TU Dresden

Precision Particle Physics with Muons

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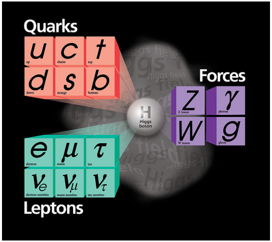
2 December 2019

Standard Model: too good to crack (so far) under experimental pressure

not good enough to make us happy

$\Rightarrow$ increase experimental scrutiny



$\Rightarrow$

$\Rightarrow$ BSM

- 1001 tests of the SM

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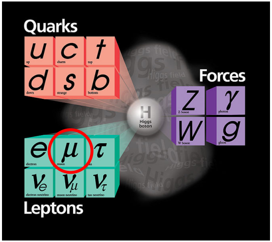


$\Rightarrow$

$\Rightarrow$ BSM

- 1001 tests of the SM
- how can the poor, little muon help ?

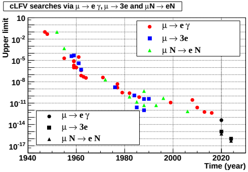
- why muons
  - what muons do and why they are interesting
- the muon Lagrangian
  - massive QED corrections and massification
- an NLO example
  - the radiative $\mu \to e\nu\bar{\nu}\gamma$ and rare $\mu \to e\nu\bar{\nu}(ee)$ muon decay
- an NNLO example
  - muon decay $\mu \to e\nu\bar{\nu}$ and looking for $\mu \to eX$
- another NNLO example: the anomalous magnetic moment
  - the trouble with hadronic contributions and a new proposal, MUonE $\mu e \to \mu e$
- yet another NNLO example: the proton radius
  - the trouble with hadronic contributions and a new proposal, MUSE $\ell p \to \ell p$
- closing remarks

- muons are fairly clean
- can be 'easily' produced in large numbers
- extremely well studied
- a very good place to look for tiny deviations from the SM
  $\Rightarrow$ we need loads of muons

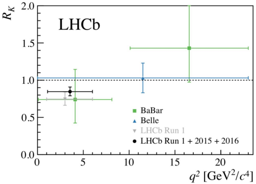
what we want muons to do: the non-expected

- charged lepton-flavour violation (cLFV, e.g. $\mu \to e\gamma, \mu \to 3e$)
- lepton-universality violation (LUV)
- other weird decays (non-SM light particles)
- the anomalous magnetic moment (AMM) a_μ

hopes and hints: the muon as trouble maker ?

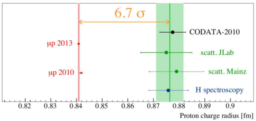


plot courtesy of A.Papa

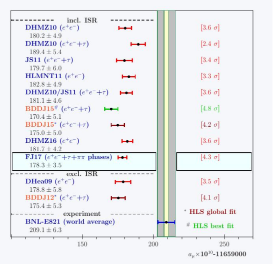


$$\frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)}$$

proton radius (2013 !?)



plot courtesy of A.Antognini



AMM: $(g-2)_\mu$

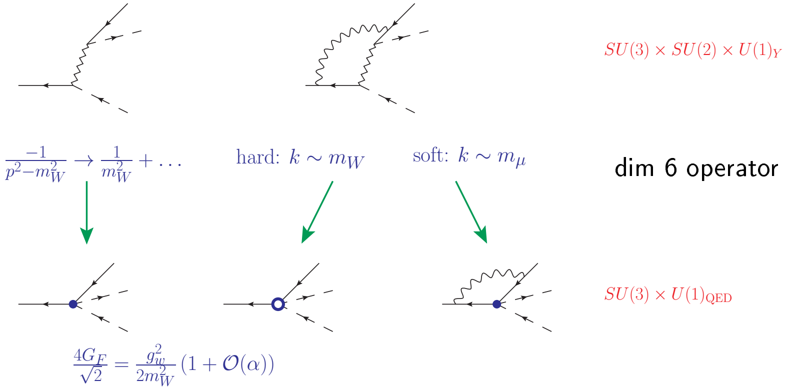
plot courtesy of F.Jegerlehner

what muons can do: decay, scatter or form bound state (and then decay)

