Precise calculations of the Higgs mass in supersymmetric models

Thomas Kwasnitza

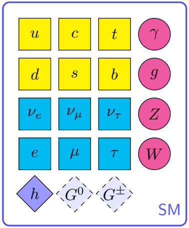
3rd December 2020

based on: [TK, D. Stöckinger, A. Voigt 20]

TECHNISCHE UNIVERSITÄT DRESDEN

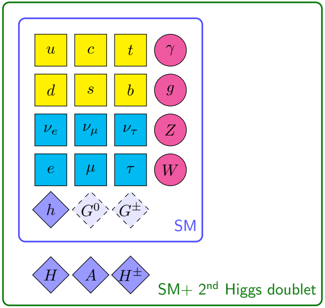
INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS

SUSY



- Supersymmetry is a new fundamental spacetime symmetry

SUSY



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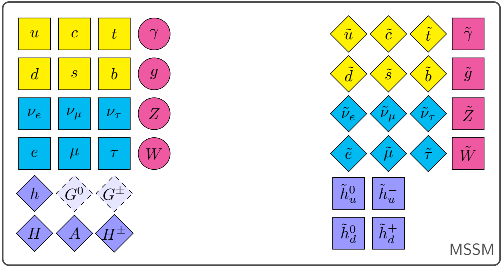

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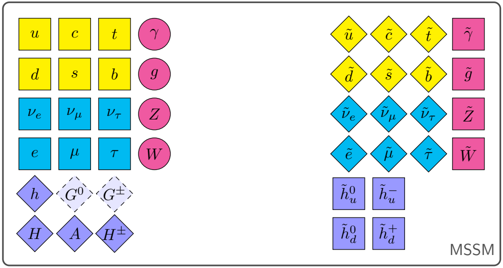
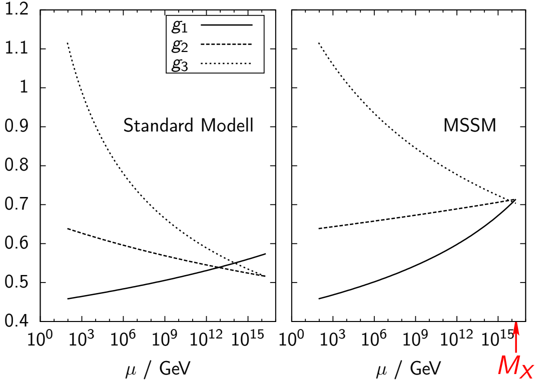


- Supersymmetry is a new fundamental spacetime symmetry
- Advantages for physics close to the EW scale
  - Dark matter
  - $g-2$ of the muon
  - Dynamical breaking of EW symmetry ($\mu^2 < 0$)
  - Predicts one light Higgs boson

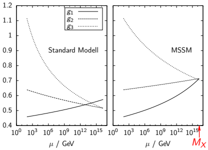
SUSY and Grand Unification



generated by FS A. Voigt

→improved unification of gauge couplings

SUSY and Grand Unification



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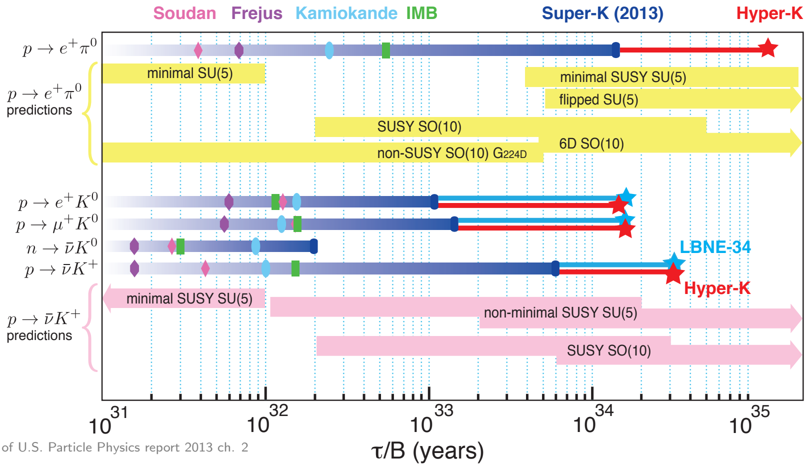
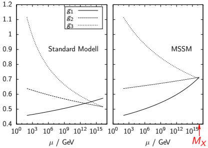


Future of U.S. Particle Physics report 2013 ch. 2

SUSY and Grand Unification



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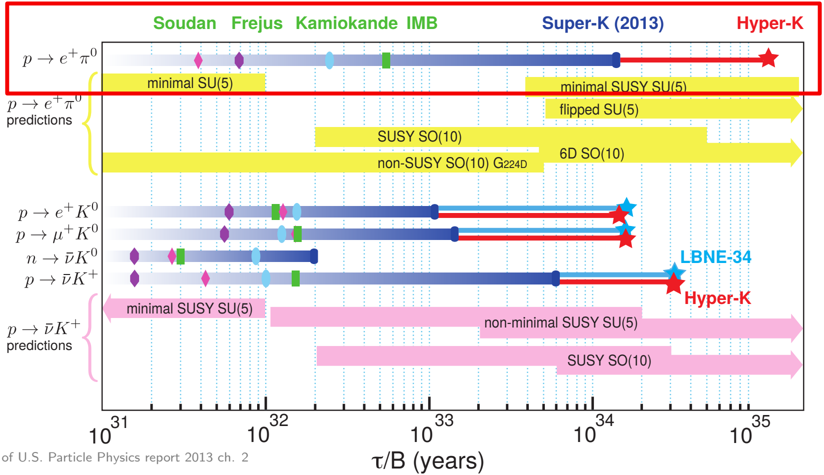

