Physics at the LHC at the precision frontier

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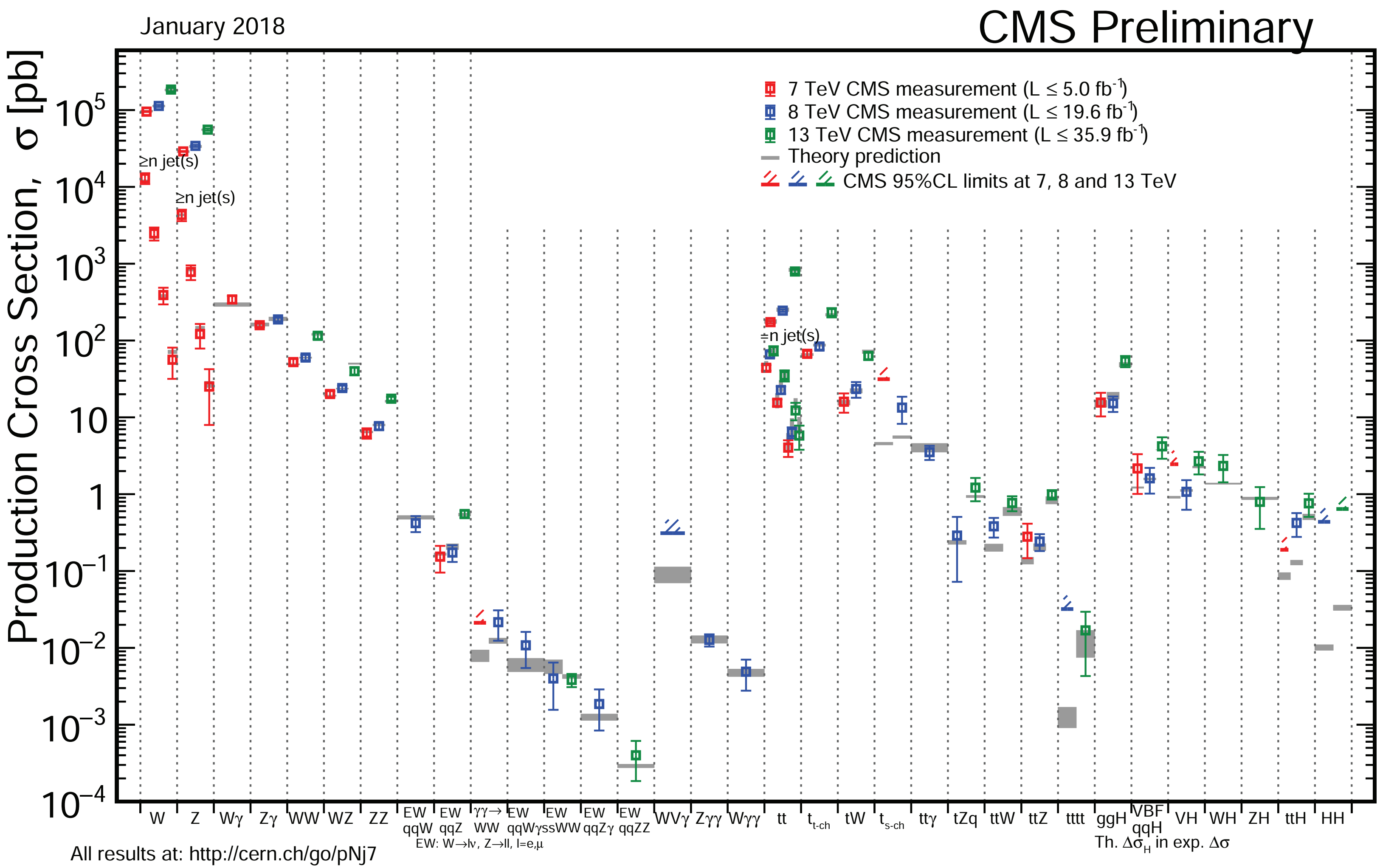
Standard Model cross sections

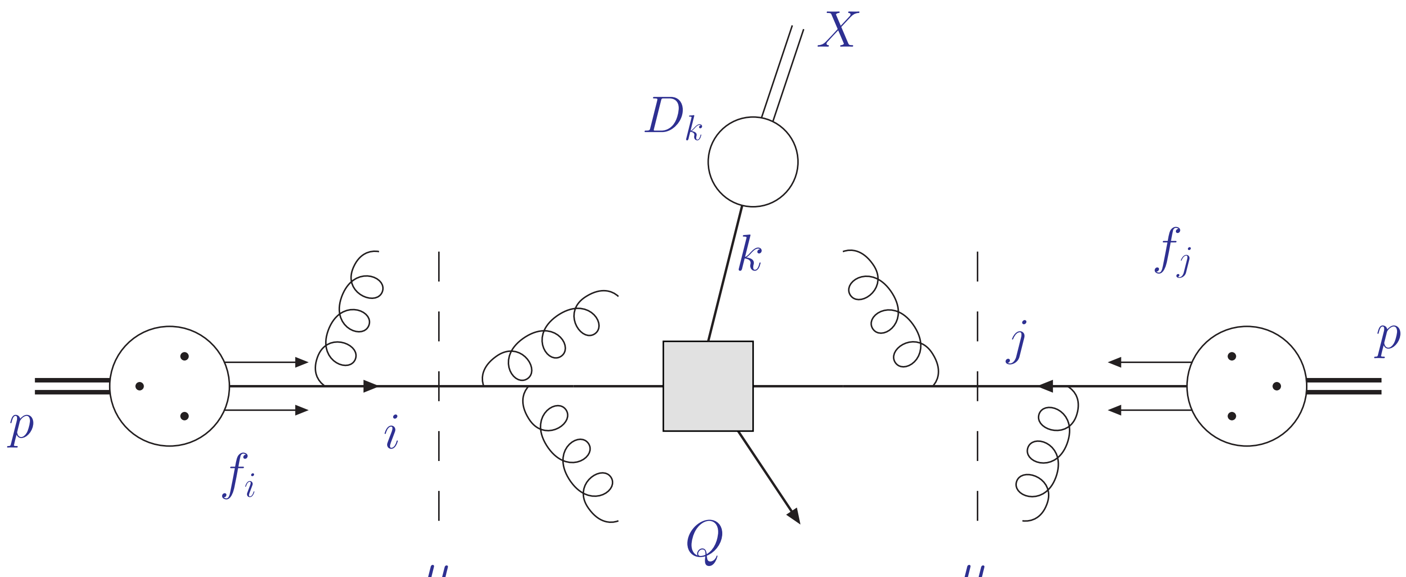
Cross sections for Standard Model processes at the LHC

- Hadroproduction of top-quarks (+ jets) and single-tops CMS coll. '18



QCD factorization

QCD factorization

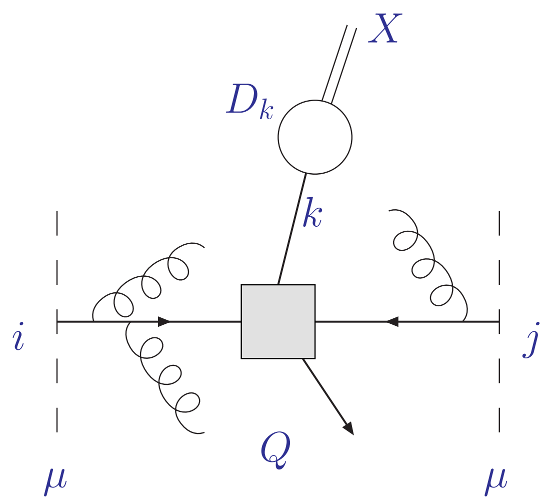


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij \to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - N^{3}LO (next-to-next-to-next-to-leading order)
  - . . .

Parton luminosity

- Long distance dynamics due to proton structure

- Cross section depends on parton distributions f_i

$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots \right]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage

PDF landscape

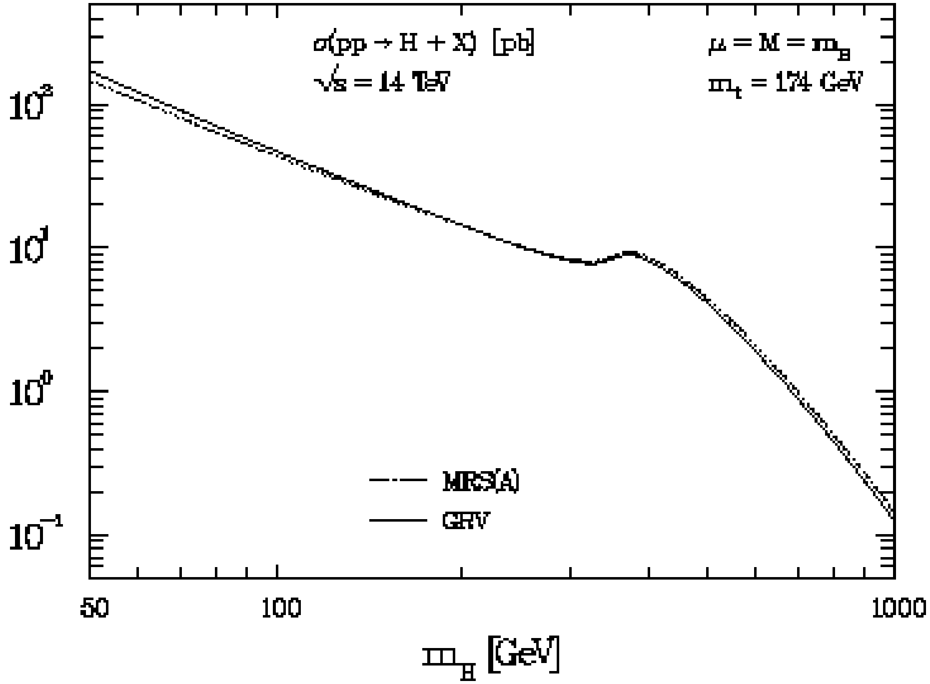
- Significant number of active groups ABMP16, CJ15, CT14, HERAPDF2.0, JR14, MMHT14, NNPDF3.1
  - PDFs accurate to NNLO in QCD, except for CJ15 (NLO)
  - different choices of data sets
  - different fitting procedures ($\Delta\chi^2$ criterium)

PDF sets	$\Delta\chi^2$ criterion	data sets used in analysis
ABMP16 arXiv:1701.05838	1	incl. DIS, DIS charm, DY, $t\bar{t}$, single t
CJ15 arXiv:1602.03154	1	incl. DIS, DY (incl. $p\bar{p} \to W^{\pm}X$), $p\bar{p}$ jets, γ+jet
CT14 arXiv:1506.07443	100	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, pp jets
HERAPDF2.0 arXiv:1506.06042	1	incl. DIS, DIS charm, DIS jets
JR14 arXiv:1403.1852	1	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, DIS jets
MMHT14 arXiv:1510.02332	$2.3 \ldots 42.3$ (dynamical)	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, pp jets, $t\bar{t}$
NNPDF3.1 arXiv:1706.00428	n.a.	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, pp jets, $t\bar{t}$, $W+$charm, Zp_T

Higgs boson production

Higgs cross section (1995)

NLO QCD corrections



MRS(A): Martin, Roberts and Stirling, Phys. Rev. D50 (1994) 6734

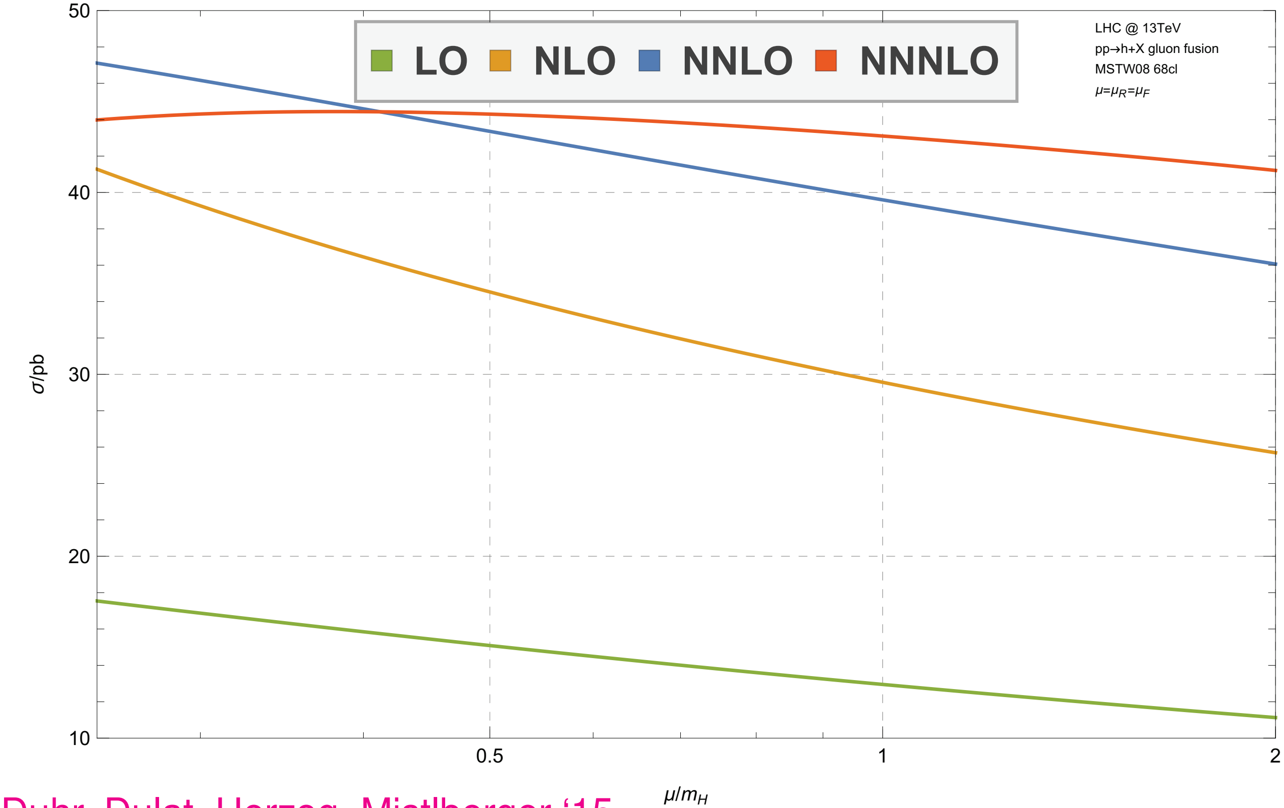
GRV: Glück, Reya and Vogt, Z. Phys. C53 (1992) 127

One of the main uncertainties *in the prediction of the Higgs production cross section is due to the* **gluon density***. [. . .] Adopting a set of representative parton distributions [. . .], we find a* **variation of about 7%** *between the maximum and minimum values of the cross section for Higgs masses above* $\sim$ *100 GeV.*

Spira, Djouadi, Graudenz, Zerwas (1995)
hep-ph/9504378

Higgs cross section (2018)