- Michel $\mu \to e\nu\bar{\nu}$, radiative $\mu \to e\nu\bar{\nu}\gamma$ and rare $\mu \to e\nu\bar{\nu}e^+e^-$ decay
  - get G_F, background to cLFV
  - test for non-SM contribution
- $\mu p \to \mu p$ and $\mu e \to \mu e$ elastic scattering
  - proton radius
  - hadronic contributions to AMM
- form muonium $M = (\mu^+ e^-)$ bound states
  - QED (and even gravity) tests
  - $M - \bar{M}$ oscillations
- muonic hydrogen $(\mu^- p)$: proton radius !
- muonic atoms: input for e.g. parity violation experiments
- get captured, $p\mu^- \to n\nu$: input to neutrino-nucleon scattering

effective (Fermi) theory quantum field theory valid for $p^2 \ll m_W^2$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}} + \frac{4G_F}{\sqrt{2}} \left(\overline{e_L}\gamma^\mu \mu_L\right)\left(\overline{\nu_L}\gamma_\mu \nu_L\right) + \text{h.c.} + \mathcal{O}(m_W^{-4})$$



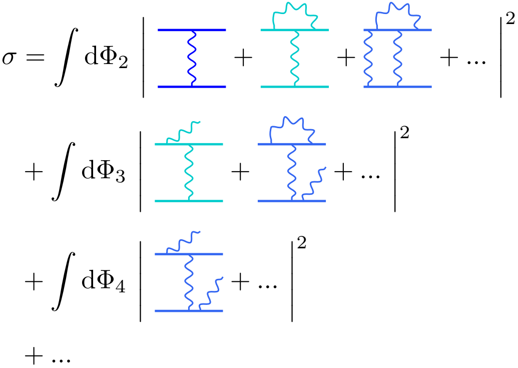
special situation: Wilson coefficient G_F does not run (“coincidence”) just do QED with:

$$\mathcal{L}_{\rm muon} = \mathcal{L}_{\rm QED} + \frac{4G_F}{\sqrt{2}} \left(\overline{e_L}\gamma^\mu \mu_L\right)\left(\overline{\nu_L}\gamma_\mu \nu_L\right) + \text{h.c.} + \mathcal{L}_{\rm dirt}$$

- compute higher-order QED corrections until you drop dead
- at some point have to deal with hadronic effects

⇒ muon is only fairly clean, not very clean

⇒ AMM and proton radius

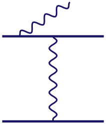


- LO trivial
- NLO
  virtual, UV IR singular
  real IR
- NNLO
  double-virtual, UV IR
  real virtual, UV IR
  double-real IR

UV $1/\epsilon$ singularities $\Rightarrow$ renormalize (only QED !)

There are IR $1/\epsilon$ singularites $\Rightarrow$ cancel between real and virtual

we use dimensional regularization throughout (no photon masses etc.)

$$\int \mathrm{d}\Phi_\gamma$$  $$\sim \quad \int_0 \mathrm{d}E_\gamma \int_0 \mathrm{d}\theta \, \frac{1}{E_\gamma(1-\beta_e\cos\theta)}$$

Two types of IR singularities

- soft: $E_\gamma \to 0$: universal behaviour $\Rightarrow \mathcal{M}^{(0)}_{n+(\gamma\to 0)} = \mathcal{E}\,\mathcal{M}^{(0)}_n$
- collinear: $\theta \to 0$ (if $\beta_e = 1 \leftrightarrow m_e = 0$)

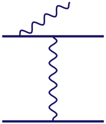
  for $m_e \neq 0$ no $1/\epsilon$ singularity, but large $\log m_e/Q$

  in QED $m_e \neq 0$ can be seen as regulator for collinear divergences

KLN theorem

- soft divergences from real emissions cancel those from loops

IR divergences

$$\int \mathrm{d}\Phi_\gamma$$  $$\sim \int_0^{E_{\text{res}}} \mathrm{d}E_\gamma \int_0^{\theta_{\text{res}}} \mathrm{d}\theta \, \frac{1}{E_\gamma(1-\beta_e\cos\theta)}$$

