



New physics explanations of a_μ in light of the FNAL muon $g - 2$ measurement

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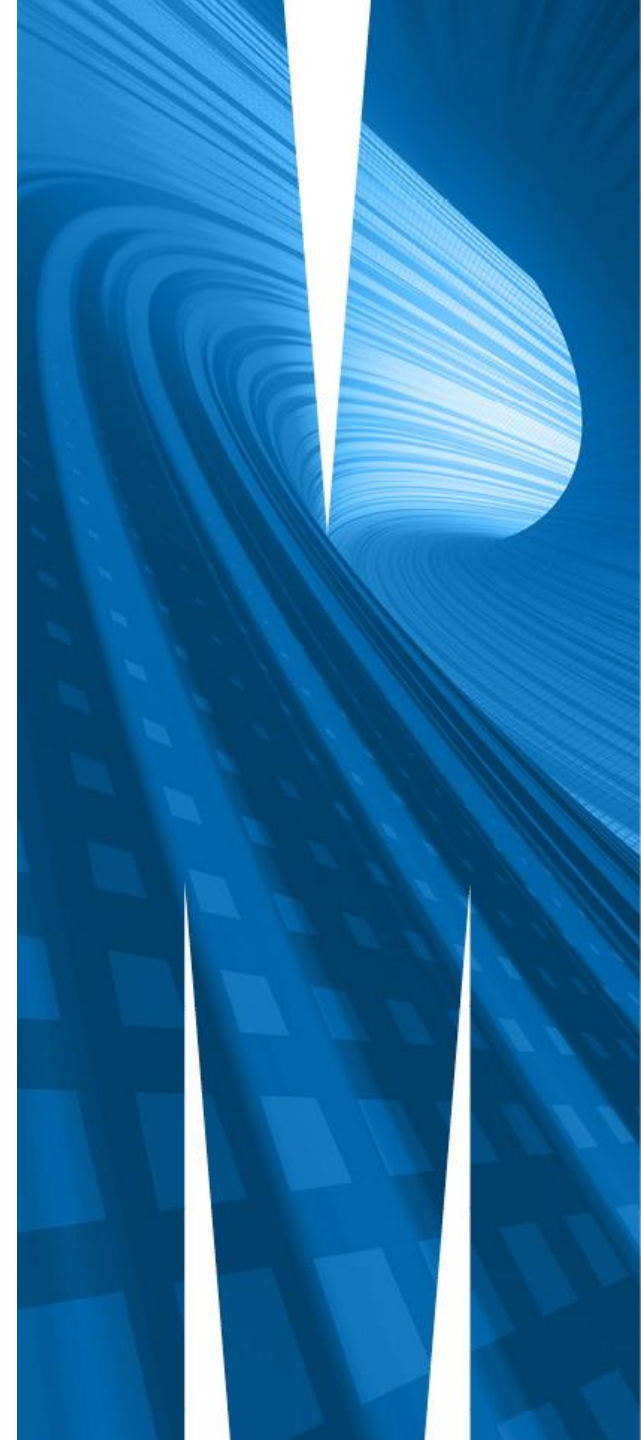
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arXiv:2104.03691



Outline

New physics explanations of a_μ in light of the FNAL muon g-2 experiment

Muon g-2

- Theoretical Prediction
- Fermilab Experiment
- 2021 Muon g-2 Discrepancy
- Phenomenological Description
- Contributions to Muon g-2

BSM Models and Results

- 1 Field Models (Leptoquarks)
- 2 Field Models with Dark Matter
- 3 Field Models with Dark Matter
- Future Developments
- Conclusions

Theoretical Prediction

Latest Theoretical Prediction

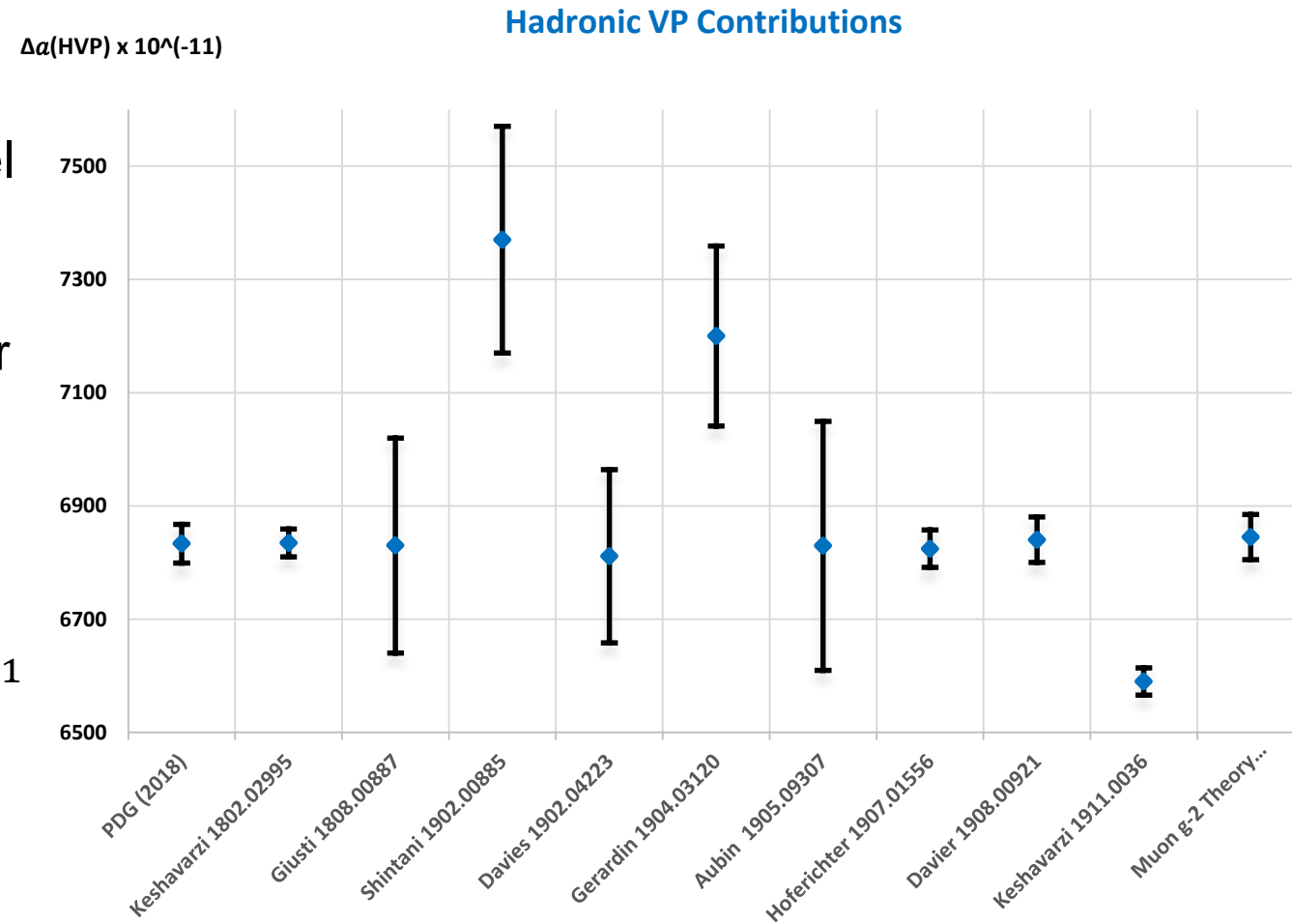
Muon g-2 Theory Initiative (arXiv:2006.04822)

Goal: Produce a single definitive standard model (SM) prediction for the value for the anomalous magnetic moment of the muon to rule them all.

Before publication, many different predictions for Hadronic contributions.

The current standard model (SM) prediction for the muon g-2 is:

$$a_{\mu}^{SM} = 116591810(1)_{EW}(40)_{HVP}(18)_{Hlbl} \times 10^{-11}$$



The Road to the Results

Muon G-2

- 2006: Brookhaven Final Results Released
- 2013: Big Move
- 2018: Run-1 Begins
- 2019: Run-2 Begins
- 2020: Muon g-2 Theory Initiative Releases White paper on SM prediction
- **2021: Results & Analysis of Run-1 Released**

Muon g-2 Discrepancy

Latest Experiment Values

SM Prediction

$$a_{\mu}^{SM} = 116591810(1)_{EW}(40)_{HVP}(18)_{Hlbl} \times 10^{-11}$$

Brookhaven Value

$$a_{\mu}^{BNL} = 116592089(54)_{stat}(33)_{sys} \times 10^{-11}$$

$$\Delta a_{\mu}^{BNL} = 279 \pm 76 \times 10^{-11}$$

Deviation:

3.7σ

Fermilab Value

$$a_{\mu}^{FNAL} = 116592040(54)_{exp} \times 10^{-11}$$

$$\Delta a_{\mu}^{FNAL} = 230 \pm 69 \times 10^{-11}$$

3.3σ

New World Average

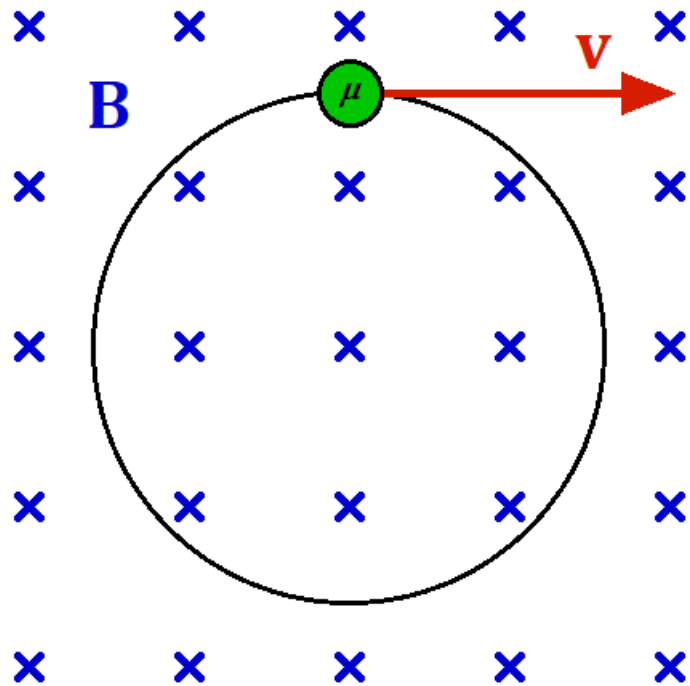
$$a_{\mu}^{2021} = 116592061(41)_{exp} \times 10^{-11}$$

$$\Delta a_{\mu}^{2021} = 251 \pm 59 \times 10^{-11}$$

$4.2\sigma!$

What is the Muon g-2?

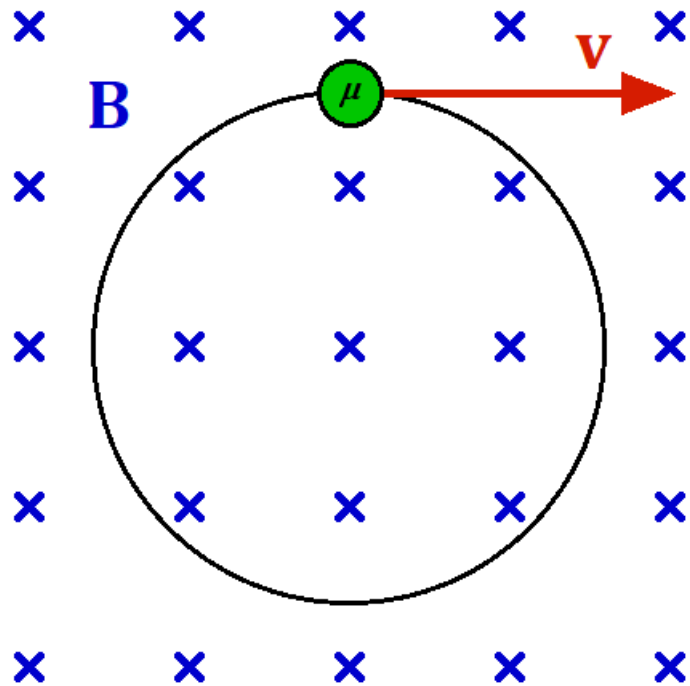
Quantum Mechanics



What is the Muon g-2?

Quantum Mechanics

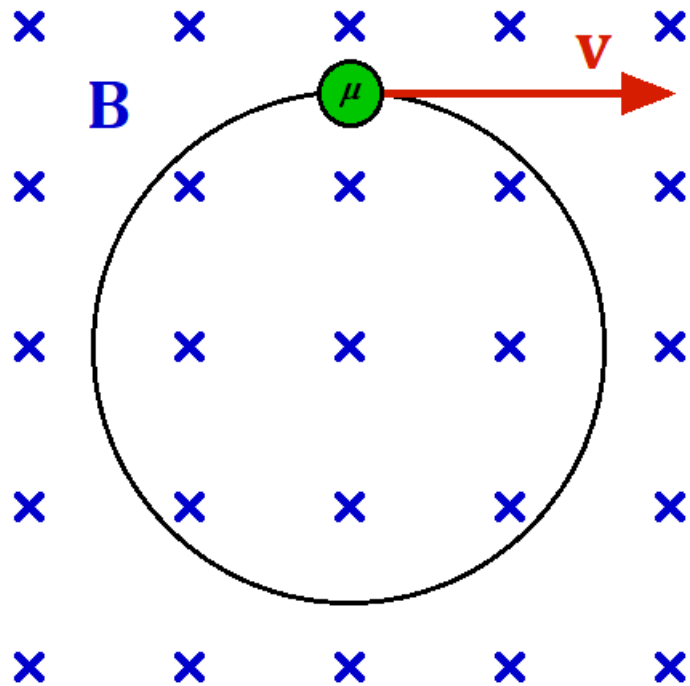
Magnetic Moment: $\vec{M} = g \frac{q}{2m} \vec{L}$



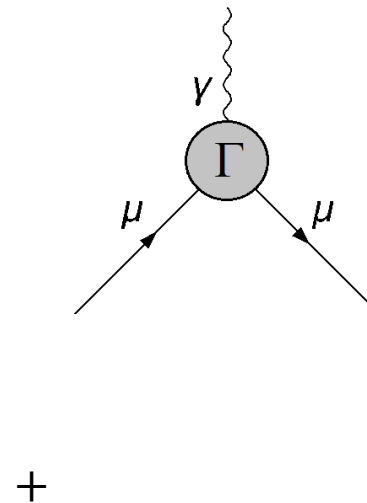
What is the Muon g-2?

Quantum Mechanics

Magnetic Moment: $\vec{M} = g \frac{q}{2m} \vec{L}$



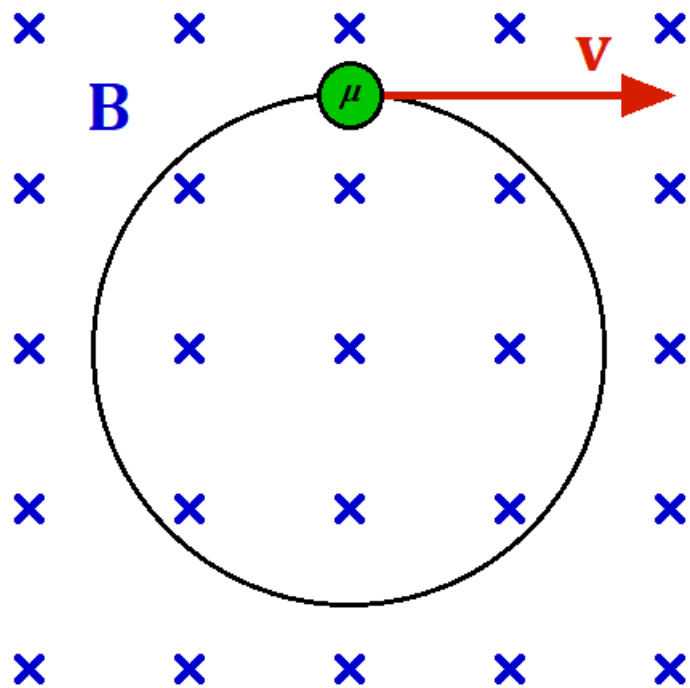
Quantum Field Theory



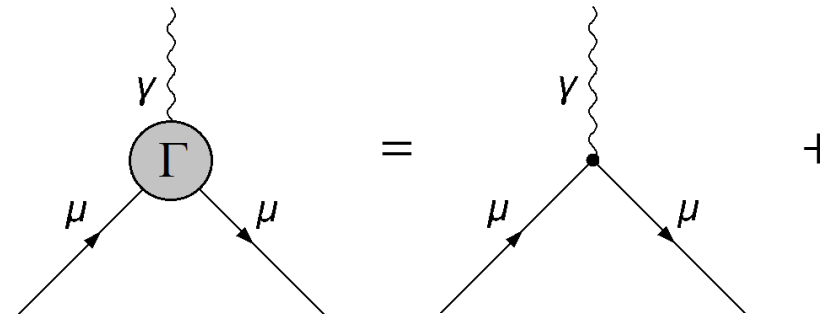
What is the Muon g-2?