Exact N^{3}LO QCD corrections

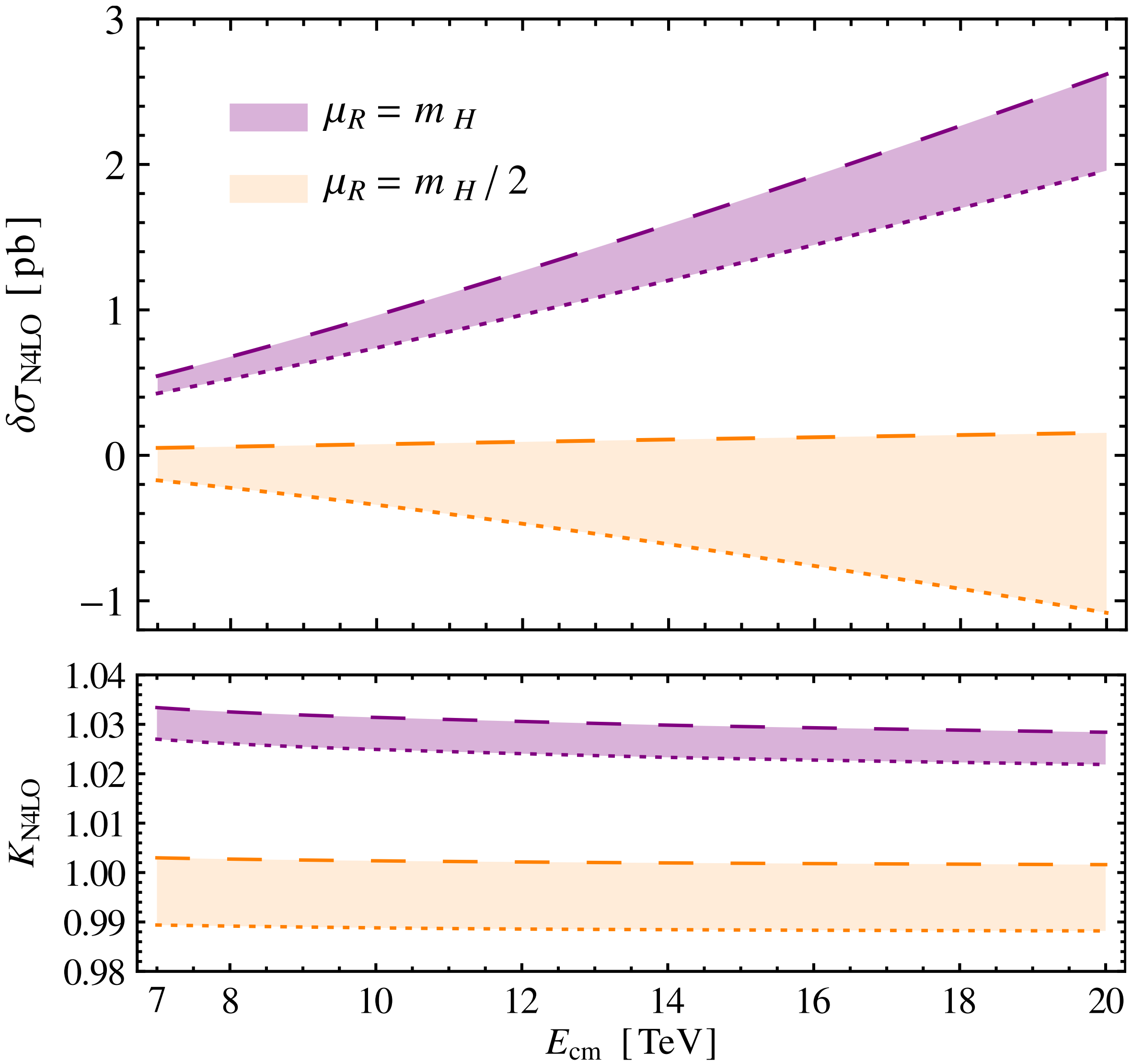


Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15

- Apparent convergence of perturbative expansion
- Scale dependence of exact N^{3}LO prediction with residual uncertainty 3%
- Minimal sensitivity at scale $\mu = m_H/2$

Approximate N^{4}LO QCD corrections

- Consistency check with approximate N^{4}LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- K-factor $\simeq 1\%$ for $\mu = m_H/2$ with at $\sqrt{s} = 13$ TeV
  de Florian, Mazzitelli, S.M., Vogt '14



Cross section dependence of PDFs

- Cross section $\sigma(H)$ at NNLO with uncertainties: $\sigma(H) + \Delta\sigma(\mathrm{PDF} + \alpha_\mathrm{s})$ for $m_H = 125.0$ GeV at $\sqrt{s} = 13$ TeV with $\mu_R, \mu_F = m_H$ and nominal α_s

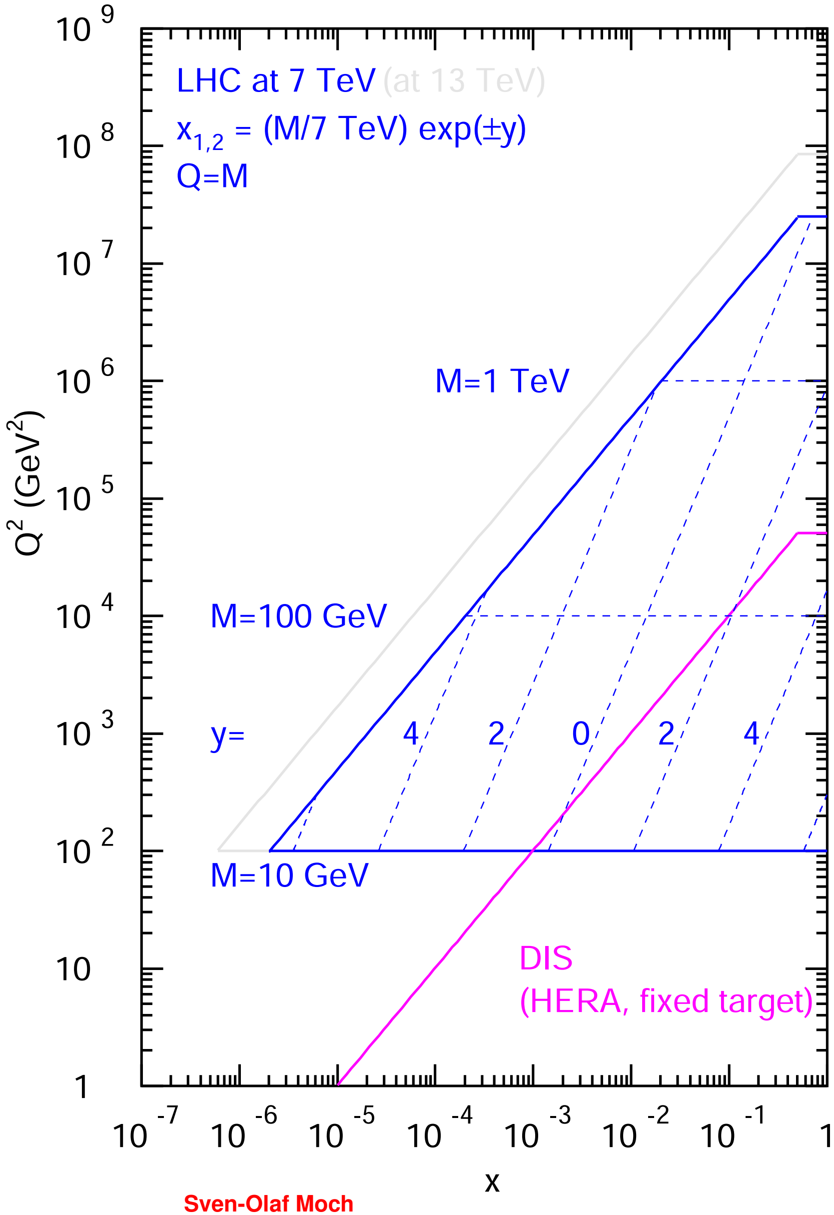
PDF sets	$\sigma(H)^{\mathrm{NNLO}}$ [pb] nominal $\alpha_s(M_Z)$
ABMP16 Alekhin, Blümlein, S.M., Placakyte '17	40.20 ± 0.63
CJ15 Accardi, Brady, Melnitchouk et al. '16	$42.45\ ^{+\ 1.73}_{-\ 1.12}$
CT14 Dulat et al. '15	$42.33\ ^{+\ 1.43}_{-\ 1.68}$
HERAPDF2.0 H1+Zeus Coll.	$42.62\ ^{+\ 0.35}_{-\ 0.43}$
JR14 (dyn) Jimenez-Delgado, Reya '14	38.01 ± 0.34
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	$42.36\ ^{+\ 0.56}_{-\ 0.78}$
NNPDF3.1 Ball et al. '17	42.98 ± 0.40
PDF4LHC15 Butterworth et al. '15	42.42 ± 0.78

- Large spread for predictions from different PDFs $\sigma(H) = 38.0 \ldots 43.0$ pb
- PDF and α_s differences between sets amount to up to 12%
  - significantly larger than residual theory uncertainty due to N^3LO QCD and NLO electroweak corrections

Parton content of the proton

Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with $x_{1,2} = M/\sqrt{S} e^{\pm y}$
  - forward rapidities sensitive to small-x
- Cross section depends on convolution of parton distributions
  - small-x part of f_i and large-x PDFs f_j

$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots \right]$$

Evolution equations

- Parton distribution functions $q_i(x, \mu^2)$, $\bar{q}_i(x, \mu^2)$ and $g(x, \mu^2)$ for quarks, antiquarks of flavour i and gluons
- Flavor non-singlet combinations with $2n_f - 1$ scalar evolution equations

$$q^{\pm}_{\mathrm{ns},ik} = (q_i \pm \bar{q}_i) - (q_k \pm \bar{q}_k) \quad \text{and} \quad q^{\mathrm{v}}_{\mathrm{ns}} = \sum_{i=1}^{n_f} (q_i - \bar{q}_i)$$

with $\displaystyle \frac{d}{d \ln \mu^2}\, q^{\pm,\mathrm{v}}_{\mathrm{ns}} = P^{\pm,\mathrm{v}}_{\mathrm{ns}} \otimes q^{\pm,\mathrm{v}}_{\mathrm{ns}}$

  - splitting functions $P^{\pm}_{\mathrm{ns}}$ and $P^{\mathrm{v}}_{\mathrm{ns}} = P^{-}_{\mathrm{ns}} + P^{\mathrm{s}}_{\mathrm{ns}}$

- Flavor singlet (2×2 matrix) evolution equations

$$\frac{d}{d \ln \mu^2} \begin{pmatrix} q_{\mathrm{s}} \\ g \end{pmatrix} = \begin{pmatrix} P_{\mathrm{qq}} & P_{\mathrm{qg}} \\ P_{\mathrm{gq}} & P_{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} q_{\mathrm{s}} \\ g \end{pmatrix} \quad \text{and} \quad q_{\mathrm{s}} = \sum_{i=1}^{n_f} (q_i + \bar{q}_i)$$

  - quark-quark splitting function $P_{\mathrm{qq}} = P^{+}_{\mathrm{ns}} + P_{\mathrm{ps}}$

- Perturbative expansion of splitting functions up to N^{3}LO

$$P_{ij} = \alpha_s P^{(0)}_{ij} + \alpha_s^2 P^{(1)}_{ij} + \alpha_s^3 P^{(2)}_{ij} + \alpha_s^4 P^{(3)}_{ij} + \ldots$$

Data in global PDF fits

Data sets considered in ABMP16 analysis

- Analysis of world data for deep-inelastic scattering, fixed-target data for Drell-Yan process and collider data ($W^{\pm}$-, Z-bosons, top-quarks)
  - inclusive DIS data HERA, BCDMS, NMC, SLAC ($NDP = 2155$)
  - semi-inclusive DIS charm-, bottom-quark data HERA ($NDP = 81$)
  - Drell-Yan data (fixed target) E-605, E-866 ($NDP = 158$)
  - neutrino-nucleon DIS (di-muon data) CCFR/NuTeV, CHORUS, NOMAD ($NDP = 232$)
  - $W^{\pm}$-, Z-boson production data D0, ATLAS, CMS, LHCb ($NDP = 172$)
  - inclusive top-quark hadro-production CDF&D0, ATLAS, CMS ($NDP = 24$)

Iterative cycle of PDF fits

i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of PDFs, strong coupling $\alpha_s(M_Z)$ and heavy quark masses m_c, m_b, m_t,

Theory considerations in PDF fits

Theory considerations in ABMP16

- Strictly NNLO QCD for determination of PDFs and α_s
- Consistent scheme for treatment of heavy quarks
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Full account of error correlations

Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- PDF parameters, α_s, m_c, m_b and m_t sensitive to
  - radiative corrections at higher orders
  - chosen scheme (e.g. ($\overline{MS}$ scheme)
  - renormalization and factorization scales μ_R, μ_F
  - ...

ABMP16 PDF ansatz

- PDFs parameterization at scale $\mu_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M., Placakyte '17
  - ansatz for valence-/sea-quarks, gluon

$$
\begin{aligned}
xq_v(x,\mu_0^2) &= \frac{2\delta_{qu}+\delta_{qd}}{N_q^v} x^{a_q}(1-x)^{b_q} x^{P_{qv}(x)} \\
xq_s(x,\mu_0^2) &= x\bar{q}_s(x,\mu_0^2) \;=\; A_{qs}(1-x)^{b_{qs}} x^{a_{qs} P_{qs}(x)} \\
xg(x,\mu_0^2) &= A_g x^{a_g}(1-x)^{b_g} x^{a_g\, P_g(x)}
\end{aligned}
$$

  - strange quark is taken in charge-symmetric form
  - function $P_p(x)$

$$
P_p(x) \;=\; (1+\gamma_{-1,p}\ln x)\left(1+\gamma_{1,p}x+\gamma_{2,p}x^2+\gamma_{3,p}x^3\right)\ ,
$$

  - 29 parameters in fit including $\alpha_s^{(n_f=3)}(\mu_0 = 3$ GeV), m_c, m_b and m_t
  - simultaneous fit of higher twist parameters (twist-4)
- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

Covariance matrix

- Positive-definite covariance matrix
  - correlations for fit parameters of ABMP16 PDFs