Two types of IR singularities

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KLN theorem

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FKS^2 subtraction scheme

idea

$$\underbrace{\int \mathrm{d}\Phi_\gamma \Big(\mathcal{M}(E_\gamma) - \mathcal{M}_{\mathsf{CT}}\Big)}_{\text{complicated \& finite} \,\to\, \text{numerically}} \qquad + \qquad \underbrace{\int \mathrm{d}\Phi_\gamma \mathcal{M}_{\mathsf{CT}}}_{\text{divergent \& easy} \,\to\, \text{analytically}}$$

idea

$$\underbrace{\int \mathrm{d}\Phi_\gamma \Big(\mathcal{M}(E_\gamma) - \mathcal{M}_{\mathsf{CT}}\Big)}_{\text{complicated \& finite} \,\to\, \text{numerically}} \qquad + \qquad \underbrace{\int \mathrm{d}\Phi_\gamma \mathcal{M}_{\mathsf{CT}}}_{\text{divergent \& easy} \,\to\, \text{analytically}}$$

FKS (NLO) [Frixione, Kunszt, Signer 95]

$$\sigma^{(1)} = \sigma^{(1)}_n(\xi_c) + \sigma^{(1)}_{n+1}(\xi_c)$$

$$\sigma^{(1)}_n(\xi_c) = \int \mathrm{d}\Phi^{d=4}_n \left(\mathcal{M}^{(1)}_n + \hat{\mathcal{E}}(\xi_c)\,\mathcal{M}^{(0)}_n\right) = \int \mathrm{d}\Phi^{d=4}_n \big(\xi_1\,\mathcal{M}^{(1)f}_n\big)$$

$$\sigma^{(1)}_{n+1}(\xi_c) = \int \mathrm{d}\Phi^{d=4}_{n+1} \left(\frac{1}{\xi_1}\right)_c \big(\xi_1\,\mathcal{M}^{(0)f}_{n+1}\big) \qquad \text{where } \xi_1 \sim E_\gamma$$

distributions: $\int d\xi \left(\frac{1}{\xi}\right)_c f(\xi) \equiv \int d\xi\, \frac{f(\xi) - f(0)\theta(\xi_c - \xi)}{\xi}$ unphysical parameter ξ_c

FKS^2 (NNLO) [Engel, AS, Ulrich 19]

$$\sigma^{(2)} = \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c)$$

$$\sigma_n^{(2)}(\xi_c) = \int \mathrm{d}\Phi_n^{d=4} \left(\mathcal{M}_n^{(2)} + \hat{\mathcal{E}}(\xi_c)\, \mathcal{M}_n^{(1)} + \frac{1}{2!}\mathcal{M}_n^{(0)} \hat{\mathcal{E}}(\xi_c)^2 \right)$$

$$\sigma_{n+1}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+1}^{d=4} \left(\frac{1}{\xi}\right)_c \left(\xi\, \mathcal{M}_{n+1}^{(1)f}(\xi_c) \right)$$

$$\sigma_{n+2}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+2}^{d=4} \left(\frac{1}{\xi_1}\right)_c \left(\frac{1}{\xi_2}\right)_c \left(\xi_1 \xi_2\, \mathcal{M}_{n+2}^{(0)f} \right)$$

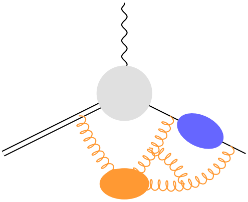
FKS$^\ell$ (N$^\ell$LO)

"easy", based on YFS: $\sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = e^{-\alpha \hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f}$

$$\sigma_{n+j}^{(\ell)}(\xi_c) = \mathrm{d}\Phi_{n+j}^{d=4} \frac{1}{j!} \left(\prod_{i=1}^{j} \left(\frac{1}{\xi_i}\right)_c \xi_i \right) \mathcal{M}_{n+j}^{(\ell-j)f}(\xi_c) \quad \text{with} \quad j = 0 \ldots \ell$$

simple process ($\mu \to e\nu\nu$)

