{1}{v}\frac{\partial \Delta V(\mu)}{\partial v} = (\mu^2)^{\text{tree}} - \frac{1}{v}\frac{\Delta V((\mu)^{\text{tree}})}{\partial v} + \frac{1}{v^2}\left(\frac{\partial^2 \Delta V}{\partial v \partial \mu}\right)\left(\frac{\partial \Delta V(\mu^{\text{tree}})}{\partial v}\right) \\ &+ \ldots\end{aligned}$$

Fortunately, in our example only m_G and m_h depend on μ, and it only enters the loop functions through those masses, so writing $T \equiv \frac{\partial \Delta V}{\partial v}$ we have

$$\mu^2 = -\lambda v^2 - \frac{1}{v} T((\mu)^{\text{tree}}) - \frac{\lambda}{v} T\left[\frac{\partial T}{\partial m_G^2} + 3\frac{\partial T}{\partial m_h^2}\right]((\mu)^{\text{tree}}) + \ldots$$

This is termed “consistently solving the tadpole equations.” But it turns out to be almost identical to our on-shell approach!

$$T \supset \frac{\lambda v}{16\pi^2} A(m_G^2) = \frac{\lambda v}{16\pi^2}\left[\underbrace{A((m_G^2)^{\text{tree}}}_{\to 0} - \underbrace{\frac{1}{v} T((\mu)^{\text{tree}}) \log \frac{(m_G^2)^{\text{tree}}}{Q^2}}_{\text{cancels divergent part at two loops}} + \underbrace{\ldots}_{3-\text{loop}}\right]$$

Differences

So what is the difference?

- In our on-shell approach, we should solve the tadpole equations iteratively. This is important for models where the vevs are small so that loop corrections to mass terms would be large – e.g. in models with a triplet scalar.
- However, a crucial part of the above working is that there are no triple Goldstone boson couplings a^{GGG} – otherwise there would be a diagram

$$\Pi_{GG} \sim (a^{GGG})^2 \int \frac{d^d q}{q^2 - (m_G^2)^{OS}} \to \infty.$$

- However, that these couplings are absent – even with CP violation – is a consequence of a Ward Identity.
- BUT, when we have a general theory, we can induce these couplings at one loop since only the full vacuum respects the broken symmetry $\rightarrow$ we use an on-shell condition in the Higgs mass calculation to set $a^{GGG} = 0$ (if we allow CPV)
- In the "consistent solutions" approach we do not need to do this.
- Also no problem with the "gaugeless limit" – our implementation in `SARAH` uses Feynman gauge at one-loop, gaugeless limit at two loops so it is not clear how to consistently iterate the tadpole solutions

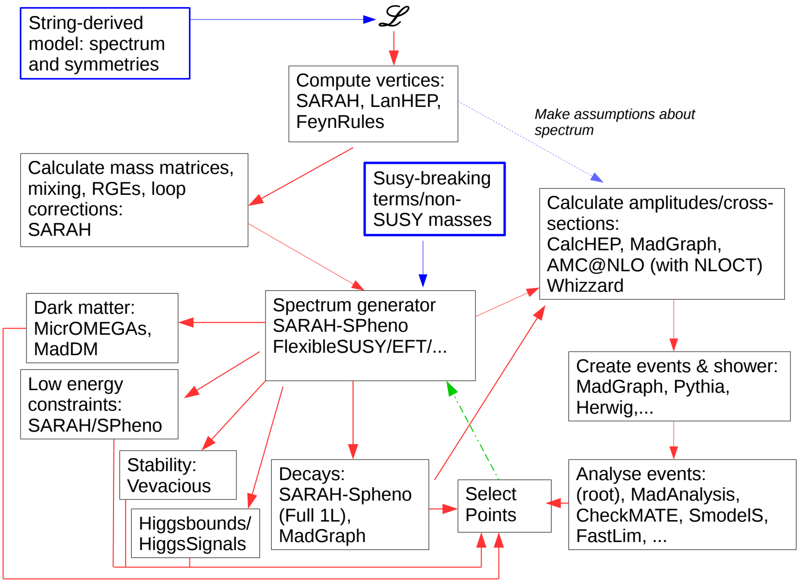
.... hence the implementation in `SARAH` uses the "consistent solutions" approach.

SARAH: a tool for BSM model builders

So what is `SARAH` ?

