# Precise calculations of the Higgs mass in supersymmetric models

Thomas Kwasnitza

#### 3rd December 2020

based on: [TK, D. Stöckinger, A. Voigt 20]







• Supersymmetry is a new fundamental spacetime symmetry



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- Supersymmetry is a new fundamental spacetime symmetry
- Advantages for physics close to the EW scale
  - Dark matter
  - ▶ g-2 of the muon
  - Dynamical breaking of EW symmetry ( $\mu^2 < 0$ )
  - Predicts one light Higgs boson

# SUSY and Grand Unification



generated by FS A. Voigt

# SUSY and Grand Unification



# SUSY and Grand Unification



 $\rightarrow \mbox{improved}$  unification of gauge couplings



No massless spin s>2 particle can couple consistently to massless particles with  $s\leq 2$ 

#### see: Prof. Dominik Stöckinger RQFT WS19/20 Lecture 21 - 23

 $Poincaré + Unitarity \implies$  the menu of **elementary** particles is given by

$$s \in \left\{ \begin{array}{cccc} 0, & & \frac{1}{2}, & & 1, & & \frac{3}{2}, & & 2 \end{array} \right\}$$

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 unique gravity

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$$s \in \left\{ \begin{array}{ccc} 0, & & \frac{1}{2}, & & 1, & & \frac{3}{2}, & & 2 \end{array} \right\}$$

$$\xrightarrow{\text{described by local SUSY}} \left( \begin{array}{c} 0, & & & 1, & & \frac{3}{2}, & & 2 \end{array} \right)$$

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# Features of Supersymmetry

#### Theorem in QFT (Weinberg Witten)

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massive case .: N. Arkani-Hamed, T. Huang, Y. Huang 2017

$$s \in \left\{ \begin{array}{cccc} 0, & & \frac{1}{2}, & & 1, & & \frac{3}{2}, & & 2 \end{array} \right\}$$

• Relation to (quantum) gravity: SUGRA, string theory, matrix models, ...

Nicht ohne SUSY Die Superstringtheorie en trisch ist – ein weiterer An Physiker auf die Entdecku

• SUSY extension predicts one light Higgs

$$m_{H,h}^2 = \frac{1}{2} \left[ m_Z^2 + M_S^2 \pm \sqrt{(m_Z^2 + M_S^2)^2 - 4m_Z^2 M_S^2 c_{2\beta}^2} \right]$$

$$m_h^2 \approx m_Z^2 c_{2\beta}^2 \le m_Z^2, \qquad \qquad m_H^2 \approx M_S^2$$

• Pole masses are PDG 2018

$$M_h = \sqrt{m_h^2 + \Delta m_h^2} \approx 125.1 \,\mathrm{GeV}$$
  
 $M_Z \approx 91.2 \,\mathrm{GeV}$ 

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• Anatomy of radiative contributions

$$\Delta m_h^2 = v^2 \kappa^n \left[ \log \frac{M_S}{v} + C_0 + C_1 \frac{v^2}{M_S^2} \right]$$

 $\rightarrow$  Use  $\left[M_{h}\stackrel{!}{=}125.1\,\mathrm{GeV}\right]$  as a constraint on new parameters

$$\Delta m_h^2 = v^2 \kappa^n \left[ \log \frac{M_S}{v} + C_0 + C_1 \frac{v^2}{M_S^2} \right]$$

• Q: Which parameter can be constrained?

$$\left\{ M_S, \qquad \tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}, \qquad \frac{x_t \text{ mixing parameter}}{M_{\tilde{t}}^2 = \begin{pmatrix} M_S^2 & x_t m_t M_S \\ (x_t m_t M_S)^* & M_S^2 \end{pmatrix}}, \cdots \right\}$$

• MSSM is consistent iff large radiative corrections exist:

$$(86\,{\rm GeV})^2 \leq \Delta m_h^2 \leq (125\,{\rm GeV})^2$$

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- ▶  $\log \frac{M_S}{v}$ : typical feature of SUSY,  $M_S$  cannot be arbitrarily high! Exception: FSSM K. Benakli et al. 2013
- Conclusive constraint requires low uncertainty
  - Good: experimental uncertainty  $\Delta M_h^{\text{exp}} \approx 0.14 \,\text{GeV}$
  - Problem: theory uncertainty  $\Delta M_h^{\rm th} \approx 1-3 \,{\rm GeV}$