Quantum Mechanics

Magnetic Moment: $\vec{M} = g \frac{q}{2m} \vec{L}$



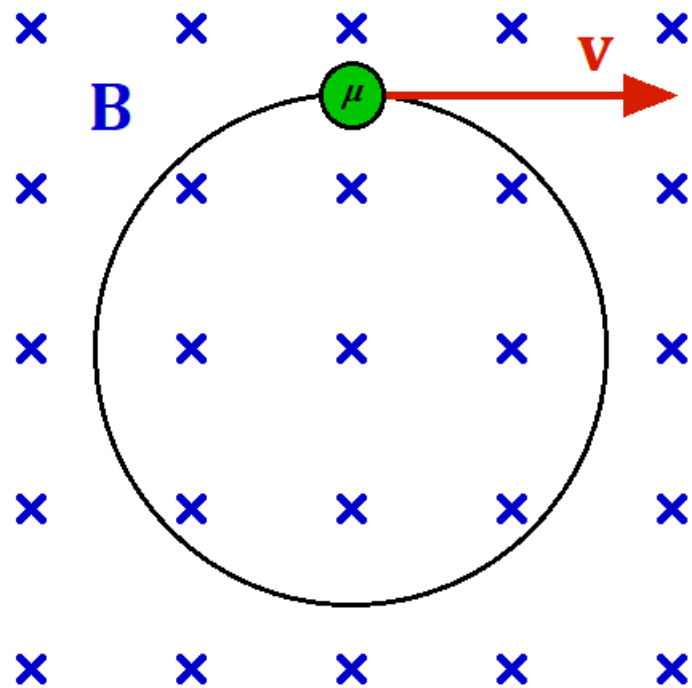
Quantum Field Theory



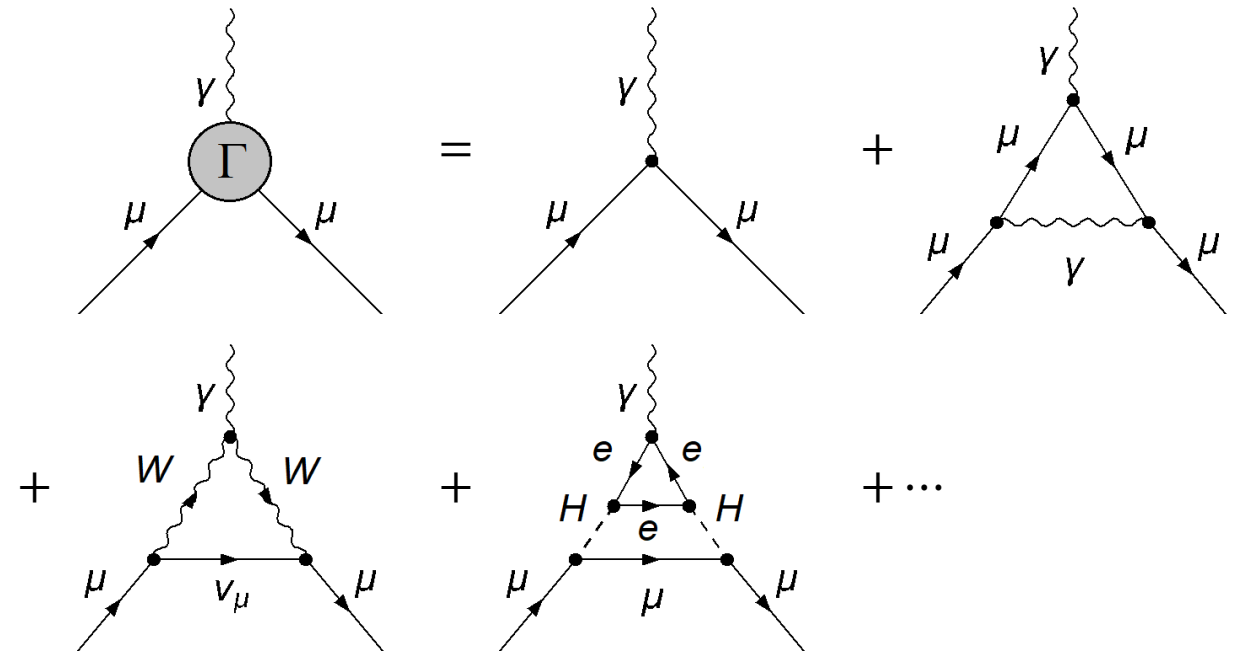
What is the Muon g-2?

Quantum Mechanics

Magnetic Moment: $\vec{M} = g \frac{q}{2m} \vec{L}$



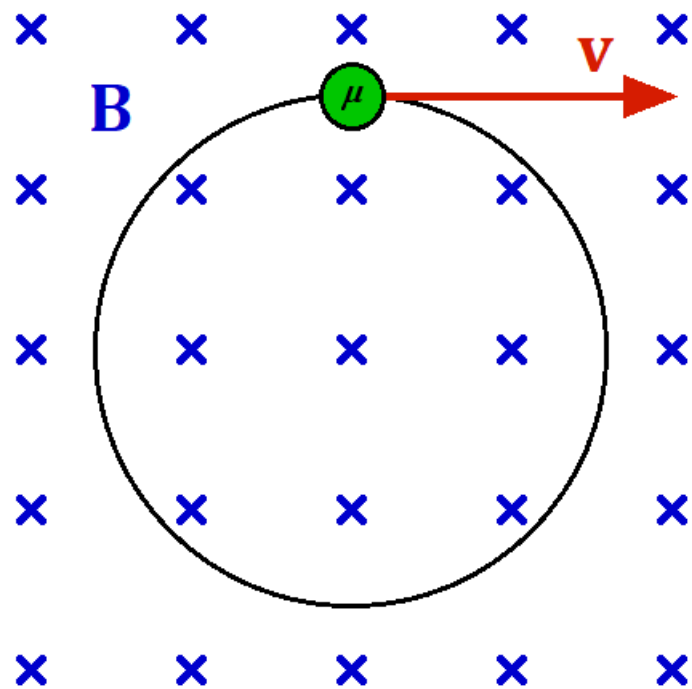
Quantum Field Theory



What is the Muon g-2?

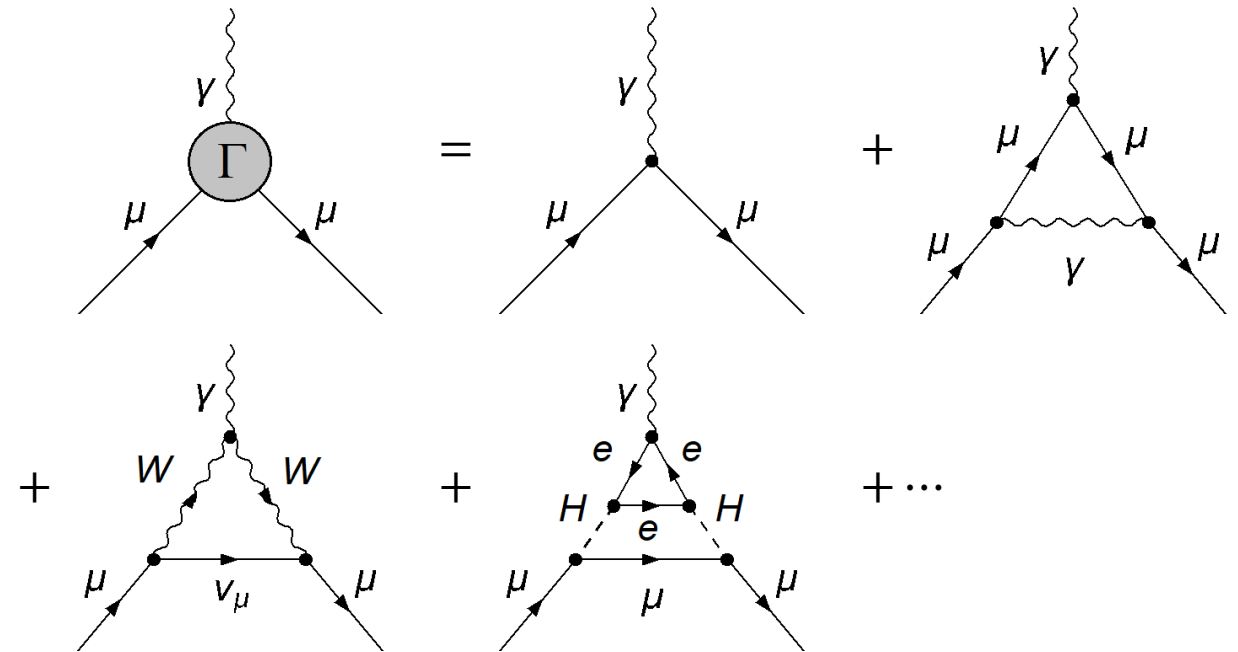
Quantum Mechanics

Magnetic Moment: $\vec{M} = g \frac{q}{2m} \vec{L}$



Quantum Field Theory

Anomalous Magnetic Moment: $a = (g - 2)/2$



Contributions to Muon $g-2$

Standard Model Contributions to Muon $g-2$

Contributions to Muon $g-2$

Standard Model Contributions to Muon $g-2$

Quantum Electrodynamics Contributions

Electroweak Contributions

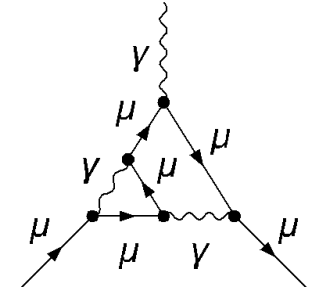
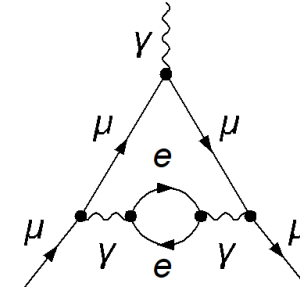
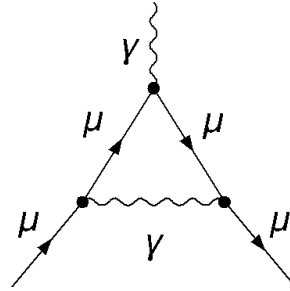
Hadronic Contributions

Contributions to Muon g-2

Standard Model Contributions to Muon g-2

Quantum Electrodynamics Contributions

$$a_{\mu}^{QED} \times 10^{11} = 116\,584\,718.931(104)$$



Electroweak Contributions

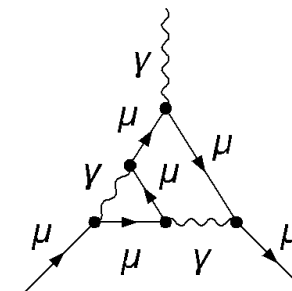
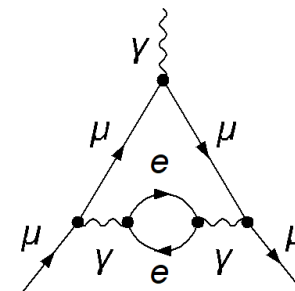
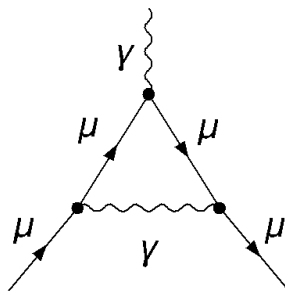
Hadronic Contributions

Contributions to Muon g-2

Standard Model Contributions to Muon g-2

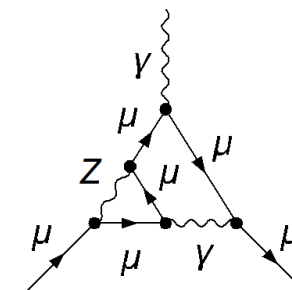
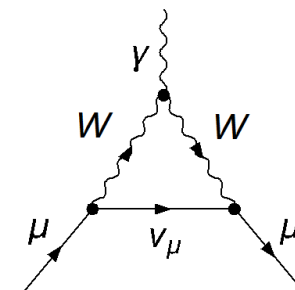
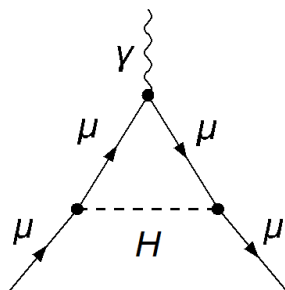
Quantum Electrodynamics Contributions

$$\alpha_{\mu}^{QED} \times 10^{11} = 116\,584\,718.931(104)$$



Electroweak Contributions

$$\alpha_{\mu}^{EW} \times 10^{11} = 153.6(1.0)$$



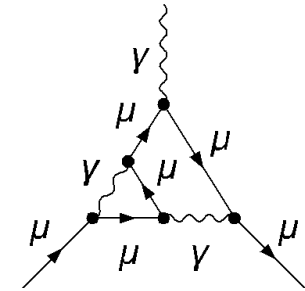
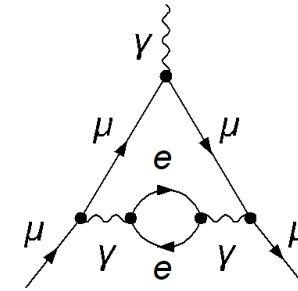
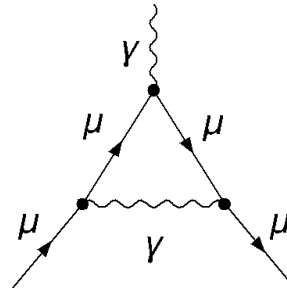
Hadronic Contributions

Contributions to Muon g-2

Standard Model Contributions to Muon g-2

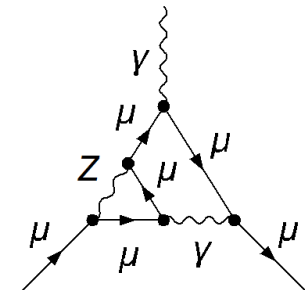
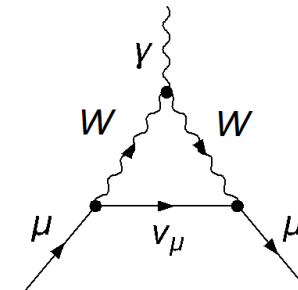
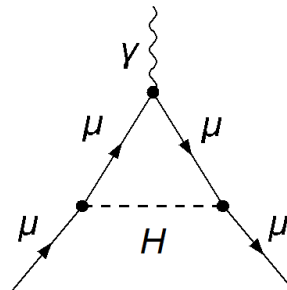
Quantum Electrodynamics Contributions

$$\alpha_{\mu}^{QED} \times 10^{11} = 116\,584\,718.931(104)$$



Electroweak Contributions

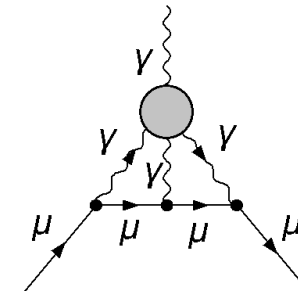
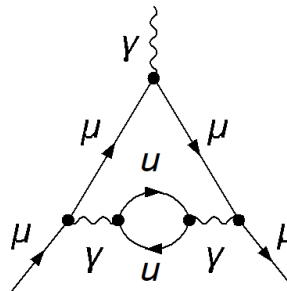
$$\alpha_{\mu}^{EW} \times 10^{11} = 153.6(1.0)$$



Hadronic Contributions

$$\alpha_{\mu}^{HVP} \times 10^{11} = 6845(40)$$

$$\alpha_{\mu}^{HLbL} \times 10^{11} = 92(18)$$

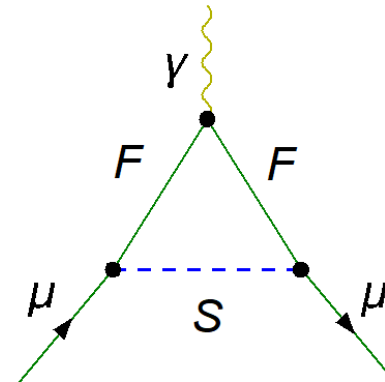


Contributions to Muon $g-2$

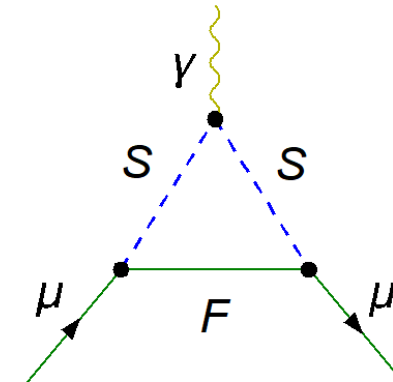
General Contributions to Muon $g-2$

All possible contributions to muon $g-2$ from 1-loop diagrams must be of these forms.