- $\mathcal{A}_\mu(m) =$
  $\mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m \log m)$
- $Z \supset \log(m)$:
  process indep. jet fct.
- $\mathcal{S} \supset \log(m)$:
  process dep. soft fct. (easy)



[Becher, Melnikov 07; Engel, Gnendiger, AS, Ulrich 18]

'massification'

simple process ($\mu \to e\nu\nu$)

- $\mathcal{A}_\mu(m) =$
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- $\mathcal{S} \supset \log(m)$:
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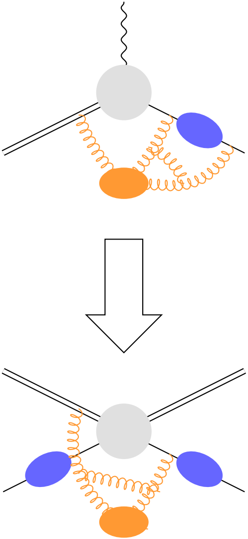
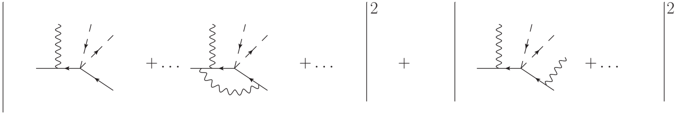
[Becher, Melnikov 07; Engel, Gnendiger, AS, Ulrich 18]

complex process ($\mu e \to \mu e$)

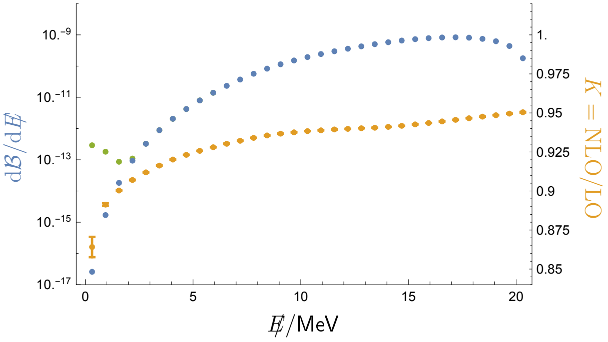
- $\mathcal{A}_{\mu e}(m) =$
  $\mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m \log m)$



radiative decay, fully differential background to $\mu \to e\gamma$ if $\not{E} = E_{2\nu} \to 0$



$\not{E}$ spectrum with experimental cuts (e.g. no $2^{\rm nd}$ γ with $E_\gamma > 2$ MeV)

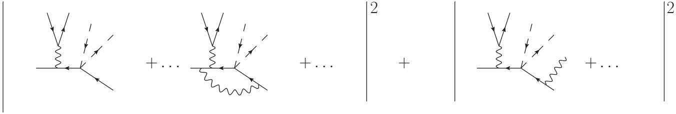


corrections large in tail $\sim \alpha \log \not{E}/M$

$\mathcal{B}_{\mathsf{NP}} \simeq 4.2 \cdot 10^{-13}$

NLO example: $\mu \to 3e + 2\nu$ in the SM

fully differential NLO calculation background to $\mu \to 3e$ if $\not{E} = E_{2\nu} \to 0$



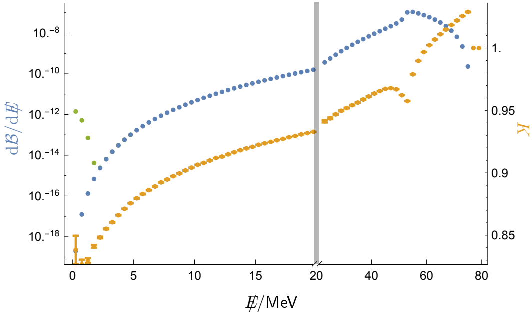
polarization: $\vec{s} = -0.85\hat{z}$ toy cuts: $E_i > 10$ MeV, $|\cos\theta_i| < 0.8$