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- 5 Uncertainty of FlexibleEFTHiggs at 3-loop





• Leading loop (in SM parameters) contributions at NLO

$$\Delta m_h^2 = \kappa y_t^2 m_t^2 \qquad \left[ 24 \log \frac{M_S}{m_t} \qquad -x_t^4 + 12x_t^2 + \frac{m_t^2}{M_S^2} \frac{15}{4} + \cdots \right]$$

$$\Delta m_h^2 \supset \xrightarrow{h} \cdots \xrightarrow{h} + \xrightarrow{h} \cdots \xrightarrow{\tilde{t}_{L,R}} + \xrightarrow{h} + \xrightarrow{\tilde{t}_{L,R}} + \xrightarrow{\tilde{t}_{L,R}} + \cdots$$

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- Large loop corrections for:
  - large mass gap  $M_S \gg m_t$
  - ▶ large off-diagonal element in the stop mass matrix:  $|x_t| \approx \sqrt{6}$







• Good: gives reliable predictions if  $M_S$  and  $m_t$  are not too far apart



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$$\begin{split} M_h &= m_h + \Delta m_h^{1\ell} + \Delta m_h^{2\ell} + \Delta m_h^{3\ell} + \cdots \\ &\approx [91 + \mathcal{O}(20 \dots 30) + \mathcal{O}(2 \dots 5) + \mathcal{O}(1 \dots 2)] \, \text{GeV} \end{split}$$







$$M_{S} \qquad M_{S} \qquad M_{L} = \lambda^{(0\ell)} + \Delta\lambda^{(1\ell)} + \Delta\lambda^{(2\ell)} + \ldots + \Delta\lambda^{(n\ell)} \\ \qquad M_{R} \qquad M_{L} = (m_{h}^{2})^{SM} + (\Delta m_{h}^{2})^{SM} \qquad M_{h}^{2} = (m_{h}^{2})^{SM} + (\Delta m_{h}^{2})^{SM} \\ \qquad \frac{(m_{h}^{2})^{SM}}{m_{t}^{2}} \supset y_{t}^{2} \left(C_{0}^{1\ell} + k_{(1,1)}\log\frac{M_{S}}{m_{t}}\right) \\ \qquad + y_{t}^{2}g_{3}^{2} \left(C_{0}^{2\ell} + k_{(1,2)}\log\frac{M_{S}}{m_{t}} + k_{(2,2)}\log^{2}\frac{M_{S}}{m_{t}}\right) \\ \qquad + y_{t}^{2}g_{3}^{4} \left(C_{0}^{3\ell} + k_{(1,3)}\log\frac{M_{S}}{m_{t}} + k_{(2,3)}\log^{2}\frac{M_{S}}{m_{t}}\right) + \cdots$$

$$M_{S} \bigwedge M_{S} \bigwedge \lambda = \lambda^{(0\ell)} + \Delta\lambda^{(1\ell)}$$

$$M_{S} \bigwedge M_{h}^{2} = (m_{h}^{2})^{SM} + (\Delta m_{h}^{2})^{SM} \text{ at NLL}$$

$$(\frac{(m_{h}^{2})^{SM}}{m_{t}^{2}} \supset y_{t}^{2} \left(C_{0}^{1\ell} + k_{(1,1)}\log\frac{M_{S}}{m_{t}}\right)$$

$$+ y_{t}^{2}g_{3}^{2} \left(C_{0}^{2\ell} + k_{(1,2)}\log\frac{M_{S}}{m_{t}} + k_{(2,2)}\log^{2}\frac{M_{S}}{m_{t}}\right)$$

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$$+ \cdots \qquad 10/21$$



#### Effective field theory approach $[\lim(v/M_S) \rightarrow 0]$



ullet Good: Predicts Higgs mass reliably for large separation of  $m_t$  and  $M_S$ 