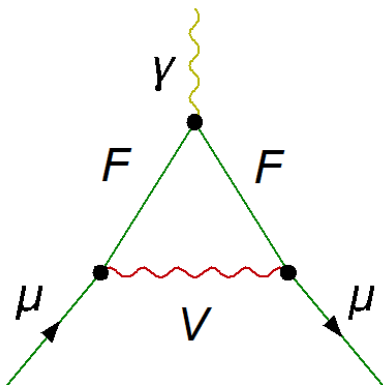
FFS Diagram



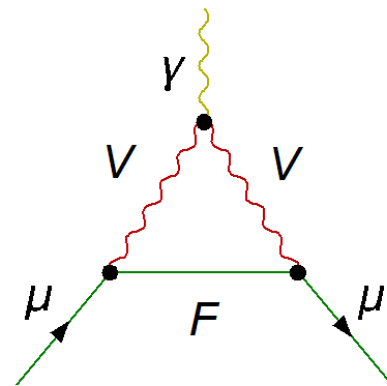
SSF Diagram



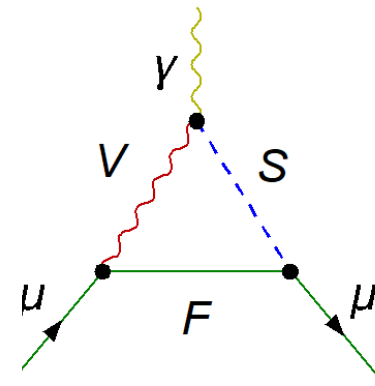
FFV Diagram



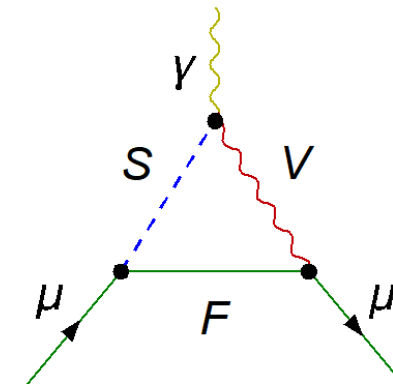
VVF Diagram



VSF Diagram



SVF Diagram



BSM Models

Simple and SUSY Explanations of Muon $g-2$

Single Field Extensions

- New Scalars
 - Scalar Leptoquarks
- New Fermions
- New Vectors

Two Field Extensions

- Fermion paired with Scalar
- Fermion paired with Vector
- Mixed Vector-like Fermion Pair

Three Field Extensions

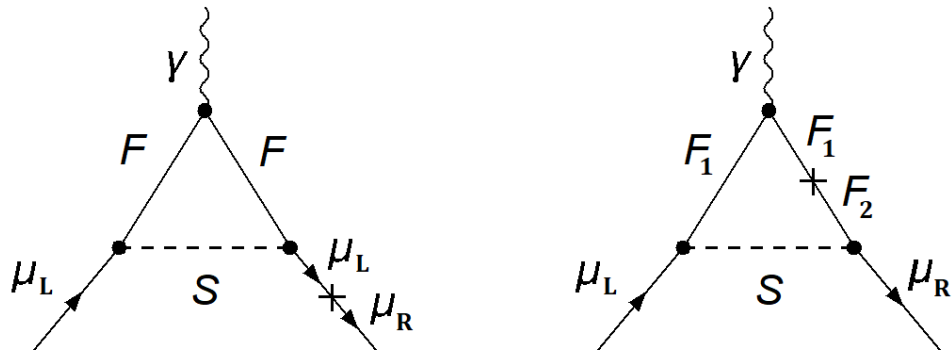
Single Field Extensions

Simple and SUSY Explanations of Muon g-2

Model	Spin	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Result for $\Delta a_\mu^{\text{BNL}}, \Delta a_\mu^{2021}$
1	0	(1, 1, 1)	Excluded: $\Delta a_\mu < 0$
2	0	(1, 1, 2)	Excluded: $\Delta a_\mu < 0$
3	0	(1, 2, -1/2)	Updated
4	0	(1, 3, -1)	Excluded: $\Delta a_\mu < 0$
5	0	($\bar{3}$, 1, 1/3)	Updated
6	0	($\bar{3}$, 1, 4/3)	Excluded: LHC searches
7	0	($\bar{3}$, 3, 1/3)	Excluded: LHC searches
8	0	(3, 2, 7/6)	Updated
9	0	(3, 2, 1/6)	Excluded: LHC searches
10	1/2	(1, 1, 0)	Excluded: $\Delta a_\mu < 0$
11	1/2	(1, 1, -1)	Excluded: Δa_μ too small
12	1/2	(1, 2, -1/2)	Excluded: LEP lepton mixing
13	1/2	(1, 2, -3/2)	Excluded: $\Delta a_\mu < 0$
14	1/2	(1, 3, 0)	Excluded: $\Delta a_\mu < 0$
15	1/2	(1, 3, -1)	Excluded: $\Delta a_\mu < 0$
16	1	(1, 1, 0)	Special cases viable
17	1	(1, 2, -3/2)	UV completion problems
18	1	(1, 3, 0)	Excluded: LHC searches
19	1	($\bar{3}$, 1, -2/3)	UV completion problems
20	1	($\bar{3}$, 1, -5/3)	Excluded: LHC searches
21	1	($\bar{3}$, 2, -5/6)	UV completion problems
22	1	($\bar{3}$, 2, 1/6)	Excluded: $\Delta a_\mu < 0$
23	1	($\bar{3}$, 3, -2/3)	Excluded: proton decay

BSM Models

Chirality Flip



Contributions from diagrams with an internal chirality flip enhanced by a factor:

$$\frac{\lambda_{BSM}^2}{\lambda_{\mu}^2}$$

Single Scalar Leptoquark

Scalar Leptoquark Singlet

Leptoquark	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
S_1	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	1/3

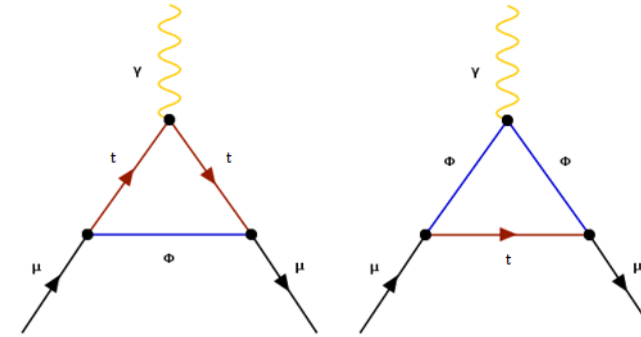
Interacts with the standard model through:

$$\mathcal{L}_{BSM} = (\lambda_{QL} Q \cdot L S_1 + \lambda_{t\mu} t \mu S_1^* + h.c.)$$

$$-M_{S_1}^2 |S_1|^2 - g_{HP} |H|^2 |S_1|^2 - \frac{\lambda_\phi}{2} |S_1|^4$$

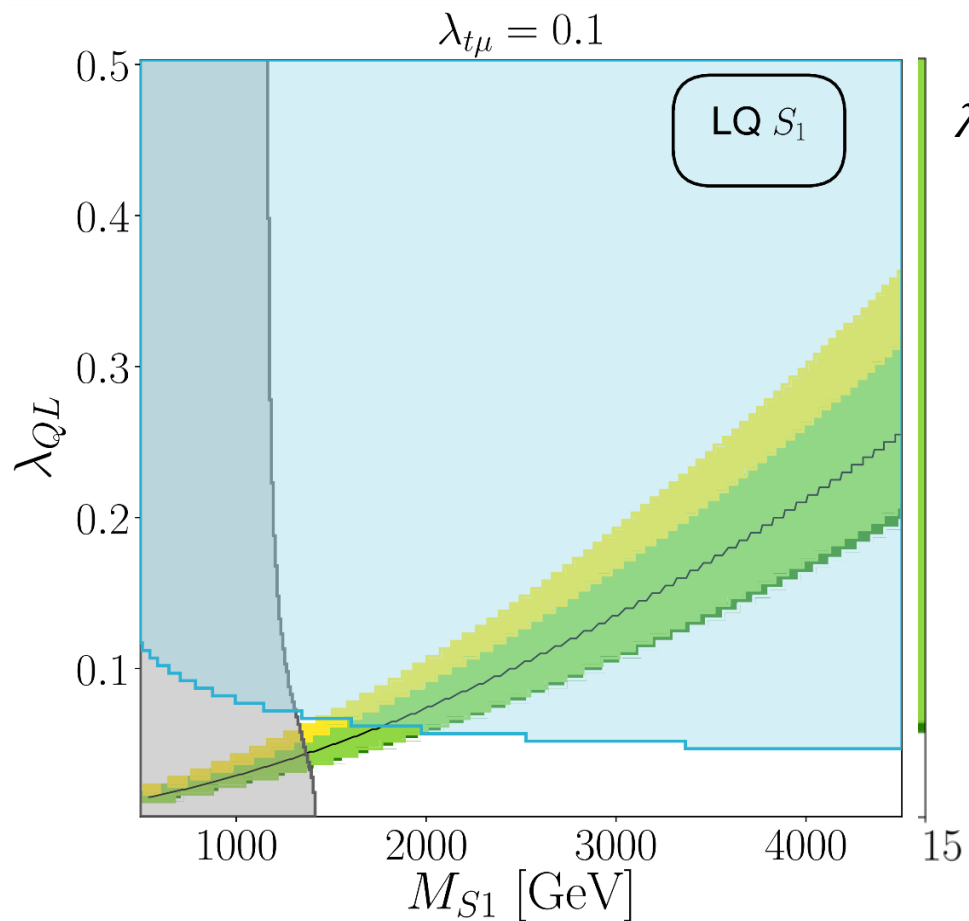
Contributes to muon g-2

S_1



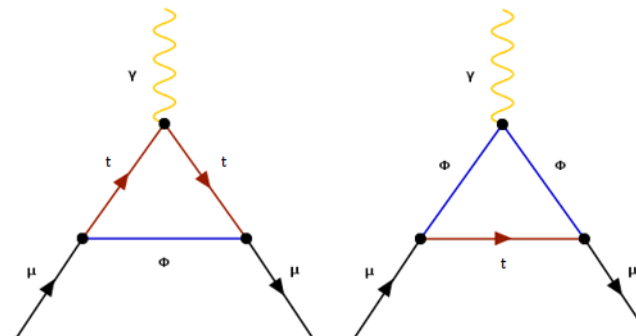
Single Scalar Leptoquark

Scalar Leptoquark Singlet



$\lambda_{QL}\lambda_{t\mu} \gtrsim 0.003$

Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

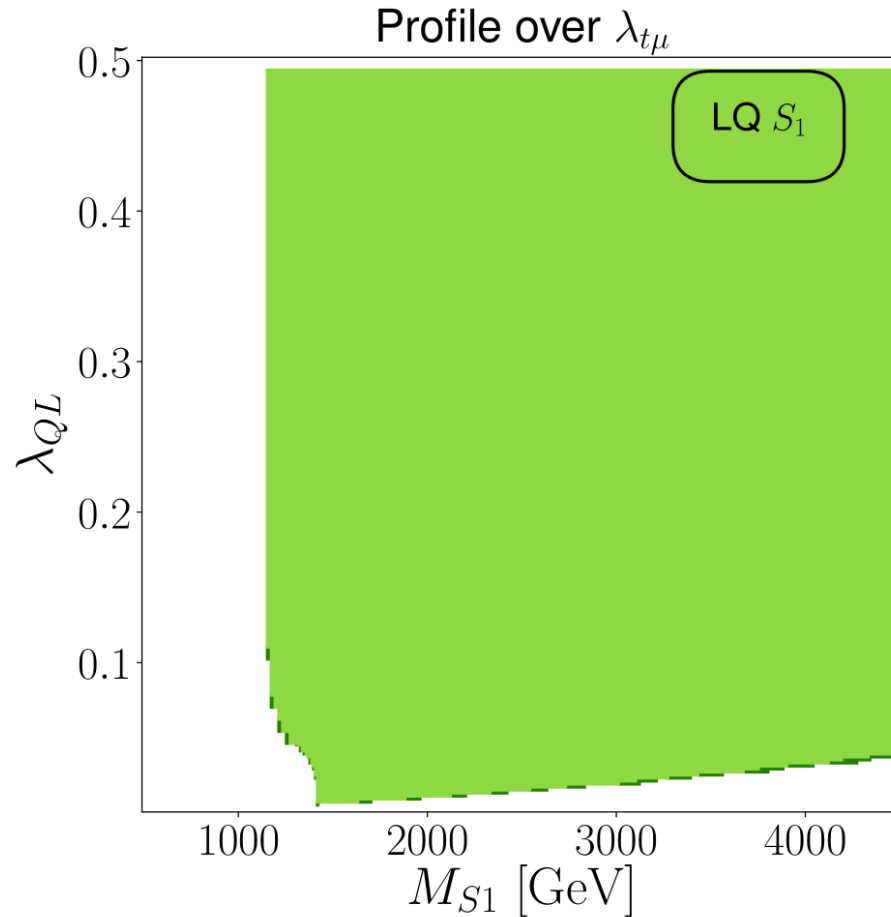
Exclusions:

LHC Searches

Fine-Tuned m_μ

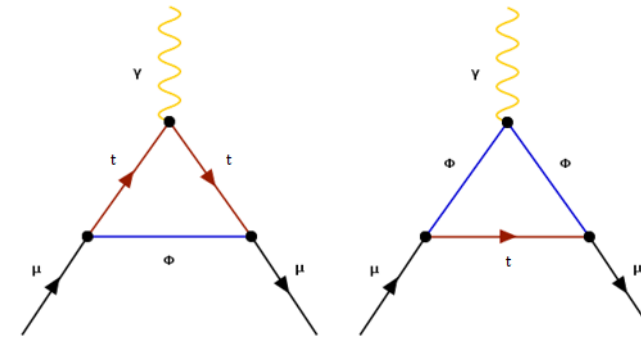
Single Scalar Leptoquark

Scalar Leptoquark Singlet



$$\lambda_{QL}\lambda_{t\mu} \gtrsim 0.003$$

Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

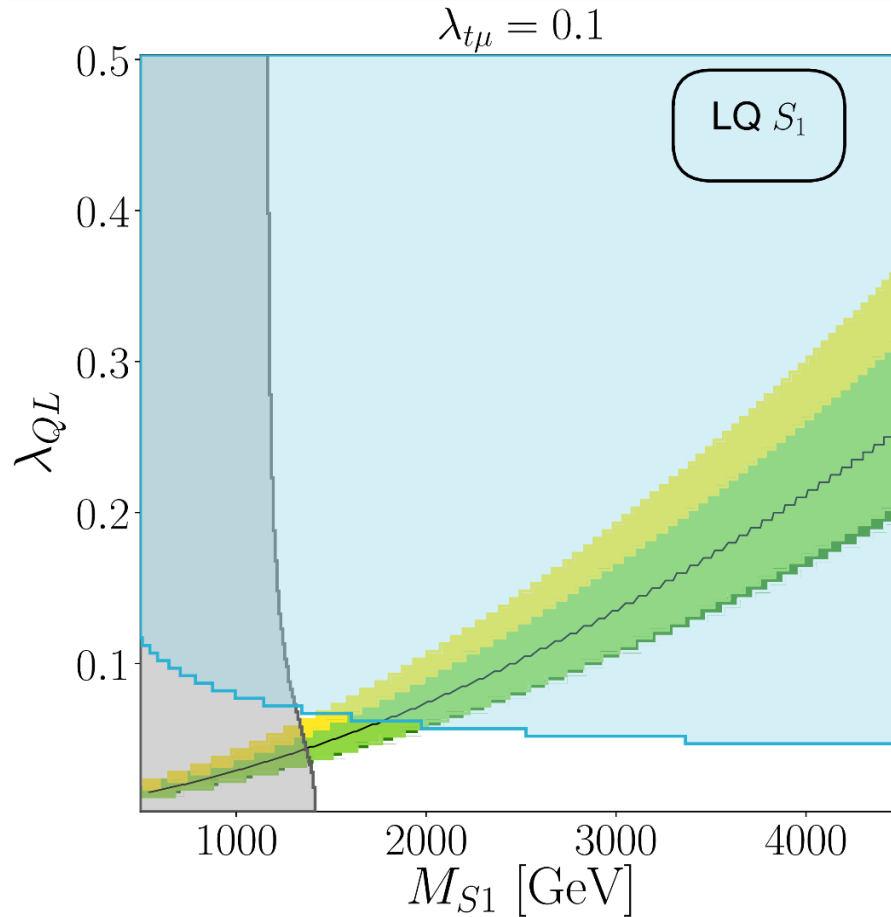
Exclusions:

LHC Searches

Fine-Tuned m_μ

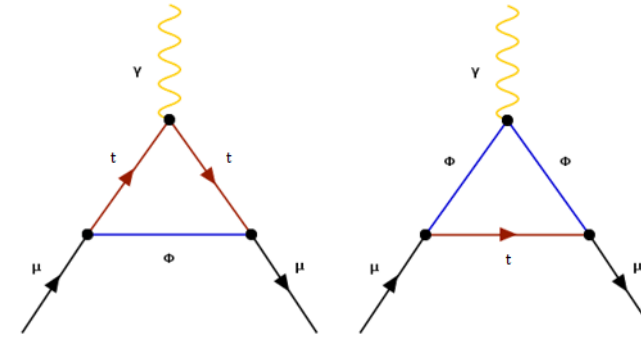
Single Scalar Leptoquark

Scalar Leptoquark Singlet



$$\lambda_{QL}\lambda_{t\mu} \gtrsim 0.003$$

Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

LHC Searches

Fine-Tuned m_μ

Single Scalar Leptoquark

Scalar Leptoquark Doublet

Leptoquark	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
$R_2 = (R_2^u, R_2^d)$	$(\mathbf{3}, \mathbf{2}, 7/6)$	$5/3, 2/3$

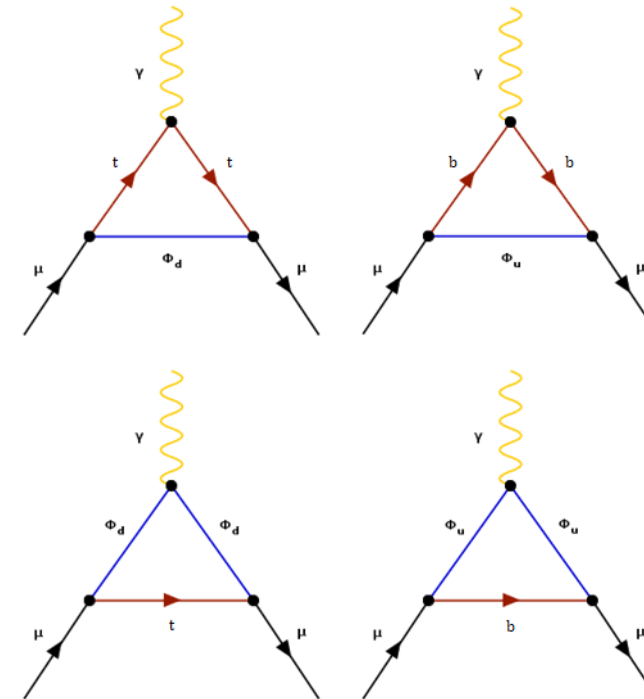
R_2

Interacts with the standard model through:

$$\mathcal{L}_{BSM} = (\lambda_{Q\mu} R_2^\dagger \mu Q + \lambda_{tL} L \cdot R_2 t + h.c.)$$

