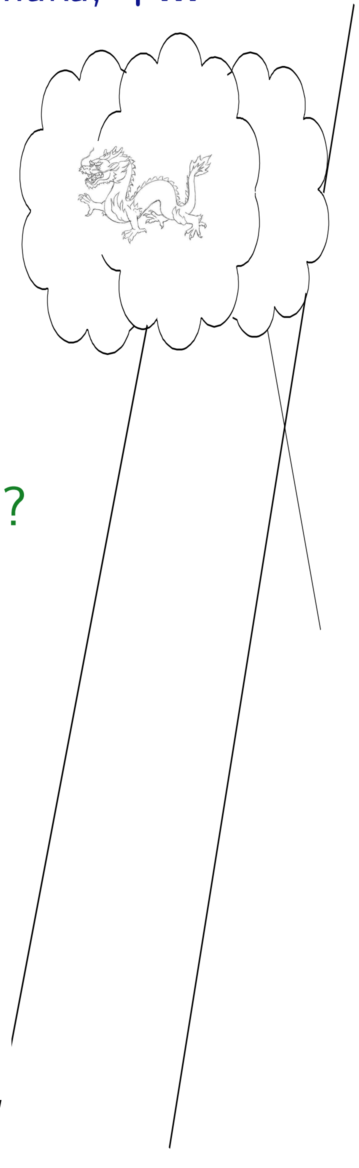
..., 1810.01884, 1710.06787, 1703.02057

To Learn about $(\mu \to e)$ Lepton Flavour Change ?

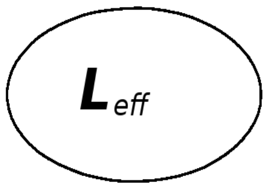
V Cirigliano, Sacha Davidson , M Gorbahn, Y Kuno, A Saporta, M Yamanaka, +...



1. Lepton Flavour Violation
   - what is it, why interesting + what do we know ?
     (exptal reach in $\mu \leftrightarrow e$ to improve by $\sim 10^4$ in a few years)

2. From an EFT perspective, what can we learn?
   - at the experimental scale (distinguishing operator coefficients)?
   - at the New Physics scale? (... RGEs)

3. Lots to do ...



(...what is not in this talk: an alternate perspective on LFV)

we know there is "New" ($=$ not Standard Model) Physics in the lepton sector, because neutrinos have tiny masses and large mixing angles

$\Rightarrow$ build beautiful, elegant, natural models that generate the observed neutrino masses, and calculate the Lepton Flavour Violation they predict!

$\Rightarrow$ compare model predictions to data $+$ select the correct model

But: been done for decades $+$ I don't know a good model from a bad one...
$\Rightarrow$ no models in this talk.

NB: same question *what is the "New Physics" in the lepton sector?*, different approach to finding solution.

What is Lepton Flavour Violation?

- in the Standard Model, there are various species and types of particles:

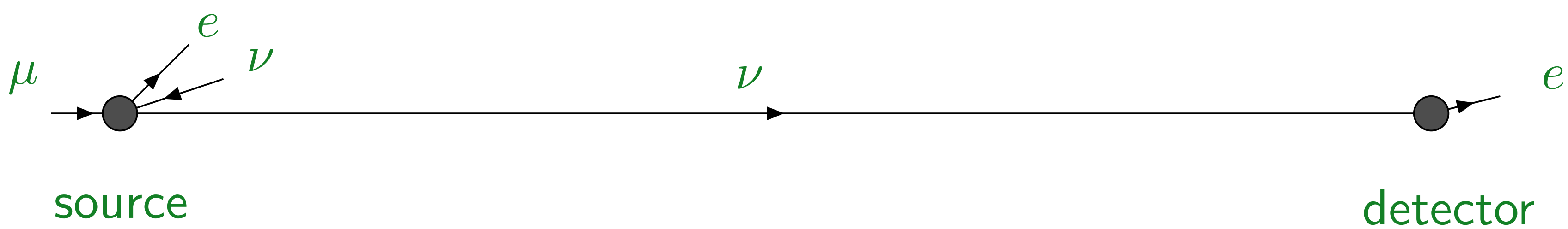
strongly interacting

$$\text{leptons} \begin{cases} \text{charged,} & e, \mu, \tau & \text{identical except for masses} \\ \text{neutral} = \text{neutrinos,} & 3\nu s & \text{shy!(cross planet without talking)} \end{cases}$$

heavy

- three lepton flavours in the Standard Model : $\boldsymbol{e}, \boldsymbol{\mu}, \tau$
(flavour $\equiv$ mass eigenstate)

- LFV $\equiv$ charged lepton flavour change, at a point.
  ν are shy, and quantum over thousands of km



$\Rightarrow$ ν oscillations don't count.

Some LFV processes and bounds

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2018, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	10^{-16} (Mu2e,COMET) 10^{-18} (PRISM/PRIME)
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)

BR $\equiv$ Branching Ratio: (rate for process)/(total decay rate)

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

What does $BR < 10^{-12}$ mean? Is it restrictive?

LFV Branching Ratios normalised to *weak* muon decay, $\tau_\mu \sim 2 \times 10^{-6}$ sec

$$BR(\mu \to e\bar{e}e) \equiv \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \quad , \quad \Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{m_\mu^5}{1536\pi^3 v^4}$$

$m_\mu = .105$ GeV
$v = 174$ GeV

$$\text{...so if } \Gamma(\mu \to e\bar{e}e) \simeq \frac{m_\mu^5}{1536\pi^3 \Lambda_{LFV}^4} \quad \text{then } BR \lesssim \begin{cases} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200 \text{ TeV} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \text{ TeV} \end{cases}$$

NB: $\Lambda_{LFV} = (16\pi^2)^n M_{LFV}/$couplings; *not* the mass scale of new particles M_{LFV}

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Compare to $\frac{(g-2)_\mu}{2} \equiv a \simeq \alpha_{em}/\pi$ (electromagnetic *amplitude*):

torque $\vec{\tau} = \vec{\mu} \times \vec{B}$; $\vec{\mu} = g\frac{e}{2m}\vec{S}$

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$$\begin{aligned} \Delta a &\equiv a^{SM} - a^{exp} \simeq 3 \times 10^{-9} \\ &\sim \frac{m_\mu^2}{16\pi^2\Lambda_{NP}^2} \end{aligned}$$

$\Rightarrow \Lambda_{NP} \sim m_t$.

More LFV processes and bounds

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
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		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb?)
$\tau \to e\phi$	$< 3.1 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^\pm e^\mp$	$< 6.9 \times 10^{-3}$	
$Z \to e^\pm\mu^\mp$	$< 7.5 \times 10^{-7}$	

LFV in EFT

1. Lepton Flavour Change is interesting:
   - none in the Standard Model with $m_\nu = 0$
   - occurs with m_ν and mixing matrix U

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   - occurs with m_ν and mixing matrix U

   m_ν renormalisable Dirac: LFV amplitudes GIM-suppressed (like quarks)

$$\mathcal{A} \propto \frac{m_\nu^2}{m_W^2} \Rightarrow BR \lesssim 10^{-48}$$

$\Rightarrow$ if see LFV, lepton flavour sector different from quarks!

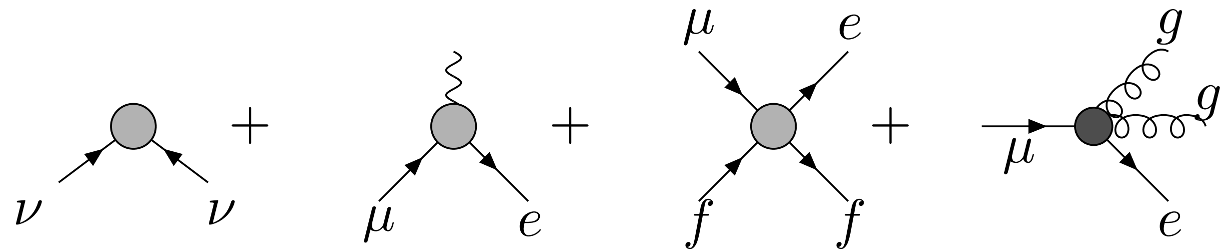
suppose m_ν NOT Dirac, New Physics heavy

2. use EFT to learn about heavy New Physics for LFV
   - parametrize LFV as contact interactions with constant coefficients
   - extract coefficients from data
   - what do coefficients tell about New Physics?

EFT: data→operator coefficients→ ?

1. parametrise LFV processes via contact interactions
   (at low E) write down all LFV 2,3,4-point functions that respect QED and QCD:

$\sum_\zeta \sum_O \Big($



$+$ h.c.$\Big)$

$\sum_\zeta$ = sum over flavours of external legs
$\sum_O$ = sum over Lorentz structure of operators $= \{m_\nu, S, P, A, V, T\}\times$chirality .
suppose constant $\{C^\zeta_O\}$ (no form factors) ⇔ New Particles are heavy

$$\delta\mathcal{L} = \sum_\zeta \sum_O \frac{C^\zeta_O}{v^n} O^\zeta + h.c. \qquad (v = 174 \text{ GeV}, \text{ ex} : \mathcal{O} \sim \bar{e}\sigma \cdot F\mu)$$

⇒ theoretical parametrisation of the data= express LFV rates in terms of $\{C^\zeta_O\}$.