Future of U.S. Particle Physics report 2013 ch. 2

Features of Supersymmetry

Theorem in QFT (Weinberg Witten)

No massless spin $s > 2$ particle can couple consistently to massless particles with $s \leq 2$

see: Prof. Dominik Stöckinger RQFT WS19/20 Lecture 21 - 23

Poincaré + Unitarity $\implies$ the menu of **elementary** particles is given by

massive case.: N. Arkani-Hamed, T. Huang, Y. Huang 2017

$$s \in \left\{ 0, \qquad \frac{1}{2}, \qquad 1, \qquad \frac{3}{2}, \qquad 2 \right\}$$

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described by gauge theories (→ 1)

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unique gravity

[Eppley, Hannah 76]

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described by local SUSY ($\frac{3}{2}$)

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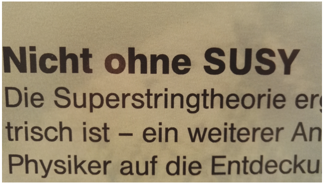
- Relation to (quantum) gravity: SUGRA, string theory, matrix models, ...



Higgs mass in the MSSM

- SUSY extension **predicts** one light Higgs

$$m_{H,h}^2 = \frac{1}{2}\left[m_Z^2 + M_S^2 \pm \sqrt{(m_Z^2 + M_S^2)^2 - 4m_Z^2 M_S^2 c_{2\beta}^2}\,\right]$$

$$m_h^2 \approx m_Z^2 c_{2\beta}^2 \le m_Z^2, \qquad\qquad m_H^2 \approx M_S^2$$

- Pole masses are PDG 2018

$$M_h = \sqrt{m_h^2 + \Delta m_h^2} \approx 125.1\,\text{GeV}$$
$$M_Z \approx 91.2\,\text{GeV}$$

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- Anatomy of radiative contributions

$$\Delta m_h^2 = v^2 \kappa^n \left[\log \frac{M_S}{v} + C_0 + C_1 \frac{v^2}{M_S^2}\right]$$

$\rightarrow$ **Use** $\left[M_h \overset{!}{=} 125.1\,\text{GeV}\right]$ **as a constraint on new parameters**

Higgs mass in the MSSM

$$\Delta m_h^2 = v^2 \kappa^n \left[\log \frac{M_S}{v} + C_0 + C_1 \frac{v^2}{M_S^2} \right]$$

- Q: Which parameter can be constrained?

$$\left\{ M_S, \qquad \tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}, \qquad \begin{array}{l} x_t \ \text{mixing parameter} \\ M_{\tilde{t}}^2 = \begin{pmatrix} M_S^2 & x_t m_t M_S \\ (x_t m_t M_S)^* & M_S^2 \end{pmatrix}, \cdots \end{array} \right\}$$

- MSSM is consistent iff **large** radiative corrections exist:

$$(86\,\text{GeV})^2 \leq \Delta m_h^2 \leq (125\,\text{GeV})^2$$

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    Exception: FSSM K. Benakli et al. 2013

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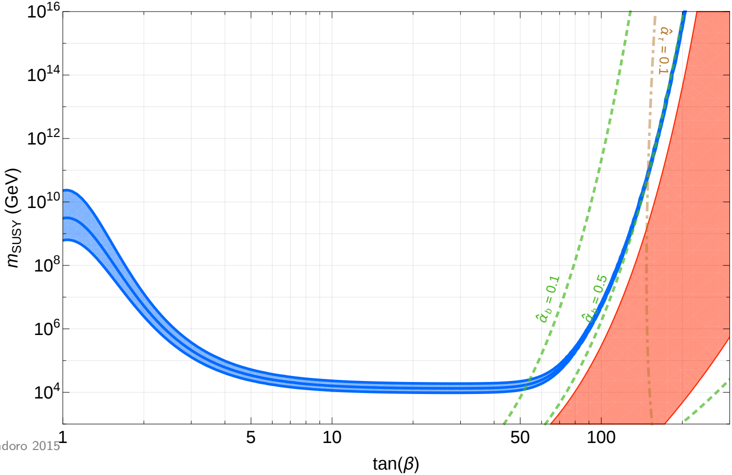
- Conclusive constraint requires low uncertainty
  - Good: experimental uncertainty $\Delta M_h^{\text{exp}} \approx 0.14\,\text{GeV}$
  - Problem: theory uncertainty $\Delta M_h^{\text{th}} \approx 1\text{-}3\,\text{GeV}$

$\left[M_h \stackrel{!}{=} 125.1 \pm 1.5\,\text{GeV}\right]$ solved for M_S



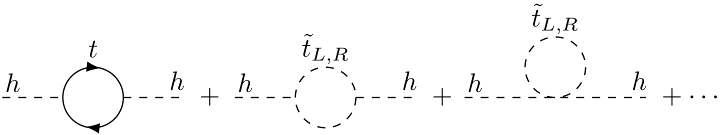
J.Vega, G. Villadoro 2015

→ Need for a high precision calculation!