$\not{E}$ spectrum

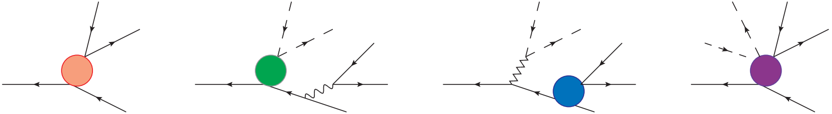
$\not{E} = m_{\mathrm{mu}} - \sum E_i$

$\mathcal{B}_{\mu\to 3e} \simeq 10^{-12}$

[Pruna,AS,Ulrich; 1611.03617]



beyond cLFV, other weird stuff



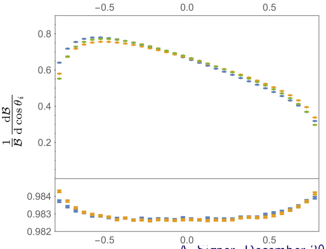
- better suited for small-coupling small-mass scenario
- simplified-model scenario better suited?
  e.g. doubly charged Higgs or dark photon or neutrinos

[Pruna,AS,Ulrich; 1611.03617]

e.g. $\cos\theta$ of hard e^+, soft e^+, e^-

no stringent cuts: $\Delta_{\text{theory}} < 0.1\%$

diagnostics @ Mu3e very useful



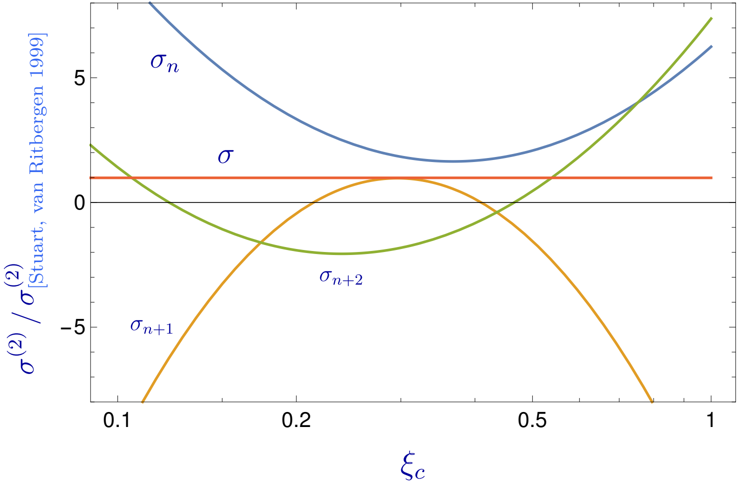
muon decay at NNLO: history calculations

- inclusive NNLO for G_F Stuart, van Ritbergen 1999
- logarithms $\log^{\{1,2\}} \frac{m_e^2}{m_\mu^2}$ of $\mathrm{d}\Gamma/\mathrm{d}E_e$ Arbuzov, Czarnecki, Gaponenko 2002, Arbuzov, Melinkov 2002
- fully inclusive, numeric energy spectrum Anastasiou, Melnikov, Petriello 2005

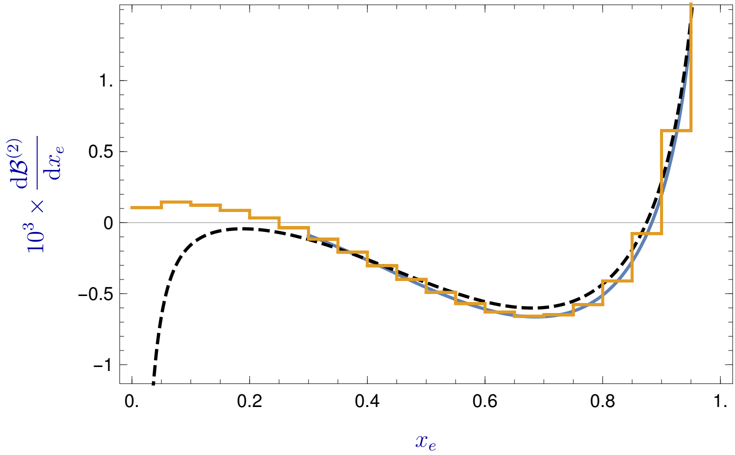
how?