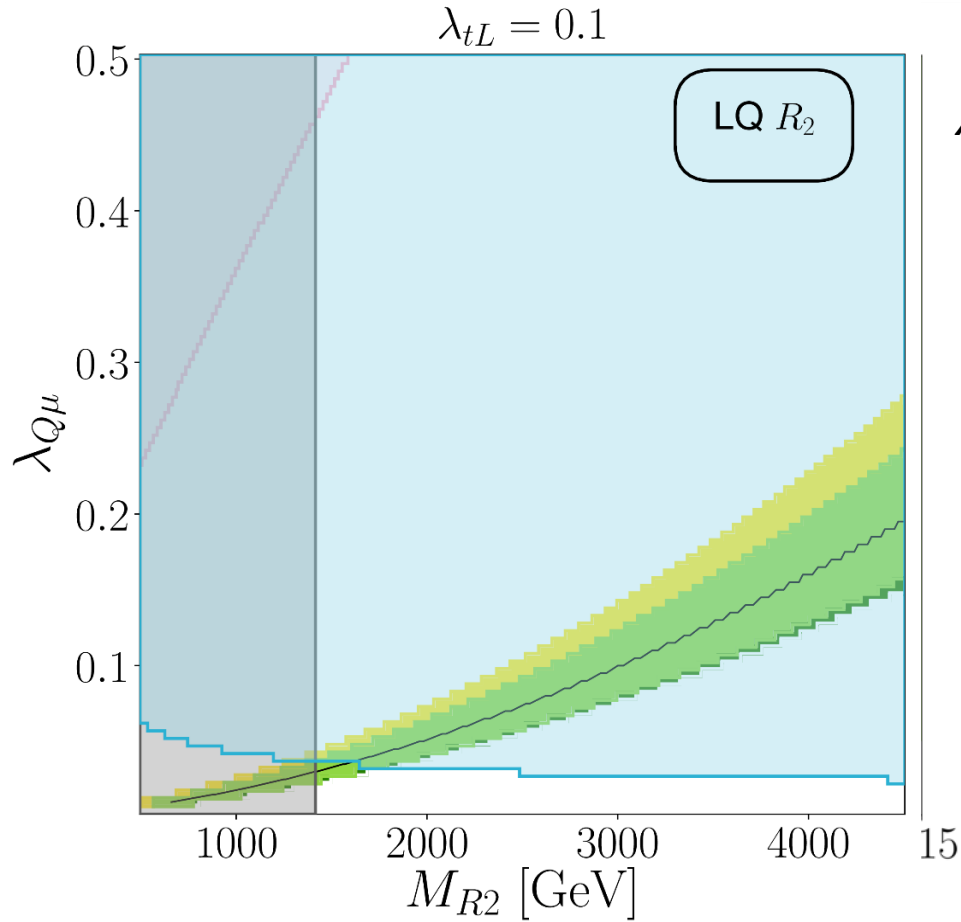
$$-M_{R_2}^2 |R_2|^2 - g_{HP} |H|^2 |R_2|^2 - \frac{\lambda_\phi}{2} |R_2|^4$$

Contributes to muon g-2



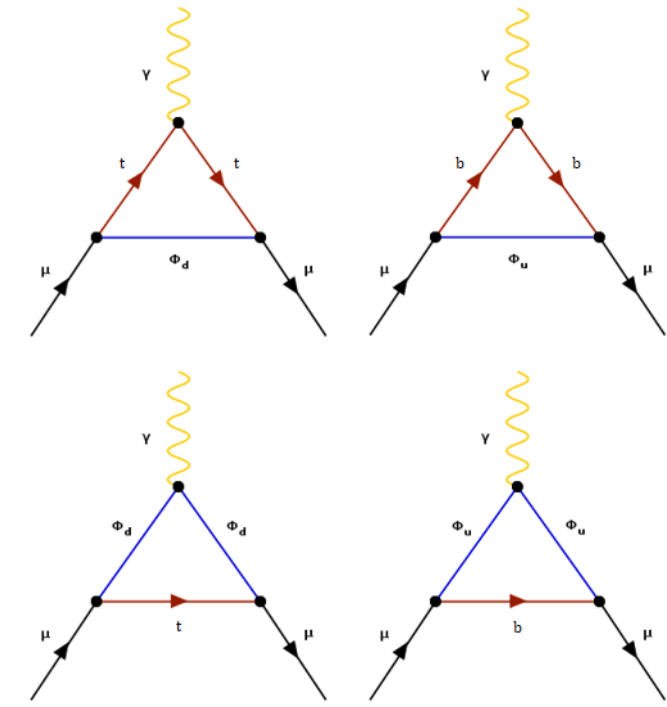
Single Scalar Leptoquark

Scalar Leptoquark Doublet



$\lambda_{Q\mu}\lambda_{tL} \gtrsim 0.003$

Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

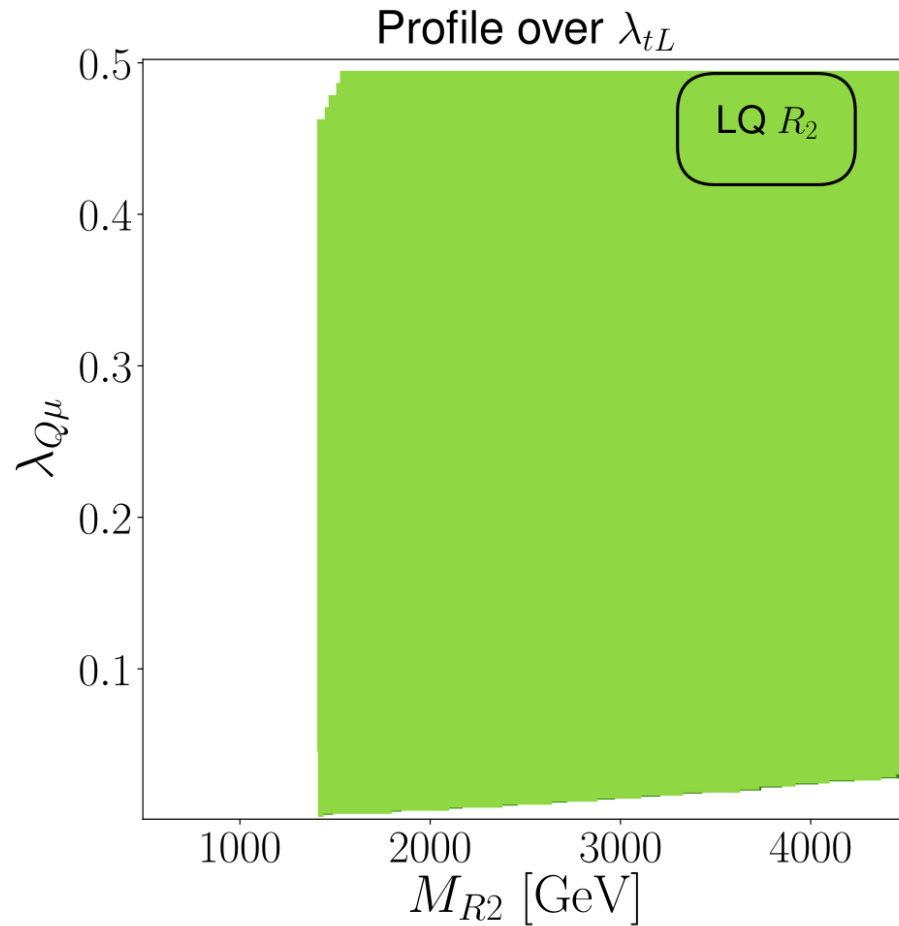
LHC Searches

Fine-Tuned m_μ

$Z \rightarrow \nu\nu$

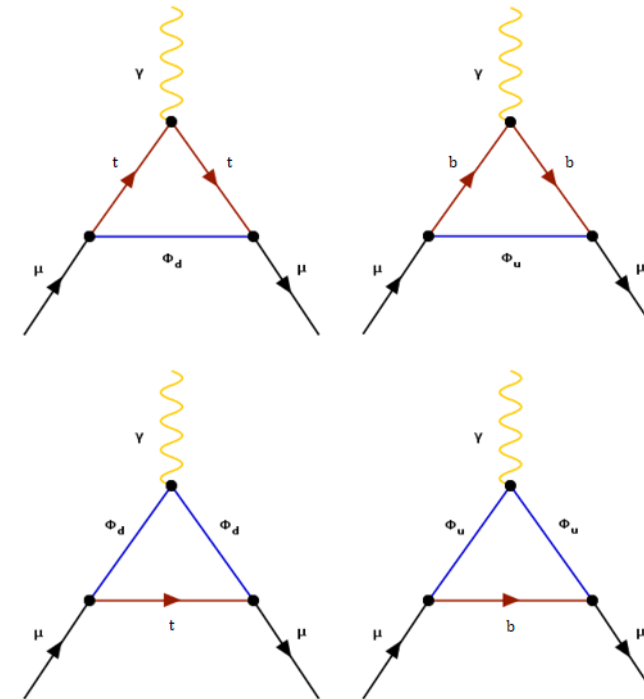
Single Scalar Leptoquark

Scalar Leptoquark Doublet



$$\lambda_{Q\mu}\lambda_{tL} \gtrsim 0.003$$

Contributes to muon $g-2$



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

LHC Searches

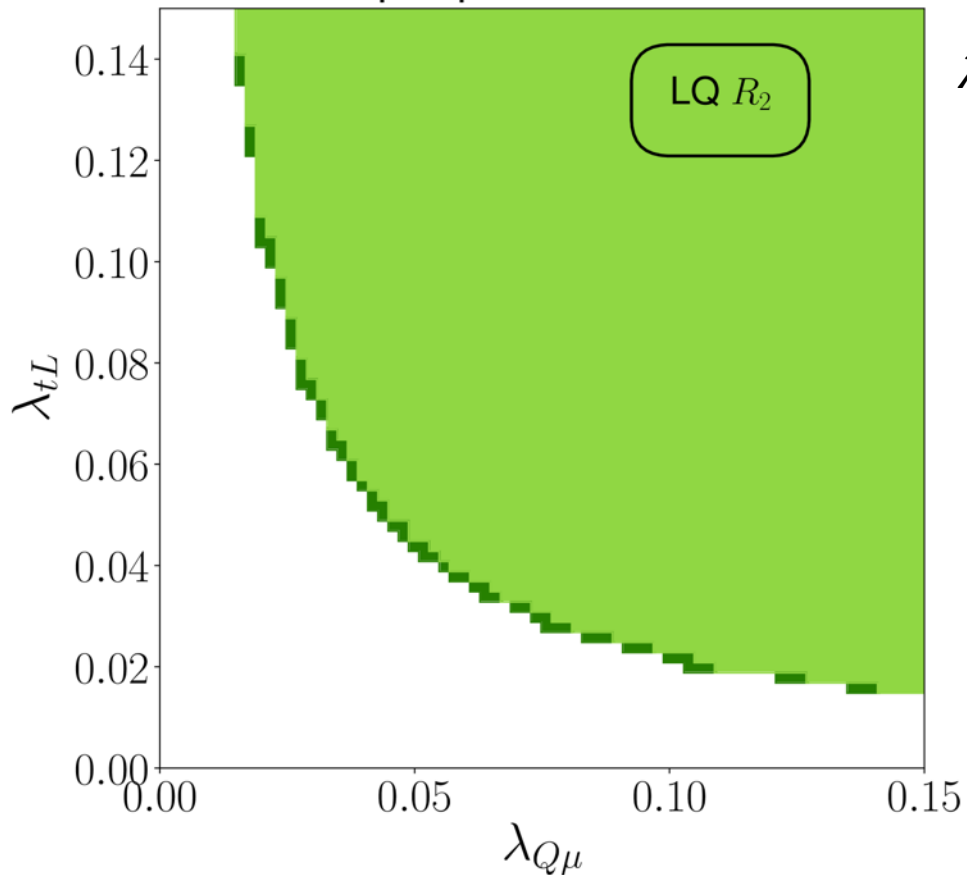
Fine-Tuned m_μ

$Z \rightarrow \nu\nu$

Single Scalar Leptoquark

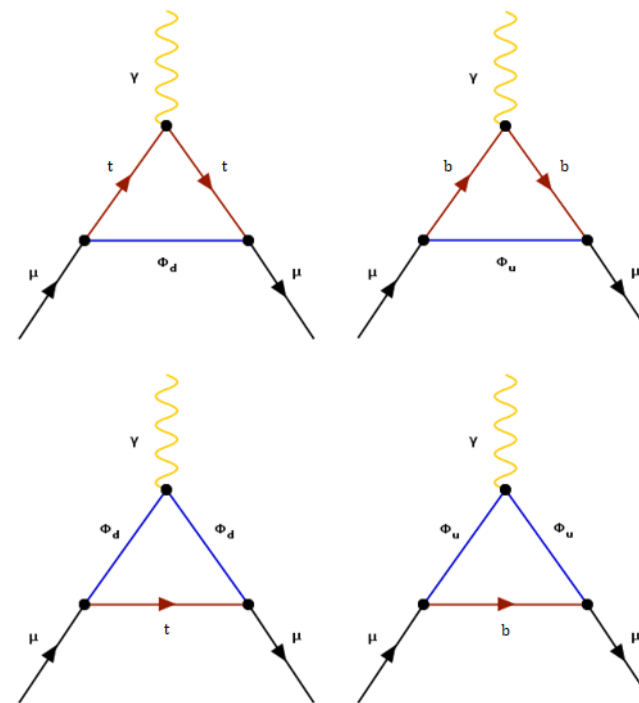
Scalar Leptoquark Doublet

Scalar Leptoquark R_2 Contributions



$$\lambda_{Q\mu}\lambda_{tL} \gtrsim 0.003$$

Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

LHC Searches

Fine-Tuned m_μ

$Z \rightarrow \nu\nu$

Two Field Extensions

Simple and SUSY Explanations of Muon g-2

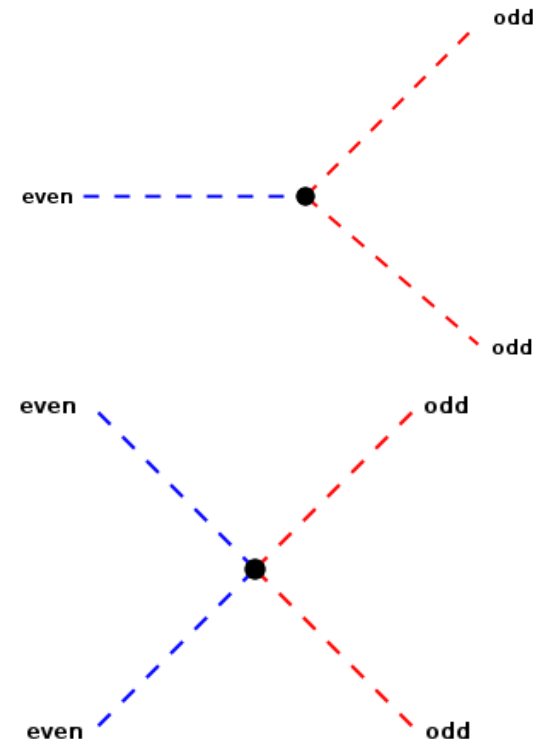
$(SU(3)_C \times SU(2)_L \times U(1)_Y)_{\text{spin}}$	$+\mathbb{Z}_2$	Result for $\Delta a_\mu^{\text{BNL}}, \Delta a_\mu^{2021}$
$(\mathbf{1}, \mathbf{1}, 0)_0 - (\mathbf{1}, \mathbf{1}, -1)_{1/2}$	No Yes	Projected LHC 14 TeV exclusion, not confirmed Updated
$(\mathbf{1}, \mathbf{1}, -1)_0 - (\mathbf{1}, \mathbf{1}, 0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{2}, -1/2)_0 - (\mathbf{1}, \mathbf{1}, 0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{1}, 0)_0 - (\mathbf{1}, \mathbf{2}, -1/2)_{1/2}$	No Yes	Excluded: LHC searches Updated
$(\mathbf{1}, \mathbf{1}, 0)_0 - (\mathbf{1}, \mathbf{1}, -1)_{1/2}$	No Yes	Excluded: LEP contact interactions Viable with under abundant DM
$(\mathbf{1}, \mathbf{1}, -1)_0 - (\mathbf{1}, \mathbf{2}, -1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{2}, -1/2)_0 - (\mathbf{1}, \mathbf{2}, -1/2)_{1/2}$	Both	Excluded: LEP search
$(\mathbf{1}, \mathbf{1}, 0)_0 - (\mathbf{1}, \mathbf{1}, -1)_{1/2}$	No Yes	Excluded: LHC searches Viable with under abundant DM
$(\mathbf{1}, \mathbf{1}, 0)_0 - (\mathbf{1}, \mathbf{1}, -1)_{1/2}$	No Yes	Excluded: LHC searches + LEP contact interactions Viable with under abundant DM
$(\mathbf{1}, \mathbf{3}, 0)_0 - (\mathbf{1}, \mathbf{2}, -1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{1}, 0)_0 - (\mathbf{1}, \mathbf{1}, -1)_{1/2}$	No Yes	Excluded: LHC searches Viable with under abundant DM
$(\mathbf{1}, \mathbf{3}, -1)_0 - (\mathbf{1}, \mathbf{2}, -1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{3}, -1)_0 - (\mathbf{1}, \mathbf{3}, 0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{1}, -1)_{1/2} - (\mathbf{1}, \mathbf{1}, 0)_1$	No	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{2}, -1/2)_{1/2} - (\mathbf{1}, \mathbf{1}, 0)_1$	No	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1}, \mathbf{2}, -1/2)_{1/2} - (\mathbf{1}, \mathbf{3}, 0)_1$	No	Excluded: LHC searches + LEP contact interactions
$(\mathbf{1}, \mathbf{1}, 0)_{1/2} - (\mathbf{1}, \mathbf{1}, 1)_1$	No	Excluded: LHC searches + LEP contact interactions
$(\mathbf{1}, \mathbf{2}, -1/2)_{1/2} - (\mathbf{1}, \mathbf{1}, -1)_1$	No	Excluded: LHC searches + LEP contact interactions
$(\mathbf{1}, \mathbf{3}, -1)_{1/2} - (\mathbf{1}, \mathbf{3}, 0)_1$	No	Excluded: $\Delta a_\mu < 0$