Contents

Radiative corrections to the Higgs boson mass

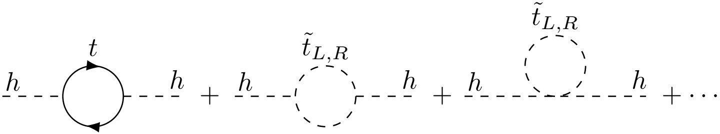
$\Delta m_h^2 \supset$



- Leading loop (in SM parameters) contributions at NLO

$$\Delta m_h^2 = \kappa y_t^2 m_t^2 \left[24 \log \frac{M_S}{m_t} - x_t^4 + 12 x_t^2 + \frac{m_t^2}{M_S^2} \frac{15}{4} + \cdots \right]$$

Radiative corrections to the Higgs boson mass

$\Delta m_h^2 \supset$ 

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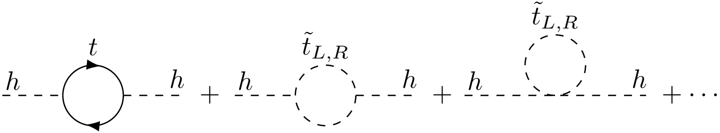
$\Delta m_h^2 \supset$ 

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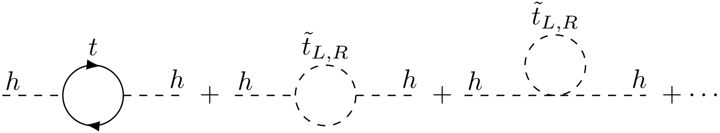
$\Delta m_h^2 \supset$ 

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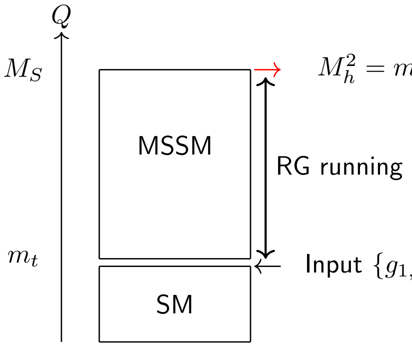
- Large loop corrections for:
  - large mass gap $M_S \gg m_t$
  - large off-diagonal element in the stop mass matrix: $|x_t| \approx \sqrt{6}$

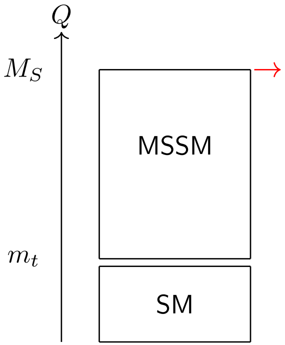
Fixed-order calculation



$$M_h^2 = m_Z^2 c_{2\beta}^2 + \Delta m_h^2\big|_{1\ell} + \Delta m_h^2\big|_{2\ell} + \cdots + \Delta m_h^2\big|_{n\ell}$$

Input $\{g_{1,2,3}, v, y_f\}$ from observ.$\{\alpha_{e,s}, G_F, M_{Z,f}\}$

Fixed-order calculation



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$+$: includes terms $\mathcal{O}(\kappa^n v^2/M_S^2)$

$-$: misses contributions $\mathcal{O}(\kappa^m \log^m M_S/m_t)$ from higher orders, $m > n$

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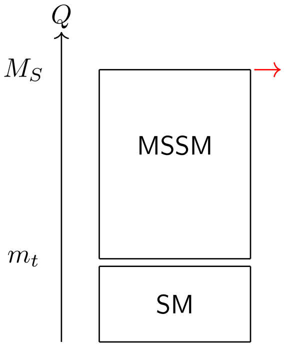

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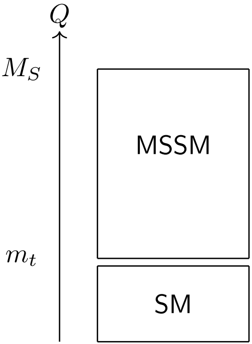

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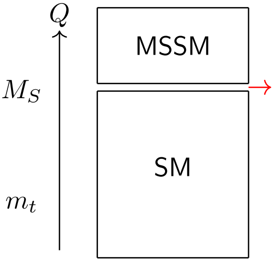
- Good: gives reliable predictions if M_S and m_t are not too far apart
- Problem: if $M_S \gg m_t \Rightarrow$ perturbative expansion converges slow

$$\begin{aligned} M_h &= m_h + \Delta m_h^{1\ell} + \Delta m_h^{2\ell} + \Delta m_h^{3\ell} + \cdots \\ &\approx [91 + \mathcal{O}(20\ldots30) + \mathcal{O}(2\ldots5) + \mathcal{O}(1\ldots2)]\,\text{GeV} \end{aligned}$$