• Problem: for  $M_S \simeq m_t \Rightarrow$  large uncertainty



- ullet Good: Predicts Higgs mass reliably for large separation of  $m_t$  and  $M_S$
- Problem: for  $M_S \simeq m_t \Rightarrow$  large uncertainty
- convergence rate of perturbative expansion  $(x_t = -\sqrt{6}, \tan \beta = 20, M_S = 3 \text{ TeV})$

$$M_h = m_h + \Delta m_h^{i\epsilon} + \Delta m_h^{j\epsilon} + \Delta m_h^{j\epsilon}$$
  
 
$$\approx \left[\mathcal{O}(117) + \mathcal{O}(7) + \mathcal{O}(1) + O(0.2)\right] \text{GeV}$$

# Comparison of $M_h$ calculations



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# Trick 2: Hybrid calculation

A: Yes e.g. FlexibleEFTHiggs [P. Athron et al. 2016], [P. Athron et al. 2017], [TK et al. 2020]



• Obtain  $\lambda$  from pole mass matching

$$(M_h^2)^{\rm SM} = (M_h^2)^{\rm MSSM}, \qquad \qquad Q = M_S$$

• Ensure that  $\log(M_S/m_t)$  cancel in  $\lambda$ 

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- Ensure that  $\log(M_S/m_t)$  cancel in  $\lambda$  igwedge M
- Good:
  - Valid at all scales
  - Automatized derivation of  $\Delta\lambda$  from self energies and tadpoles

# Hybrid calculation FlexibleEFTHiggs at N<sup>3</sup>LO and N<sup>3</sup>LL



 $M_h^2$  at  $3\ell$ 

 $\mathcal{O}(1\ell + (\alpha_t + \alpha_b)\alpha_s + (\alpha_t + \alpha_b + \alpha_\tau)^2 + \alpha_s^2\alpha_t)$ 

[0105096, 0112177, 0212132, 0206101, 0305127, 1205.6497, 1407.4336, 1708.05720]

RGE at  $4\ell$  [1201.5868, 1205.2892, 1212.6829, 1303.4364, 1604.00853]

 $lpha_s^{\mathsf{SM}(5)}$  at  $2\ell$  [9305305, 9707474]  $M_t$  at  $3\ell$   $\mathcal{O}(lpha_s^2 + lpha_s^3)$  [9912391, 0507139, 9912391]

• 3L self energy from FlexibleSUSY+Himalaya obtained without terms of  $\mathcal{O}(\kappa^3 v^2/M_S^2)$ 

# Hybrid calculation FlexibleEFTHiggs at N<sup>3</sup>LO and N<sup>3</sup>LL

• Interpolation of Hybrid FEFT between FO and EFT:



• Discrepancy is dominated by highest power  $x_t$  contributions included in FEFT

• **Observation 1**: the presented EFT calc. and FlexibleEFTHiggs work in different parametrizations!

 $\Rightarrow$  expand both matching conditions to 2-loop

$$\begin{split} \Delta\lambda^{\mathsf{FEFT}} = & \kappa(y_t^{\mathsf{MSSM}})^4 \bigg[ 6x_t^2 & -\frac{1}{2}x_t^4 \bigg] + \mathcal{O}\left(\frac{m_t^2}{M_S^2}\right) \\ & + \kappa^2 g_3^2 (y_t^{\mathsf{MSSM}})^4 \bigg[ c_{(0,2)} + c_{(1,2)}x_t + \cdots & + c_{(4,2)}x_t^4 \bigg] + \mathcal{O}\left(\frac{m_t^2}{M_S^2}\right) \\ \Delta\lambda^{\mathsf{EFT}} = & \kappa(y_t^{\mathsf{SM}})^4 \bigg[ 6x_t^2 & -\frac{1}{2}x_t^4 \bigg] \\ & + \kappa^2 g_3^2 (y_t^{\mathsf{SM}})^4 \bigg[ \tilde{c}_{(0,2)} + \tilde{c}_{(1,2)}x_t + \cdots & + \tilde{c}_{(4,2)}x_t^4 + \tilde{c}_{(5,2)}x_t^5 \bigg] \end{split}$$

• **Q**: How is the  $x_t^5$  term reproduced in FEFT?

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• **Q**: How is the  $x_t^5$  term reproduced in FEFT?

• A: The reparametrization of  $y_t$  contains contributions  $\Delta_t \propto g_3^2 x_t$ 

$$y_t^{SM} = y_t^{MSSM} \left[ 1 - \Delta_t^{1\ell} \right] \Rightarrow y_t^{MSSM} \approx y_t^{SM} \left[ 1 + \Delta_t^{1\ell} \right]$$
  
Parametrizations are equivalent at 2-loop!