EFT: data→operator coefficients→ ?

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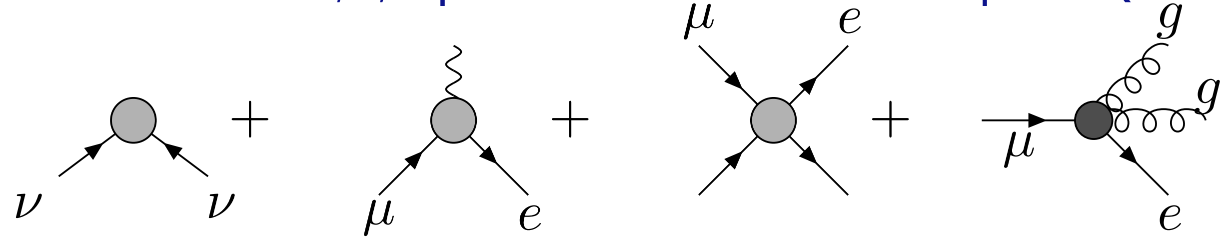

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   $\sum_\zeta$ = sum over flavours of external legs
   $\sum_O$ = sum over Lorentz structure of operators $= \{m_\nu, S, P, A, V, T\}\times$chirality .

   suppose constant $\{C_O^\zeta\}$ (no form factors) $\Leftrightarrow$ New Particles are heavy

$$\delta\mathcal{L} = \sum_\zeta \sum_O \frac{C_O^\zeta}{v^n} O^\zeta + h.c. \qquad (v = 174\,GeV)$$

   $\Rightarrow$ theoretical parametrisation of the data= express LFV rates in terms of $\{C_O^\zeta\}$.

2. extract coefficients from data.

   $\Rightarrow$ how many operator coefficients can be constrained?

3. **Then ask questions...** what can I learn about $\mathcal{L}_{BSM}$ from the coefficients?

   use SM RGEs to translate constraints from exptal scale to NP scale

lets try to do this with $\mu \to e\gamma, \mu \to e\bar{e}e$ and $\boldsymbol{\mu \to e}$ conversion...

An operator basis for $\mu \to e$ conversion, $\mu \to e\bar{e}e$, $\mu \to e\gamma$

At Λ_{expt}, μ interaction with nucleon $N \in \{n, p\}$ parametrised by 20 4-f operators :

$$
\begin{array}{lccl}
S,\ V & \overline{e}P_X\mu\overline{N}N & \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha N & X \in \{L, R\} \\
A,\ T & \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha\gamma_5 N & \overline{e}\sigma^{\alpha\beta}P_X\mu\overline{N}\sigma_{\alpha\beta}N & \\
P,\ Der & \overline{e}P_X\mu\overline{N}\gamma_5 N & \overline{e}\gamma^\alpha P_X\mu(\overline{N}i\overleftrightarrow{\partial}_\alpha\gamma_5 N) &
\end{array}
$$

Matching in χPT gives Derivative. But absorb in matching into $G_O^{N,q}$= quark matrix elements in nucleons. and 2 dipoles

$$
D \qquad \overline{e}\sigma^{\alpha\beta}P_X\mu F_{\alpha\beta}
$$

which also contribute in $\mu \to e\gamma$, $\mu \to e\bar{e}e$. For $\mu \to e\bar{e}e$

$$
\begin{array}{lcc}
V & (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) & (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e) \\
S & (\overline{e}P_Y\mu)(\overline{e}P_Y e) &
\end{array}
$$

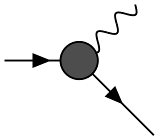
chiral basis for the lepton current (relativistic e), but not for the non-rel. nucleons.

add to $\mathcal{L}$ as :

$$
2\sqrt{2}G_F\sum_O\left(m_\mu\tilde{C}_O\mathcal{O}_O\right)
$$

Constraining the operator-zoo with 3 processes: $\mu \to e\gamma$

Two dipole operators contribute to $\mu \to e\gamma$:



$$\begin{aligned}
\delta\mathcal{L}_{meg} &= -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R}\overline{\mu_R}\sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_{D,L}\overline{\mu_L}\sigma^{\alpha\beta} e_R F_{\alpha\beta} \right) \\
BR(\mu \to e\gamma) &= 384\pi^2 (|C_{D,R}|^2 + |C_{D,L}|^2) < 5.7 \times 10^{-13} \\
&\Rightarrow |C_X^D| \lesssim 10^{-8}
\end{aligned}$$

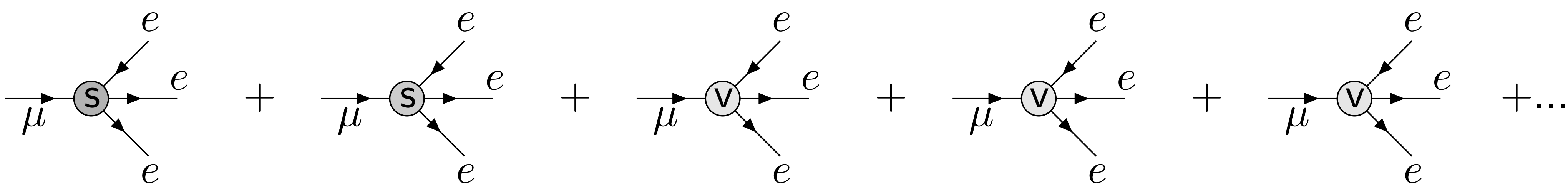
MEG expt, PSI

How big does one expect $C_{D,X}$ to be? Suppose operator coefficient

$$ec\frac{m_\mu}{v^2} \sim \frac{em_\mu}{(16\pi^2)^n \Lambda^2} \qquad \Rightarrow \qquad \begin{array}{ccc} & n = 1 & n = 2 \\ \text{probes} & \Lambda \lesssim 100 \text{ TeV} & 10 \text{ TeV} \end{array}$$

$\Rightarrow \mu \to e$ expts probe multi-loop effects in NP theories with $\Lambda_{NP} \gg$ reach of LHC

Constraining the operator-zoo with 3 processes: $\mu \to e\bar{e}e$



$(\overline{e}P_L\mu)(\overline{e}P_Le)$ $(\overline{e}P_R\mu)(\overline{e}P_Re)$ $(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_Le)$ $(\overline{e}\gamma P_R\mu)(\overline{e}\gamma P_Re)$ $(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_Re)$

In $\mu \to e\bar{e}e$, interference between operators $\propto m_e^2/m_\mu^2$

$$
\begin{aligned}
BR(\mu \to e\bar{e}e) \quad &= \quad \frac{|C_{S,LL}|^2 + |C_{S,RR}|^2}{8} + 2|C_{V,RR}|^2 + 2|C_{V,LL}|^2 \\
&\quad + |C_{V,LR}|^2 + |C_{V,RL}|^2 \qquad \Rightarrow \quad \boldsymbol{|C_X| \lesssim 10^{-6}\sqrt{\frac{BR}{10^{-12}}}}
\end{aligned}
$$

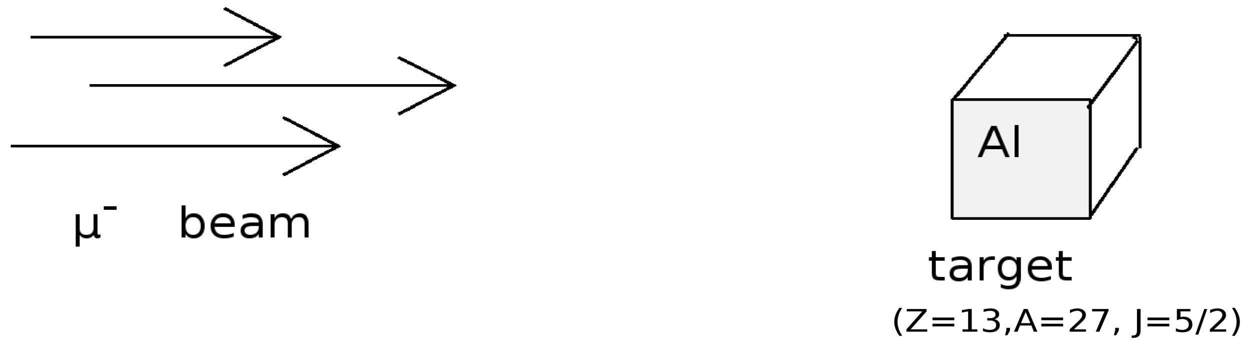
(set dipole contributions $\to 0$)

see nothing $\Rightarrow$ all Cs small

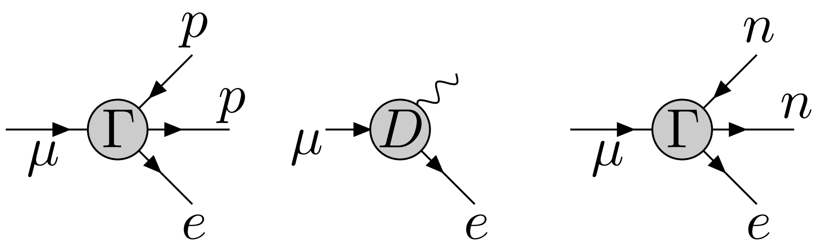
recall $2\sqrt{2}G_F C_X = 1/\Lambda^2 \Rightarrow \boldsymbol{\Lambda \gtrsim 2000}$TeV

see something $\Rightarrow$ distinguish operator via angular distributions?