Trick 1: Resummation of large logarithms

Effective field theory approach $[\lim(v/M_S) \to 0]$



$$\lambda = \lambda^{(0\ell)} + \Delta\lambda^{(1\ell)} + \Delta\lambda^{(2\ell)} + \ldots + \Delta\lambda^{(n\ell)}$$

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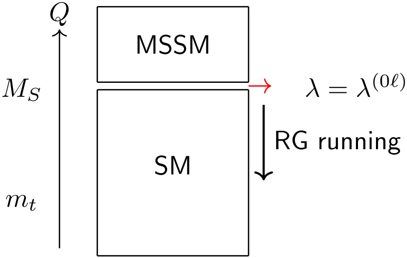

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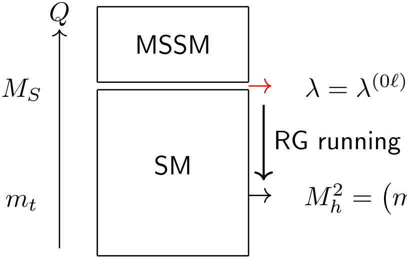

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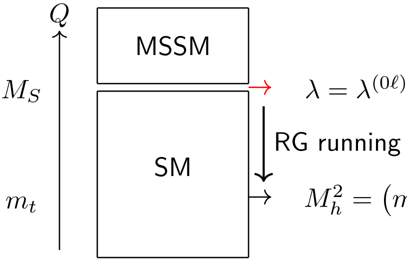

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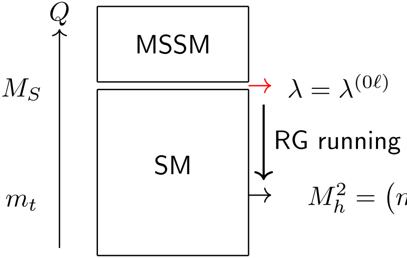

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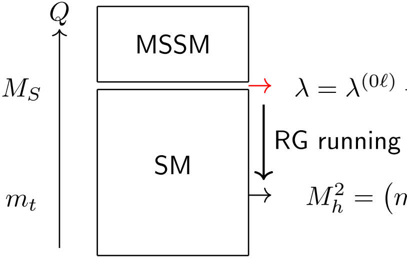

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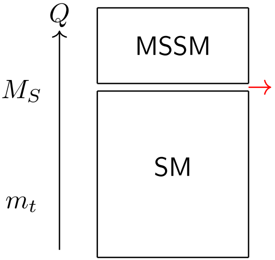
& + y_t^2 g_3^2 \left(C_0^{2\ell} + k_{(1,2)} \log \frac{M_S}{m_t} + k_{(2,2)} \log^2 \frac{M_S}{m_t} \right) \\
& + y_t^2 g_3^4 \left(C_0^{3\ell} + k_{(1,3)} \log \frac{M_S}{m_t} + k_{(2,3)} \log^2 \frac{M_S}{m_t} + k_{(3,3)} \log^3 \frac{M_S}{m_t} \right) \\
& + \cdots
\end{aligned}$$

Trick 1: Resummation of large logarithms

Effective field theory approach $[\lim(v/M_S) \to 0]$



$$\lambda = \lambda^{(0\ell)} + \Delta\lambda^{(1\ell)} + \Delta\lambda^{(2\ell)} + \Delta\lambda^{(3\ell)}$$

$+$: resummed logarithms of M_S/m_t

$-$: misses suppressed terms of $\mathcal{O}(v^2/M_S^2)$

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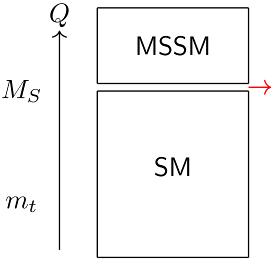

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- Good: Predicts Higgs mass reliably for large separation of m_t and M_S
- Problem: for $M_S \simeq m_t \Rightarrow$ large uncertainty

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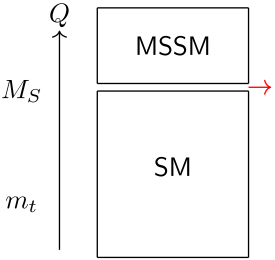

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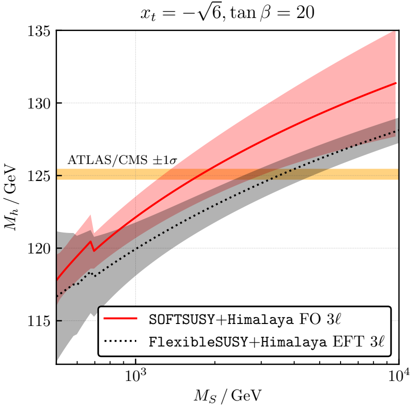
- Good: Predicts Higgs mass reliably for large separation of m_t and M_S
- Problem: for $M_S \simeq m_t \Rightarrow$ large uncertainty
- convergence rate of perturbative expansion
  $(x_t = -\sqrt{6}, \tan\beta = 20, M_S = 3\,\text{TeV})$

$$\begin{aligned} M_h &= m_h + \Delta m_h^{1\ell} + \Delta m_h^{2\ell} + \Delta m_h^{3\ell} \\ &\approx [\mathcal{O}(117) + \mathcal{O}(7) + \mathcal{O}(1) + O(0.2)]\,\text{GeV} \end{aligned}$$