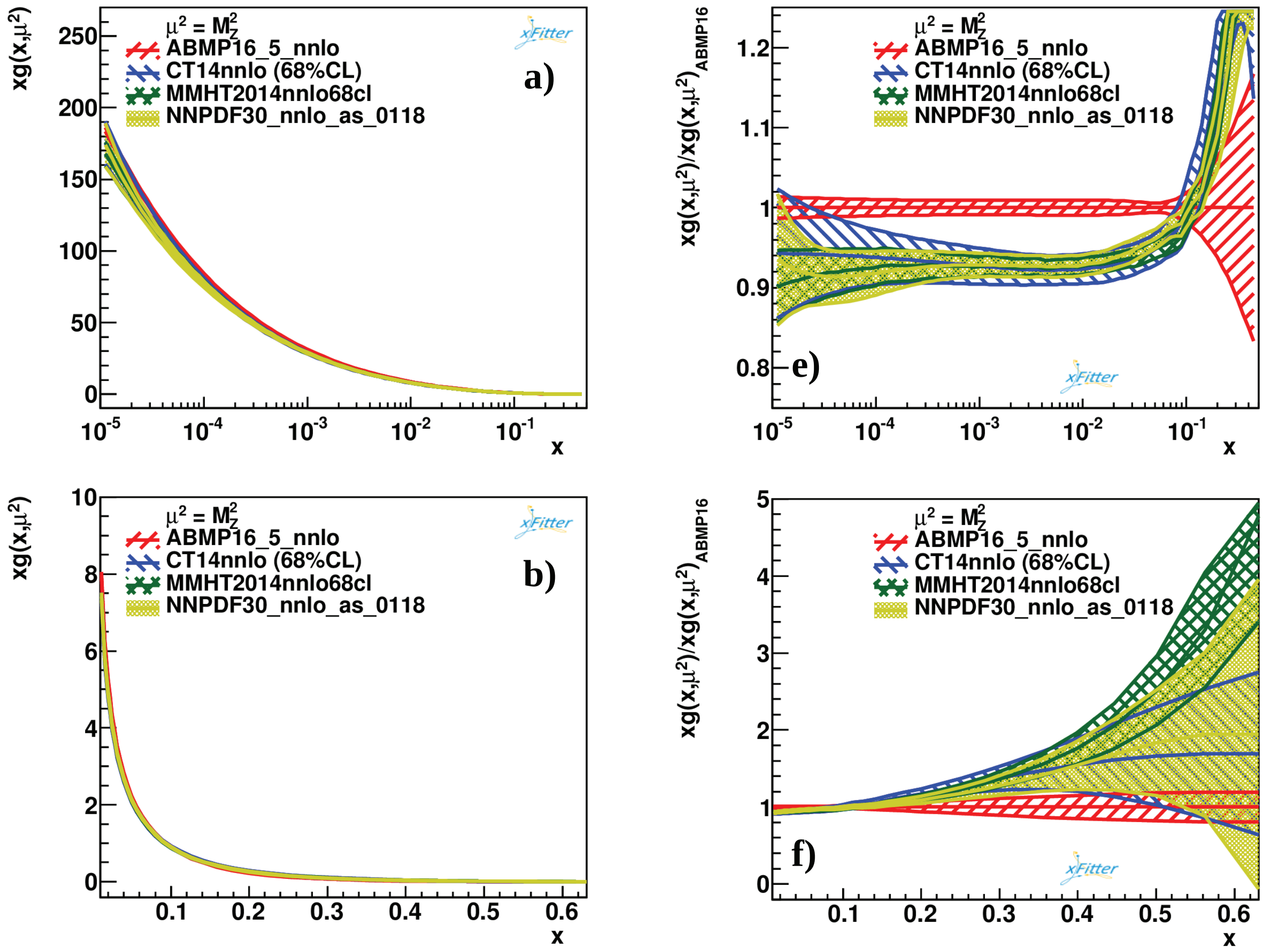
	a_u	b_u	$\gamma_{1,u}$	$\gamma_{2,u}$	$\gamma_{3,u}$	a_d	b_d	$\gamma_{1,d}$	$\gamma_{2,d}$	$\gamma_{3,d}$
a_u	1.0	0.7617	0.9372	-0.5078	0.4839	0.4069	0.3591	0.4344	-0.3475	0.0001
b_u	0.7617	1.0	0.6124	-0.1533	-0.0346	0.3596	0.2958	0.3748	-0.2748	0.0001
$\gamma_{1,u}$	0.9372	0.6124	1.0	-0.7526	0.7154	0.2231	0.2441	0.2812	-0.2606	0.0001
$\gamma_{2,u}$	-0.5078	-0.1533	-0.7526	1.0	-0.9409	0.2779	0.2276	0.2266	-0.1860	0.0
$\gamma_{3,u}$	0.4839	-0.0346	0.7154	-0.9409	1.0	-0.1738	-0.1829	-0.1327	0.1488	0.0
a_d	0.4069	0.3596	0.2231	0.2779	-0.1738	1.0	0.7209	0.9697	-0.6529	0.0001
b_d	0.3591	0.2958	0.2441	0.2276	-0.1829	0.7209	1.0	0.7681	-0.9786	-0.0001
$\gamma_{1,d}$	0.4344	0.3748	0.2812	0.2266	-0.1327	0.9697	0.7681	1.0	-0.7454	0.0002
$\gamma_{2,d}$	-0.3475	-0.2748	-0.2606	-0.1860	0.1488	-0.6529	-0.9786	-0.7454	1.0	-0.0002
$\gamma_{3,d}$	0.0001	0.0001	0.0001	0.0	0.0	0.0001	-0.0001	0.0002	-0.0002	1.0
a_{us}	-0.0683	-0.0081	-0.2094	0.3881	-0.3206	0.2266	0.1502	0.2000	-0.1293	0.0
b_{us}	-0.3508	-0.3089	-0.3462	0.0906	-0.0537	-0.1045	-0.2000	-0.2241	0.2798	0.0
$\gamma_{-1,us}$	0.2296	0.1387	0.3367	-0.4043	0.3474	-0.1171	-0.1127	-0.0810	0.0767	0.0
$\gamma_{1,us}$	-0.4853	-0.4119	-0.3844	-0.0365	0.0064	-0.4380	-0.3592	-0.4957	0.3771	-0.0001
A_{us}	0.0506	0.0807	-0.0949	0.3198	-0.2560	0.2527	0.1648	0.2350	-0.1509	0.0
a_{ds}	-0.0759	-0.0443	-0.0951	0.0263	-0.0382	-0.2565	-0.2541	-0.2666	0.2380	0.0
b_{bs}	0.0452	-0.0197	0.0345	-0.0589	0.0683	-0.2084	0.0190	-0.1841	-0.0522	0.0
$\gamma_{1,ds}$	-0.0492	-0.0809	0.0101	-0.1791	0.1309	-0.5576	-0.2029	-0.4584	0.0946	0.0
A_{ds}	-0.1980	-0.1262	-0.2349	0.1526	-0.1428	-0.1113	-0.2167	-0.1739	0.2407	0.0
a_{ss}	-0.2034	-0.1285	-0.2362	0.2328	-0.2080	0.0960	0.1596	0.0661	-0.1054	0.0
b_{ss}	-0.1186	-0.0480	-0.1532	0.1549	-0.1536	0.0486	0.1508	0.0267	-0.1161	0.0
A_{ss}	-0.1013	-0.0411	-0.1458	0.1802	-0.1625	0.1216	0.1678	0.0924	-0.1196	0.0
a_g	0.0046	-0.0374	0.1109	-0.1934	0.1653	-0.0288	-0.0122	0.0053	0.0059	0.0
b_g	0.2662	0.3141	0.1579	-0.0050	-0.0207	0.0973	0.0870	0.0646	-0.0666	0.0
$\gamma_{1,g}$	0.2008	0.2274	0.0706	0.0876	-0.0835	0.0919	0.0574	0.0493	-0.0364	0.0
$\alpha_s^{(n_f=3)}(\mu_0)$	0.1083	-0.0607	0.0848	-0.0250	0.0765	0.0763	-0.0306	0.0725	0.0243	0.0
$m_c(m_c)$	-0.0006	0.0170	-0.0104	0.0206	-0.0201	-0.0123	-0.0161	-0.0114	0.0108	0.0
$m_b(m_b)$	0.0661	0.0554	0.0605	-0.0367	0.0287	-0.0116	0.0029	-0.0074	-0.0051	0.0
$m_t(m_t)$	-0.1339	-0.2170	-0.0816	0.0081	0.0250	-0.0616	-0.0813	-0.0491	0.0736	0.0

	a_{us}	b_{us}	$\gamma_{-1,us}$	$\gamma_{1,us}$	A_{us}	a_{ds}	b_{bs}	$\gamma_{1,ds}$	A_{ds}	a_{ss}
a_u	-0.0683	-0.3508	0.2296	-0.4853	0.0506	-0.0759	0.0452	-0.0492	-0.1980	-0.2034
b_u	-0.0081	-0.3089	0.1387	-0.4119	0.0807	-0.0443	-0.0197	-0.0809	-0.1262	-0.1285
$\gamma_{1,u}$	-0.2094	-0.3462	0.3367	-0.3844	-0.0949	-0.0951	0.0345	0.0101	-0.2349	-0.2362
$\gamma_{2,u}$	0.3881	0.0906	-0.4043	-0.0365	0.3198	0.0263	-0.0589	-0.1791	0.1526	0.2328
$\gamma_{3,u}$	-0.3206	-0.0537	0.3474	0.0064	-0.2560	-0.0382	0.0683	0.1309	-0.1428	-0.2080
a_d	0.2266	-0.1045	-0.1171	-0.4380	0.2527	-0.2565	-0.2084	-0.5576	-0.1113	0.0960
b_d	0.1502	-0.2000	-0.1127	-0.3592	0.1648	-0.2541	0.0190	-0.2029	-0.2167	0.1596
$\gamma_{1,d}$	0.2000	-0.2241	-0.0810	-0.4957	0.2350	-0.2666	-0.1841	-0.4584	-0.1739	0.0661
$\gamma_{2,d}$	-0.1293	0.2798	0.0767	0.3771	-0.1509	0.2380	-0.0522	0.0946	0.2407	-0.1054
$\gamma_{3,d}$	0.0	0.0	0.0	-0.0001	0.0	0.0	0.0	0.0	0.0	0.0
a_{us}	1.0	-0.3156	-0.8947	-0.5310	0.9719	0.2849	0.0241	-0.0470	0.2983	0.4131
b_{us}	-0.3156	1.0	0.1372	0.8258	-0.3995	0.0467	-0.0221	-0.1190	0.1856	0.0291
$\gamma_{-1,us}$	-0.8947	0.1372	1.0	0.2611	-0.7829	-0.1695	0.0156	0.0501	-0.2117	-0.7191
$\gamma_{1,us}$	-0.5310	0.8258	0.2611	1.0	-0.6479	0.0086	0.0076	0.1460	0.0781	-0.0010
A_{us}	0.9719	-0.3995	-0.7829	-0.6479	1.0	0.2983	0.0515	-0.0404	0.3055	0.2811
a_{ds}	0.2849	0.0467	-0.1695	0.0086	0.2983	1.0	-0.1608	0.0719	0.9152	-0.2941
b_{bs}	0.0241	-0.0221	0.0156	0.0076	0.0515	-0.1608	1.0	0.7834	-0.3022	-0.0390
$\gamma_{1,ds}$	-0.0470	-0.1190	0.0501	0.1460	-0.0404	0.0719	0.7834	1.0	-0.1838	-0.1373
A_{ds}	0.2983	0.1856	-0.2117	0.0781	0.3055	0.9152	-0.3022	-0.1838	1.0	-0.1833
a_{ss}	0.4131	0.0291	-0.7191	-0.0010	0.2811	-0.2941	-0.0390	-0.1373	-0.1833	1.0
b_{ss}	0.2197	0.0643	-0.4479	0.1286	0.1193	-0.1579	-0.0260	0.0169	-0.0896	0.6522
A_{ss}	0.3627	0.0261	-0.6319	0.0102	0.2412	-0.2688	-0.0180	-0.0960	-0.1797	0.9280
a_g	-0.2570	0.0001	0.2196	0.0039	-0.2493	-0.2190	-0.0454	-0.1031	-0.2571	0.0626
b_g	-0.1419	0.1266	0.0694	0.2648	-0.1715	-0.0515	0.0917	0.2130	-0.0469	-0.0092
$\gamma_{1,g}$	-0.0241	0.0332	-0.0226	0.1296	-0.0489	-0.0137	0.0503	0.1409	0.0022	-0.0279
$\alpha_s^{(n_f=3)}(\mu_0)$	0.0954	-0.2866	-0.0341	-0.3493	0.1110	-0.0604	-0.1265	-0.1811	-0.1330	-0.0841
$m_c(m_c)$	0.0704	-0.0093	-0.0033	-0.0462	0.1182	0.0849	0.0547	0.0413	0.1193	-0.0728
$m_b(m_b)$	-0.0183	-0.0132	0.0044	0.0209	-0.0298	-0.0006	0.0332	0.0695	-0.0432	-0.0159
$m_t(m_t)$	0.0641	-0.1841	-0.0408	-0.2635	0.0755	-0.0573	-0.1067	-0.2003	-0.0869	0.0169

	b_{ss}	A_{ss}	a_g	b_g	$\gamma_{1,g}$	$\alpha_s^{(n_f=3)}(\mu_0)$	$m_c(m_c)$	$m_b(m_b)$	$m_t(m_t)$
a_u	-0.1186	-0.1013	0.0046	0.2662	0.2008	0.1083	-0.0006	0.0661	-0.1339
b_u	-0.0480	-0.0411	-0.0374	0.3141	0.2274	-0.0607	0.0170	0.0554	-0.2170
$\gamma_{1,u}$	-0.1532	-0.1458	0.1109	0.1579	0.0706	0.0848	-0.0104	0.0605	-0.0816
$\gamma_{2,u}$	0.1549	0.1802	-0.1934	-0.0050	0.0876	-0.0250	0.0206	-0.0367	0.0081
$\gamma_{3,u}$	-0.1536	-0.1625	0.1653	-0.0207	-0.0835	0.0765	-0.0201	0.0287	0.0250
a_d	0.0486	0.1216	-0.0288	0.0973	0.0919	0.0763	-0.0123	-0.0116	-0.0616
b_d	0.1508	0.1678	-0.0122	0.0870	0.0574	-0.0306	-0.0161	0.0029	-0.0813
$\gamma_{1,d}$	0.0267	0.0924	0.0053	0.0646	0.0493	0.0725	-0.0114	-0.0074	-0.0491
$\gamma_{2,d}$	-0.1161	-0.1196	0.0059	-0.0666	-0.0364	0.0243	0.0108	-0.0051	0.0736
$\gamma_{3,d}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
a_{us}	0.2197	0.3627	-0.2570	-0.1419	-0.0241	0.0954	0.0704	-0.0183	0.0641
b_{us}	0.0643	0.0261	0.0001	0.1266	0.0332	-0.2866	-0.0093	-0.0132	-0.1841
$\gamma_{-1,us}$	-0.4479	-0.6319	0.2197	0.0694	-0.0226	-0.0341	-0.0034	0.0044	-0.0408
$\gamma_{1,us}$	0.1286	0.0102	0.0039	0.2648	0.1296	-0.3493	-0.0462	0.0209	-0.2635
A_{us}	0.1193	0.2412	-0.2493	-0.1715	-0.0489	0.1110	0.1182	-0.0298	0.0755
a_{ds}	-0.1579	-0.2688	-0.2190	-0.0515	-0.0137	-0.0604	0.0849	-0.0006	-0.0573
b_{bs}	-0.0260	-0.0180	-0.0454	0.0917	0.0503	-0.1265	0.0547	0.0332	-0.1067
$\gamma_{1,ds}$	0.0169	-0.0960	-0.1031	0.2130	0.1409	-0.1811	0.0413	0.0695	-0.2003
A_{ds}	-0.0896	-0.1797	-0.2571	-0.0469	0.0022	-0.1330	0.1193	-0.0432	-0.0869
a_{ss}	0.6522	0.9280	0.0626	-0.0092	-0.0279	-0.0841	-0.0728	-0.0159	0.0169
b_{ss}	1.0	0.6427	-0.0179	0.1967	0.1164	-0.2390	-0.0965	0.0169	-0.1675
A_{ss}	0.6427	1.0	-0.0211	0.1403	0.0997	-0.1385	0.0216	0.0072	-0.1109
a_g	-0.0179	-0.0211	1.0	-0.5279	-0.8046	0.1838	-0.2829	0.0076	0.3310
b_g	0.1967	0.1403	-0.5279	1.0	0.8837	-0.5124	0.1438	0.1255	-0.7275
$\gamma_{1,g}$	0.1164	0.0997	-0.8046	0.8837	1.0	-0.2511	0.1829	0.0814	-0.5180
$\alpha_s^{(n_f=3)}(\mu_0)$	-0.2390	-0.1385	0.1838	-0.5124	-0.2511	1.0	-0.1048	0.0423	0.6924
$m_c(m_c)$	-0.0965	0.0216	-0.2829	0.1438	0.1829	-0.1048	1.0	0.0328	-0.1577
$m_b(m_b)$	0.0169	0.0072	0.0076	0.1255	0.0814	0.0423	0.0328	1.0	-0.0900
$m_t(m_t)$	-0.1675	-0.1109	0.3310	-0.7275	-0.5180	0.6924	-0.1577	-0.0900	1.0