$\mu \to e$ conversion



- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon capture $\mu + p \to \nu + n$
- bound μ interacts with nucleus, converts to e $(E_e \approx m_\mu)$



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ , A\ , T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over nucleons $\propto$ spin of only unpaired nucleon)

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Constraints on the nucleon operators from $\mu \to e$ conversion

DavidsonKunoSaporta

$$\mathrm{BR}_{SD}(A\mu \to Ae) \sim \left|\widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R}\right|^2 + \left|\widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L}\right|^2 \qquad (N \text{ odd})$$

$$\mathrm{BR}_{SI}(A\mu \to Ae) \propto |\widetilde{C}^{pp}_{V,R} V_A^{(p)} + \widetilde{C}'^{pp}_{S,L} S_A^{(p)} + \widetilde{C}^{nn}_{V,R} V_A^{(n)} + \widetilde{C}'^{nn}_{S,L} S_A^{(n)} + C_{D,L} D|^2 + \{L \leftrightarrow R\}$$

$$\sim \quad Z^2\left|\vec{C}_R \cdot \hat{v}_A\right|^2 + Z^2\left|\vec{C}_L \cdot \hat{v}_A\right|^2 \qquad \vec{v}_A \equiv \left(V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}, D_A\right)$$

Can distinguish SD vs SI, L vs R. But if observe SI conversion, how to know if is due to scalar/vector operator on n or p?

KitanoKoikeOkada

$$S_A^{(p)}, V_A^{(p)} \sim \int d^3x \widetilde{\psi}_\mu^{1s} |f_p(x)|^2 \widetilde{\psi}_e^* (\bar{p}\{1, \gamma_0\} p)$$

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different "target vectors" $\vec{v}_A$ for different nuclear targets

target vectors "live" in coefficient space, like $\vec{C} = (\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S, (D))$

1.1st exptal search (eg Gold) probes $\vec{C} \parallel \vec{v}_{Au}$

2.next target, suff large component $\perp$ Gold

$\Rightarrow$ three (suitable) nuclear targets (+improve theory caln) could probe 3 combinations of $\{\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S\}$

What to learn at Λ_{exp}: setting constraints from $\mu A \to eA$, $\mu \to e\gamma$ + $\mu \to e\bar{e}e$

parametrise with 20 nucleon ops (8 SI: S,V) + (12 SD: P,A,T)
+2 dipole operators
+6 four-lepton operators

1. constrain 2 dipoles +6 4ℓ coeffs with $\mu \to e\gamma + \mu \to e\bar{e}e$

2. SI now: constrain 4 combinations of 8 $\{S, V\}$ coefficients

SI future: constrain 6 combinations of 8 $\{S, V\}$ coefficients

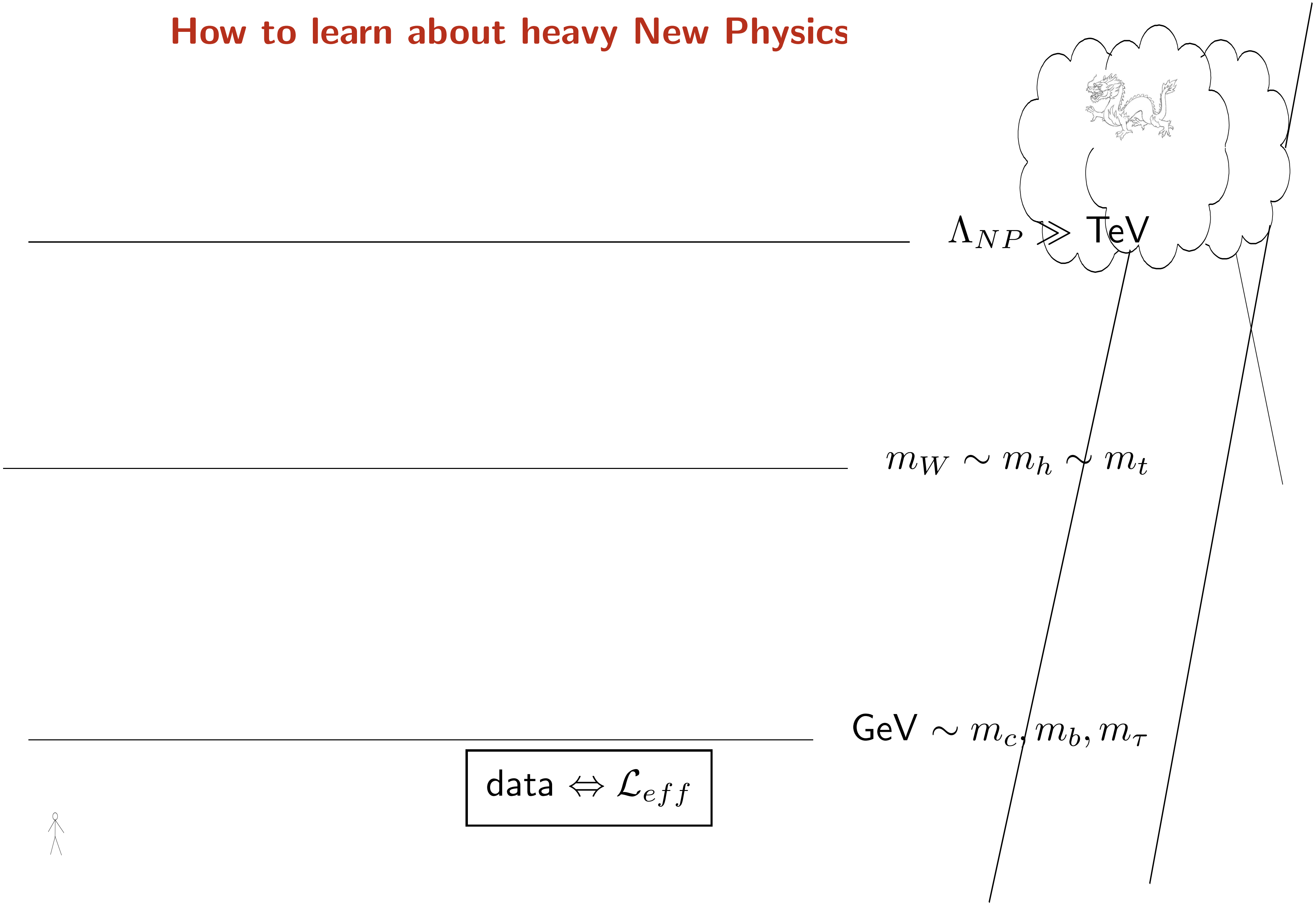
3. Spin-Dependent, now: (?) 2 counstraints? (Ti?)

future: $4 \to 8$ constraints ?

n vs p by comparing odd-p, A vs T vs P $\Leftrightarrow$ dedicated nucl.caln.)