Comparison of M_h calculations



B. Allanach, A. Voigt 2018

	low M_S $M_S \lesssim 1.3$ TeV	high M_S $M_S \gtrsim 1.3$ TeV
fixed-order (FO)	✓	✗
EFT	✗	✓

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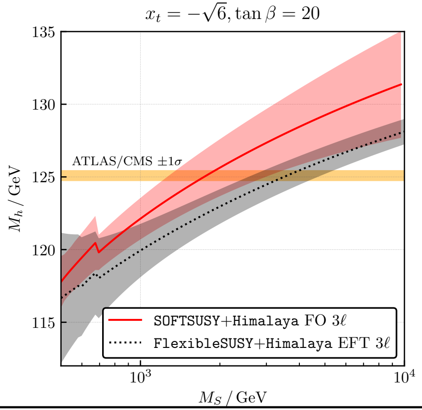

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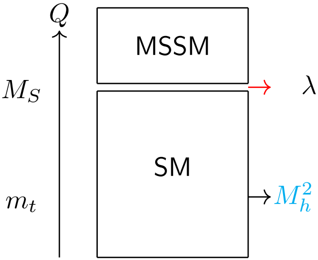
	low M_S $M_S \lesssim 1.3\,\text{TeV}$	high M_S $M_S \gtrsim 1.3\,\text{TeV}$
fixed-order (FO)	✓	✗
EFT	✗	✓
Q: FO and EFT combined?	✓	✓

Trick 2: Hybrid calculation

A: Yes e.g. FlexibleEFTHiggs [P. Athron et al. 2016], [P. Athron et al. 2017], [TK et al. 2020]



$$\lambda = \frac{1}{v^2}\left[(M_h^2)^{\text{MSSM}} - \Delta m_h^{\text{SM}}\right]$$

- Obtain λ from pole mass matching

$$(M_h^2)^{\text{SM}} = (M_h^2)^{\text{MSSM}}, \qquad\qquad Q = M_S$$

- Ensure that $\log(M_S/m_t)$ cancel in λ ⚠

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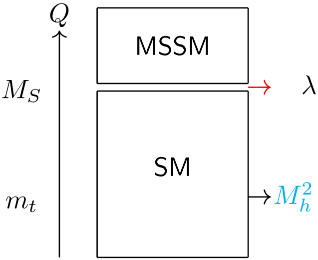

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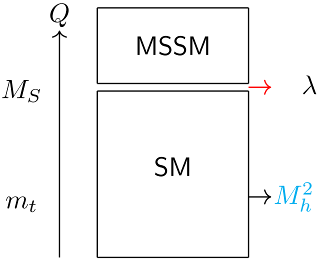

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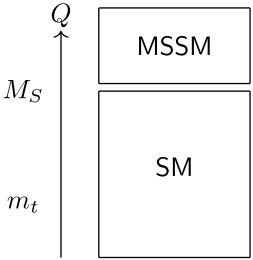
$$(M_h^2)^{\text{SM}} = (M_h^2)^{\text{MSSM}}, \qquad Q = M_S$$

- Ensure that $\log(M_S/m_t)$ cancel in λ ⚠
- Good:
  - Valid at all scales
  - Automatized derivation of $\Delta\lambda$ from self energies and tadpoles

Hybrid calculation FlexibleEFTHiggs at N^{3}LO and N^{3}LL



M_h^2 at 3ℓ

$\mathcal{O}(1\ell + (\alpha_t + \alpha_b)\alpha_s + (\alpha_t + \alpha_b + \alpha_\tau)^2 + \alpha_s^2\alpha_t)$

[0105096, 0112177, 0212132, 0206101, 0305127, 1205.6497, 1407.4336, 1708.05720]

RGE at 4ℓ [1201.5868, 1205.2892, 1212.6829, 1303.4364, 1604.00853]

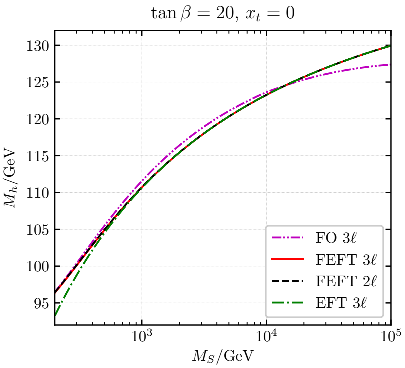
$\alpha_s^{\text{SM}(5)}$ at 2ℓ [9305305, 9707474]