• Observation 2: Use the exact (implemented) relation

$$\begin{split} y_t^{\text{MSSM}} &= \frac{y_t^{\text{SM}}}{\left[1 - \Delta_t^{1\ell}\right]} = y_t^{\text{SM}} \sum_n (\Delta_t^{1\ell})^n \\ (\Delta \lambda^{\text{FEFT}})^{(1\ell)} &= \kappa (y_t^{\text{MSSM}})^4 \begin{bmatrix} 6x_t^2 & -\frac{1}{2}x_t^4 \end{bmatrix} \\ &= \kappa (y_t^{\text{SM}})^4 \begin{bmatrix} 6x_t^2 & -\frac{1}{2}x_t^4 \end{bmatrix} \\ &+ \kappa^2 g_3^2 (y_t^{\text{SM}})^4 \begin{bmatrix} \cdots & +\hat{c}_{(4,2)}x_t^4 + \hat{c}_{(5,2)}x_t^5 \end{bmatrix} \\ &+ \kappa^3 g_3^4 (y_t^{\text{SM}})^4 \begin{bmatrix} \cdots & +\hat{c}_{(4,3)}x_t^4 + \hat{c}_{(5,3)}x_t^5 + \hat{c}_{(5,3)}x_t^6 \end{bmatrix} \\ &\cdots \end{split}$$

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- Direct calculation reveals
  - $\hat{c}_{(5,2)}x_t^5 = \tilde{c}_{(5,2)}x_t^5 \checkmark$
  - $\hat{c}_{(4,2)}x_t^4 \not$ , gets contributions from 2-loop Feynman diagrams
- Q: What can be said about higher orders?

**A**: An inspection of multi-loop diagrams reveals that the highest power contributions  $x_t^{\max}$  receive no further corrections

Theorem TK, D. Stöckinger

In MSSM parameters:  $\Delta \lambda \not\supseteq (y_t^{\text{MSSM}})^4 g_3^{2n} x_t^{(5 \text{ or higher})}$  for  $n \ge 0$ .

$$\begin{split} \frac{\Delta\lambda}{y_t^{\text{SM}})^4} &\supset \kappa g_3^2 \left( \propto x_t^{\leq 4} + c_{(4+1,1)} x_t^{4+1} \right) \\ &+ \kappa^2 g_3^4 \left( \propto x_t^{\leq 4} + c_{(4+1,2)} x_t^{4+1} + c_{(4+2,2)} x_t^{4+2} \right) \\ &+ \kappa^3 g_3^6 \left( \propto x_t^{\leq 4} + c_{(4+1,3)} x_t^{4+1} + c_{(4+2,3)} x_t^{4+2} \right) + c_{(4+3,3)} x_t^{4+3} \right) \\ &+ \cdots \end{split}$$

- Constraint can be extended to orders with combinations of  $g_3$  with other couplings, i.e.  $\Delta\lambda=y_t^2g_{1,2}^2g_3^{2n}+y_t^6g_3^{2n}$
- Predicted correctly  $\Delta\lambda=y_t^2g_{1,2}^2g_3^2x_t^3$  contribution from Bagnaschi et al. 2019

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# Impact of $x_t$ resummation



• Good: fast convergence of perturbation theory for all  $x_t \in [-3,3]$ 

$$\begin{split} |M_h^{\mathsf{FEFT \ 1L}} - M_h^{\mathsf{FEFT \ 3L}}| \lesssim 0.35 \, \mathrm{GeV} \\ |M_h^{\mathsf{FEFT \ 2L}} - M_h^{\mathsf{FEFT \ 3L}}| \lesssim 0.05 \, \mathrm{GeV} \end{split}$$