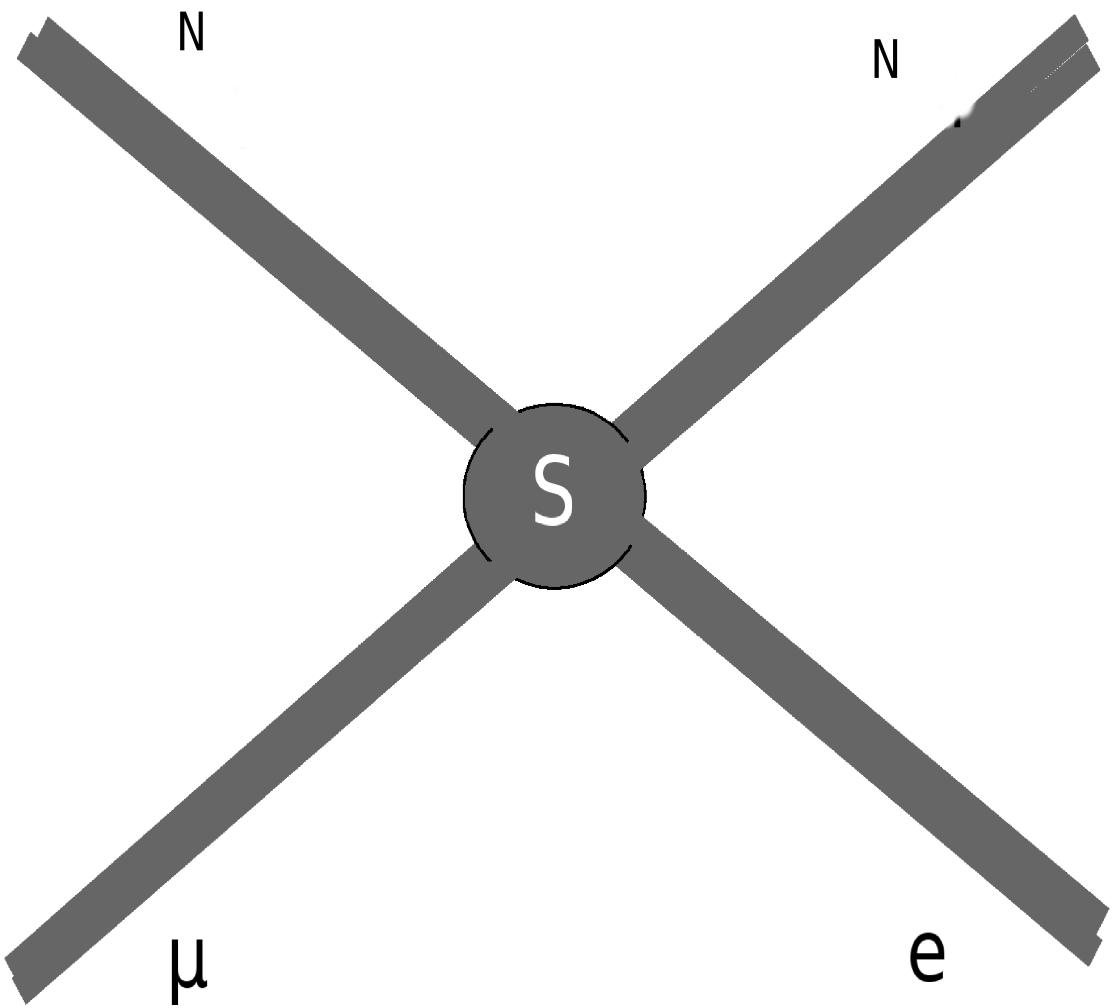
$\Rightarrow$ 28 coefficients, $\left\{ \begin{array}{cc} \text{now} & 12 \to 14 \\ \text{future} & 18 \to 22 \end{array} \right\}$ constraints

How to learn about heavy New Physics



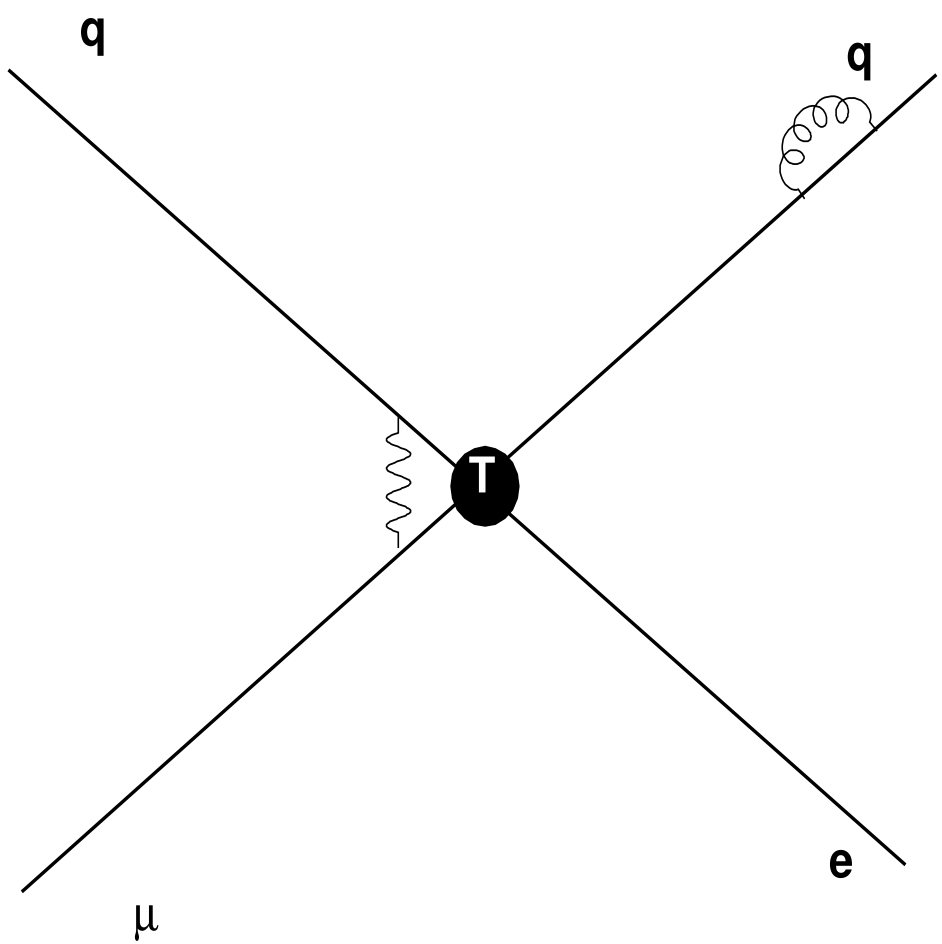
Peeling off the SM loop corrections

expt measures operator coefficient $\widetilde{C}(\mu_{exp})$, at exptal energy scale $\sim m_\mu \to m_\tau$, among external legs at same scale...



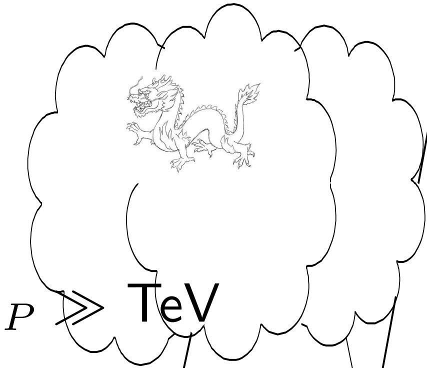
Peeling off SM loops

But if I look on shorter distance scale ($\sim 1/m_W$) I might see



Georgi, EFT, ARNPP 43(93) 209

How to learn about heavy New Physics



$\Lambda_{NP} \gg$ TeV

$\{Z, W, h, t, +\gamma, g, \{f\}\}$

$m_W \sim m_h \sim m_t$

$\{\gamma, e, \mu, \tau, g, u, d, c, s, b\}$

GeV $\sim m_c, m_b, m_\tau$

$\{\gamma, e, \mu, p, n, (\pi)\}$

$\boxed{\text{data} \Leftrightarrow \mathcal{L}_{eff}}$

The operator basis below m_W : 82 operators

Add QCD$\times$QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad \textit{dim } 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad \textit{dim } 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$$

$$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$

$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f) \qquad f \in \{u, d, s, c, b, \tau\}$$

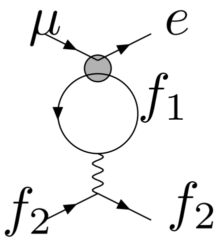
$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad \textit{dim } 7$$

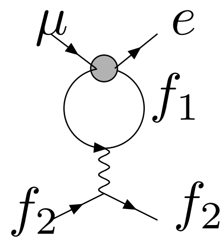
$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

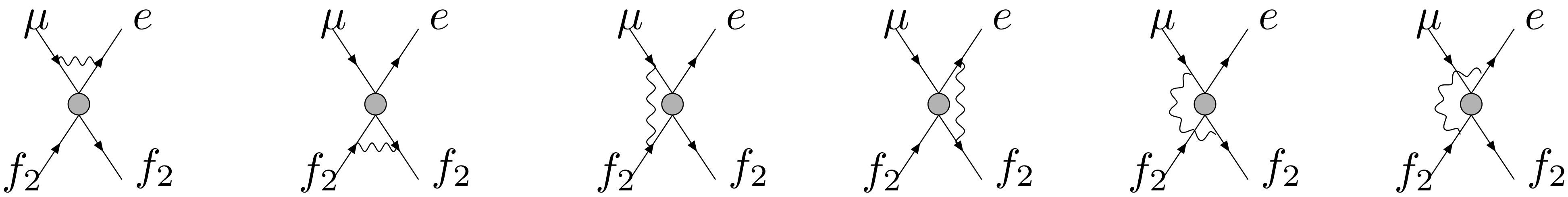
...zzz...but 82 coeffs!
(recall: 12-22 constraints...
...what to do?

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.



Run with QED + QCD between m_μ and m_W





$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_s}{4\pi} \vec{C} \mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

QCD: not mix ops, should resum $\Rightarrow$ multiplicative renorm S,T ops

QED: *does* mix ops, but $\alpha_{em} \ll$

But QED loops are $\mathcal{O}(\alpha/4\pi)$... surely negligeable?