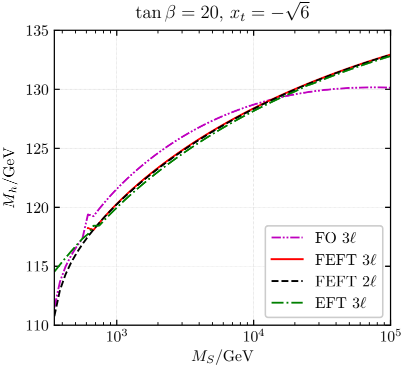
M_t at 3ℓ $\mathcal{O}(\alpha_s^2 + \alpha_s^3)$ [9912391, 0507139, 9912391]

- 3L self energy from FlexibleSUSY+Himalaya obtained without terms of $\mathcal{O}(\kappa^3 v^2/M_S^2)$

Hybrid calculation FlexibleEFTHiggs at N^{3}LO and N^{3}LL

- Interpolation of Hybrid FEFT between FO and EFT:





for $x_t = 0$ exact

for $x_t = -\sqrt{6}$ discrepancy

- Discrepancy is dominated by highest power x_t contributions included in FEFT

Trick 3: Resummation of leading x_t powers

- **Observation 1**: the presented EFT calc. and FlexibleEFTHiggs work in different parametrizations!
  $\Rightarrow$ expand both matching conditions to 2-loop

$$
\begin{aligned}
\Delta\lambda^{\text{FEFT}} =& \kappa(y_t^{\text{MSSM}})^4\left[6x_t^2 \qquad\qquad -\frac{1}{2}x_t^4\right] + \mathcal{O}\left(\frac{m_t^2}{M_S^2}\right) \\
&+ \kappa^2 g_3^2 (y_t^{\text{MSSM}})^4\left[c_{(0,2)} + c_{(1,2)}x_t + \cdots \quad + c_{(4,2)}x_t^4\right] + \mathcal{O}\left(\frac{m_t^2}{M_S^2}\right) \\
\Delta\lambda^{\text{EFT}} =& \kappa(y_t^{\text{SM}})^4\left[6x_t^2 \qquad\qquad -\frac{1}{2}x_t^4\right] \\
&+ \kappa^2 g_3^2 (y_t^{\text{SM}})^4\left[\tilde{c}_{(0,2)} + \tilde{c}_{(1,2)}x_t + \cdots \quad + \tilde{c}_{(4,2)}x_t^4 + \tilde{c}_{(5,2)}x_t^5\right]
\end{aligned}
$$

- **Q**: How is the x_t^5 term reproduced in FEFT?

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\Delta\lambda^{\text{FEFT}} =& \kappa(y_t^{\text{MSSM}})^4 \left[6x_t^2 - \frac{1}{2}x_t^4\right] + \mathcal{O}\left(\frac{m_t^2}{M_S^2}\right) \\
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\end{aligned}
$$

- **Q**: How is the x_t^5 term reproduced in FEFT?
- **A**: The reparametrization of y_t contains contributions $\Delta_t \propto g_3^2 x_t$

$$
y_t^{\text{SM}} = y_t^{\text{MSSM}}\left[1 - \Delta_t^{1\ell}\right] \qquad \Rightarrow \qquad y_t^{\text{MSSM}} \approx y_t^{\text{SM}}\left[1 + \Delta_t^{1\ell}\right]
$$

Parametrizations are equivalent at 2-loop!

Trick 3: Resummation of leading x_t powers

- **Observation 2**: Use the exact (implemented) relation

$$y_t^{\mathsf{MSSM}} = \frac{y_t^{\mathsf{SM}}}{\left[1 - \Delta_t^{1\ell}\right]} = y_t^{\mathsf{SM}} \sum_n (\Delta_t^{1\ell})^n$$

$$\begin{aligned}
(\Delta\lambda^{\mathsf{FEFT}})^{(1\ell)} =& \kappa (y_t^{\mathsf{MSSM}})^4 \left[6 x_t^2 \qquad - \frac{1}{2} x_t^4 \right] \\
=& \kappa (y_t^{\mathsf{SM}})^4 \left[6 x_t^2 \qquad - \frac{1}{2} x_t^4 \right] \\
&+ \kappa^2 g_3^2 (y_t^{\mathsf{SM}})^4 \left[\cdots \qquad + \hat{c}_{(4,2)} x_t^4 + \hat{c}_{(5,2)} x_t^5 \right] \\
&+ \kappa^3 g_3^4 (y_t^{\mathsf{SM}})^4 \left[\cdots \qquad + \hat{c}_{(4,3)} x_t^4 + \hat{c}_{(5,3)} x_t^5 + \hat{c}_{(5,3)} x_t^6 \right] \\
&\cdots
\end{aligned}$$

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&\ldots
\end{aligned}$$

- Direct calculation reveals
  - $\hat{c}_{(5,2)} x_t^5 = \tilde{c}_{(5,2)} x_t^5$ ✓
  - $\hat{c}_{(4,2)} x_t^4$ ✗, gets contributions from 2-loop Feynman diagrams
- **Q: What can be said about higher orders?**

Trick 3: Resummation of leading x_t powers

A: An inspection of multi-loop diagrams reveals that the highest power contributions x_t^{max} receive no further corrections