Z2 Symmetry

Z2-odd fields interact only in pairs:

$$\psi_{\text{even}} \rightarrow \psi_{\text{even}}$$

$$\psi_{\text{odd}} \rightarrow \psi_{\text{odd}} e^{i\pi}$$



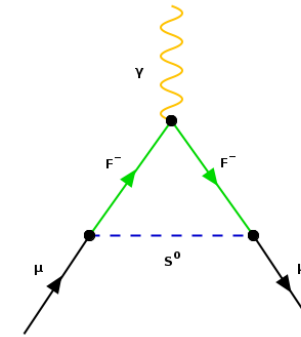
Two Fields with Dark Matter

New Fermion and Scalar Coupling to Left-Handed Muon

New Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
$\psi_d = (\psi_d^+, \psi_d^0)$	$(\mathbf{1}, \mathbf{2}, 1/2)$	1, 0
ϕ	$(\mathbf{1}, \mathbf{1}, 0)$	0



Contributes to muon g-2



Interacts with the standard model through:

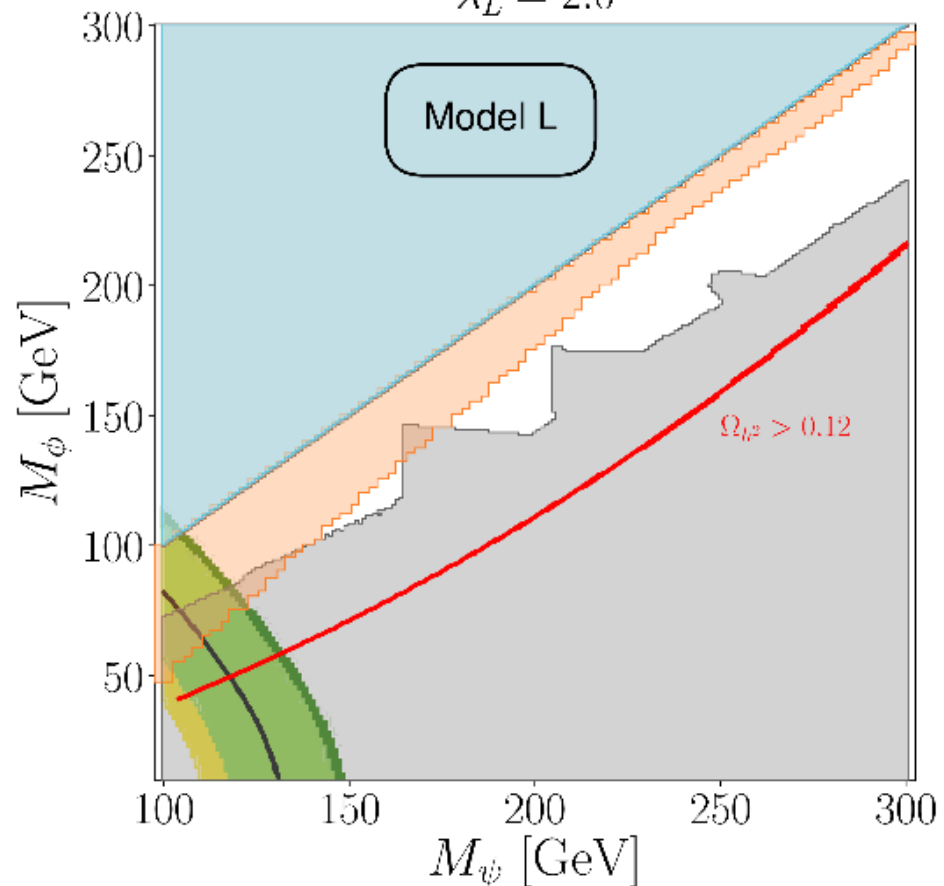
$$\mathcal{L}_{BSM} = (\lambda_L L_L \cdot \psi_d \phi - M_\psi \psi_d^c \psi_d + h.c.) - \frac{M_\phi^2}{2} \phi^2$$

Source: 1804.00009

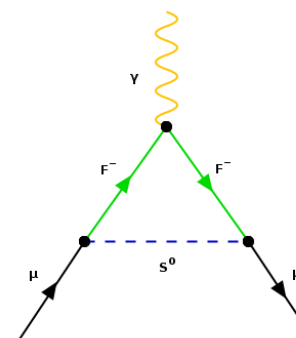
Two Fields with Dark Matter

New Fermion and Scalar Coupling to Left-Handed Muon

$$\lambda_L = 2.0$$



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

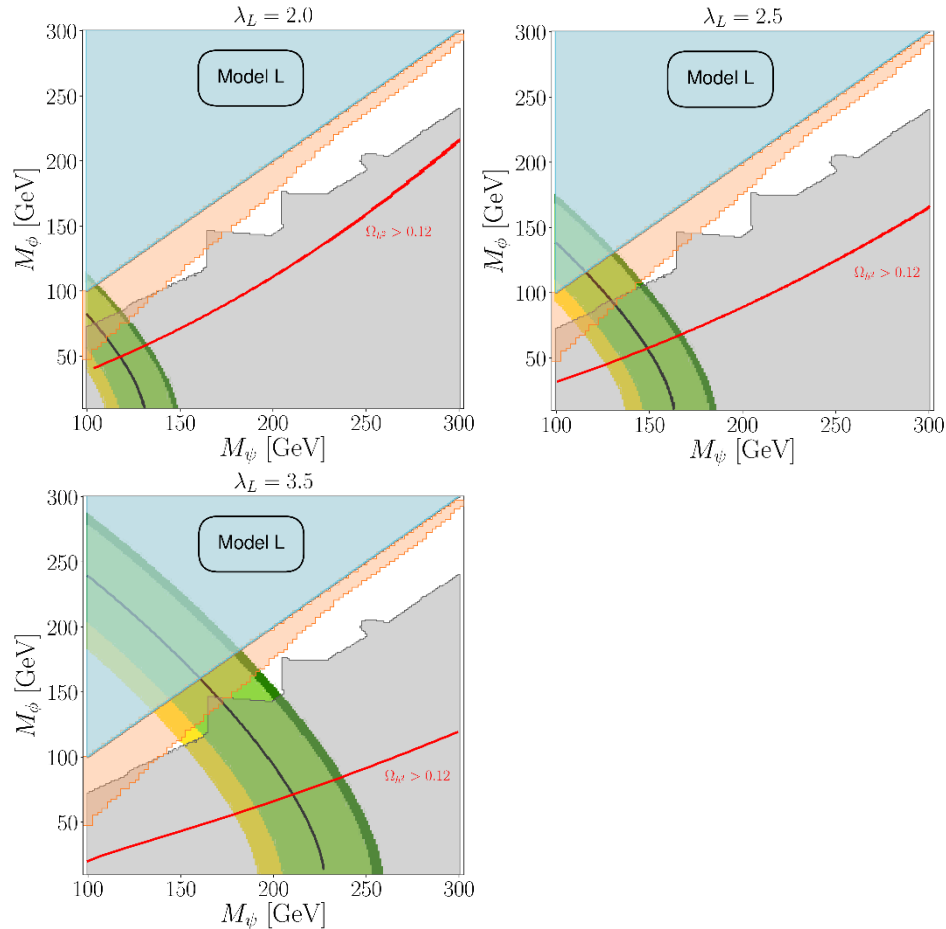
Exclusions:

LHC Searches

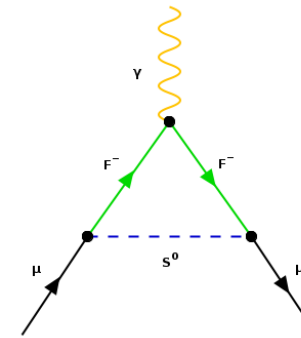
Cmp. Spectra

Two Fields with Dark Matter

New Fermion and Scalar Coupling to Left-Handed Muon



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

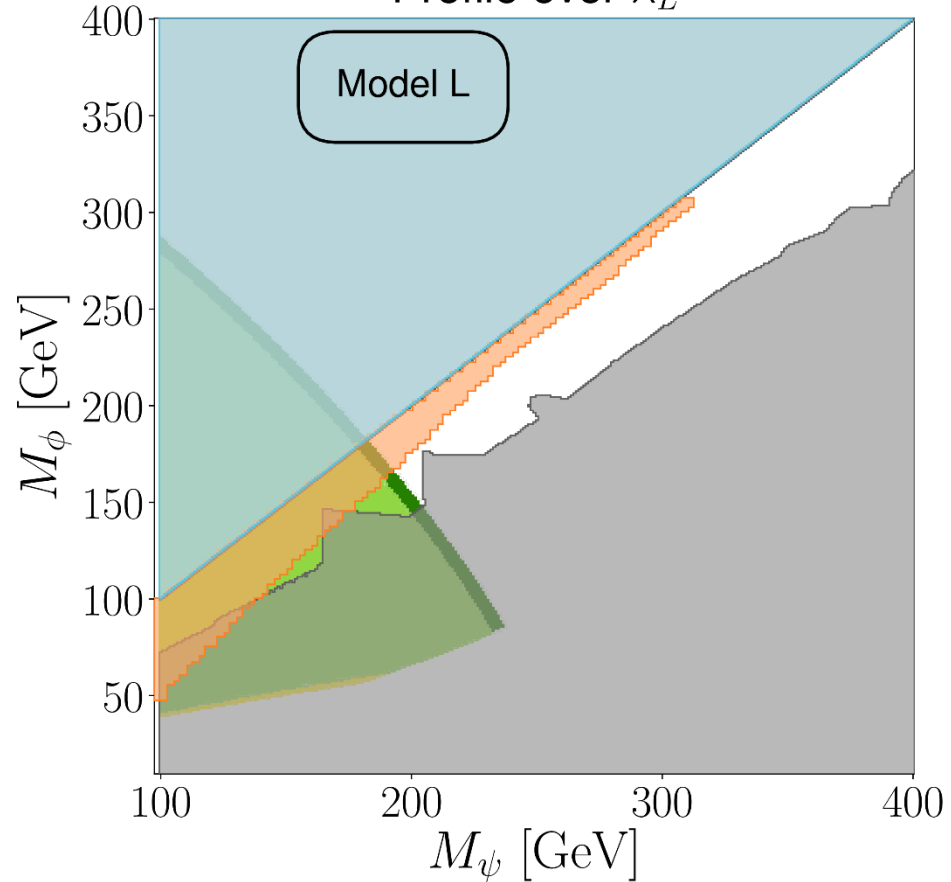
LHC Searches

Cmp. Spectra

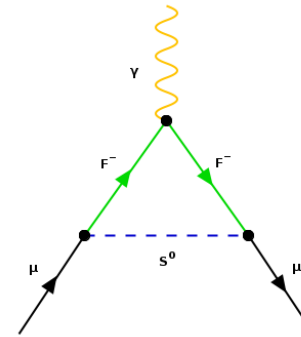
Two Fields with Dark Matter

New Fermion and Scalar Coupling to Left-Handed Muon

Profile over λ_L



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

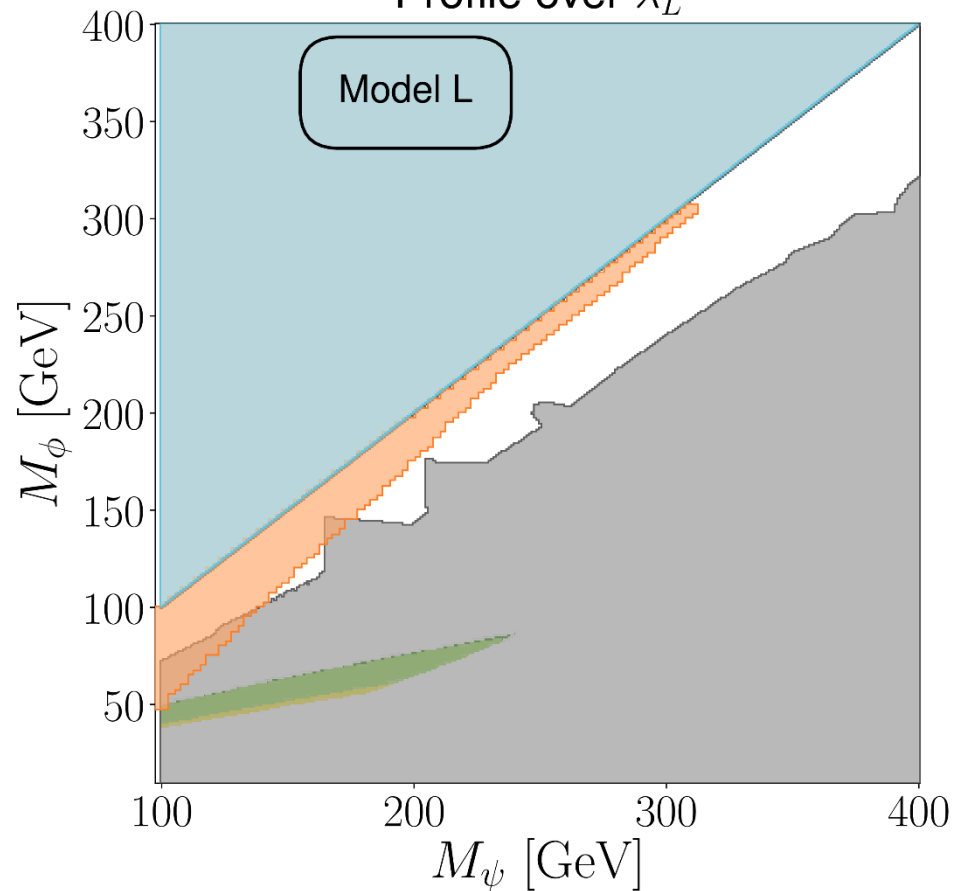
LHC Searches

Cmp. Spectra

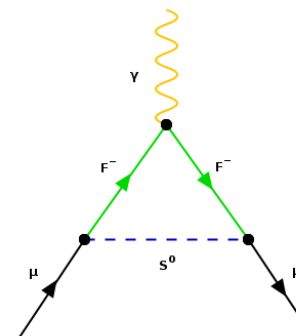
Two Fields with Dark Matter

New Fermion and Scalar Coupling to Left-Handed Muon

Profile over λ_L



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

LHC Searches

Cmp. Spectra

Two Fields with Dark Matter

New Fermion and Scalar Coupling to Right-Handed Muon

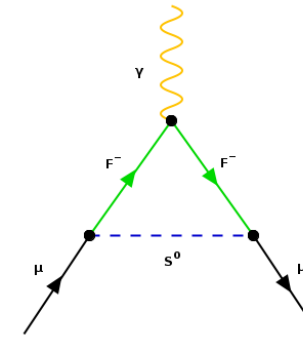
New Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
$\psi_s = \psi_s^-$	$(\mathbf{1}, \mathbf{1}, -1)$	-1
ϕ	$(\mathbf{1}, \mathbf{1}, 0)$	0

Interacts with the standard model through:

$$\mathcal{L}_{BSM} = (\lambda_R \mu \phi \psi_s - M_\psi \psi_s^c \psi_s + h.c.) - \frac{M_\phi^2}{2} \phi^2$$