Work top-down = suppose a model that gives only tensor operator at m_W:

$$2\sqrt{2}G_F\ C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$$

1: forget RGEs Match to nucleons $N \in \{n,p\}$ as $\widetilde{C}_T^{NN} = \langle N|\bar{u}\sigma u|N\rangle C_T^{uu} \lesssim \frac{3}{4}C_T^{uu}$

$$\Rightarrow \boldsymbol{BR \approx BR_{SD} \approx \frac{1}{2}|C_T|^2}$$

nuclear matrix elements:
EngelRTO, KlosMGS

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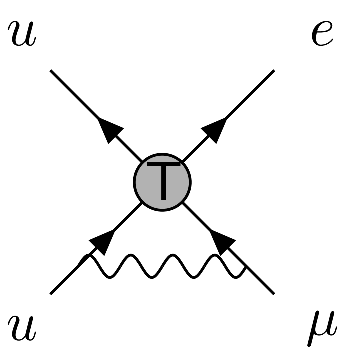
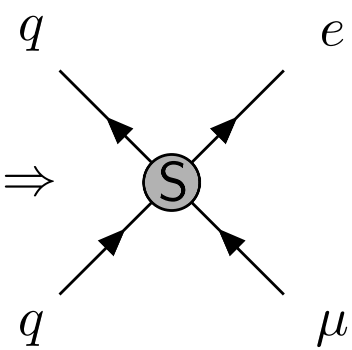
nuclear matrix elements:
EngelRTO, KlosMGS

2: include RGEs

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$



$+ \ldots \quad \Rightarrow$



$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y\mu)$$

$$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7}C_T^{uu}(m_W)$$

Then match to nucleons: $\widetilde{C}_S^{NN} = \langle N|\bar{u}u|N\rangle \Delta C_S^{uu} \sim C_T^{uu}$ so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$,

$$\boldsymbol{BR \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 8Z^2 BR_{SD}}$$

$\Rightarrow$ loop effects mix tensor to scalar.. change $BR(\mu A \to eA)$ by $\mathcal{O}(10^3)$

“peeling off SM loops” causes more coefficients to contribute

At tree level/at 2 GeV, 14 quark coefficients (+dipoles and di-gluons) contribute to SI $\boldsymbol{\mu \to e}$ conversion: ($|\vec{C} \cdot \hat{v}|$ for Al)

$$\sqrt{\frac{BR^{exp}_{Al}}{33}} \gtrsim \left|3C^{uu}_{V,L} + 3C^{dd}_{V,L} + 11C^{uu}_{S,R} + 11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R}\right|$$

also constraint on coeffs with $L \leftrightarrow R$ (the chirality of e)

quark coefficients at 2 GeV(lattice matching $\{G^{Nq}_S\}$)

at one loop, 44 (2 dipoles+2digluons) of 82 operators contribute to $\mu \to e$ conversion

$$
\begin{aligned}
\sqrt{\frac{BR^{exp}_{Al}}{33}} \quad \gtrsim \quad & \Big| 3C^{uu}_{V,L} + 3C^{dd}_{V,L} + \frac{\alpha}{\pi}\Big[3C^{dd}_{A,L} - 6C^{uu}_{A,L}\Big] \log \\
& + \frac{\alpha}{3\pi}[C^{ee}_{V,L} + C^{\mu\mu}_{V,L}] \log - \frac{\alpha}{3\pi}[C^{ee}_{A,L} + C^{\mu\mu}_{A,L}] \log \\
& - \frac{2\alpha}{3\pi}\Big[2(C^{uu}_{V,L} + C^{cc}_{V,L}) - (C^{dd}_{V,L} + C^{ss}_{V,L} + C^{bb}_{V,L}) - (C^{ee}_{V,L} + C^{\mu\mu}_{V,L} + C^{\tau\tau}_{V,L})\Big] \log \\
& + \lambda^{-a_S}\Big(11C^{uu}_{S,R} + 11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R}\Big) \\
& + \lambda^{-a_S}\frac{\alpha}{\pi}\Big[\frac{13}{6}(11C^{uu}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R}) + \frac{5}{3}(11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R})\Big] \log \\
& - \lambda^{a_T} f_{TS}\frac{8\alpha}{\pi}\Big[22C^{uu}_{T,R} + \frac{8m_N}{27m_c}C^{cc}_{T,R} - 11C^{dd}_{T,R} - 0.84C^{ss}_{T,R} - \frac{4m_N}{27m_b}C^{bb}_{T,R})\Big] \log \Big|
\end{aligned}
$$

also constraint on coeffs with $L \leftrightarrow R$ (the chirality of e)
quark coefficients at m_W
$\log \equiv \log(m_W/2\text{GeV}) \simeq 3.7$,
$\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

peeling loops off $\mu \to e\gamma$, $\mu \to e\bar{e}e$ $\Leftrightarrow$ sensitivity to 4-l tensors+scalars,+vectors
$\Rightarrow$ **$\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu \to e$ conversion *sensitive* to (almost) all operators**

Summary

Lepton Flavour Violation is the transformation of a charged lepton (e, μ or τ) into another, at a point. It is "New Physics" from "Beyond the Standard Model" that must exist (because of observed neutrino masses and mixing angles).
If observed, it would imply that the lepton flavour sector is different from the quarks, and give information on the required New Physics.

Sensitivive probes of $\mu \leftrightarrow e$ flavour change, are $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\boldsymbol{\mu \to e}$ conversion (= conversion of a μ, in the $1s$ state of a nucleus, into an electron who escapes with $E_e \sim m_\nu$). Current bounds probe an LFV mass scale $\Lambda_{LFV} \lesssim 10^3$ TeV at tree level. New expts will reach to $\Lambda_{LFV} \lesssim 10^4$ TeV in the next few years.

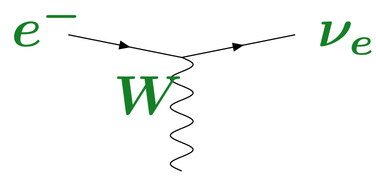
There are 82 three- or four-particle contact interactions, that involves $m < m_W$ particles, change $\mu \leftrightarrow e$, and are otherwise flavour diagonal. $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\boldsymbol{\mu \to e}$ conversion are sensitive, at tree level or via one-loop diagrams, to (almost) all these interactions (operators like $(\bar{s}\gamma P_L b)(\bar{e}\gamma P_X \mu)$ are constrained by meson decays).

However, these processes only set 12-20 constraints on the 82 operator coefficients. To reconstruct (heavy) New Physics model from operator coefficients, need to restrict allowed ranges of all operator coefficients...
$\Rightarrow$ find more (restrictive) constraints?
$\Rightarrow$ think about defining "fine-tuning" in EFT?

BackUp

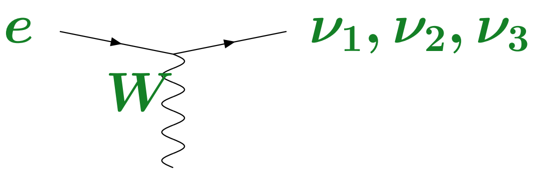
LFV in EFT

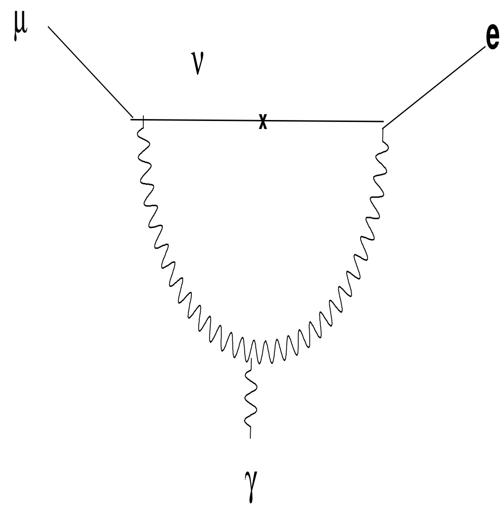
1. Lepton Flavour Change is interesting:
   - none in the Standard Model with $m_\nu = 0$
   - occurs with m_ν and mixing matrix U



1. (recall) flavour $\equiv$ mass eigenstate
2. charged leptons talk to νs at the W vertex
$\Rightarrow$ if $m_\nu = 0$, the define ν_e such that

...but when $m_\nu \neq 0$, $e + W$ could turn into any ν mass eigenstate:





LFV in EFT

1. Lepton Flavour Change is interesting:
   - none in the Standard Model with $m_\nu = 0$
   - occurs with m_ν and mixing matrix U

   m_ν renormalisable Dirac: LFV amplitudes GIM-suppressed (like quarks)

$$\mathcal{A} \propto \frac{m_\nu^2}{m_W^2} \Rightarrow BR \lesssim 10^{-48}$$

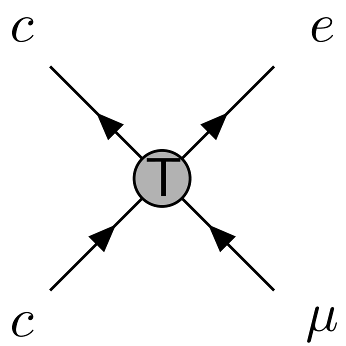
$\Rightarrow$ if see LFV, lepton flavour sector different from quarks!