# Uncertainty of FlexibleEFTHiggs at 3-loop





• Low uncertainty:  $\Delta M_h^{\rm th} \lesssim 1 \, {\rm GeV}$ 

- FlexibleEFTHiggs 3-loop (N<sup>3</sup>LL + N<sup>3</sup>LO) a state-of-the-art calculation valid for all scales  $M_S$
- Hybrid approach is elegant and can be applied to other BSM
- New resummation technique developed: It is possible to resum squark mixing parameter  $x_f \in \{\tan \beta, x_t\}$  in the Higgs mass through 1-loop threshold corrections
  - $\Rightarrow$  Resummation accounts for 500 MeV in realistic scenarios
- $\bullet\,$  Theoretical uncertainty reduced to  $\Delta M_h^{\rm th} \lesssim 1\,{\rm GeV}$  in single-scale scenarios
- Agreement with  $M_h = 125.1 \,\text{GeV}$  requires  $M_S > 1 \,\text{TeV}$
- Outlook:
  - Application to non-minimal SUSY extensions of the SM

### Backup

### ATLAS results $M_S$

#### ATLAS SUSY Searches\* - 95% CL Lower Limits

#### ATLAS Preliminary $\sqrt{s} = 7, 8, 13 \text{ TeV}$

July 2018

	Model	$e, \mu, \tau, \gamma$	Jets	$E_{\rm T}^{\rm miss}$	∫£ dt[fb	-') Mas	ss limit		$\sqrt{s} = 7, 8 \text{ TeV}$	$\sqrt{s} = 13 \text{ TeV}$	Reference
Inclusive Searches	$\tilde{q}\tilde{q}, \tilde{q} \rightarrow q \tilde{t}_1^0$	0 mono-jet	2-6 jets 1-3 jets	Yes Yes	36.1 36.1	<pre># [2x, 8x Degen] # [1x, 8x Degen]</pre>	0.43 0.71	0.9	1.55	m({t_1}^0)<100 GeV m(q)-m(t_1^0)=5 GeV	1712.02332 1711.03301
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow q \tilde{q} \tilde{t}_1^0$	0	2-6 jets	Yes	36.1	8 8	Forbi	dden	2.0	m( $\ell_1^0$ )<200 GeV m( $\ell_1^0$ )=900 GeV	1712.02332 1712.02332
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow q\tilde{q}(\ell \ell)\tilde{\chi}_{1}^{0}$	3 e, μ ee, μμ	4 jets 2 jets	Yes	36.1 36.1	8 8			1.85	m( $\hat{r}_1^0$ )=800 GeV m( $\hat{r}_1$ )=50 GeV	1708.03731 1805.11381
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow qqWZ\tilde{k}_{1}^{0}$	0 3 e, µ	7-11 jets 4 jets	Yes	36.1 36.1	8 8		0.98	1.8	m( $\hat{t}_1^0$ ) <400 GeV m( $\hat{t}$ )=200 GeV	1708.02794 1706.03731
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow t\tilde{t}\tilde{\chi}_1^0$	0-1 e.μ 3 e.μ	3 b 4 jets	Yes	36.1 36.1	8 8			2.0	m( <sup>§</sup> )-200 GeV m(§)-m( <sup>§</sup> )=300 GeV	1711.01901 1706.03731
3 <sup>rd</sup> gen. squarks direct production	$b_1b_1, b_1{\rightarrow} b \tilde{\chi}_1^0 / t \tilde{\chi}_1^*$		Multiple Multiple Multiple		36.1 36.1 36.1	δ <sub>1</sub> Forbidden δ <sub>1</sub> δ <sub>2</sub>	Forbidden 0.58-0.1 Forbidden 0.7	0.9 12	m(t <sup>0</sup> )= m(t <sup>0</sup> )=200 G	$m(\hat{\xi}_1^0)=300 \text{ GeV}, BR(\delta \hat{\xi}_1^0)=1$ $300 \text{ GeV}, BR(\delta \hat{\xi}_1^0)=BR(\ell \hat{\xi}_1^+)=0.5$ $\text{IeV}, m(\hat{\xi}_1^+)=300 \text{ GeV}, BR(\ell \hat{\xi}_1^+)=1$	1708.09288, 1711.03301 1708.09288 1706.03731