- analytic two-loop integrals Chen 2018 and form factors Engel, Gnendiger, AS, Ulrich 2018
- fully differential Monte Carlo Engel, AS, Ulrich, 2019 using FKS^2

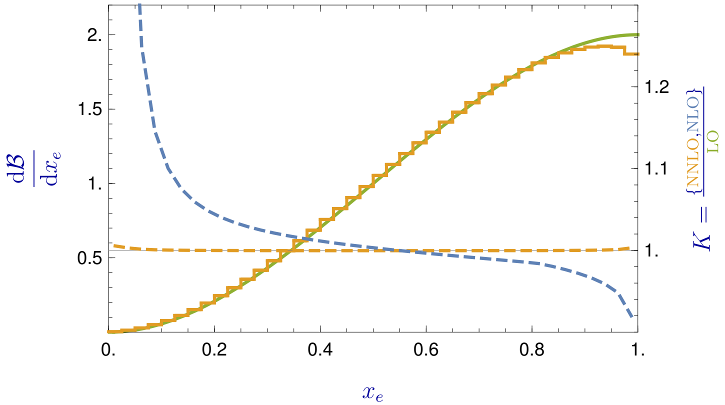
inclusive decay: check ξ_c (in)dependence



electron energy spectrum



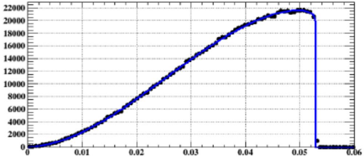
our result, Anastasiou et al, logarithms total photon energy within $\cos\sphericalangle(\vec{p}_e, p_\gamma) > 0.8$ is $\sum E_\gamma < 10\text{MeV}$



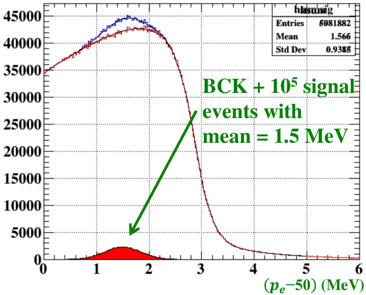
NNLO K-factor dashed, large logs in tail $\Rightarrow$ resummation

$\mu \to eX$ to look for X: a (usually very) light neutral boson

find bump in positron energy spectrum: $$E_e = \frac{M^2 + m^2 - m_X^2}{2M}$$



plots from talk of Fabrizio Cei (MEG)



m_X not too small: no theory needed; Twist

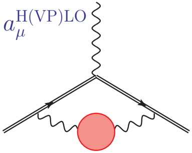
$m_X \to 0$: theory needs to provide precise spectrum at end point!
N. . .NLO plus resummation!

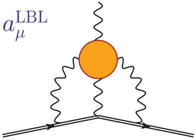
AMM: a_μ

BNL E821 (2006): $a_\mu^{\text{exp}} = 116592091(63) \times 10^{-11}$

many (!!) theorists : $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim 300(80) \times 10^{-11}$

Fermilab E989 : $\delta a_\mu^{\text{exp}} \to \sim 15 \times 10^{-11}$ also J-PARC





'problematic' hadronic contributions to a_μ^{SM}

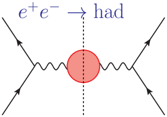
a_μ^{HLO} : needs exp. (or lattice) input currently largest uncertainty
$\{6931(34), 6933(25), 6881(41)\} \times 10^{-11}$
Davier et al; Keshavarzi et al; Jegerlehner, 2017/18

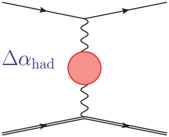
$a_\mu^{\text{HN...NLO}}$: $\sim -85(2) \times 10^{-11}$

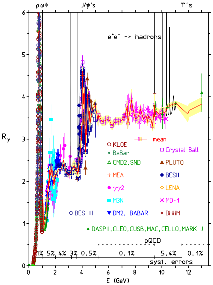
a_μ^{LBL} : most difficult (but smaller) $\sim 100(30) \times 10^{-11}$

$$a_\mu^{\mathsf{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\sigma_{\mathsf{had}}$$