suppose m_ν NOT Dirac, New Physics heavy

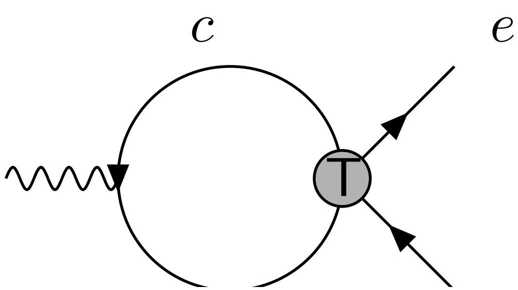
Does one need the loops, part 3? Of the tensor and the dipole...

suppose at $\sim m_W$: $\delta\mathcal{L} \supset C_T^{cc}(\bar{c}\sigma^{\alpha\beta}P_L c)(\bar{e}\sigma_{\alpha\beta}P_L\mu) + \ldots$
(eg from doublet leptoquark S with interactions $\lambda_L(\overline{\nu}s_L^c - \overline{\mu}c_L^c)S + \lambda_R\overline{e}c_R^c S$)

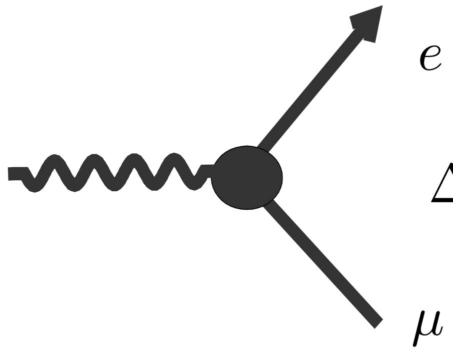
?How to observe that operator at tree level??



$\Rightarrow$



$\frac{16 m_c}{e m_\mu}\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\; C_T^{cc} m_\mu(\overline{e}\sigma\cdot F P_L\mu)$



$\Delta c_{D,L} \sim 1.2 C_T^{cc} m_\mu(\overline{e}\sigma\cdot F P_L\mu)$

recall MEG bound : $c_{D,Y} \lesssim 10^{-8}$ at m_μ

at m_W : $|C_{D,L} - C_{T,L}^{cc} + C_{T,L}^{\tau\tau} + 1.8 C_{T,L}^{bb} + \mathcal{O}(10^{-3})C| \lesssim 10^{-8}$

excellent sensitivity of $\mu \to e\gamma$ to mid-weight-fermion tensor operators

To calculate the $\mu \to e$ conversion rate (like WIMP scattering on nuclei)

build the nucleus as a bound state of nucleons, ($|f_N(x)|^2$= distribution of N in nucleus A)
bind muon in $1s$ state. For 4-ferm operators :

SI overlap int: KitanoKoikeOkada

$$\mathcal{M} \sim \sum_{N,O} \widetilde{C}_O(\bar{u}_e \Gamma_O u_\mu) \int d^3x \widetilde{\psi}_\mu^{1s} |f_N(x)|^2 e^{-iqx} (\bar{N}\Gamma_O N)$$

SD overlap int: guess from SD DM targets

For light nuclei ($Z \lesssim 30$), $\widetilde{\psi}_\mu^{1s} \simeq$ constant in nucleus, $\Rightarrow$ use WIMP results.
eg for **Spin-Dependent**: ($S_N^A \equiv$ spin expect. value of nucleon N in nucleus A of spin J_A. $S_N^A \simeq 1/2$).

$$\sum_{N\in A} \int d^3x |f_N(\vec{x})|^2 (\overline{u}_N \gamma^k \gamma_5 u_N) = 4 m_N S_N^A \frac{J_A^k}{|J_A|} \quad ,$$

Engel,...

also at $q^2 \to 0$: $\overline{N}\sigma N = 2\overline{N}\gamma\gamma_5 N$, $\overline{N}\gamma_5 N \to 0$ so with $\widetilde{C}'^{pp}_{A,L} \equiv \widetilde{C}^{pp}_{A,L} + 2\widetilde{C}'^{pp}_{T,R}$:

$$\frac{\Gamma_{SD}}{\Gamma_{capt}} \simeq \frac{8 G_F^2 m_\mu^5}{\Gamma_{capt}\pi^2} (Z\alpha)^3 \frac{J_A + 1}{J_A} \frac{S_A(m_\mu)}{S_A(0)} \left[\left| S_p^A \widetilde{C}'^{pp}_{A,L} + S_n^A \widetilde{C}'^{nn}_{A,L} \right|^2 + \{L \leftrightarrow R\} \right]$$

$$BR_{SD} \sim \left| \widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R} \right|^2 + \left| \widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L} \right|^2$$

CiriglianoDavidsonKuno

$S_A(q)$ finite momentum transfer correction (exists only for Axial) for Al $\simeq 0.29$ EngelRTO,KlosMGS
(also can make WIMP approx for low-Z SI $\mu \to e$ conversion)

The Spin-Independent $\mu \to e$ conversion rate

build the nucleus as a bound state of nucleons,
bind muon in $1s$ state. For 4-ferm operators :

SI overlap int: KitanoKoikeOkada

$$\mathcal{M} \sim \sum_{N,O} \widetilde{C}_O(\bar{u}_e \Gamma_O u_\mu) \int d^3x \widetilde{\psi}_\mu^{1s} |f_N(x)|^2 \widetilde{\psi}_e^* (\bar{N}\Gamma_O N)$$

SD overlap int: guess from SD DM targets

For heavy nuclei, $\widetilde{\psi}_\mu^{1s} \simeq$ varies in nucleus, $\Rightarrow$ evaluate overlap integrals.
For **Spin Independent** operators (D,S,V) KKO calculated "overlap integrals" of wavefns $\widetilde{\psi}_\mu^{1s}$, $\widetilde{\psi}_e \sim e^{iqx}$ and (for 4-f ops) operator $\times$nucleon density ($|f_N(x)|^2$):

$$S^{(p)}, V^{(p)} \quad \sim \quad \int d^3x \widetilde{\psi}_\mu^{1s} |f_p(x)|^2 \widetilde{\psi}_e^* (\bar{p}\{1, \gamma_0\}p)$$

Distortion of $\widetilde{\psi}_e$ at high Z causes $V^{(N)} > S^{(N)}$

$$\mathrm{BR}_{SI} \; = \; \frac{32 G_F^2 m_\mu^5}{\Gamma_{cap}} \Big[|\widetilde{C}_{V,R}^{pp} V^{(p)} + \widetilde{C}_{S,L}^{\prime pp} S^{(p)} + \widetilde{C}_{V,R}^{nn} V^{(n)} + \widetilde{C}_{S,L}^{\prime nn} S^{(n)} + C_{D,L} D|^2 + \{L \leftrightarrow R\} \Big]$$

$$\simeq \quad Z^2 |\sum \tilde{C}|^2 \quad \sim \quad Z^2 \frac{|\sum \tilde{C}_{SI}|^2}{|\sum \tilde{C}_{SD}|^2} BR_{SD}$$

caveats to (our) Spin Dep Estimates

make approximation
A,T overlap integrals $\leftrightarrow$ nuclear expectation value of spin current
SD $\boldsymbol{\mu \to e}$ conversion DM WIMP scattering

1. to use SD WIMP results, must be able to factor ψ_μ out of overlap integral
   but for "heavier" nuclei, $R_{nucleus} > R_{\psi_\mu} \sim \alpha/m_\mu$

2. SD WIMP results for Axial currents of nucleons
   at $q^2 = 0$, pseudoscalar vanishes and tensor current $\propto$ axial:

$$\begin{aligned}
\overline{u}^o_N(P_f)\gamma_5 u^t_N(P_i) &\to 2\vec{q}\cdot\vec{S}_N \\
\overline{u}^o_N(P_f)\gamma^j\gamma_5 u^t_N(P_i) &\to 4m_N S^j_N \\
\overline{u}^o_N(P_f)\sigma_{ik} u^t_N(P_i) &\to 4m_N \epsilon_{ikj} S^j_N
\end{aligned}$$

   spin vector of the nucleon: $2\vec{S}_N = u^\dagger_N \vec{\Sigma} u_N / 2E_N$
   rotation generator : $S^{ij} = \frac{i}{4}[\gamma^i, \gamma^j] = \frac{1}{2}\epsilon^{ijk}\Sigma^k$.

   But $q^2 = m^2_\mu$...what about P, and $A \neq T$ because no pion exchange to T.