Results for parton distributions

- PDFs with 1σ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Gluon $g(x)$



Strong coupling constant

Strong coupling constant (1992)

	$\alpha_s(\mathrm{M}_\mathrm{Z}^2)$
R_τ	$0.117\ {}^{+\ 0.010}_{-\ 0.016}$
DIS	0.112 ± 0.007
Υ Decays	0.110 ± 0.010
$R_{e^+e^-}(s < 62$ GeV$)$	0.140 ± 0.020
$p\bar{p} \to W + jets$	0.121 ± 0.024
$\Gamma(Z \to \mathrm{hadrons})/\Gamma(Z \to l\bar{l})$	0.132 ± 0.012
Jets at LEP	0.122 ± 0.009
Average	0.118 ± 0.007

G. Altarelli (1992)
in *QCD - 20 Years Later*,
CERN-TH-6623-92

Essential facts

- World average 1992 $\alpha_s(M_Z) = 0.118 \pm 0.007$
- Central value at NLO QCD
  - still right, but for very different reasons
- Error at NLO QCD
  - now down to $\sim 0.0050 - 0.0040$ (theory scale uncertainty)

$\alpha_s(M_Z)$ in PDFs

PDF sets	$\alpha_s(M_Z)$	method of determination
ABMP16 Alekhin, Blümlein, S.M., Placakyte '17	0.1147 ± 0.0008	fit at NNLO
CJ15 Accardi, Brady, Melnitchouk et al. '16	0.118 ± 0.002	fit at NLO
CT14 Dulat et al. '15	0.118	assumed at NNLO
HERAPDF2.0 H1+Zeus Coll.	$0.1183\ ^{+0.0040}_{-0.0034}$	fit at NLO
JR14 Jimenez-Delgado, Reya '14	0.1136 ± 0.0004	dynamical fit at NNLO
	0.1162 ± 0.0006	standard fit at NNLO
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	0.118	assumed at NNLO
	0.1172 ± 0.0013	best fit at NNLO
NNPDF3.1 Ball et al. '17	0.118	assumed at NNLO
Ball et al. '18	0.1185 ± 0.0012	best fit at NNLO
PDF4LHC15 Butterworth et al. '15	0.118	assumed at NLO
	0.118	assumed at NNLO

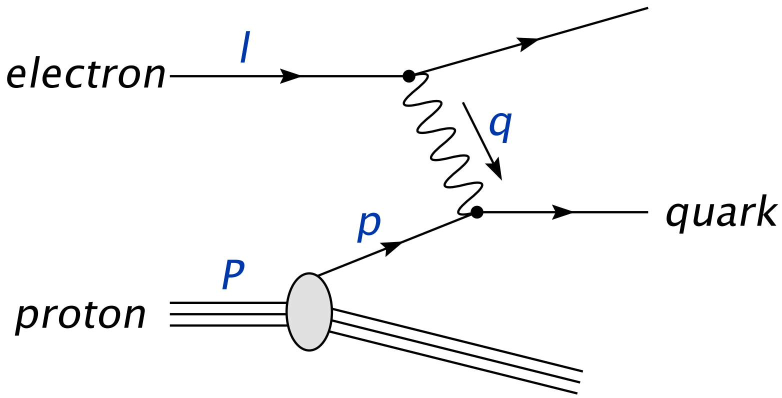
- Values of $\alpha_s(M_Z)$ often assumed and not fitted (no correlations)
- Large spread of fitted values at NNLO: $\alpha_s(M_Z) = 0.1136 \ldots 0.1185$
- PDF4LHC: order independent recommendation
  - use $\alpha_s(M_Z) = 0.118$ at NLO and NNLO

Strong coupling constant (2018)

BBG	$0.1134\ ^{+\ 0.0019}_{-\ 0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti '06
JR08	0.1128 ± 0.0010	dynamical approach	Jimenez-Delgado, Reya '08
	0.1162 ± 0.0006	including NLO jets	
ABKM09	0.1135 ± 0.0014	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. '09
	0.1129 ± 0.0014	HQ: BSMN	
MSTW	0.1171 ± 0.0014		Martin, Stirling, Thorne, Watt '09
Thorne	0.1136	[DIS+DY, HT*] (2013)	Thorne '13
$ABM11_J$	$0.1134 \ldots 0.1149\ \pm 0.0012$	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. '11
NN21	0.1173 ± 0.0007	(+ heavy nucl.)	NNPDF '11
ABM12	0.1133 ± 0.0011		Alekhin, Blümlein, S.M. '13
	0.1132 ± 0.0011	(without jets)	
CT10	0.1140	(without jets)	Gao et al. '13
CT14	$0.1150\ ^{+\ 0.0060}_{-\ 0.0040}$	$\Delta\chi^2 > 1$ (+ heavy nucl.)	Dulat et al. '15
JR14	0.1136 ± 0.0004	dynamical approach	Jimenez-Delgado, Reya '14
	0.1162 ± 0.0006	standard approach	
MMHT	0.1172 ± 0.0013	(+ heavy nucl.)	Martin, Motylinski, Harland-Lang, Thorne '15
ABMP16	0.1147 ± 0.0008		Alekhin, Blümlein, S.M., Placakyte '17
NN31	0.1185 ± 0.0012	including NLO jets	NNPDF '18

- Measurements at NNLO (last ~ 10 years) from DIS data
- Large spread of fitted values at NNLO: $\alpha_s(M_Z) = 0.1128 \ldots 0.1185$
- [yellow box] taken for 2017 PDG average: $\alpha_s(M_Z) = 0.1156 \pm 0.0021$

Theory description of deep-inelastic scattering



Kinematic variables

- momentum transfer $Q^2 = -q^2$
- Bjorken variable $x = Q^2/(2p \cdot q)$

- Structure functions (up to order $\mathcal{O}(1/Q^2)$)

$$F_a\left(x, Q^2\right) = \sum_i \left[C_{a,i}\left(\alpha_s(\mu^2), \mu^2/Q^2\right) \otimes PDF\left(\mu^2\right)\right](x)$$

- Perturbative expansion of coefficient functions up to N^{3}LO

$$C_{a,i} = \alpha_s^n \left(c_{a,i}^{(0)} + \alpha_s c_{a,i}^{(1)} + \alpha_s^2 c_{a,i}^{(2)} + \alpha_s^3 c_{a,i}^{(3)} + \dots\right)$$

- Application to DIS data requires careful consideration of kinematic region in Q^2 and x
  - invariant mass of the hadronic system $W^2 = M_P^2 + Q^2(1-x)/x$
  - cuts $W^2 \geq 12.5\ \mathrm{GeV}^2$ and $Q^2 \geq 2.5 \div 10\ \mathrm{GeV}^2$

- Additional corrections for $F_a\left(x, Q^2\right)$ necessary dependent on cuts
  - higher twist and target mass corrections

Higher twist

- Operator product expansion predicts inifinte tower of $(1/Q^2)^n$ of power corrections (higher twist terms)
- Physical interpretation as multi-parton correlations
- Higher twist terms modify structure functions (up to order $\mathcal{O}(1/Q^4)$)

$$F_i^{\rm ht}(x,Q^2) \;=\; F_i^{\rm TMC}(x,Q^2) + \frac{H_i^{\tau=4}(x)}{Q^2}\,, \qquad\qquad i \,=\, 2, T$$

Target mass corrections

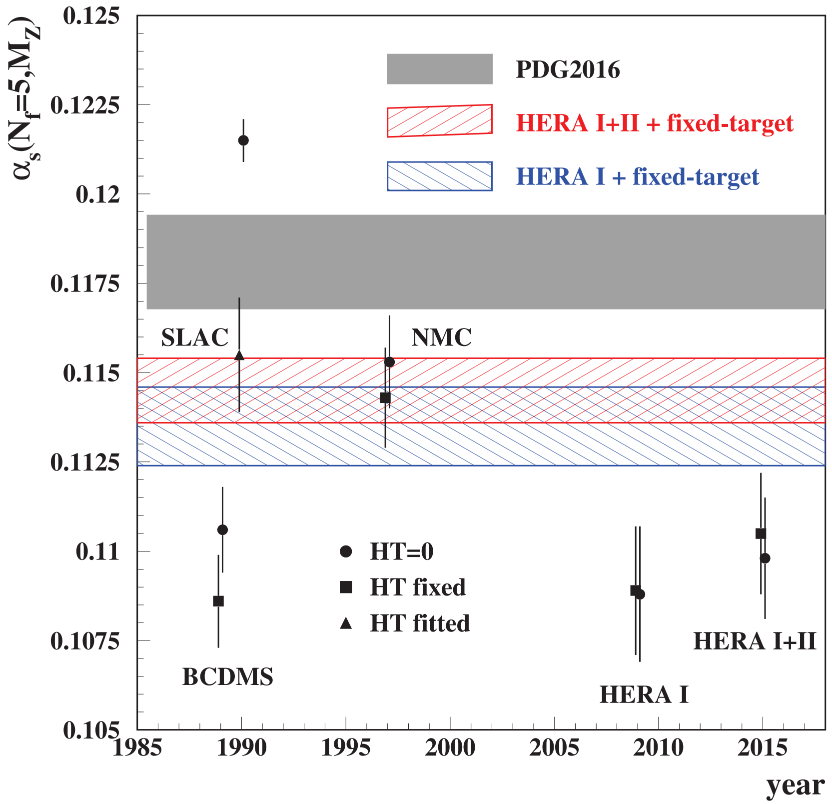
- Finite nucleon mass leads to target mass corrections up to $\mathcal{O}(M_N^2/Q^2)$

$$F_2^{\rm TMC}(x,Q^2) \;=\; \frac{x^2}{\xi^2\gamma^3}\,F_2(\xi,Q^2) + 6\,\frac{x^3M_N^2}{Q^2\gamma^4}\int\limits_{\xi}^{1}\frac{d\xi'}{\xi'^2}\,F_2(\xi',Q^2)$$

  - kinematic variable $\xi = 2x/(1+\gamma)$
  - Nachtmann variable $\gamma = (1+4x^2M_N^2/Q^2)^{1/2}$

Impact on α_s determinations

- Correlation of errors among different data DIS sets
- Target mass corrections (powers of nucleon mass M_N^2/Q^2)
- Variants with no higher twist give larger α_s values Alekhin, Blümlein, S.M. ‘17

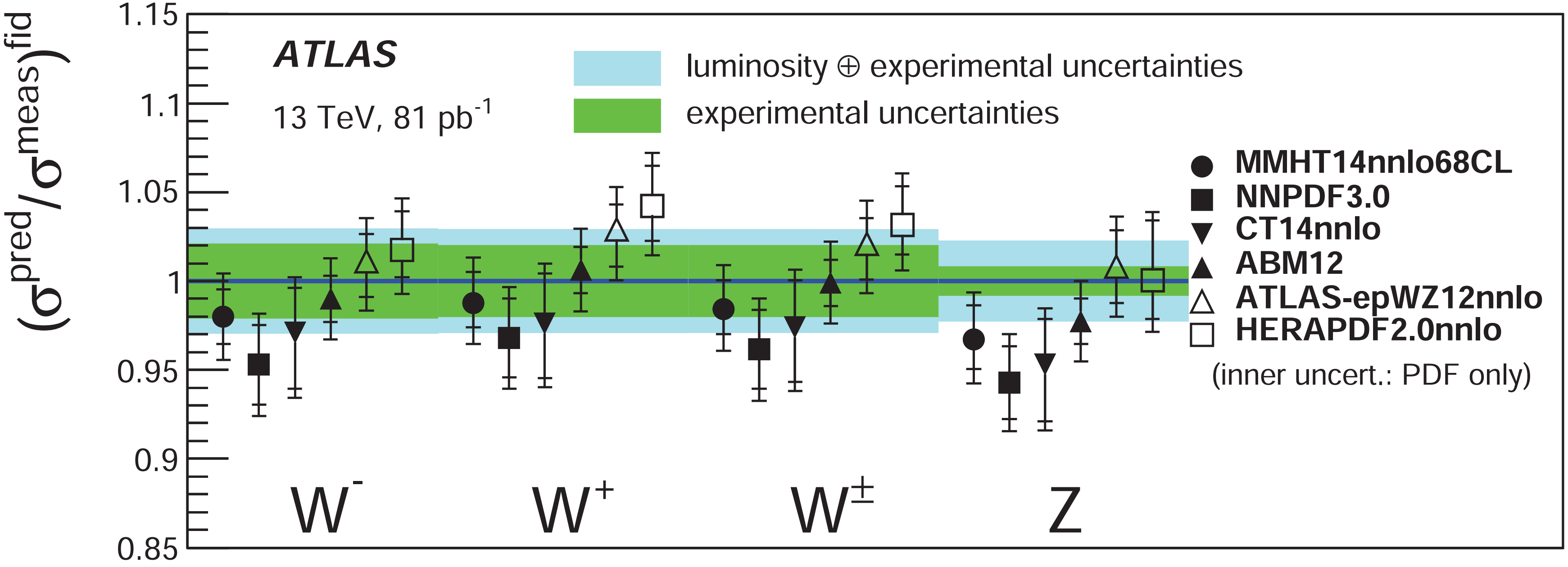


- Theoretical uncertainty of α_s at NNLO from DIS data $\gtrsim \mathcal{O}(1\ldots2)\%$