Theorem TK, D. Stöckinger

In MSSM parameters: $\Delta\lambda \not\supset (y_t^{\mathit{MSSM}})^4 g_3^{2n} x_t^{(5 \text{ or higher})}$ *for* $n \geq 0$.

$$\frac{\Delta\lambda}{(y_t^{\text{SM}})^4} \supset \kappa g_3^2 \left(\propto x_t^{\leq 4} + c_{(4+1,1)} x_t^{4+1} \right)$$
$$+ \kappa^2 g_3^4 \left(\propto x_t^{\leq 4} + c_{(4+1,2)} x_t^{4+1} + c_{(4+2,2)} x_t^{4+2} \right)$$
$$+ \kappa^3 g_3^6 \left(\propto x_t^{\leq 4} + c_{(4+1,3)} x_t^{4+1} + c_{(4+2,3)} x_t^{4+2} + c_{(4+3,3)} x_t^{4+3} \right)$$
$$+ \cdots$$

- Constraint can be extended to orders with combinations of g_3 with other couplings, i.e. $\Delta\lambda = y_t^2 g_{1,2}^2 g_3^{2n} + y_t^6 g_3^{2n}$
- Predicted correctly $\Delta\lambda = y_t^2 g_{1,2}^2 g_3^2 x_t^3$ contribution from Bagnaschi et al. 2019

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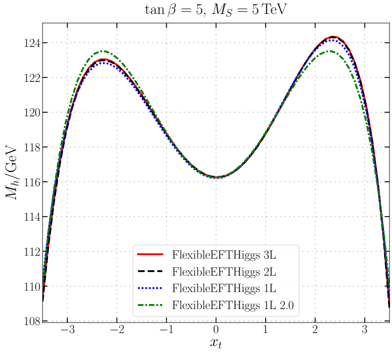
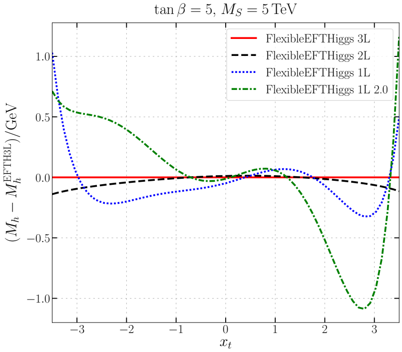
Impact of x_t resummation

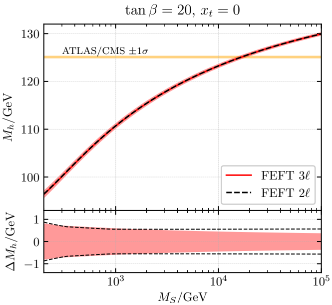


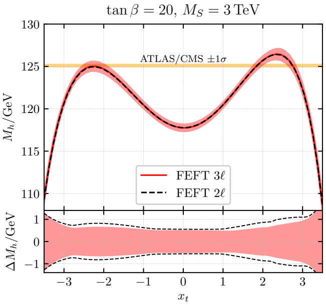


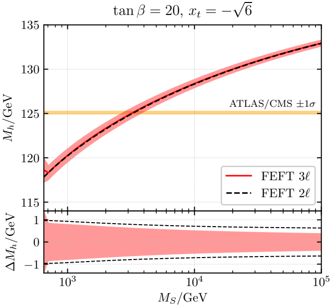
- Good: fast convergence of perturbation theory for all $x_t \in [-3, 3]$

$$|M_h^{\text{FEFT 1L}} - M_h^{\text{FEFT 3L}}| \lesssim 0.35\,\text{GeV}$$
$$|M_h^{\text{FEFT 2L}} - M_h^{\text{FEFT 3L}}| \lesssim 0.05\,\text{GeV}$$

Uncertainty of FlexibleEFTHiggs at 3-loop







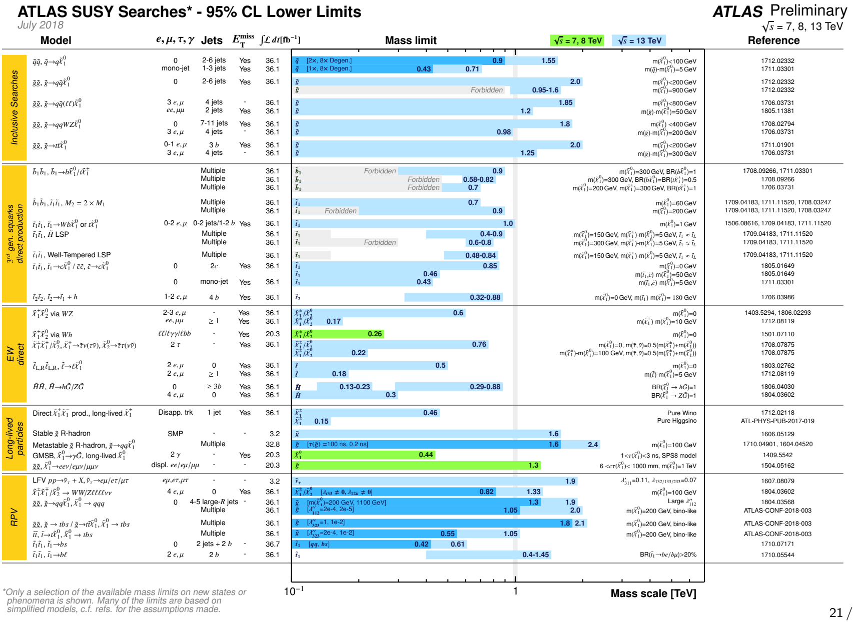
- Low uncertainty: $\Delta M_h^{\text{th}} \lesssim 1\,\text{GeV}$