Quantifying which targets give independent information

1. neglect Dipole (better sensitivity of $\mu \to e\gamma$ (MEGII) and $\mu \to e\bar{e}e$(Mu3e).
   remain to determine: $\vec{C} \equiv (\widetilde{C}^{pp}_{VR}, \widetilde{C}^{pp}_{SL}, \widetilde{C}^{nn}_{VR}, \widetilde{C}^{nn}_{SL})$

2. recall that
$$BR_{SI}(A\mu \to Ae) \propto \left| \vec{C} \cdot \vec{v}_A \right|^2$$
   where target vector for nucleus A
$$\vec{v}_A \equiv \left(V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)} \right)$$

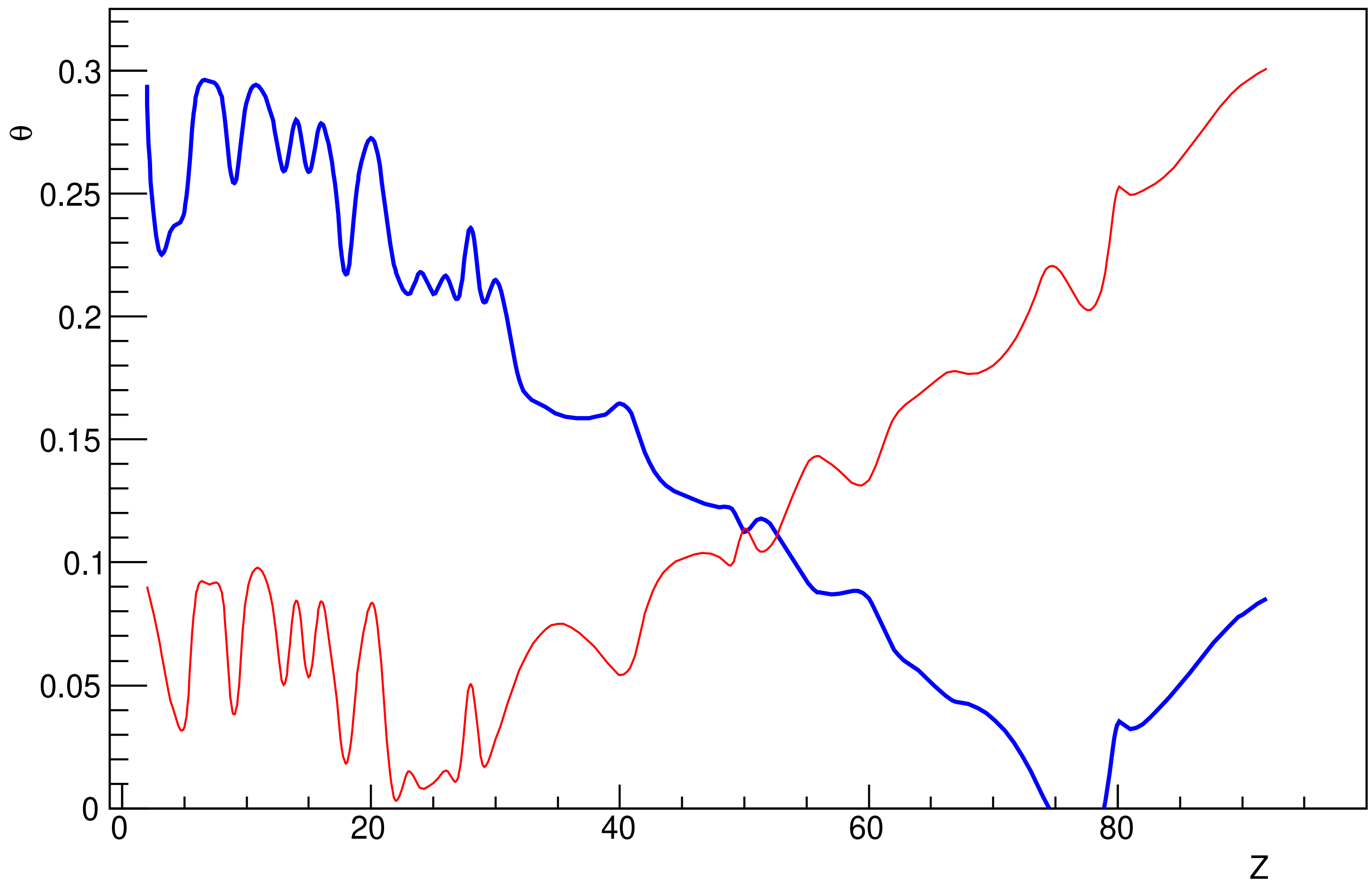
3. So first experimental search (eg on Aluminium) probes projection of $\vec{C}$ of $\vec{v}_{Al}$
   ... next target needs to have component $\perp$ to Aluminium!
   $\Leftrightarrow$ plot misalignment angle θ between target vectors

4. how big does θ need to be?
   overlap integrals have theory uncertainty: $\Delta\theta \begin{cases} \text{nuclear} & \sim 5\%(KKO) \\ NLO\ \chi\text{PT} & \sim 10\%(?) \end{cases}$
   Both vectors uncertain by $\Delta\theta$; need misaligned by $2\Delta\theta \approx 10 \to 20\%$

Current data+ theory uncertainty $\sim 10\%$: two targets give $\Delta\theta > 0.2$

$$\boldsymbol{BR(\mu Au \to e Au) \leq 7 \times 10^{-13}} \qquad (Au : Z = 79)$$

$$BR(\mu Ti \to e Ti) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$



$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis

In the future...with a 5% theory uncertainty:

First target of Mu2e, COMET: Aluminium (Z=13, A=27)

$$\hat{v}_{Al} \approx \tfrac{1}{2}(1,1,1,1)$$

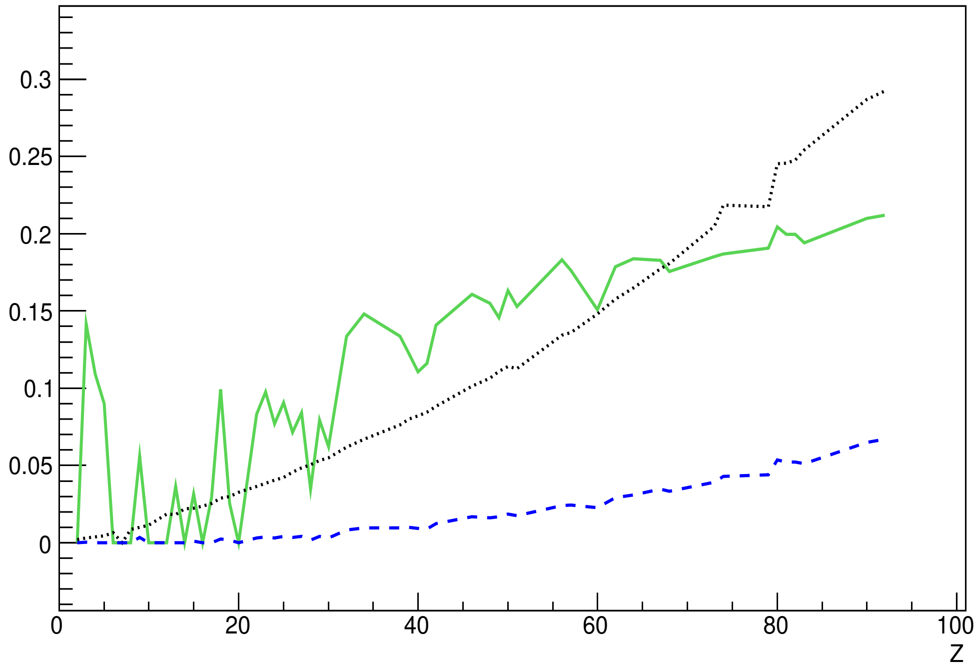
(recall $\tilde{C}_V^{pp}, \tilde{C}_S^{pp}, \tilde{C}_V^{nn}, \tilde{C}_S^{nn}$)

basis of three other "directions":