$W^{\pm}$- and Z-boson production

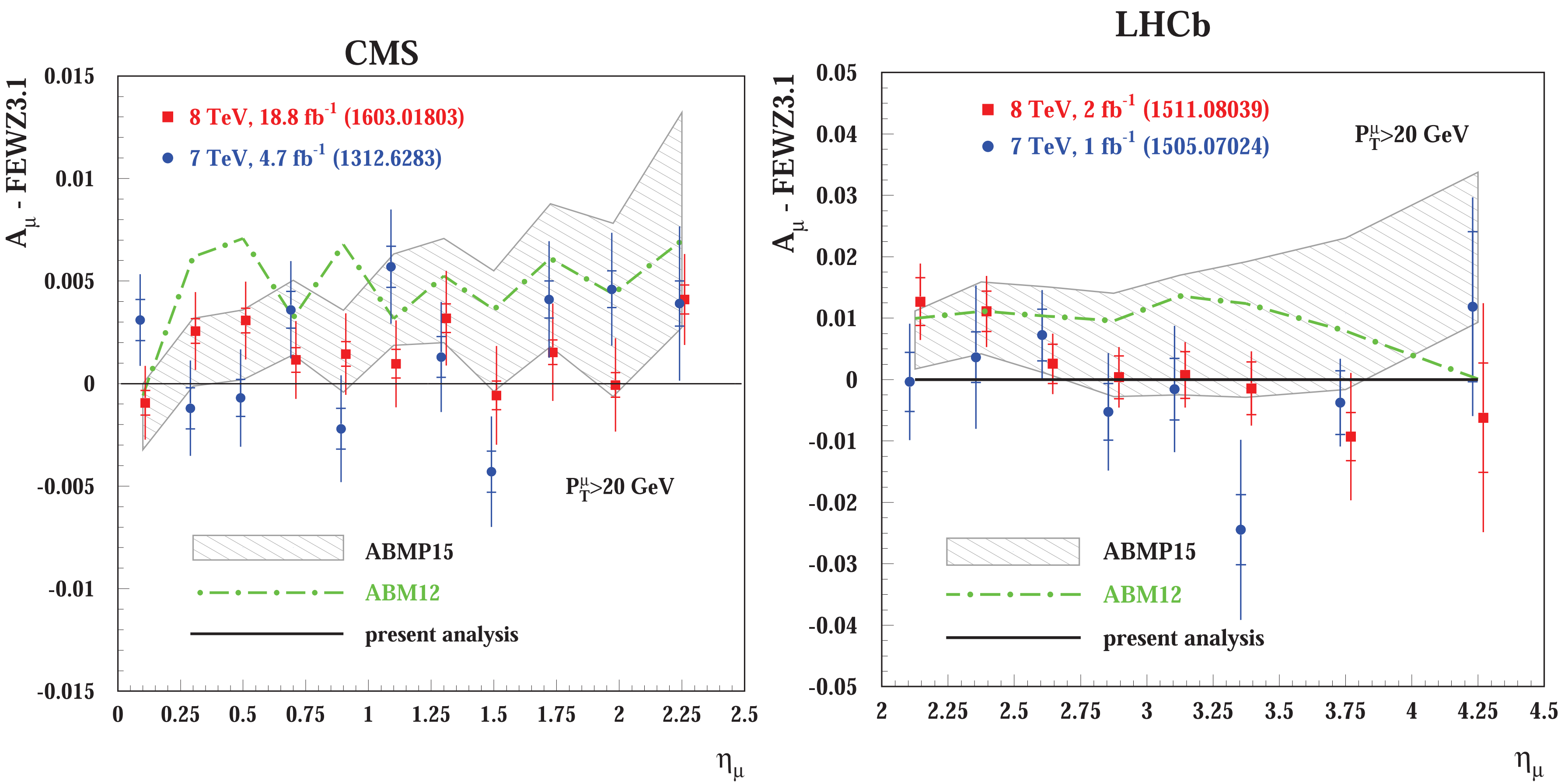
W- and Z-boson cross sections

- High precision data from LHC ATLAS, CMS, LHCb and Tevatron D0
  - differential distributions extend to forward region
  - sensitivity to light quark flavors at $x \simeq 10^{-4}$
  - statistically significant: $NDP = 172$ in ABMP16
- ATLAS measurement at $\sqrt{s} = 13$ TeV from arXiv:1603.09222



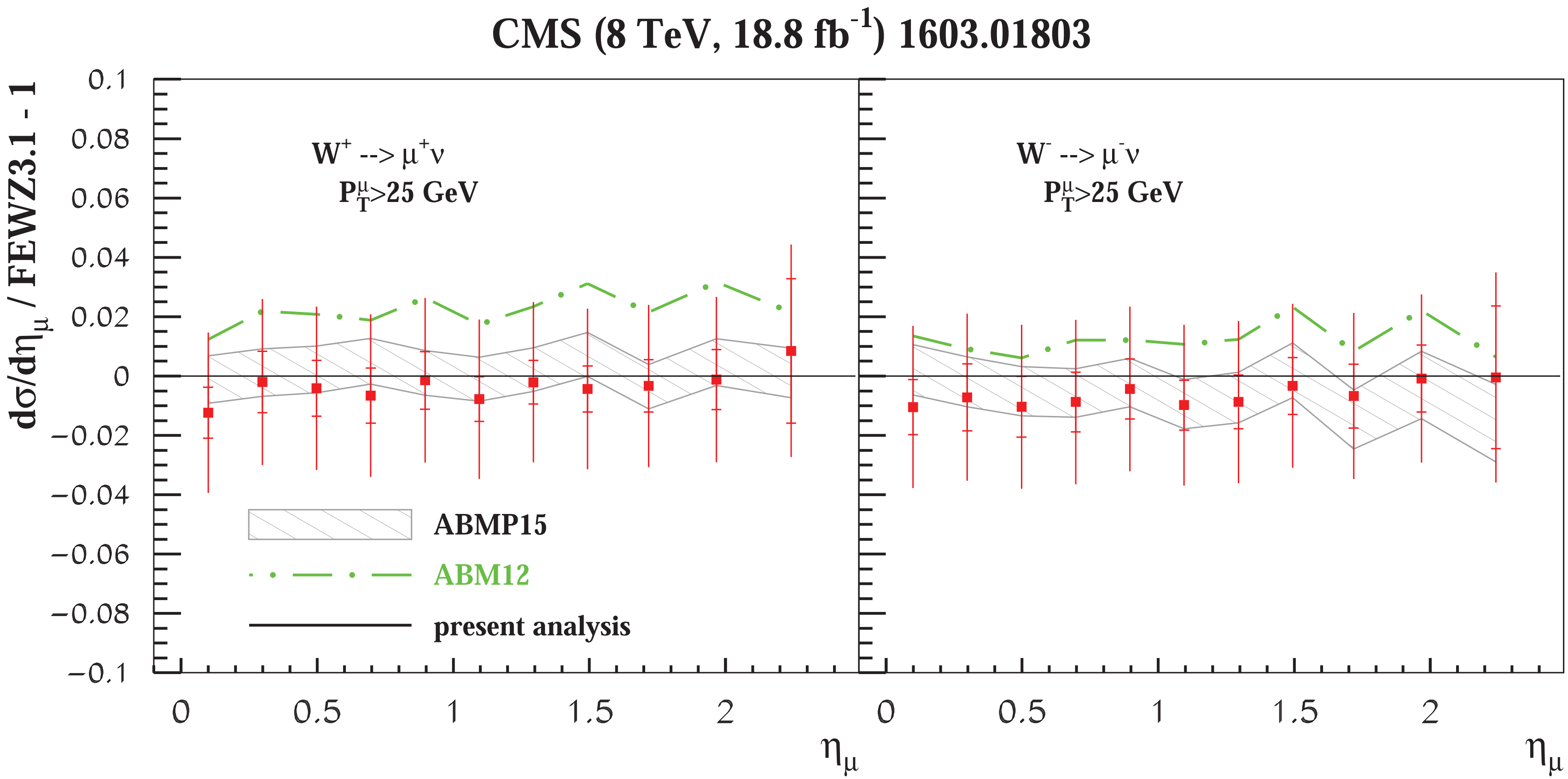
- Spread in predictions from different PDFs significantly larger than experimental precision

Muon charge asymmetry from LHC



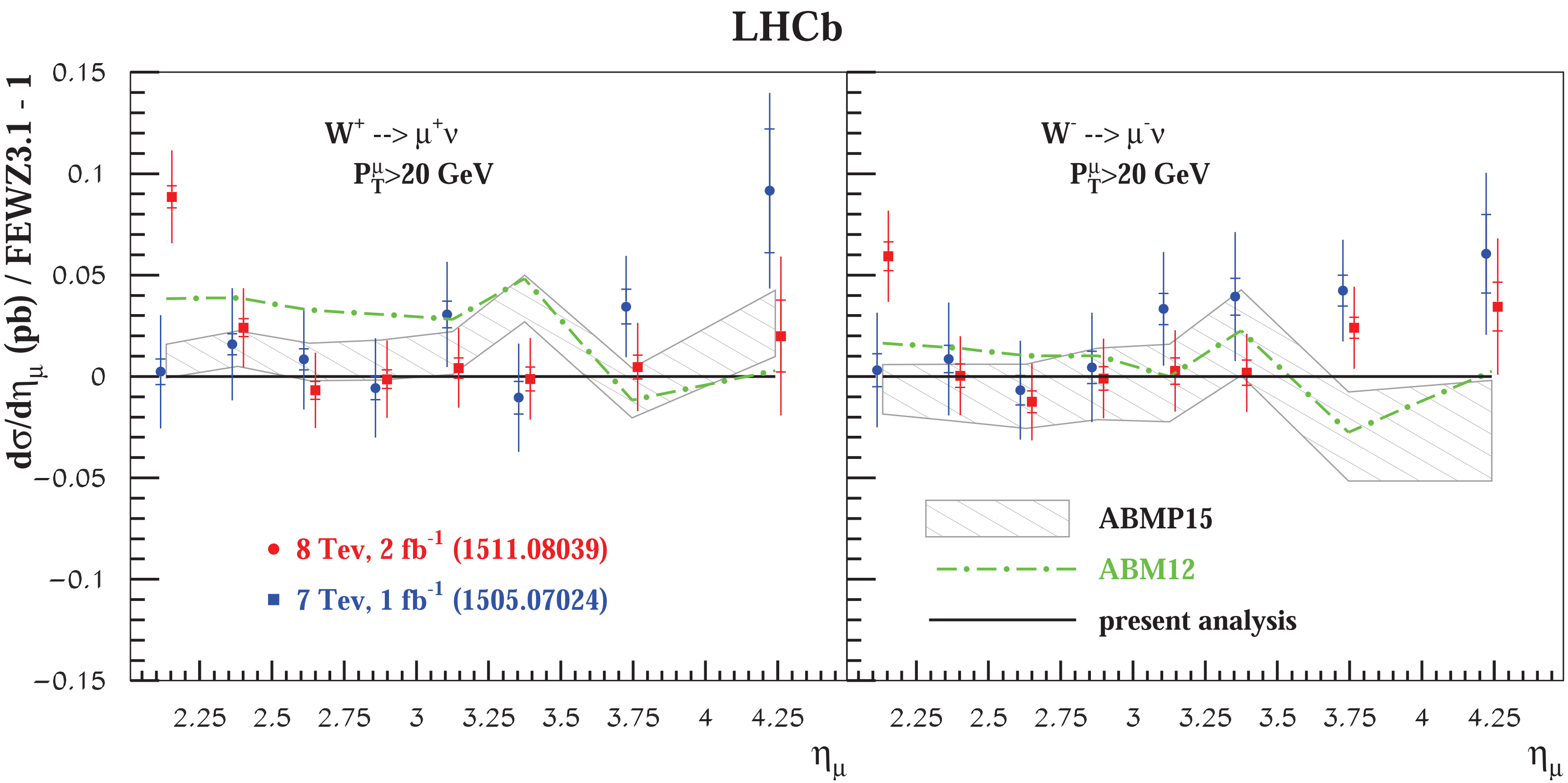
- CMS and LHCb data for $pp \to W^{\pm} + X \to \mu^{\pm}\nu + X$ at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV
  - comparison of ABM12, ABMP15 and ABMP16 fits
- Problematic data point at $\eta_{\mu} = 3.375$ for $\sqrt{s} = 7$ TeV in LHCb data are omitted in fit

$W^{\pm}$-boson production from LHC (I)



- CMS data on cross section of inclusive $W^{\pm}$-boson production at $\sqrt{s} = 8$ TeV
  - channel $W^{\pm} \to \mu^{\pm}\nu$

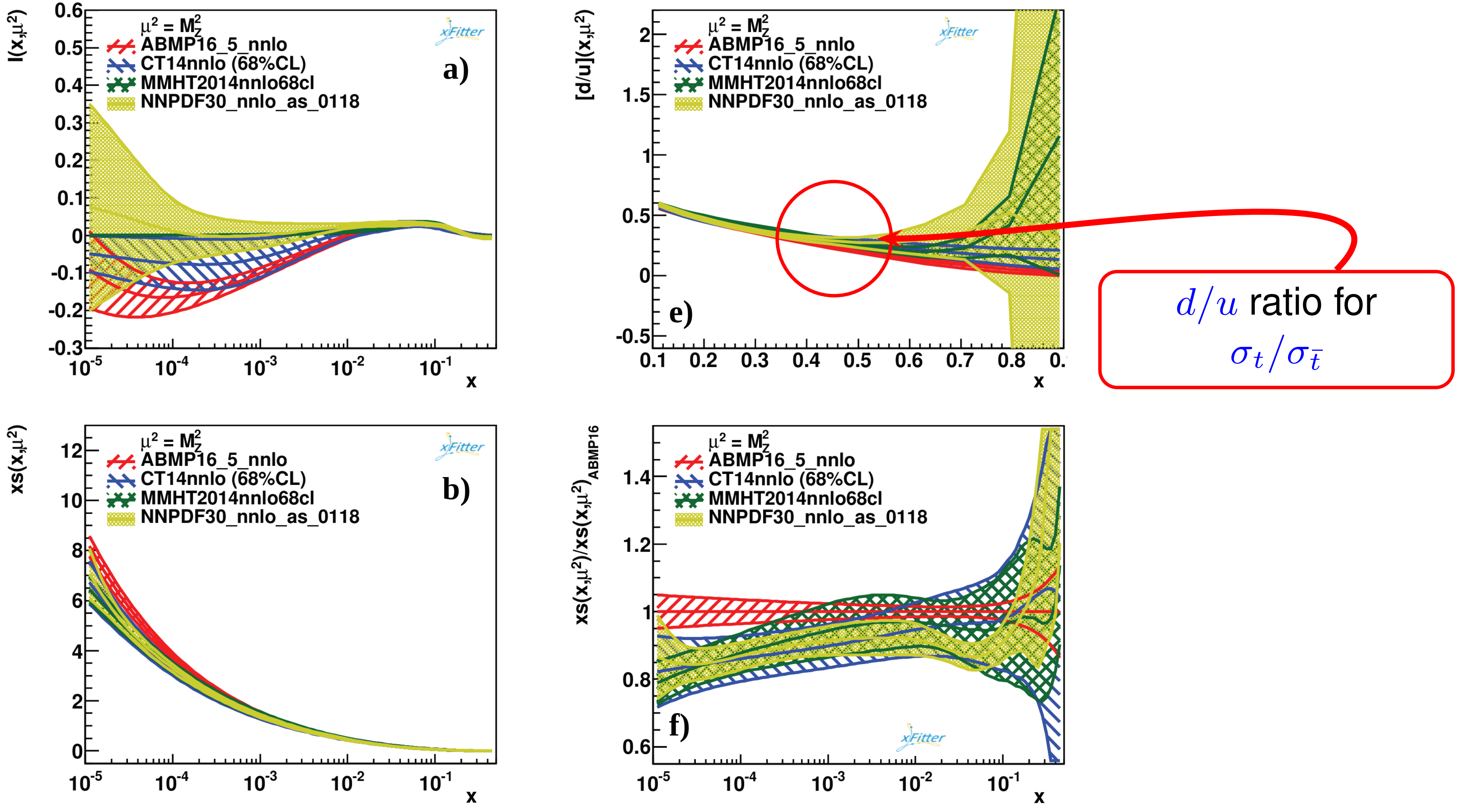
$W^{\pm}$-boson production from LHC (II)