Summary

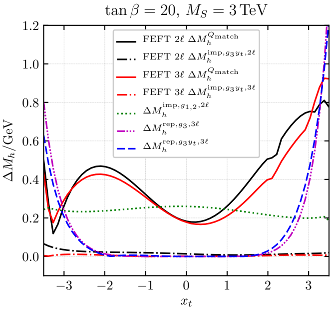
- **FlexibleEFTHiggs** 3-loop ($N^3LL + N^3LO$) a state-of-the-art calculation valid for all scales M_S
- Hybrid approach is elegant and can be applied to other BSM
- **New resummation technique developed:** It is possible to resum squark mixing parameter $x_f \in \{\tan\beta, x_t\}$ in the Higgs mass through 1-loop threshold corrections
  $\Rightarrow$ Resummation accounts for 500 MeV in realistic scenarios
- Theoretical uncertainty reduced to $\Delta M_h^{\text{th}} \lesssim 1\,\text{GeV}$ in single-scale scenarios
- Agreement with $M_h = 125.1\,\text{GeV}$ requires $M_S > 1\,\text{TeV}$
- **Outlook:**
  - Application to non-minimal SUSY extensions of the SM

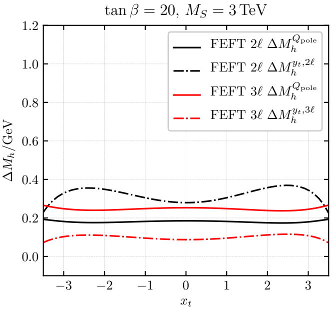
Backup

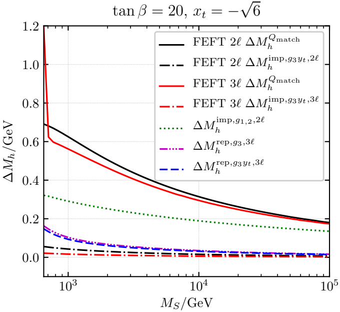
ATLAS results M_S

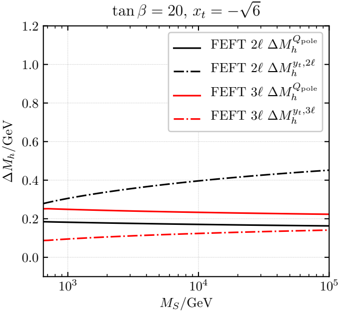


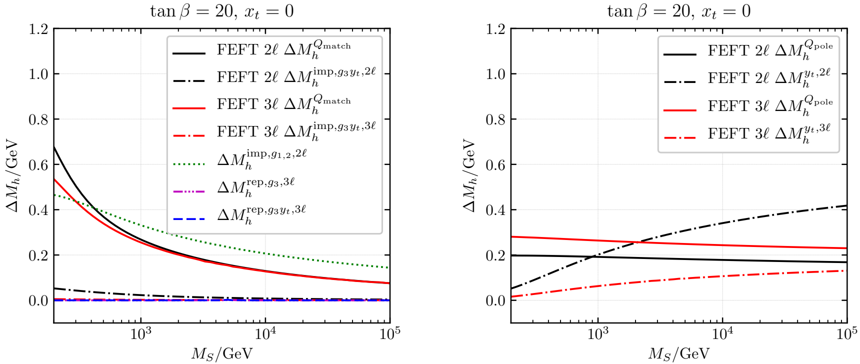
Uncertainty M_S











$\tan\beta = 20,\ x_t = 0$
$\Delta M_h/\mathrm{GeV}$
M_S/GeV
FEFT 2ℓ $\Delta M_h^{Q_\mathrm{match}}$
FEFT 2ℓ $\Delta M_h^{\mathrm{imp},g_3 y_t,2\ell}$
FEFT 3ℓ $\Delta M_h^{Q_\mathrm{match}}$
FEFT 3ℓ $\Delta M_h^{\mathrm{imp},g_3 y_t,3\ell}$
$\Delta M_h^{\mathrm{imp},g_{1,2},2\ell}$
$\Delta M_h^{\mathrm{rep},g_3,3\ell}$
$\Delta M_h^{\mathrm{rep},g_3 y_t,3\ell}$
$\tan\beta = 20,\ x_t = 0$
$\Delta M_h/\mathrm{GeV}$
M_S/GeV
FEFT 2ℓ $\Delta M_h^{Q_\mathrm{pole}}$
FEFT 2ℓ $\Delta M_h^{y_t,2\ell}$
FEFT 3ℓ $\Delta M_h^{Q_\mathrm{pole}}$
FEFT 3ℓ $\Delta M_h^{y_t,3\ell}$