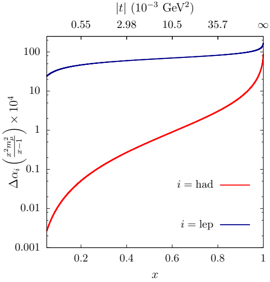
$$a_\mu^{\mathsf{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\Delta\alpha_{\mathsf{had}}$$







Jegerlehner:1511.04473

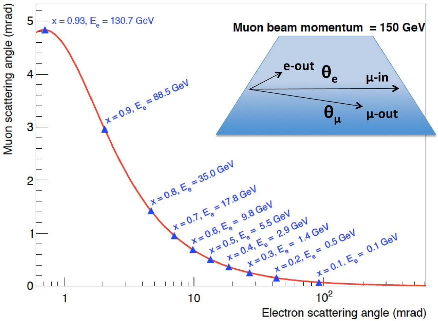


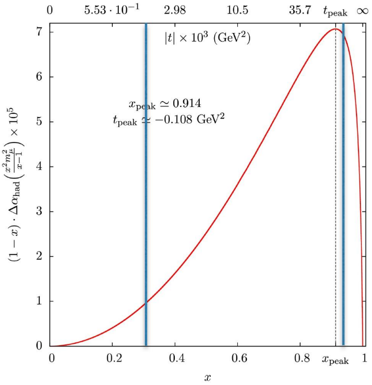
Abbiendi et al:1609.08987

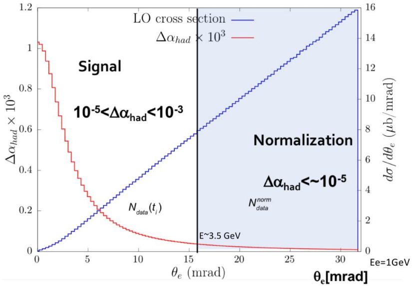
new proposal [Abbiendi et al.]: elastic scattering $\mu e \to \mu e$

150 GeV μ beam (CERN M2 beam) on e at rest (Beryllium target)

$\to$ nearly full coverage of integrand [plots from talks by M.Passera and G.Venanzoni]

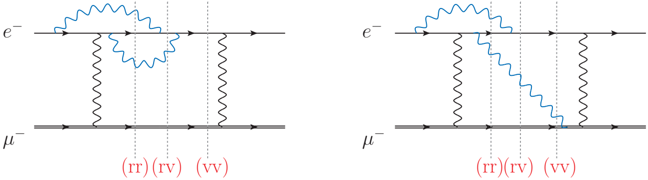




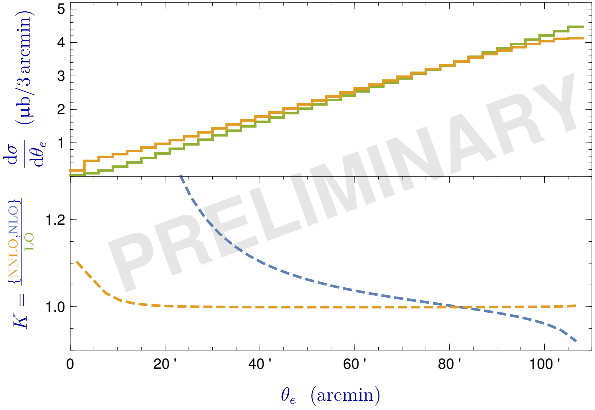


- need ~ 10 ppm determination of cross section
- signal region: high e energy, small angle

Need NNLO QED and hadronic corrections + resummation of logarithms



- first results: NNLO due to emission from e line only
- (dominant) gauge-invariant subset Banerjee, Engel, AS, Ulrich



resummation wil be needed, add accomplanarity cut Alacevich et al. (2018)