- LHCb data on cross section of inclusive $W^{\pm}$-boson production at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV
  - channel $W^{\pm} \to \mu^{\pm}\nu$
- Points at $\eta_\mu = 2.125$ for $\sqrt{s} = 8$ TeV are not used in fit

Results for parton distributions

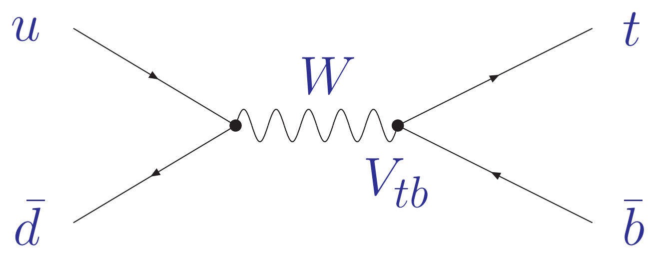
- PDFs with 1σ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Iso-spin asymmetry $x(\bar{d}(x) - \bar{u}(x))$; ratio $d(x)/u(x)$; strange $s(x)$



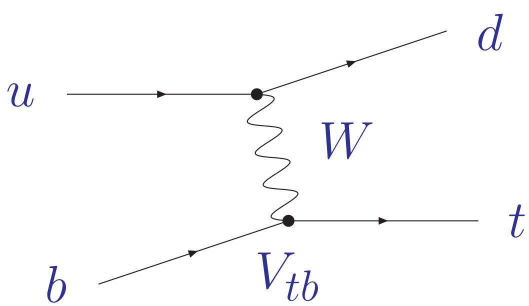
Single top-quark production

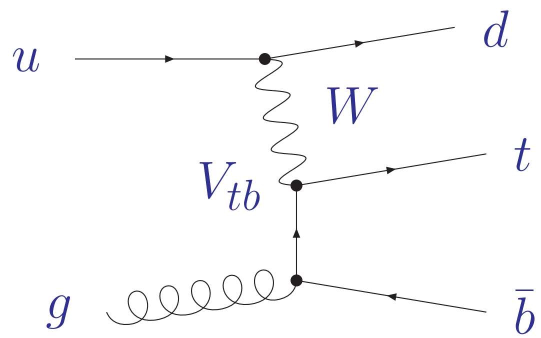
Single top-quark production

- Study of charged-current weak interaction of top quark
- s-channel production

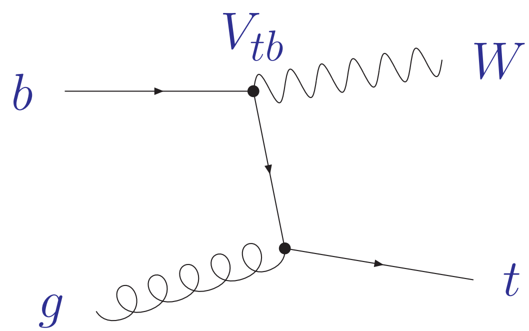


- t-channel production
  - sensitivity to light flavor PDFs
  - bg-channel at NLO enhanced by gluon luminosity



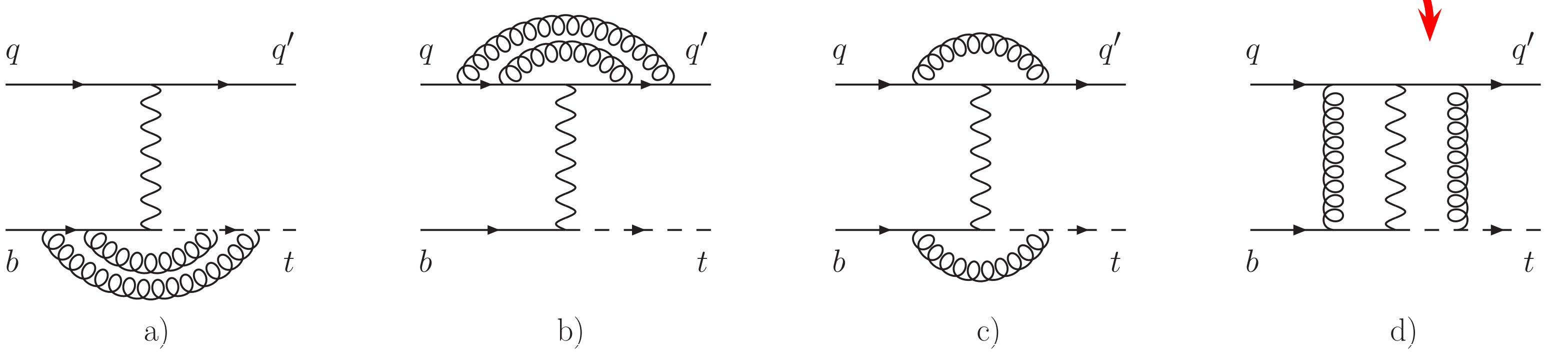


- Wt-production
  - contributes at LHC (small at Tevatron)



QCD corrections at NNLO

- Computation of NNLO QCD corrections Brucherseifer, Caola, Melnikov ‘14
  - fully differential, with cuts on p_T
- QCD corrections treated in structure function approach
  - non-factorizable contributions neglected
    (neglected diagrams $\mathcal{O}(1/N_c^2)$ supressed)

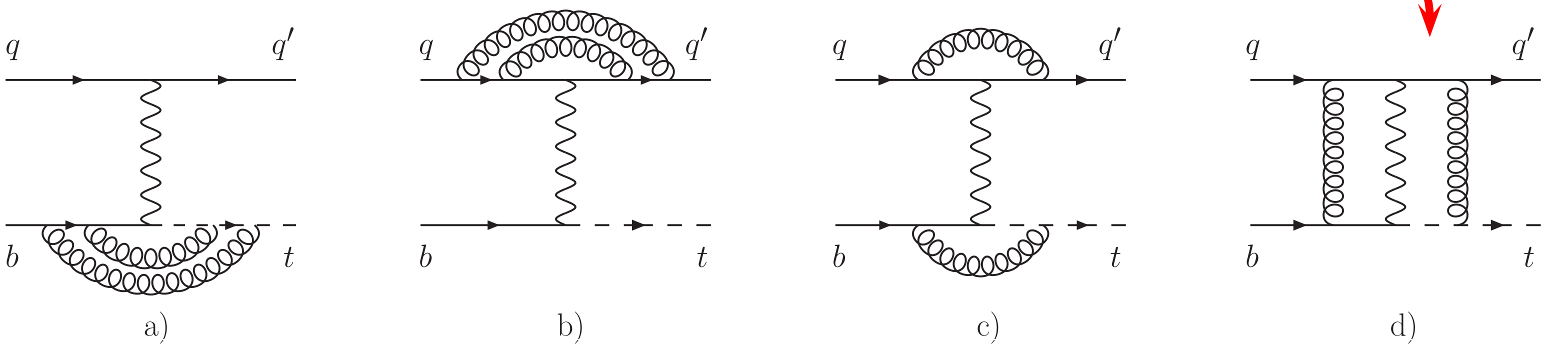


- QCD corrections to t-channel single top quark production at LHC8

$p_\perp$	$\sigma_{\rm LO}$, pb	$\sigma_{\rm NLO}$, pb	$\delta_{\rm NLO}$	$\sigma_{\rm NNLO}$, pb	$\delta_{\rm NNLO}$
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	$+2.4\%$	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	$+4.9\%$	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	$+9.3\%$	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	$+13.6\%$	$25.4^{-0.1}_{+0.2}$	$+1.6\%$

QCD corrections at NNLO

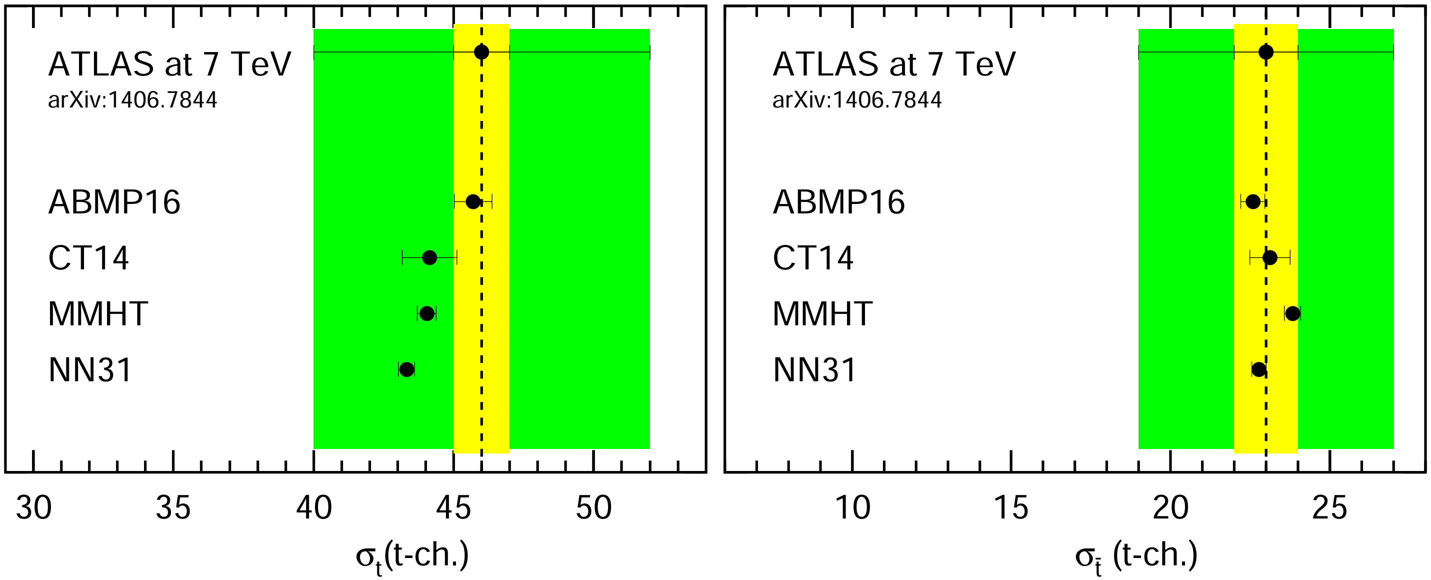
- Computation of NNLO QCD corrections Brucherseifer, Caola, Melnikov '14
  - fully differential, with cuts on p_T
- QCD corrections treated in structure function approach
  - non-factorizable contributions neglected (neglected diagrams $\mathcal{O}(1/N_c^2)$ supressed)



- QCD corrections to t-channel single anti-top quark production at LHC8

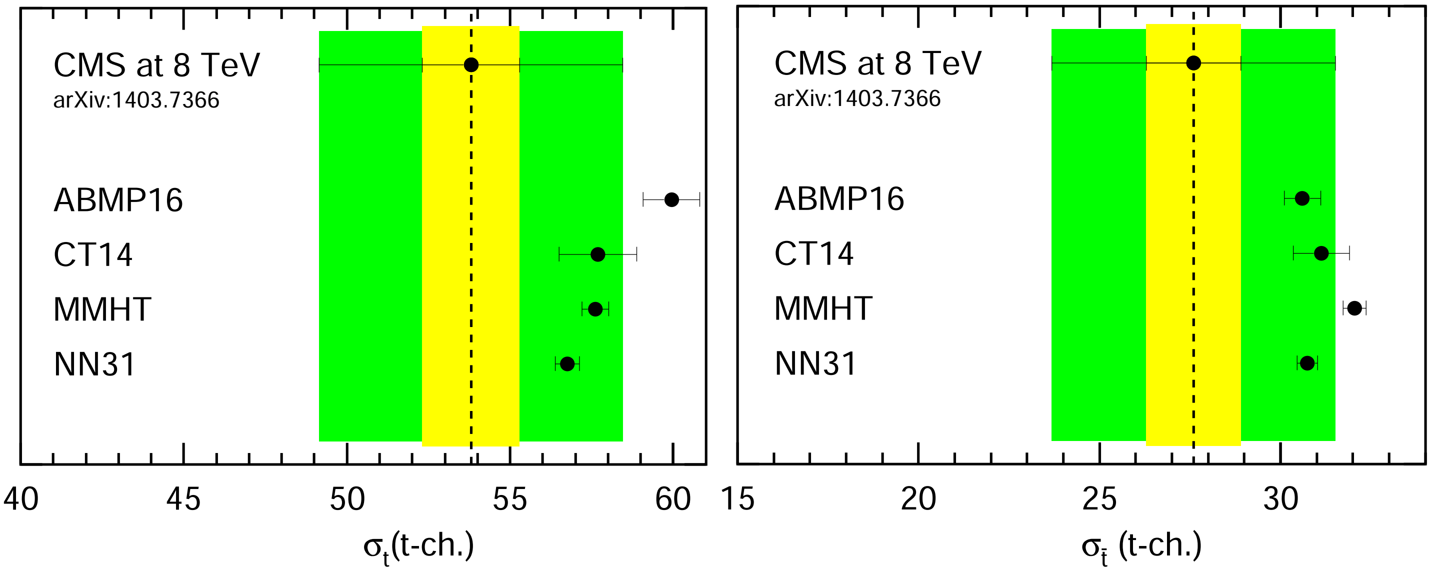
$p_\perp$	$\sigma_{\rm LO}$, pb	$\sigma_{\rm NLO}$, pb	$\delta_{\rm NLO}$	$\sigma_{\rm NNLO}$, pb	$\delta_{\rm NNLO}$
0 GeV	$29.1^{+1.7}_{-2.4}$	$30.1^{+0.9}_{-0.5}$	$+3.4\%$	$29.7^{+0.3}_{-0.1}$	-1.3%
20 GeV	$24.8^{+1.4}_{-2.0}$	$26.3^{+0.7}_{-0.3}$	$+6.0\%$	$26.2^{-0.01}_{-0.1}$	-0.4%
40 GeV	$17.1^{+0.9}_{-1.3}$	$19.1^{+0.3}_{+0.1}$	$+11.7\%$	$19.3^{-0.2}_{+0.1}$	$+1.0\%$
60 GeV	$10.8^{+0.5}_{-0.7}$	$12.7^{+0.03}_{+0.2}$	$+17.6\%$	$12.9^{-0.2}_{+0.2}$	$+1.6\%$

Inclusive cross sections (I)



- Cross sections for t-channel production of single (anti)top-quarks at LHC with 1σ PDF uncertainties
  - computation of hard cross section to NLO in QCD with Hathor for $\overline{\mathrm{MS}}$ mass $m_t(m_t) = 163$ GeV at scale $\mu_R = \mu_F = m_t(m_t)$
- Data at $\sqrt{s} = 7$ TeV from ATLAS
  - inner (yellow) band for statistical uncertainty and outer (green) band for combined statistics and systematics uncertainty

Inclusive cross sections (II)