	$\tilde{b}_1\tilde{b}_1,\tilde{t}_1\tilde{t}_1,M_2=2\times M_1$		Multiple Multiple		36.1 36.1	is is Forbidden	0.7	0.9		m(t <sup>0</sup> <sub>1</sub> )=60 GeV m(t <sup>0</sup> <sub>1</sub> )=200 GeV	1709.04183, 1711.11520, 1708.03247 1709.04183, 1711.11520, 1708.03247
	$\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow Wh \tilde{\chi}_1^0 \text{ or } t \tilde{\chi}_1^0$ $\tilde{t}_1 \tilde{t}_1, \tilde{H} LSP$	0-2 e,µ (	0-2 jets/1-2 Multiple Multiple	b Yes	36.1 36.1 36.1	1 1 1 1 Forbidden	0. 0.6-0.	1.0 I-0.9 B	m(t <sup>0</sup> <sub>1</sub> )=150 0 m(t <sup>0</sup> <sub>1</sub> )=300 0	$m(\hat{\xi}_{1}^{0})=1 \text{ GeV}$ $3eV, m(\hat{\xi}_{1}^{0})-m(\hat{\xi}_{1}^{0})=5 \text{ GeV}, \tilde{r}_{1} = \tilde{r}_{L}$ $3eV, m(\hat{\xi}_{1}^{0})-m(\hat{\xi}_{1}^{0})=5 \text{ GeV}, r_{1} = r_{L}$	1506.08816, 1700.04183, 1711.11520 1709.04183, 1711.11520 1709.04183, 1711.11520
	III, Well-Tempered LSP		Multiple		36.1	i,	0.48-0.	84	m(2 <sup>0</sup> )=150 0	SeV, $m(\tilde{k}_1^*)$ - $m(\tilde{k}_1^0)$ =5 GeV, $\tilde{r}_1 = \tilde{r}_L$	1709.04183, 1711.11520
	$\tilde{t}_1 \tilde{t}_1, \tilde{t}_1 \rightarrow c \tilde{\chi}_1^0 / \tilde{c}\tilde{c}, \tilde{c} \rightarrow c \tilde{\chi}_1^0$	0	2c	Yes	36.1	1, 1, 1	0.46	85		m( $\hat{k}_{1}^{0}$ )=0 GeV m( $\hat{k}_{1}^{2}$ )-m( $\hat{k}_{1}^{0}$ )=50 GeV	1805.01649 1805.01649 1711.00901
	55 5	1.2 c #	4.6	Vor	36.1	1	0.43		m15 <sup>6</sup>	m(r_, r)-m(r_) = 180 GeV	1706 03066
	C*C <sup>0</sup>	23.6.11		Var	26.1	12 24 a <sup>24</sup>	0.6		inde 1	modet, mptpmptp= tab det	1403 5294, 1806 02293
EW direct	A 142 Will W Z	ee, µµ	$\geq 1$	Yes	36.1	$\tilde{\chi}_{1}^{4}/\tilde{\chi}_{2}^{4} = 0.17$				$m(\tilde{\epsilon}_1^*)-m(\tilde{\epsilon}_1^*)=10 \text{ GeV}$	1712.08119
	$\hat{\chi}_1^* \hat{\chi}_2^0 $ via Wh $\hat{\chi}_1^* \hat{\chi}_2^0 $ via Wh	<i>tt/tγγ/tbb</i>	1.1	Yes	20.3	x <sup>+</sup> <sub>1</sub> /x <sup>+</sup> <sub>2</sub> 0.26	0.76			0=( <sup>2</sup> <sub>1</sub> )m( <sup>2</sup> )	1501.07110
	$\chi_1\chi_1/\chi_2,\chi_1 \rightarrow rr(rr),\chi_2 \rightarrow rr(rr)$			iea	50.1	$\hat{x}_{1}^{+} \hat{x}_{2}^{+}$ 0.22	0.70		m( $\hat{r}_1^n$ )-m( $\hat{r}_1^n$ )=100	$r_1 = 0, m(\tau, \tau) = 0.5(m(\lambda_1) + m(\lambda_2))$ $0 \text{ GeV}, m(\tau, \tau) = 0.5(m(\lambda_1^{\circ}) + m(\lambda_2^{\circ}))$	1708.07875
	$\tilde{\ell}_{L,R}\tilde{\ell}_{L,R}, \tilde{\ell} \rightarrow \ell \tilde{\ell}_1^0$	2 e, μ 2 e, μ	0 ≥ 1	Yes Yes	36.1 36.1	2 0.18	0.5			m(ℓ̂_1)=0 m(ℓ̂)-m(ℓ̂_1)=5 GeV	1803.02762 1712.08119