$$\hat{v}_{np} \equiv \frac{1}{2}(-1,-1,1,1)$$

$$\hat{v}_{VS} \equiv \frac{1}{2}(1,-1,1,-1)$$

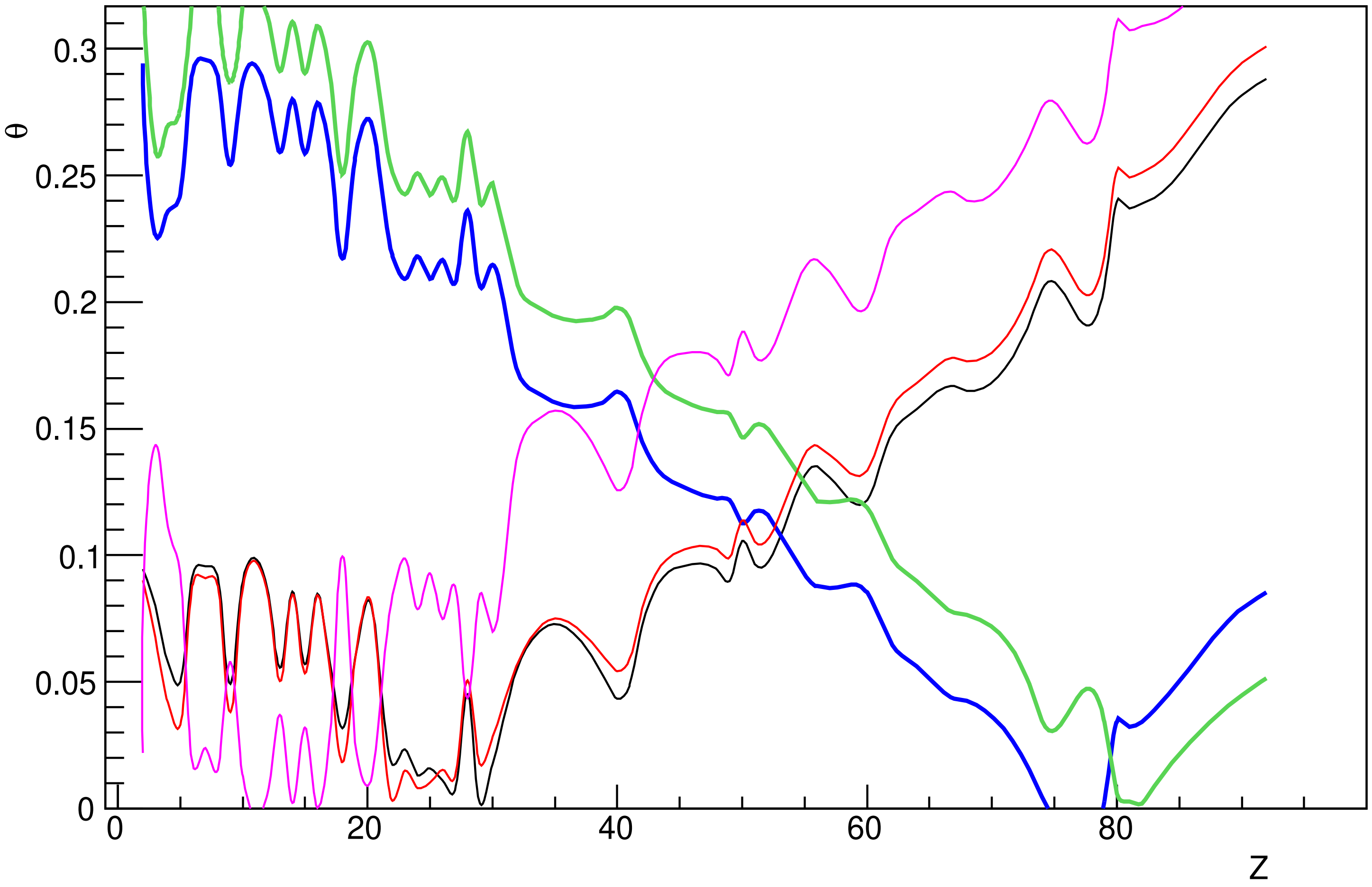
$$\hat{v}_{IsoSV} \equiv \frac{1}{2}(-1,1,1,-1)$$



probe 3 combinations of SI coeffs

All current data... $\boldsymbol{BR(\mu Au \to eAu) \le 7 \times 10^{-13}}$ $(Au : Z = 79)$

$BR(\mu Ti \to eTi) \le 4.3 \times 10^{-12}$ $(Ti : Z = 22)$



$\boldsymbol{BR(\mu Pb \to ePb) \le 4.6 \times 10^{-11}}$

$BR(\mu S \to eS) \le 7 \times 10^{-11}$ S = Sulpher, Z = 16

$BR(\mu Cu \to eCu) \le 1.6 \times 10^{-8}$ Cu = Copper, Z = 29

in practise: need to "match" and "run"

need a recipe to relate EFTs at different scales

1. when change EFTs (eg $N \leftrightarrow q$ at 2 GeV):
   $match$ ($=$ set equal) Greens functions in both EFTs at the matching scale
   match quark operators onto nucleon ($N \in \{n, p\}$) operators:

$$\bar{q}(x)\Gamma_O q(x) \to G_O^{N,q}\bar{N}(x)\Gamma_O N(x)$$

   eg, $\langle N|\bar{q}(x)q(x)|N\rangle = G_O^{N,q}\langle N|\bar{N}(x)N(x)|N\rangle = G_O^{N,q}\overline{u_N}(P_f)u_N(P_i)e^{-i(P_f-P_i)x}$
   So obtain, eg $\widetilde{C}^p_{S,L} = \sum_q G_S^{p,q}c^{qq}_{S,L}$

2. Within an EFT: Lagrangian parameters $(\alpha_s(\mu), \phi(\mu), C_I(\mu), ...)$ evolve with scale (due to loops).Described by Renormalisation Group Eqns. For $\{C_I\}$ below m_W:
   Davidson,CrivellinDPS

$$\mu\frac{\partial}{\partial\mu}(C_I, ...C_J, ...) = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}^e$$

   line up operator coefficients in $\vec{C}$, $\mathbf{\Gamma}$ = anomalous dimension matrix :
   $\mathbf{\Gamma}^s \leftrightarrow$ rescales coefficients, $\mathbf{\Gamma}^e \leftrightarrow$ transform one coeff to another...
   Above m_W : $\mathbf{\Gamma}$ for $SU(3) \times SU(2) \times U(1)$ JenkinsManoharTrott

But to reconstruct New Physics, need constraints not sensitivities...

sensitivity: range of parameter that $could$ see

(in best of all possible worlds)

Outside **thick** red line

constraint: outside ellipse incompatible with data

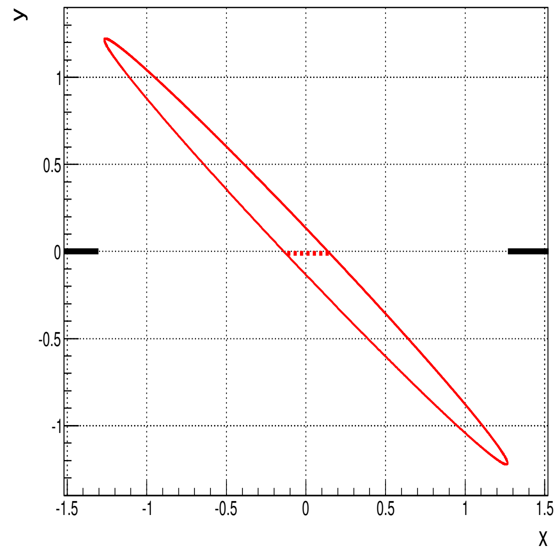
thick black line

to reconstruct NP, need to know ellipse
inside which sit $\{C\}$.

82 parameters, 12-22 constraints...what to do?

a) find more constraints ?

b) what cancellations are "natural" in EFT?



How much cancellation to beleive? ("fine-tuning of coefficients"?)

suppose $\{C(\Lambda_{expt})\}$ parametrise renormalisable, natural high-scale model.

1. allow arbitrary cancellations among $\{C(\Lambda_{NP})\}$

   ($\{C(\Lambda_{NP})\}$ unknown functions of the model parameters, symmetry-based cancellations could appear fortuitous?)

2. assume model not know Λ_{expt} (despite that is determined by mass ratios which models knows)

   so coefficients not cancel against logs

$\Rightarrow$ allow: $|C_1 + nC_2| = 0$, not allow: $|C_1 + n\alpha_{em}C_2 \log| = 0$

• QCD-running of scalars and tensors $\Leftrightarrow$ not cancel S vs T vs V to more than one sig fig

$$\Rightarrow |\textstyle\sum_j C_{V,j} + \lambda^{a_S}\sum_k C_{S,k} + \lambda^{a_T}\sum_i C_{T,i}| < \epsilon \longrightarrow \begin{cases} |\sum_j C_{V,j}| < 10\epsilon \\ |\lambda^{a_S}\sum_k C_{S,k}| < 10\epsilon \\ |\lambda^{a_T}\sum_i C_{T,i}| < 10\epsilon \end{cases}$$

Then within each subset, at each order in $\alpha_{em}\log$, allow cancellations up to next order $\sim \mathcal{O}(\frac{\alpha_{em}}{4\pi}\log)$:

$$\text{if} \quad |\Sigma_j n_j C_j| < \epsilon \Rightarrow C_j \lesssim \frac{4\pi}{\alpha_{em}}\epsilon$$

Which coefficients are missing? Why?

axial operators $(\overline{e}\gamma^\alpha P_Y \mu)(\overline{f}\gamma_\alpha f)$ for $f \in \{\tau, c, s, b\}$

pseudoscalar operators $(\overline{e}P_Y \mu)(\overline{f}\gamma_5 f)$ for $f \in \{\tau, u, d, c, s, b\}$

diphotons $+$ CPV digluons