- Cross sections for t-channel production of single (anti)top-quarks at LHC with 1σ PDF uncertainties
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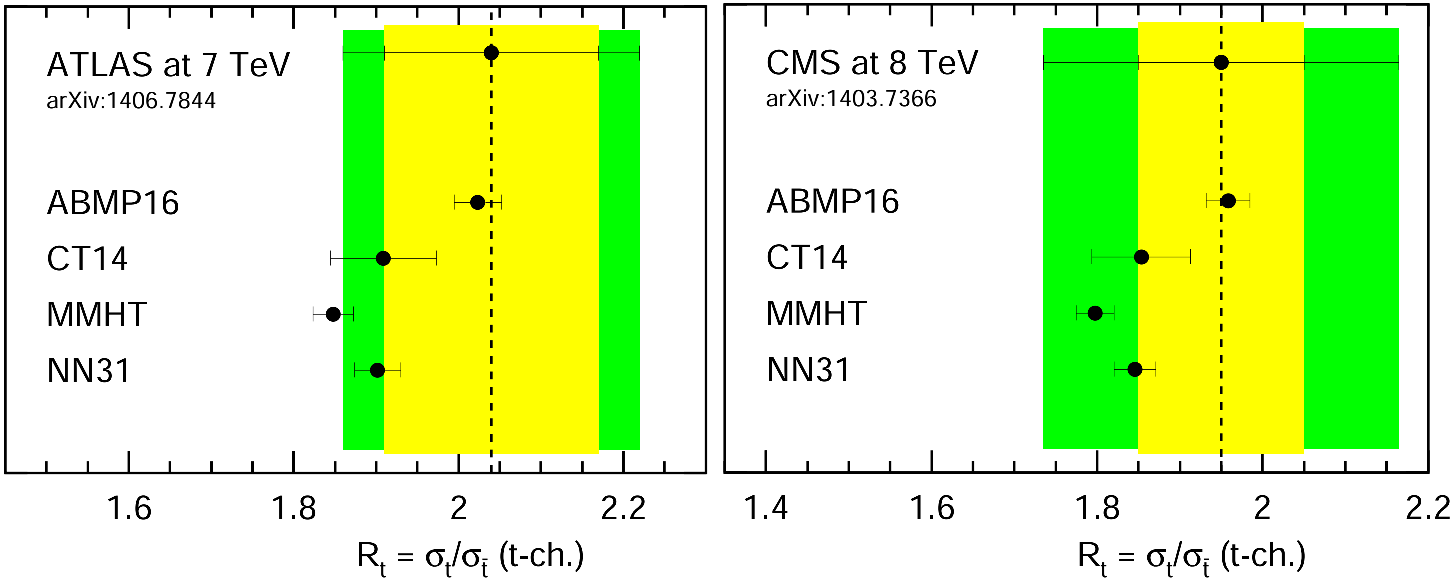
- Data at $\sqrt{s} = 8$ TeV from CMS
  - inner (yellow) band for statistical uncertainty and outer (green) band for combined statistics and systematics uncertainty

Cross section ratio



- Cross section ratio $R_t = \sigma_t/\sigma_{\bar{t}}$ is very sensitive probe
  - data from ATLAS and CMS dominated by inner (yellow) band for statistical uncertainty, systematics largey cancel
- Theory predictions sensitive to ratio d/u of PDFs
  - 1σ PDF uncertainties in R_t small

Upshot

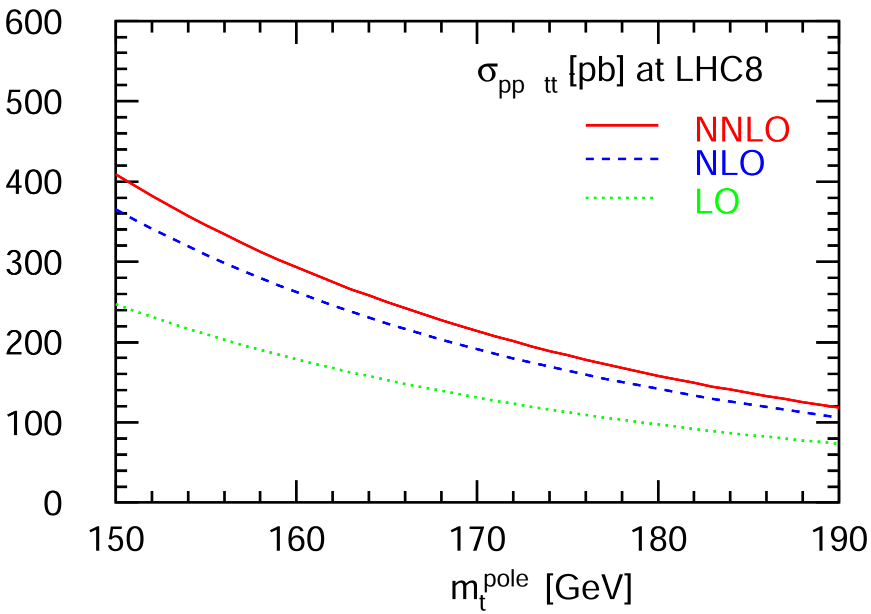
- Production of single top-quarks at LHC can serve as standard candle for the light quark flavor content of proton

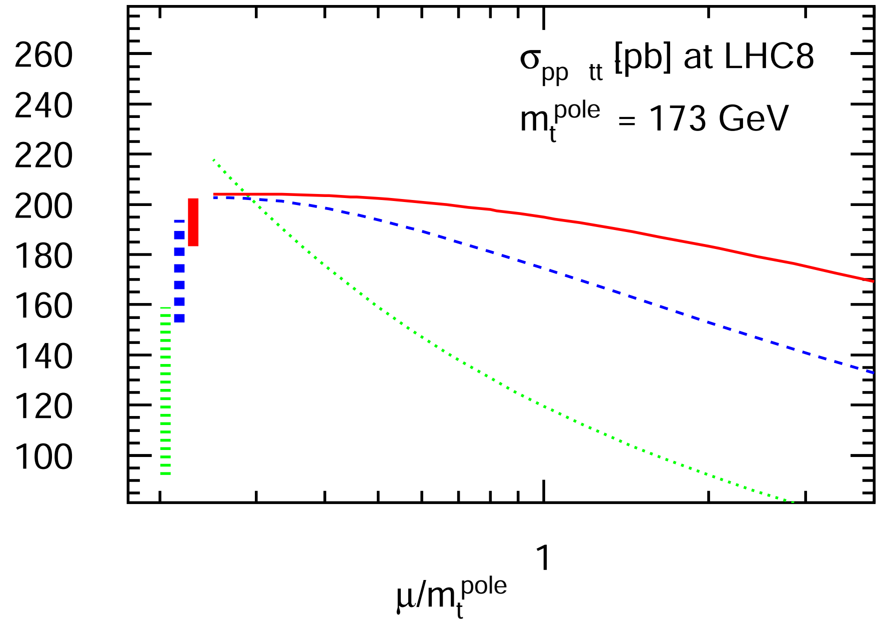
Top-quark pair production

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13





- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$; scale stability of $\mathcal{O}(\pm 5\%)$
- Beyond NNLO
  - theory improvements with soft gluon resummation [many people]
  - K-factor (NNLO $\to$ resummed) small; scale stability further improved

Top-quark mass from total cross section

- Cross section for $t\bar{t}$-production with parametric dependence

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X} \left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

  - PDFs f_i, strong coupling α_s, masses m_X
- PDFs and $\alpha_s(M_Z)$ already well constrained by global fit
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$

Top-quark mass determination

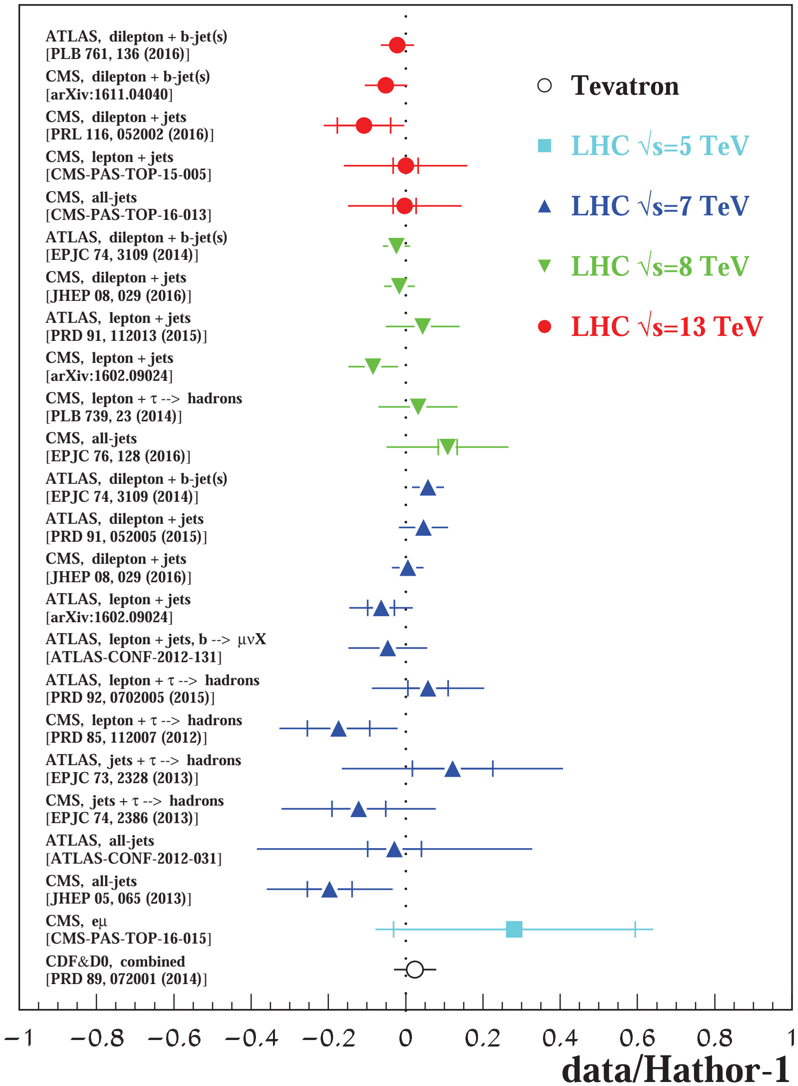
- Choice of renormalization scheme for treatment of heavy quarks
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
- Intrinsic limitation of sensitivity in total cross section