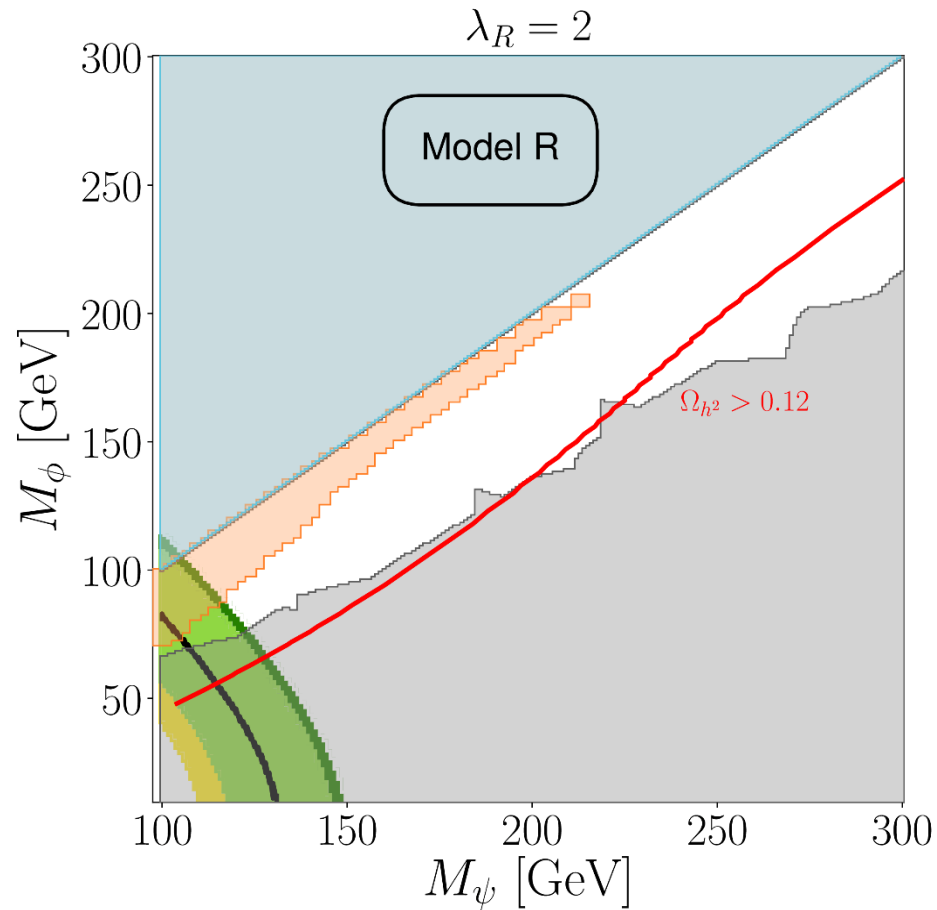
Contributes to muon g-2



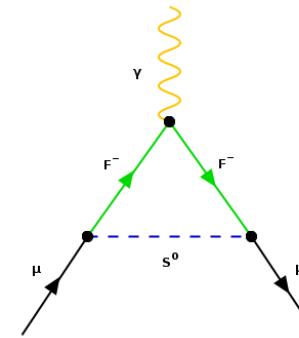
Source: 1804.00009

Two Fields with Dark Matter

New Fermion and Scalar Coupling to Right-Handed Muon



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

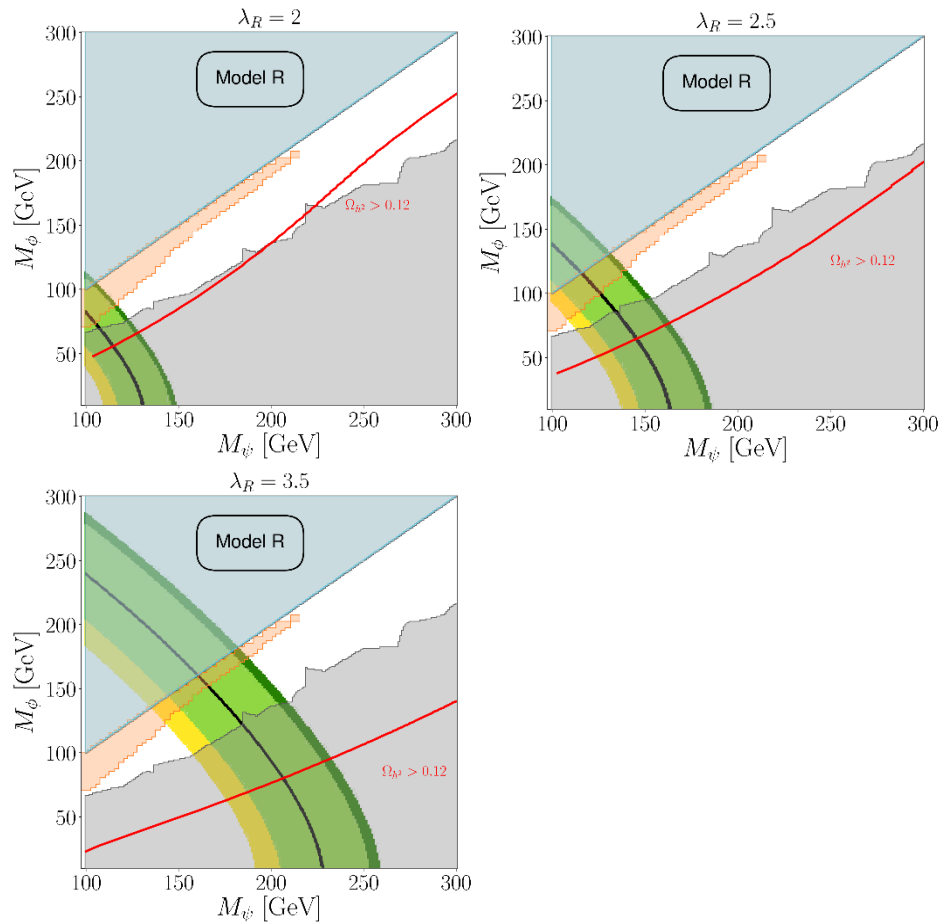
Exclusions:

LHC Searches

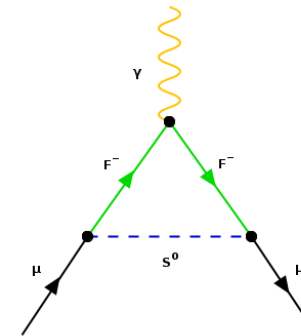
Cmp. Spectra

Two Fields with Dark Matter

New Fermion and Scalar Coupling to Right-Handed Muon



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

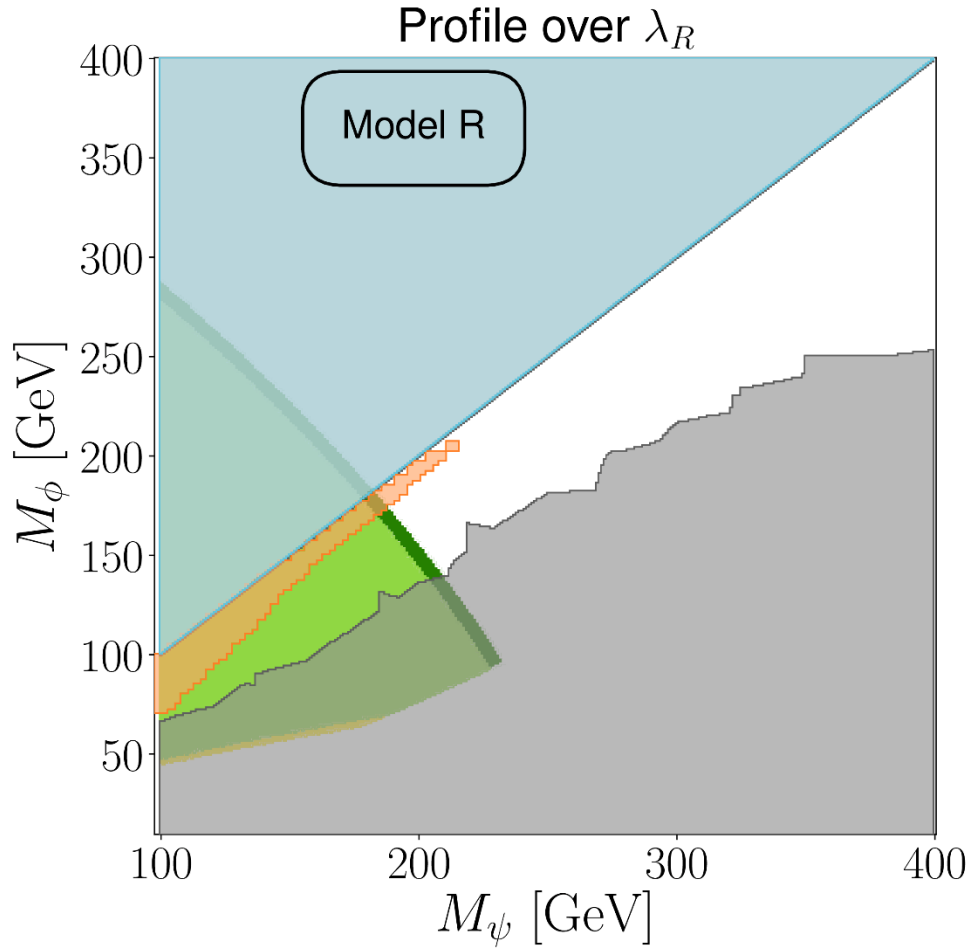
Exclusions:

LHC Searches

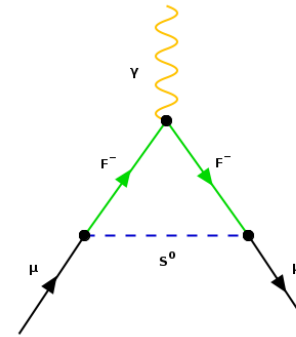
Cmp. Spectra

Two Fields with Dark Matter

New Fermion and Scalar Coupling to Right-Handed Muon



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

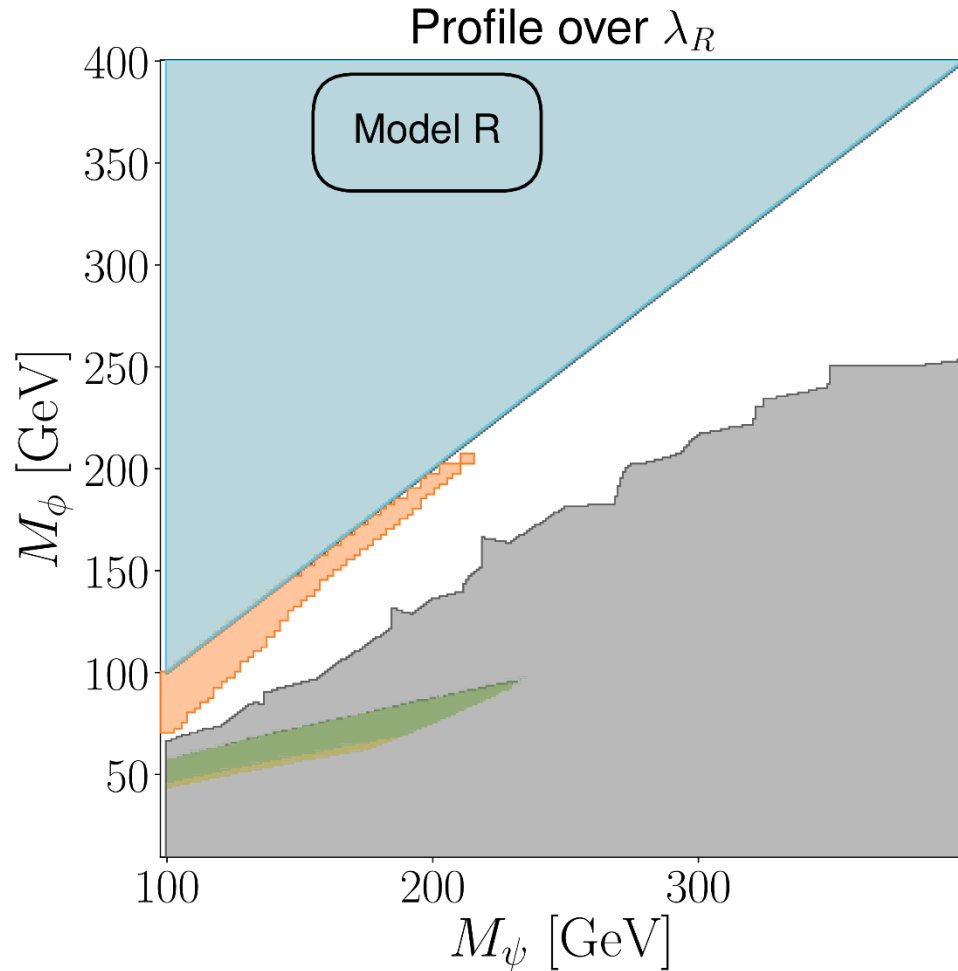
Exclusions:

LHC Searches

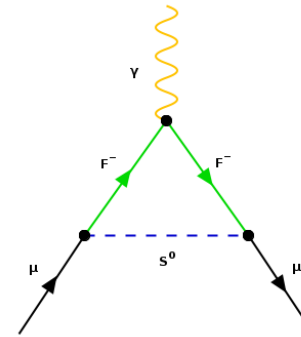
Cmp. Spectra

Two Fields with Dark Matter

New Fermion and Scalar Coupling to Right-Handed Muon



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

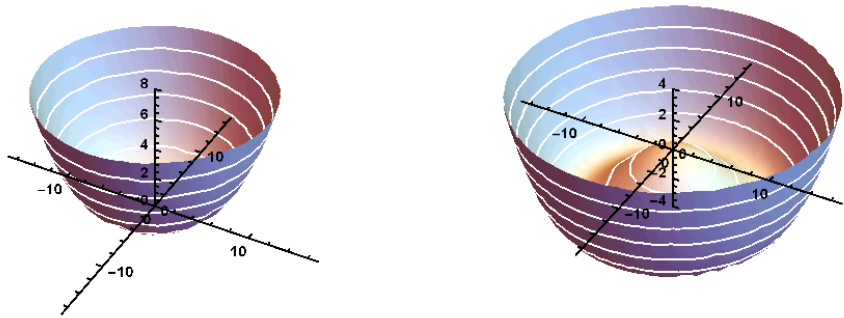
Exclusions:

LHC Searches

Cmp. Spectra

BSM Models

Mixing Fermions and Scalars



After EWSB, mixed fields with identical electric charges mix:

$$\begin{aligned}\psi_s^0, \psi_d^0 &\rightarrow \psi_1^0, \psi_2^0 \\ \psi_s^-, \psi_d^- &\rightarrow \psi_1^-, \psi_2^-\end{aligned}$$

Three Fields with Dark Matter

Pair of New Scalars + Fermion

New Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
$\psi_s = \psi_s^{-\dagger}$	$(\mathbf{1}, \mathbf{1}, 1)$	1
$\phi_s = \phi_s^0$	$(\mathbf{1}, \mathbf{1}, 0)$	0
$\phi_d = (\phi_d^0, \phi_d^-)$	$(\mathbf{1}, \mathbf{2}, -1/2)$	0, -1

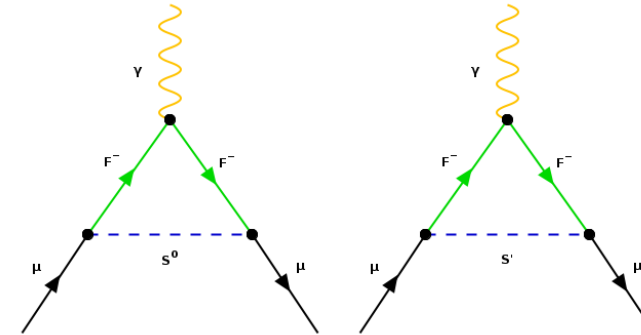


Interacts with the standard model through:

$$\mathcal{L}_{BSM} = (a_H H \cdot \phi_d \phi_s + \lambda_L L_L \cdot \phi_d \psi_s + \lambda_R \phi_s e_R^\dagger \psi_s^c$$

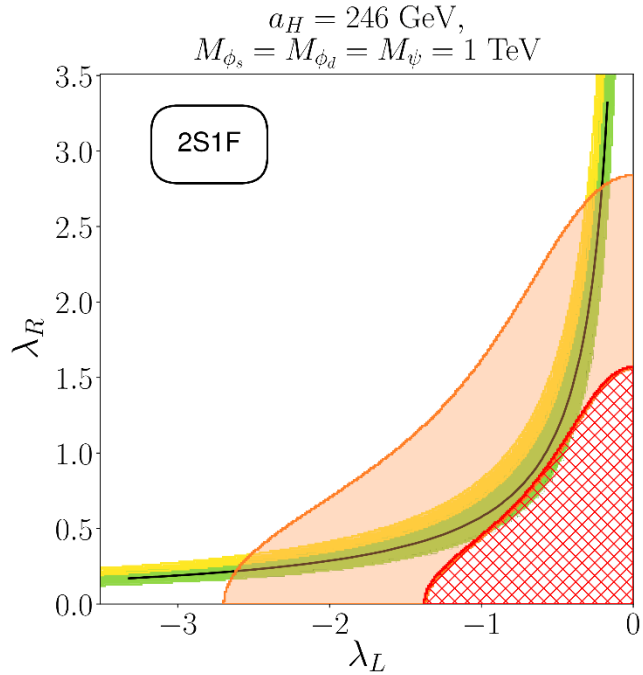
$$- M_\psi \psi_s^c \psi_s + h.c.) - \frac{M_{\phi_d}}{2} |\phi_d|^2 - M_{\phi_s}^2 |\phi_s|^2$$

Contributes to muon g-2

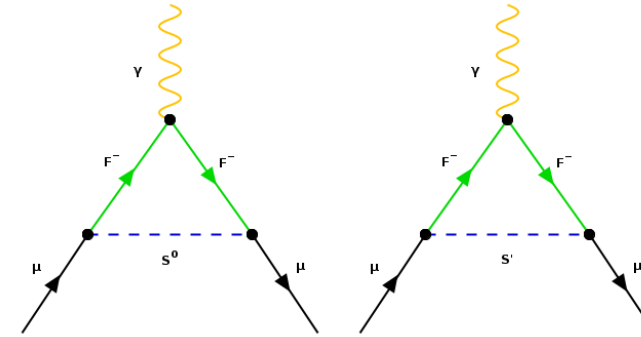


Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

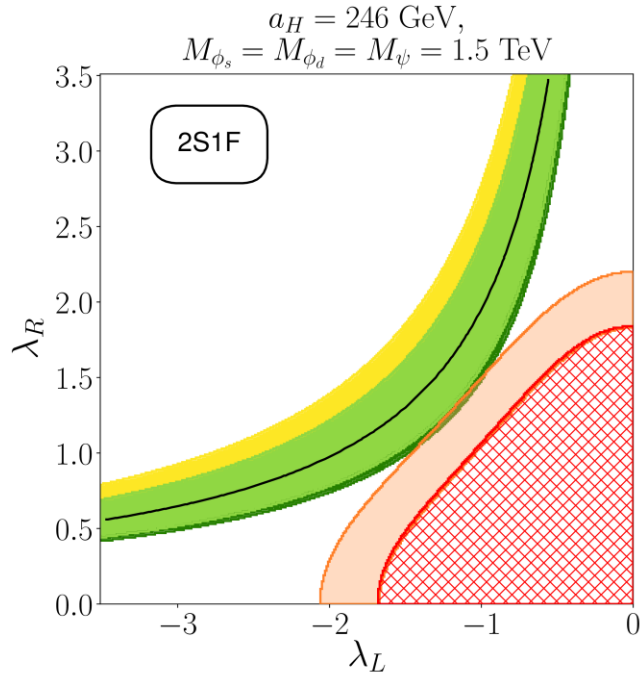
Exclusions:

~~Over Abundant~~

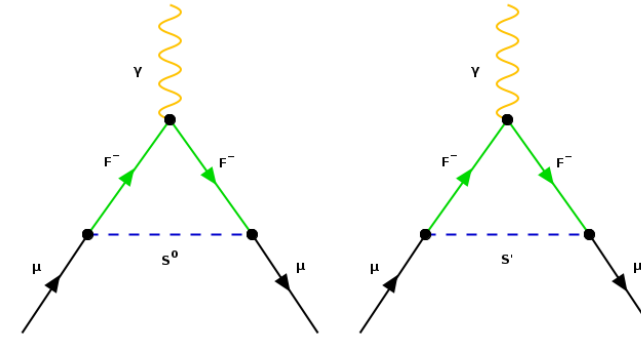
Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