	$\hat{H}\hat{H}, \hat{H} \rightarrow h\hat{G}/Z\hat{G}$	0 4 e, µ	≥ 3 <i>b</i> 0	Yes Yes	36.1 36.1	<u>й</u> 0.13-0.23 Н 0.3	0.29-	0.88		$\begin{array}{c} BR(\hat{\tilde{r}}_{j}^{0} \rightarrow h\tilde{G}){=}1\\ BR(\hat{\tilde{r}}_{1}^{-} \rightarrow Z\tilde{G}){=}1 \end{array}$	1806.04030 1804.03602
Long-lived particles	$\operatorname{Direct} \tilde{\chi}_1^* \tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^*$	Disapp. trk	1 jet	Yes	36.1	x <sup>±</sup> x <sup>±</sup> <sub>1</sub> 0.15	0.46			Pure Wino Pure Higgsiro	1712.02118 ATL-PHYS-PUB-2017-019
	Stable g R-hadron	SMP			3.2	ž		_	1.6		1606.05129
	Metastable $\tilde{g}$ R-hadron, $\tilde{g} \rightarrow qq \tilde{\chi}_1^0$ GMSR $\tilde{g}^0 \rightarrow q \tilde{\chi}_1^0$ loop-lived $\tilde{g}^0$	2 *	Multiple	Ver	32.8	g [r(g) =100 ns, 0.2 ns]	0.44		1.6 2.4	m(%)=100 GeV	1/10.04901, 1604.04520
	$\tilde{g}\tilde{g}, \tilde{\chi}^0_1 \rightarrow eev/e\muv/\mu\muv$	displ. ee/eµ/µ	μ -		20.3	8	0.44		1.3 6.	cr(k <sup>0</sup> <sub>1</sub> )< 1000 mm, m(k <sup>0</sup> <sub>1</sub> )=1 TeV	1504.05162
ЧЧ	LFV $pp \rightarrow \hat{v}_{\tau} + X, \hat{v}_{\tau} \rightarrow e\mu/e\tau/\mu\tau$	еµ,ет,µт			3.2	Ϋ́,			1.9	X_{311}=0.11, X_{332/333/233}=0.07	1607.08079
	$\tilde{\chi}_1^* \tilde{\chi}_1^* / \tilde{\chi}_2^0 \rightarrow WW/Z\ell\ell\ell\ell_{VV}$	4 e, µ	0	Yes	36.1	$\tilde{\lambda}_{1}^{\pm}/\tilde{\lambda}_{2}^{\pm} = [\lambda_{131} \neq 0, \lambda_{124} \neq 0]$	0.0	12	1.33	m(R <sup>0</sup> )=100 GeV	1804.03602
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow qq\tilde{t}_{1}^{0}, \tilde{t}_{1}^{0} \rightarrow qqq$	0 4	5 large-R j Multiple	ets -	36.1			1.0	1.3 1.9	Large J <sup>*</sup> <sub>112</sub>	1804.03568 ATLAS CONE 2018 002
		Multiple			00.1	₹ [#" =1 1e.2]			10.04	m(4))#200 GeV, BHS-IKe	AT 10 00015 0010 000
	$gg, g \rightarrow m_3 / g \rightarrow d\ell_1, \ell_1 \rightarrow lbs$ $\pi \to \tilde{m}^0 \tilde{\nu}^0 \rightarrow the$		Multiple		36.1	F [X], =20-4, 10-2]	0.55	1.0	1.0 2.1	mit 1 made Gev, BHD-like	ATLAS.CONF.2018-003
	$\tilde{h}_{1}^{1} \rightarrow h c_{1}^{1} \rightarrow h s$	0	2 jets + 2 i	ь.	36.7	i [ag, by]	0.42 0.61			mit place des, and the	1710.07171
	$I_1I_1, I_1 \rightarrow b\ell$	2 e, µ	2 b		36.1	i,			0.4-1.45	$BR(\tilde{t}_1 \rightarrow hv/b\mu) > 20\%$	1710.05544
* O-1-		an filming and s				0-1					
"Comy a selection of the available mass immus on new states or 10" 1 Mass scale [TeV]											

phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

# Uncertainty $M_S$





# Uncertainty $M_S$