$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

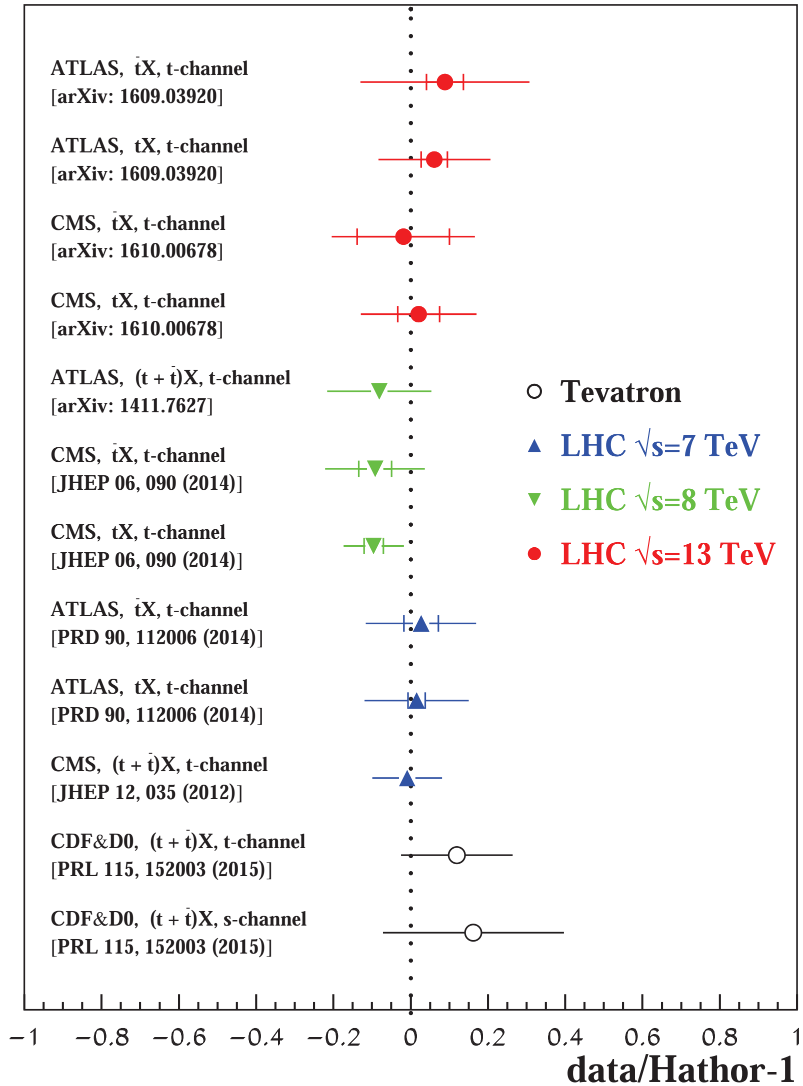
Data on top-quark cross sections

- Pulls for $t\bar{t}$- and single-t inclusive cross sections in ABMP16

$\sigma(t\bar{t}X)$

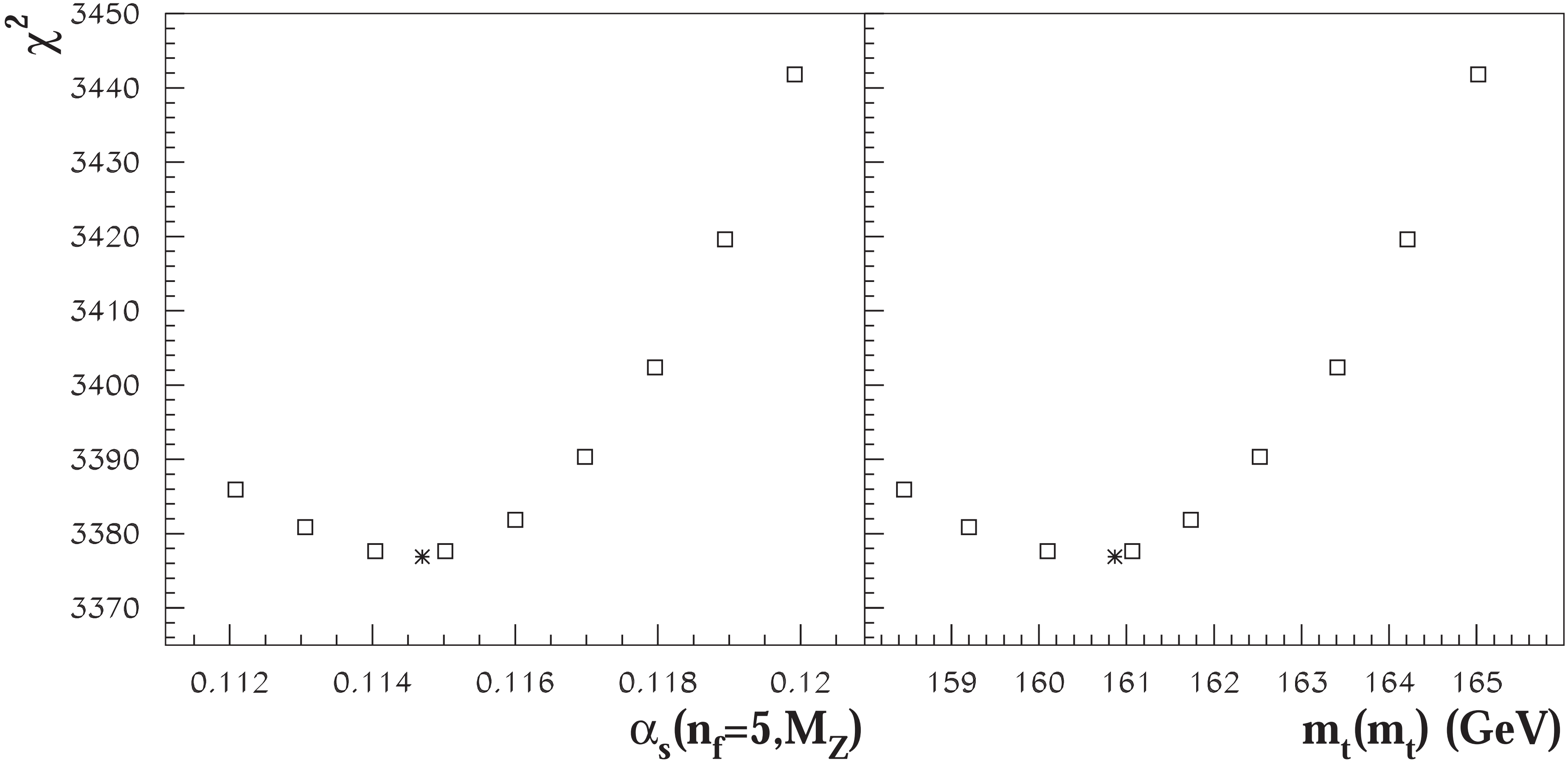


$\sigma(t/\bar{t}\ X)$



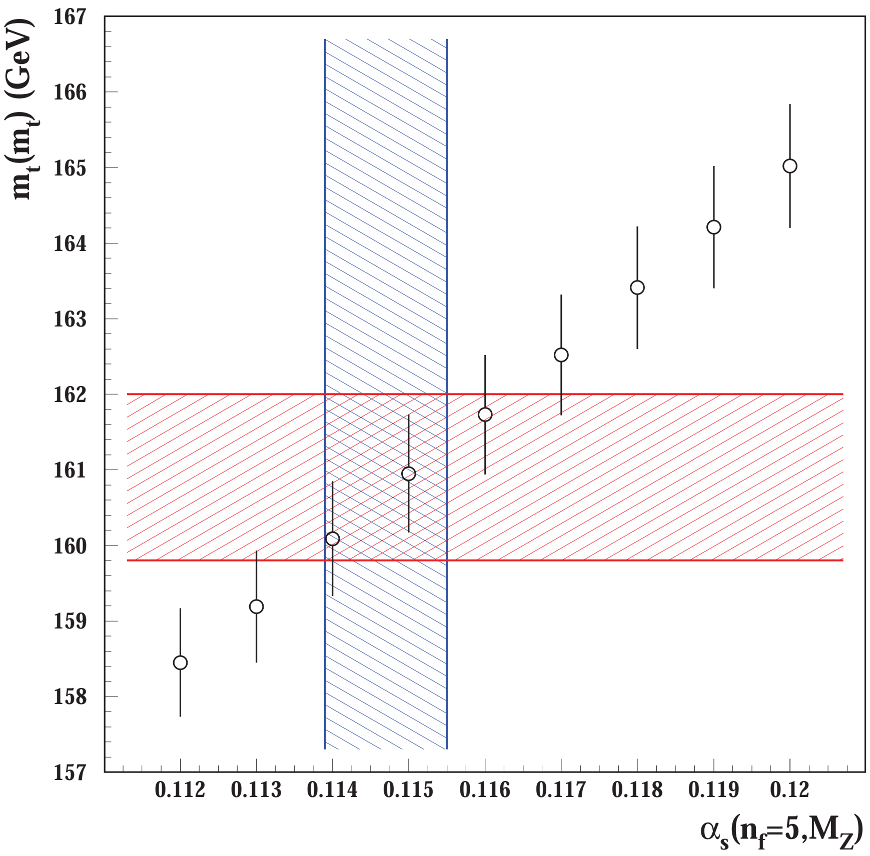
Fit quality

- Goodness-of-fit estimator χ^2 for extracted $\alpha_s(M_Z)$ and $m_t(m_t)$ values
  - χ^2 of global fit with $NDP = 2834$
  - data on top-quark production with $NDP = 36$ D0, ATLAS, CMS, LHCb



Correlations

- Correlations between gluon PDF $g(x)$, $\alpha_s(M_Z)$ and $m_t(m_t)$



- Fits with fixed values of m_t and $\alpha_S(M_Z)$ carry significant bias

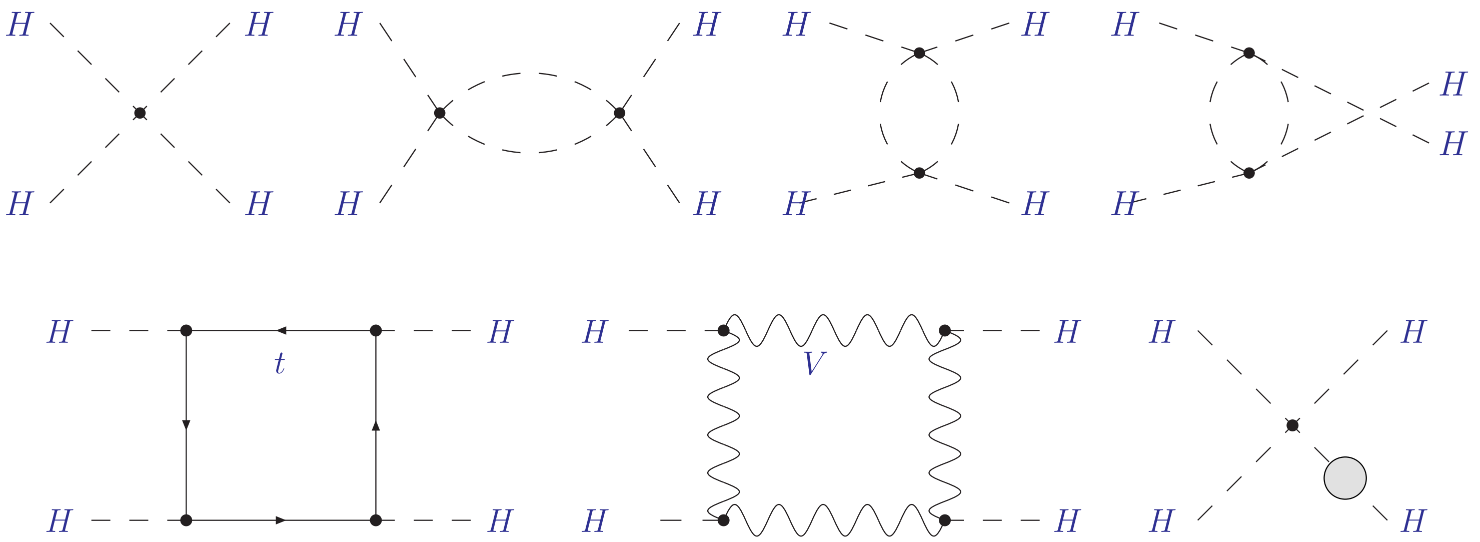
Implications on electroweak vacuum

Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left| \Phi^\dagger \Phi - \frac{v}{2} \right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \ldots$$



Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3 m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

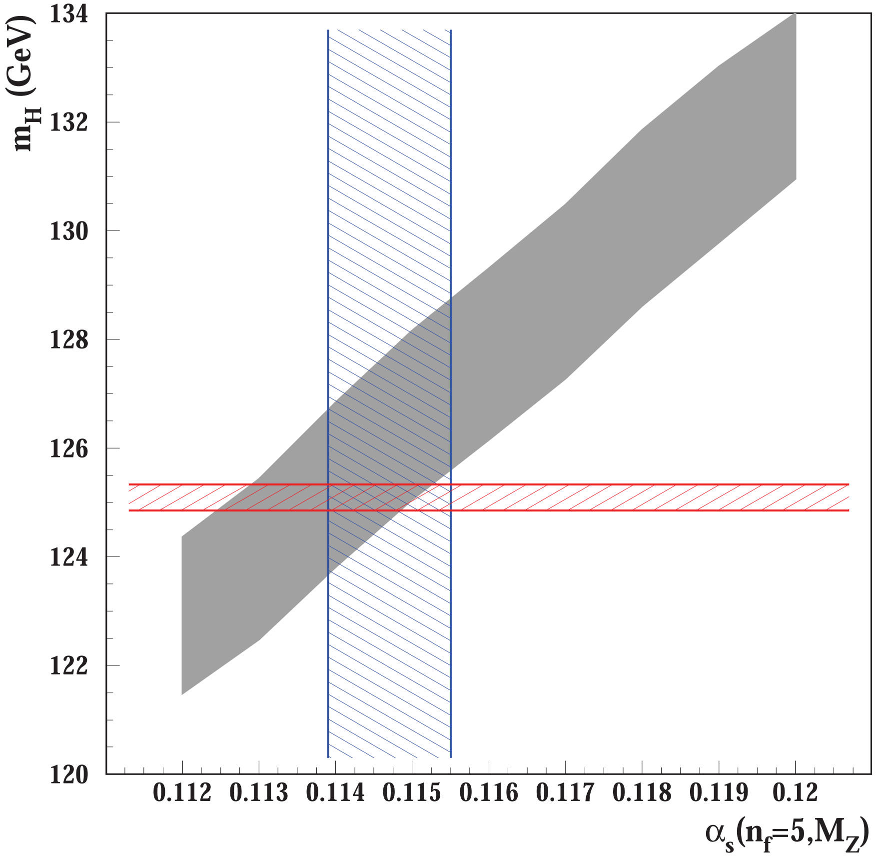
$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

Implications on electroweak vacuum

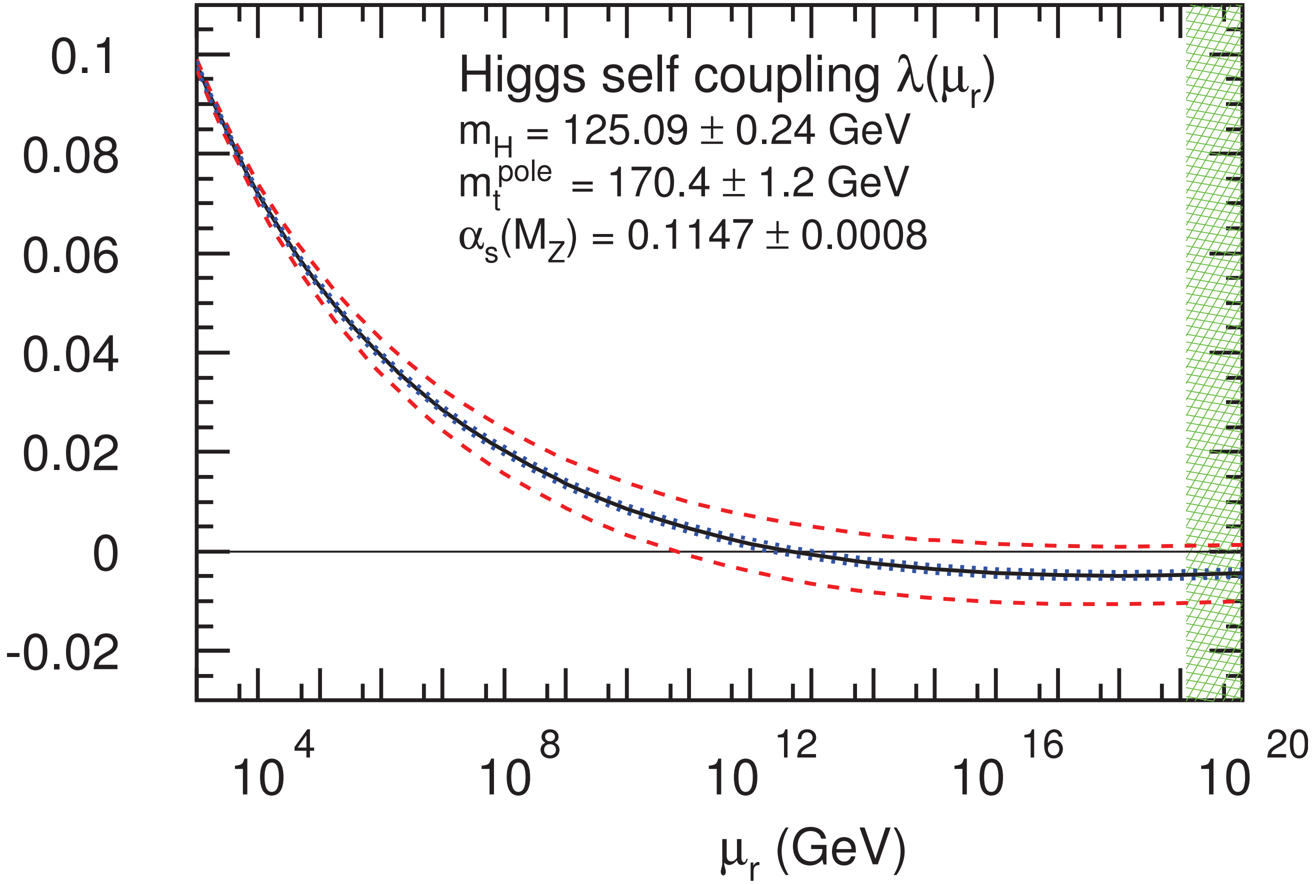
- Condition of absolute stability of electroweak vacuum at Planck scale M_{Planck} requires Higgs self-coupling $\lambda(\mu_r) \geq 0$
  - correlation between Higgs mass m_H, m_t and $\alpha_s(M_Z)$ at $\mu = M_{\mathrm{Planck}}$

$$m_H \;\geq\; 129.6 + 2.0 \times \left(m_t^{\mathrm{pole}} - 173.34\ \mathrm{GeV} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.3\ \ \mathrm{GeV}$$



- NNLO analyses
  Bezrukov, Kalmykov, Kniehl, Shaposhnikov ‘12; Degrassi et al. ‘12; Buttazzo et al. ‘13; Bednyakov, Kniehl, Pikelner, Veretin ‘15

Higgs self-coupling



- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s up to $\mu_r = M_{\rm Planck}$ (using program mr Kniehl, Pikelner, Veretin '16)
  - top-quark mass least precise parameter
- $\lambda(\mu_r = M_{\rm Planck}) \simeq 0$ implies "fate of universe" may not be fatal, after all

Summary

- Experimental precision of $\lesssim 1\%$ makes theoretical predictions at NNLO in QCD mandatory
- Values of $\alpha_s(M_Z)$ at NNLO from measurements at colliders lower than world average
  - $\alpha_s(M_Z) = 0.118$ at NNLO not preferred by data
  - details of kinematic cuts, treatment of higher twist, target mass corrections are essential
- LHC data for $W^{\pm}$- and Z-boson production gives valuable information on light flavor PDFs u, d and s over wide range of x
  - important constraints on single-top production
  - single-top production has potential to become standard candle process
- Top-quark pair production provides precision determination of top-quark mass m_t
  - correlations with PDFs, strong coupling constant $\alpha_s(M_Z)$ are essential and need to be taken into account
- Values of m_t and $\alpha_s(M_Z)$ are crucial for decisive statement on electroweak vacuum