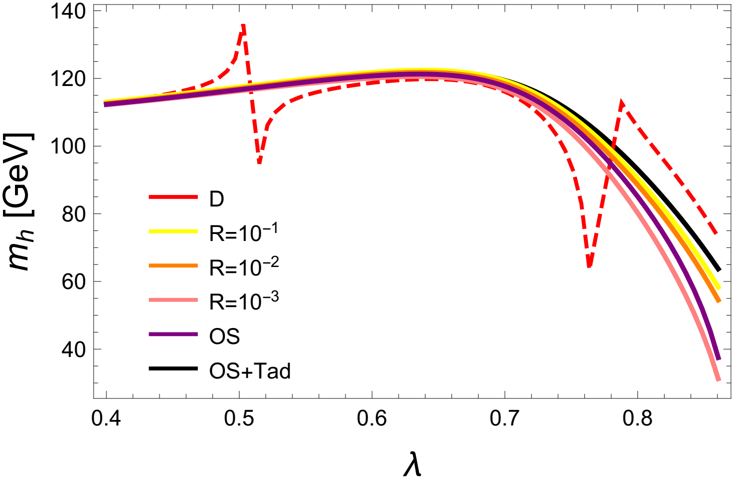
- `Mathematica` package created by F. Staub, with now many contributions from MDG.
- Takes an input model file for any SUSY or non-SUSY model.
- Specify: gauge groups, matter content, superpotential/couplings in Lagrangian.
- Relevant for this talk: spectrum generation with `SPheno`. Produces fortran code which compiles against the `SPheno` library to generate spectrum and precision observables etc for the model.
- Can specify input parameters at any scale: TeV, ~~SUSY~~ scale, GUT scale ...
- Will calculate two-loop RGEs, one-loop masses for all particles in $\overline{\mathrm{DR}}'$ (SUSY) or $\overline{MS}$ (non-SUSY) models, one-loop decays, ...
- Have now implemented two-loop neutral scalar masses including the above GBC solution, allowing Non-SUSY models to be studied.

A web of codes from the top down



First applications

There are a huge number of possible applications of these results – since we can now study non-SUSY models for the first time – and we are only just starting.
We already saw the Standard Model itself and the $\mathbb{Z}_2$ singlet model
But we can also solve the problem beyond the MSSM, e.g. in the NMSSM:

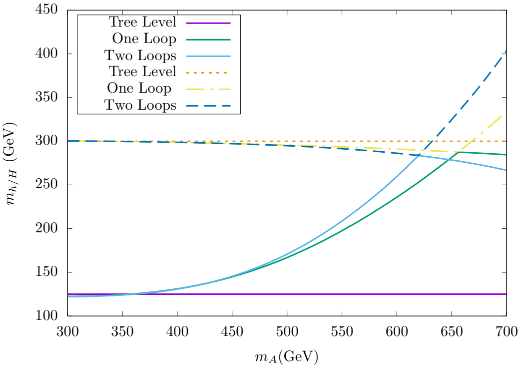


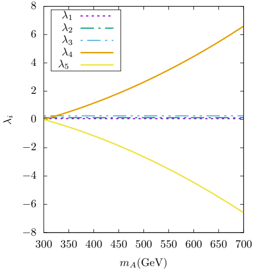
THDM

For a non-SUSY model, consider the Two-Higgs-Doublet-Model, taking as often done the inputs

$$m_h, m_H, m_{H\pm}, m_A^2, m_{12}^2, \tan\alpha, \tan\beta$$

from which we determine $\lambda_i, i = 1..5$. If we enforce the alignment limit of $\tan\alpha = -1/\tan\beta$, we can scan over the other parameters. If we take all of the Heavy Higgs masses to be 300 GeV and scan only over e.g. m_A we find:





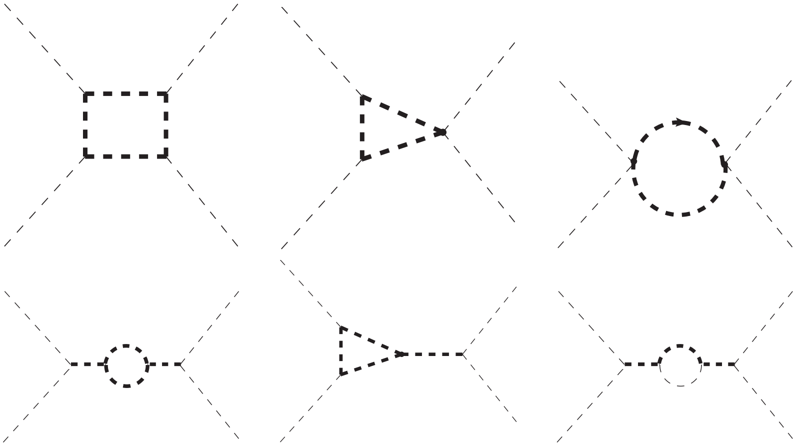
EFT approaches

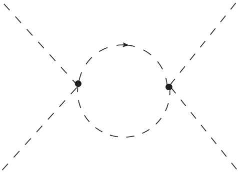
When matching a UV theory onto an EFT, there are several possible approaches:

- Use the path integral to integrate out the heavy fields to derive a Wilsonian action:
$$\int [d\phi_H][d\phi_L] e^{iS[\phi_L,\phi_H]} \rightarrow \int [d\phi_L] e^{i\tilde{S}[\phi_L]}$$
This generates a large set of diagrams to compute
- Match pole masses/physical quantities in both theories $\rightarrow$ can extract quartic couplings from only two-point amplitudes!
- Match the effective actions of both theories using the equations of motion on the full effective action to integrate out the heavy fields $\rightarrow$ still need to compute four-point diagrams, but takes care of combinatorics

... all should give the same results!

Pure diagrams



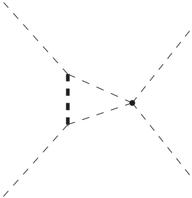
Matches against  in both theories.

Infra-red catastrophes in the EFT approach

- So far, few computations have been done. But at one loop it is not hard to derive the generic expressions. At one-loop, provided there are now couplings of the form $\frac{1}{2} a^{HLL} \phi_H \phi_L^2$, there are few subtleties (except for the mixing between heavy and light states).
- But when we allow these couplings, we first see that they generate tree-level contributions to the quartics:

$$\mathcal{L} \supset -\frac{1}{24} \lambda \phi_L^4 \rightarrow \delta\lambda \supset -3 \frac{(a^{HLL})^2}{m_H^2}$$

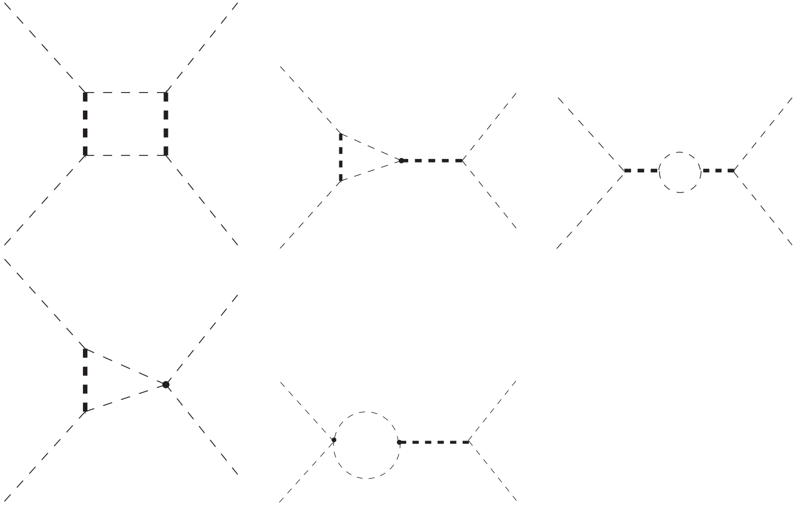
- ... and we then find that the results at one loop are not infra-red safe! E.g.



$$\sim \int d^4 q \frac{1}{q^2 + m_H^2} \frac{1}{q^4} \sim \frac{1}{m_H^2} \int d^4 q \frac{1}{q^4}$$

IR divergent pieces

So the set of relevant terms are:



- The problem can of course be solved perturbatively, by writing $m_H^2 = (m_H^2)^{\text{tree}} + \Delta m_h^2$ etc
- ... but there is also an elegant relation with the GBC and Π_g, which after all involves splitting the EFT into “heavy” and “light” states in the loop!
- It will then be a challenge to extend this to two loops, to put the EFT and conventional calculations on the same footing.

Conclusions

- It is now possible to study the phenomenology of any renormalisable BSM theory with high precision
- For example, `SARAH` now gives the most precise value of the Higgs mass in the NMSSM, and even for the MSSM with CPV is the only code appropriate for top-down analysis (DR' as opposed to on-shell scheme inputs).
- From the top-down perspective, can potentially rule models out based on field content, choices of couplings and mass scales.
- Ongoing work will further refine the precision: EFT approach, including EW contributions, ...
- I haven't talked about Higgs couplings, decays etc but these can also be studied.