- extract form factors at high precision at low Q^2 (MESA, MUSE, PRad, QWeak, ...)
- QED corrections (cf. at NLO Gramolin 2014, Akushevich et al. 2015 and NNLO Bucoveanu, Spiesberger 2018)
- emission-from-ℓ-line-only relatively straightforward

$$\propto \bar{u}(m_\mu)\gamma_\mu u(m_\mu)$$

$$\propto \bar{u}(m_p)\Big(F_1\gamma^\mu + F_2\,\frac{i\sigma^{\mu\nu}q_\nu}{2m}\Big)u(m_p)$$

- source of all trouble: proton is not point like

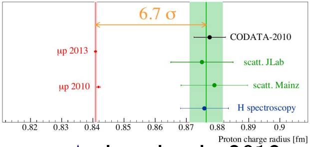
$$\langle N(p)|J^\mu|N(p)\rangle = \bar{u}(p') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M} \right] u(p)$$

- electric form factor

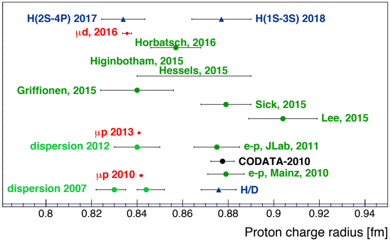
$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2}F_2(q^2)$$
$$= \int d^3r\, \rho(r) e^{-iqr} = \int d^3r\, \rho(r) \left(1 - q^2\frac{r^2}{6} + \dots\right)$$

- proton radius: $\langle r_E^2 \rangle = \int d^3r\, r^2 \rho(r) = -6 \frac{dG_E(q^2)}{dq^2}\Big|_{q^2=0}$
- need slope of $G_E(q^2)$ at $q^2 = 0$
- or spectroscopy; $(e^- p)$ or $(\mu^- p)$ bound states

confusing situation, solution in sight?

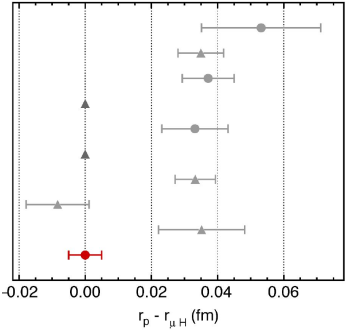


⇑ situation in 2013



⇑ situation in 2018

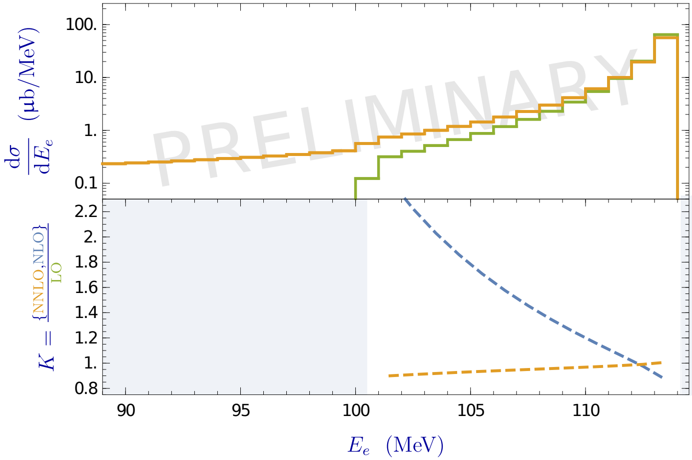
plot courtesy of A.Antognini



e^- and μ^- spectroscopy
and e scattering

⇐ another approach: MUSE
do $e^\pm$ and $\mu^\pm$ scattering on p
$q^2 = 0.002 - 0.07$ GeV^2

MUSE cuts $20° < \theta < 100°$, $p_{\text{in}} = 115$ MeV



- shaded region kinematically forbidden at tree-level muons can be exciting (powerful model killers)

- cLFV
- anomalous magentic moment
- search for other (mainly) low mass BSM

muons can be helpful (not really covered here)

- measurement of proton radius (muonic hydrogen)
- measurement of charge radius of radium (for atomic parity violation experiments)
- test QED (and even gravity) with muonium
- measurement of nucleon axial radius through capture rate (input for neutrino-nucleon scattering)