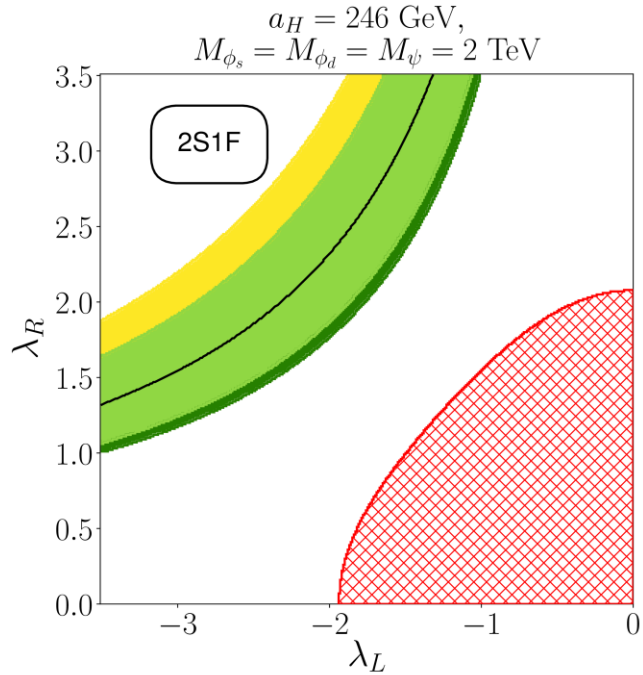
Exclusions:

~~Over Abundant~~

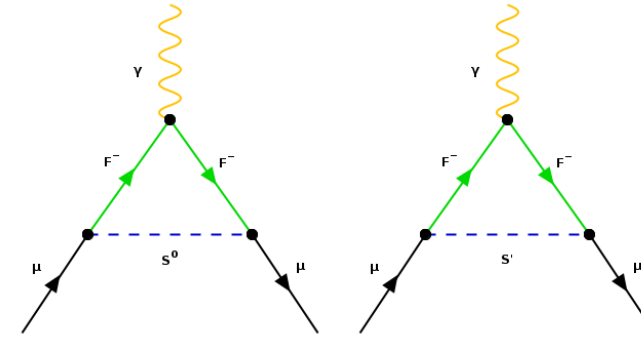
Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

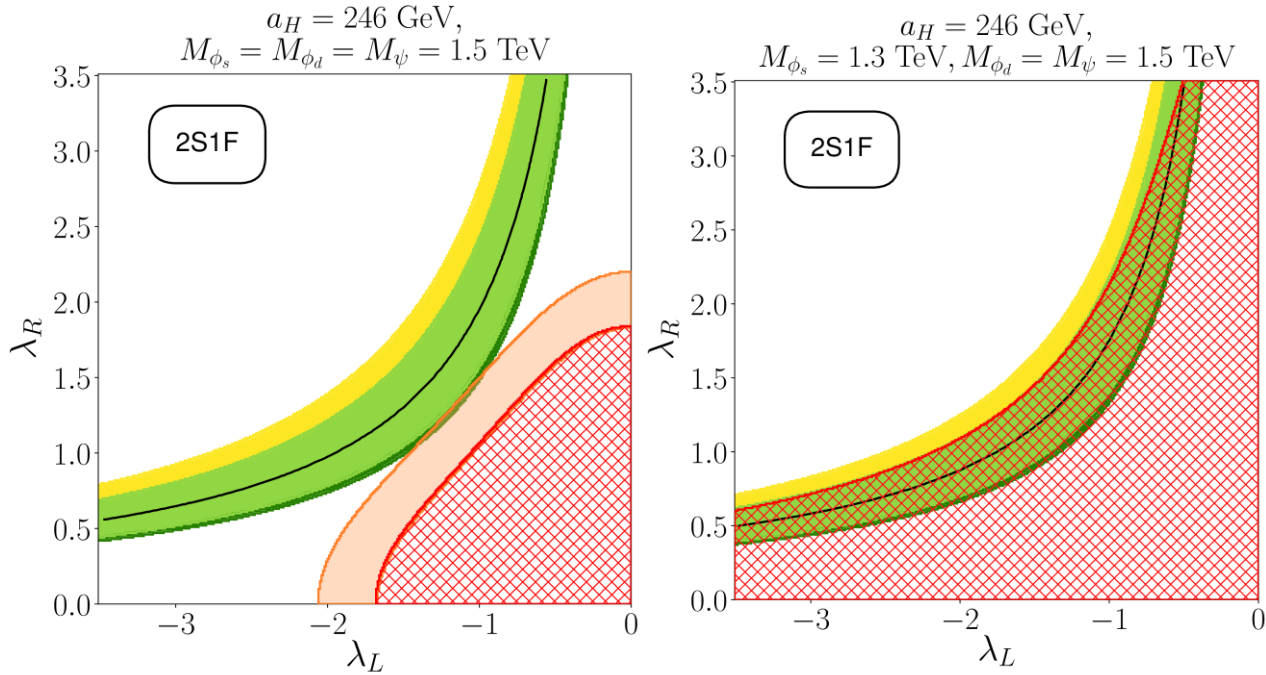
Exclusions:

~~Over Abundant~~

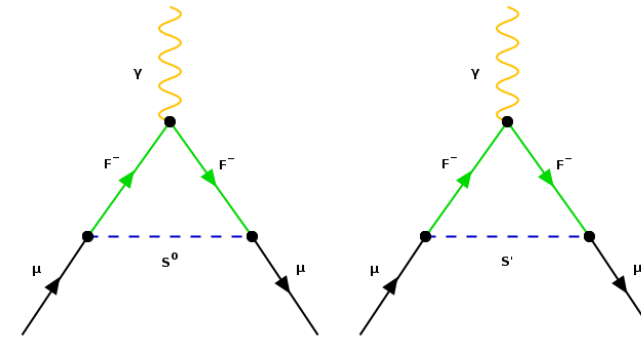
Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

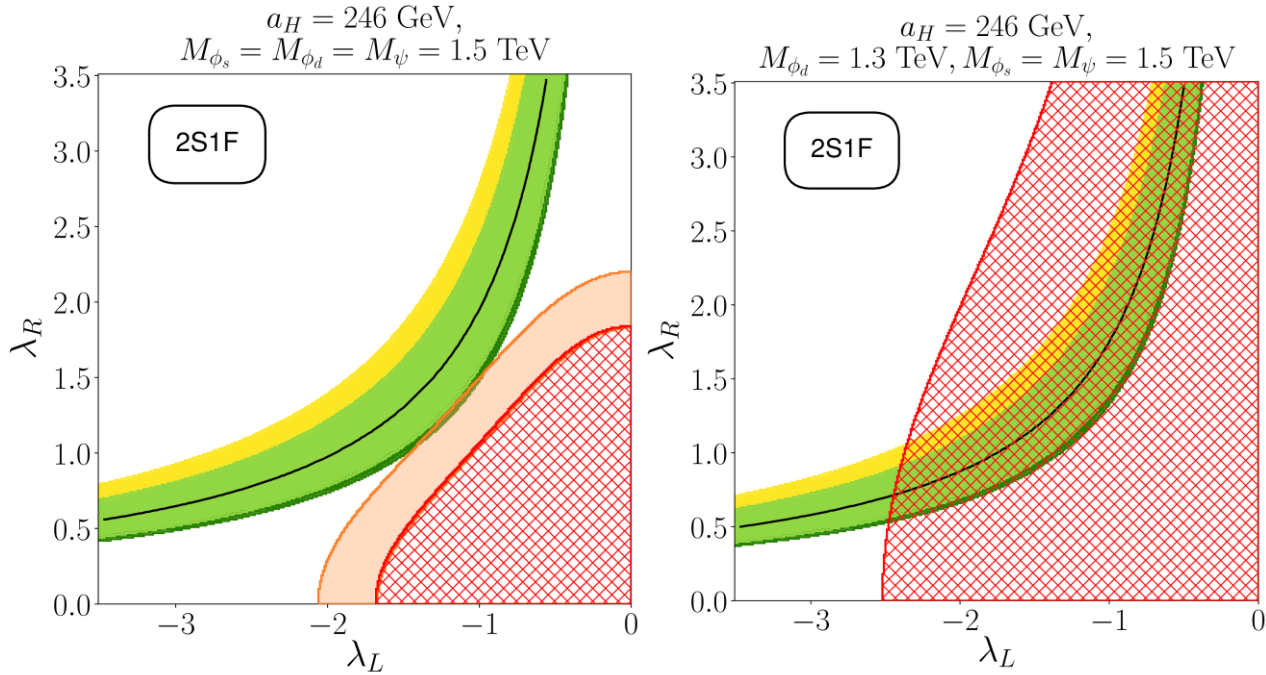
Exclusions:

Over Abundant

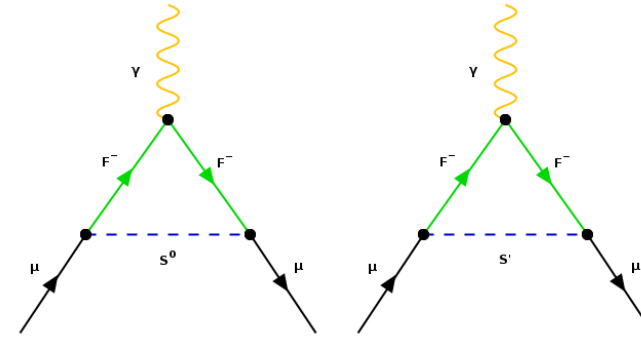
Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

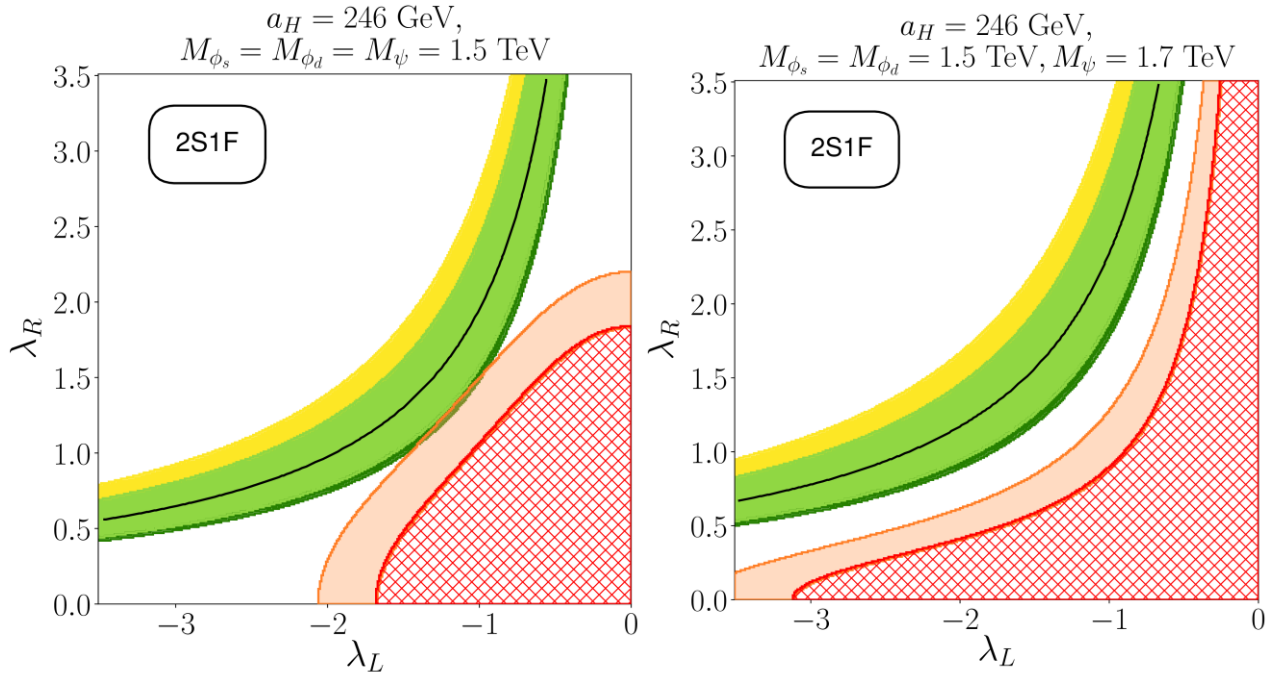
Exclusions:

Over Abundant

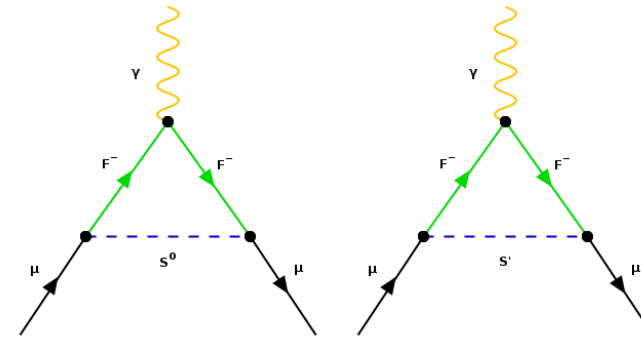
Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

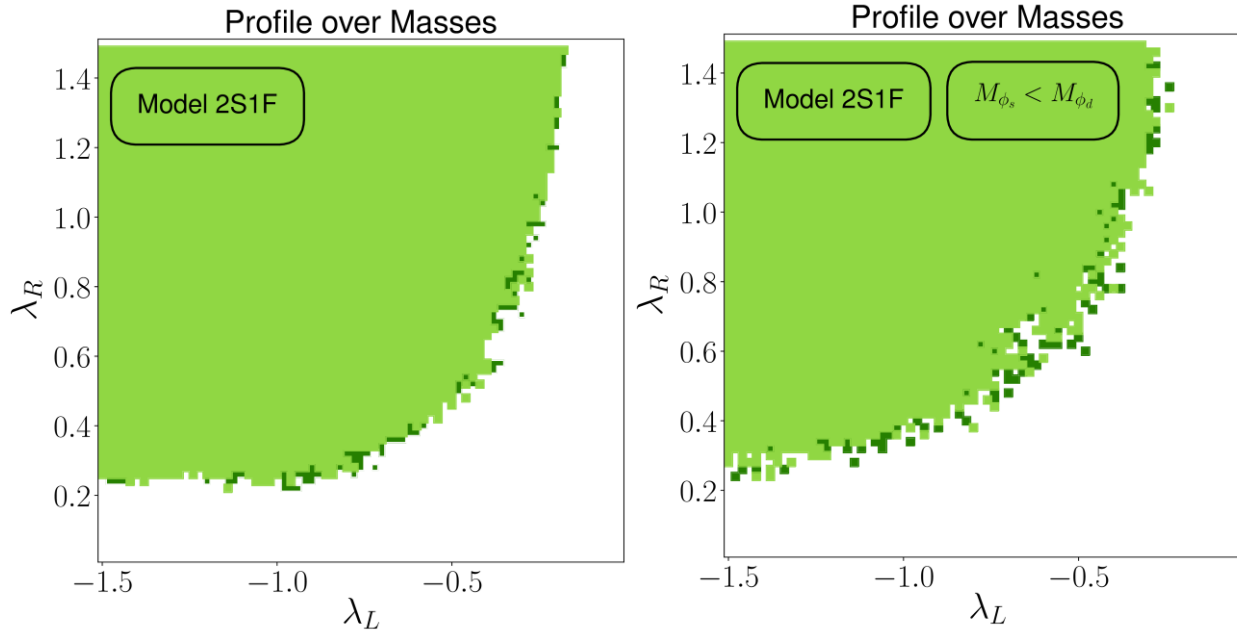
Exclusions:

Over Abundant

Direct Detection

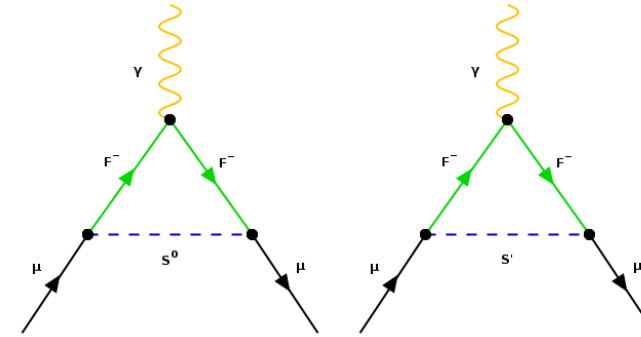
Three Fields with Dark Matter

Pair of New Fermions + Scalar



$$|\lambda_L \lambda_R| \gtrsim 0.22$$

Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

~~Over Abundant~~

Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar

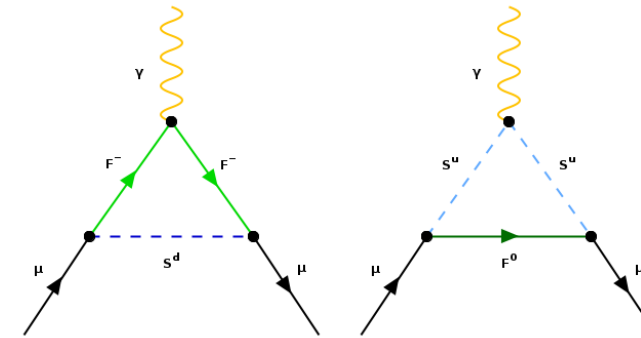
New Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
$\psi_d = (\psi_d^0, \psi_d^-)$	$(\mathbf{1}, \mathbf{2}, -1/2)$	0, -1
$\psi_s = \psi_s^0$	$(\mathbf{1}, \mathbf{1}, 0)$	0
$\phi_d = (\phi_d^+, \phi_d^0)$	$(\mathbf{1}, \mathbf{2}, 1/2)$	1, 0



Interacts with the standard model through:

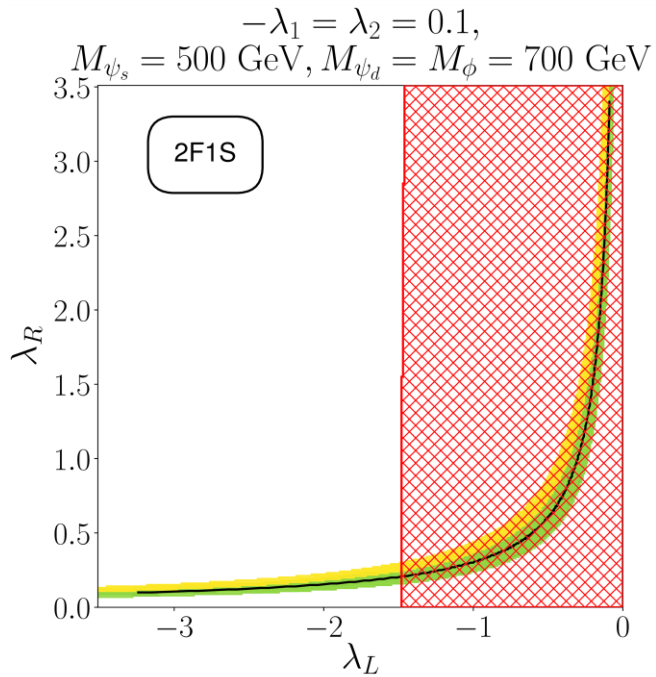
$$\mathcal{L}_{BSM} = (\lambda_{H1} H \cdot \psi_d \psi_s + \lambda_{H2} H^\dagger \cdot \psi_d^c \psi_s + \lambda_L L_L \cdot \phi_d \psi_s + \lambda_R \psi_d e_R^\dagger \phi_d^\dagger - M_{\psi_d} \psi_d^c \psi_d + h.c.) - \frac{M_{\psi_s}}{2} \psi_s \psi_s - M_\phi^2 |\phi_d|^2$$

Contributes to muon g-2

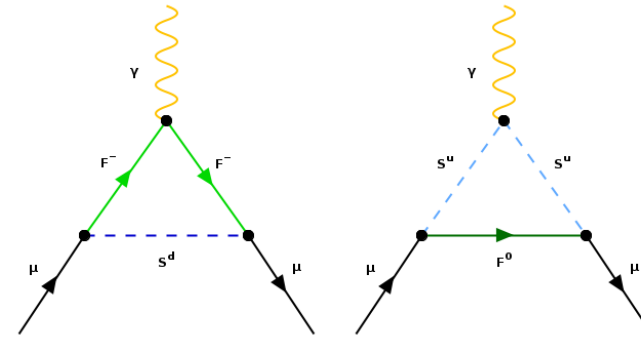


Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

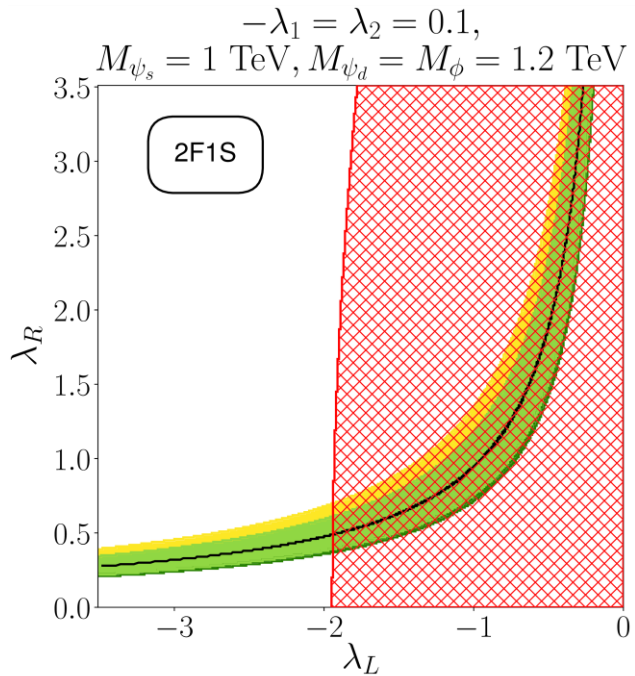
Exclusions:

~~Over Abundant~~

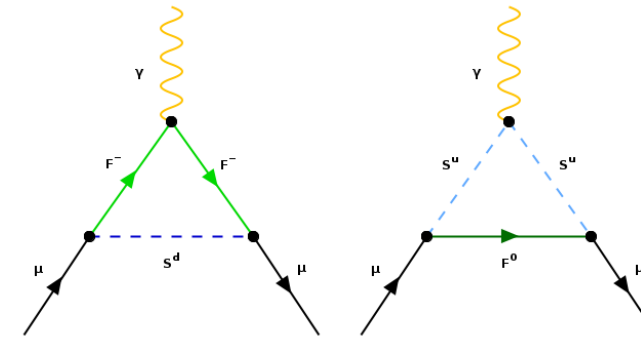
Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

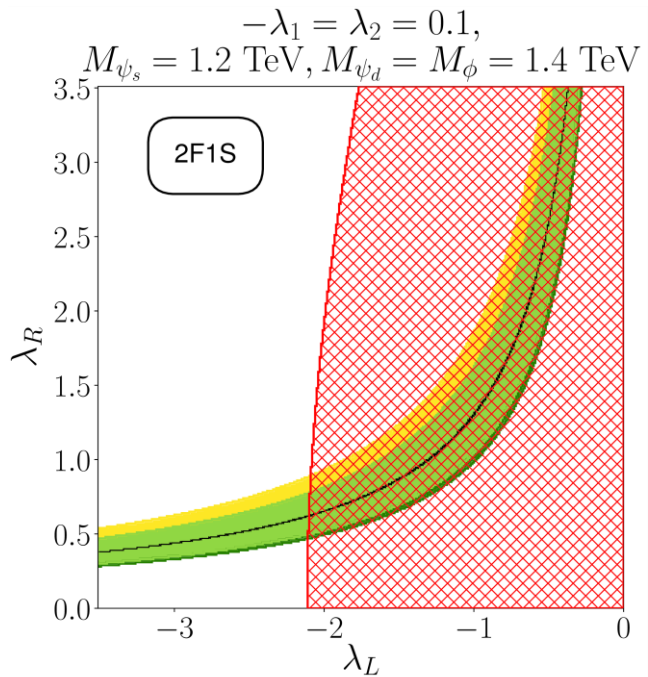
Exclusions:

~~Over Abundant~~

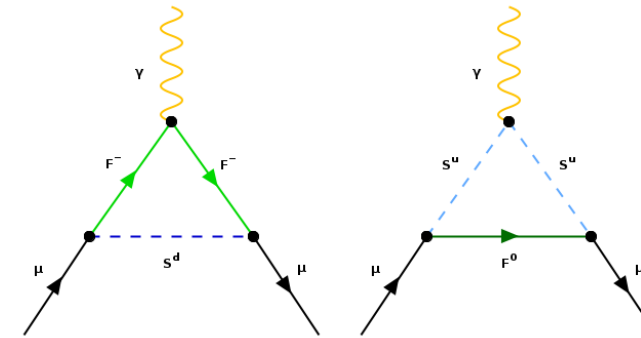
Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2




Now Ruled Out

Still Viable

Newly Viable

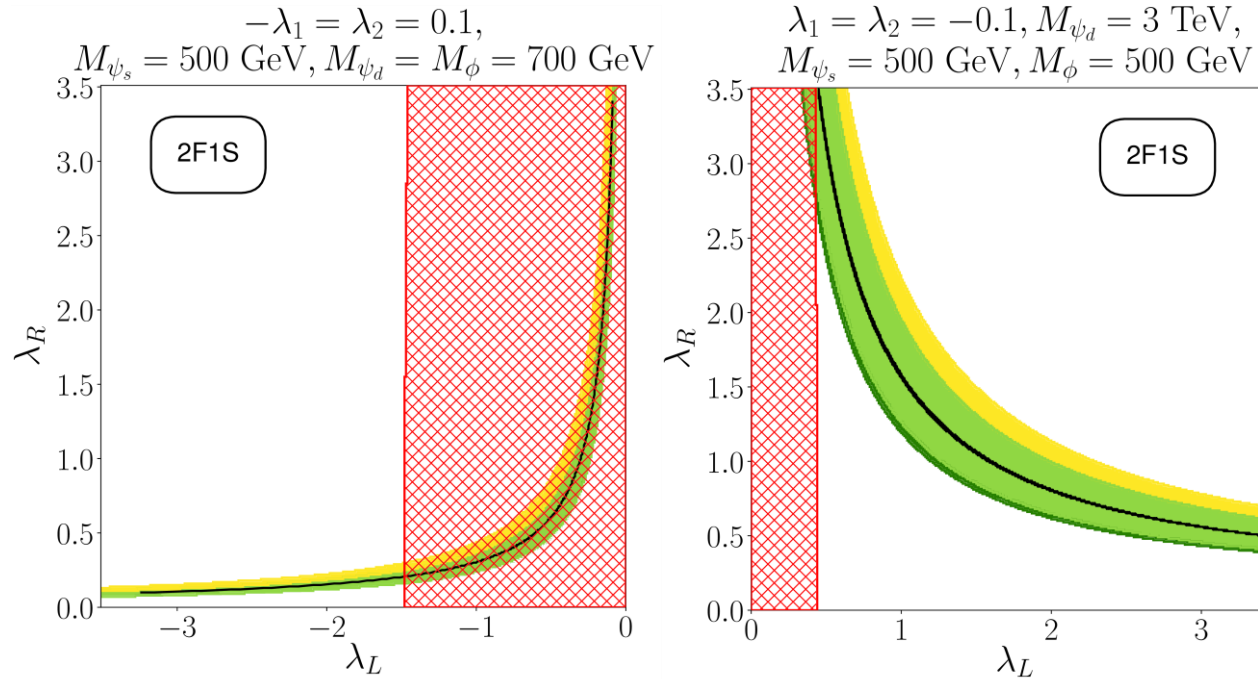
Exclusions:

 Over Abundant

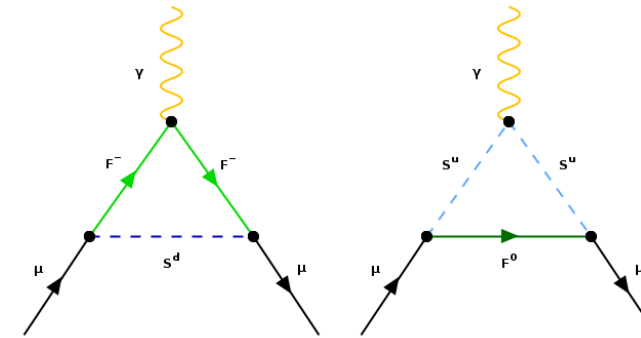
 Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2




Now Ruled Out

Still Viable

Newly Viable

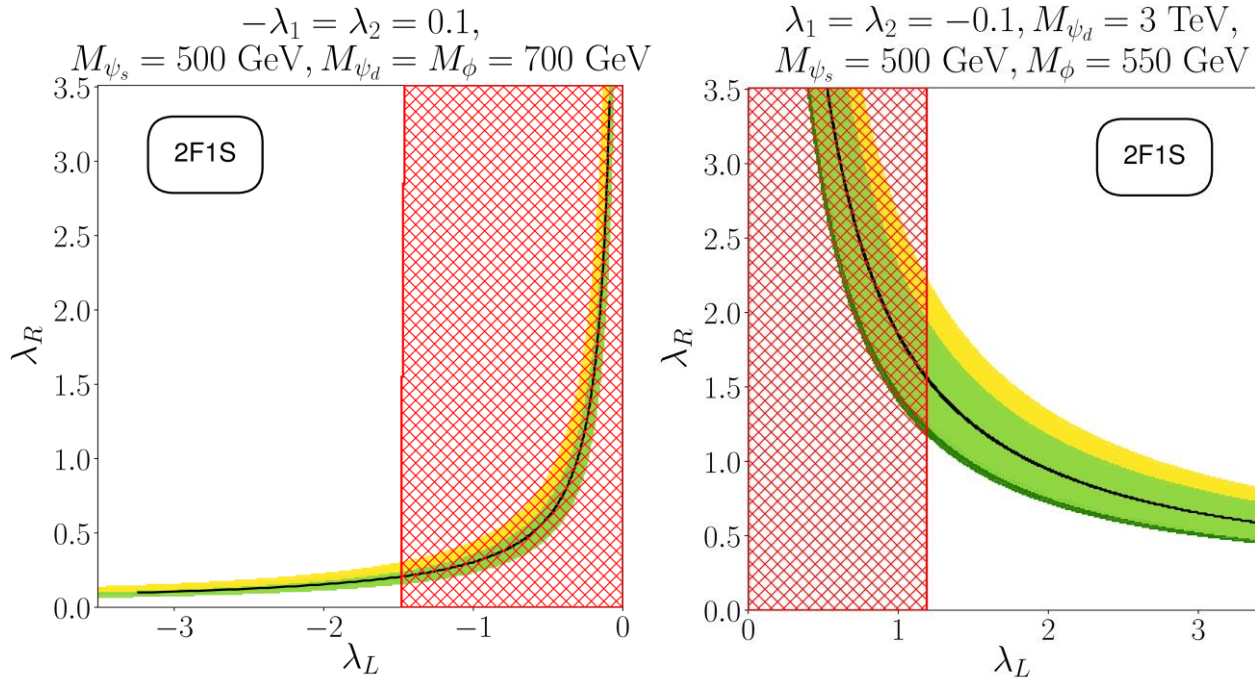
Exclusions:

 Over Abundant

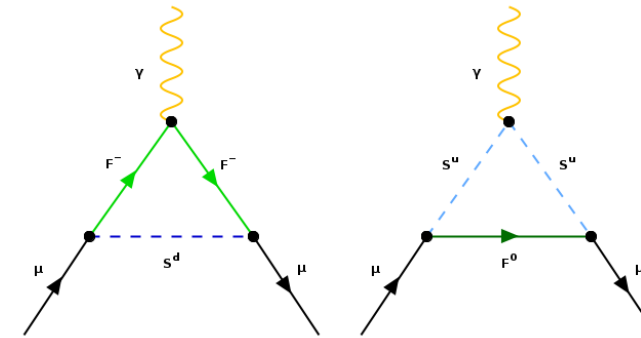
 Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2




Now Ruled Out

Still Viable

Newly Viable

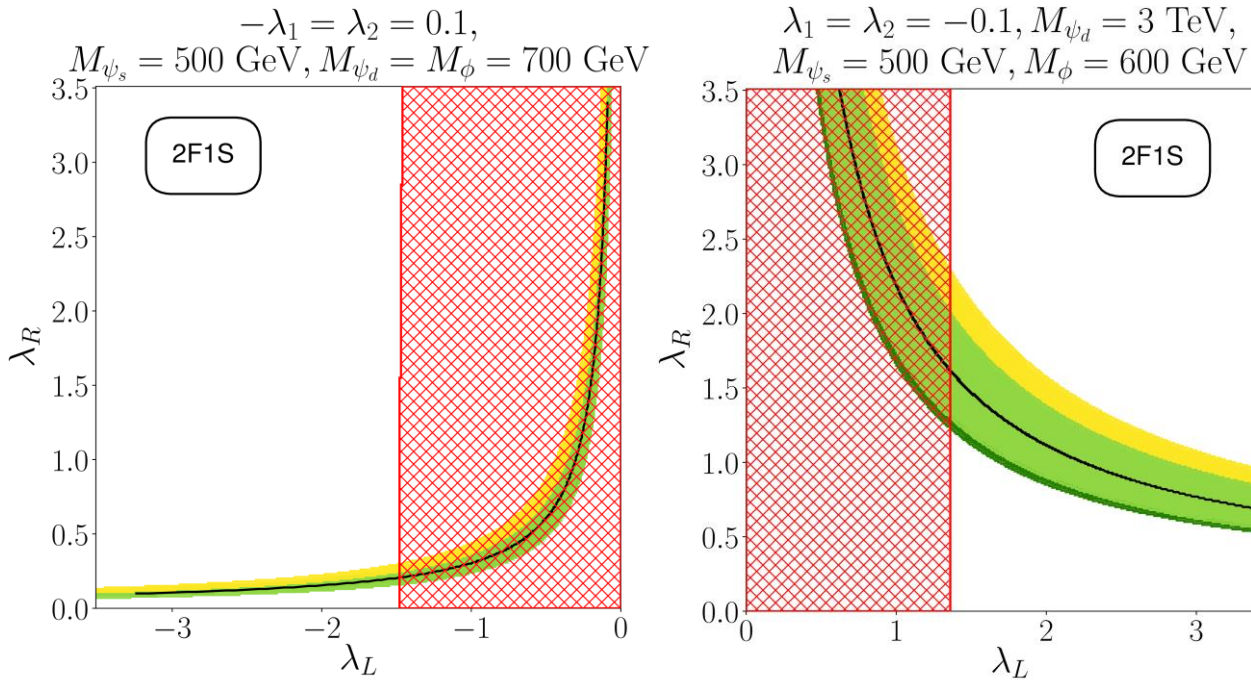
Exclusions:

 Over Abundant

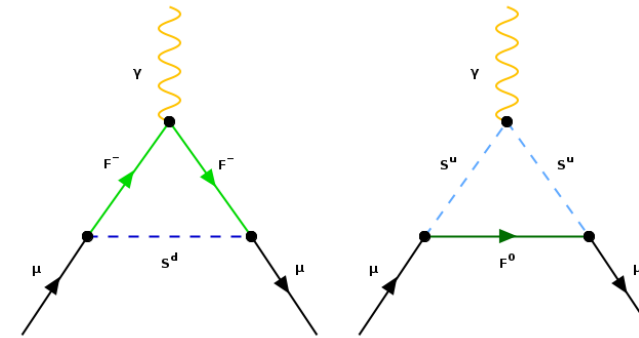
 Direct Detection

Three Fields with Dark Matter

Pair of New Fermions + Scalar



Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

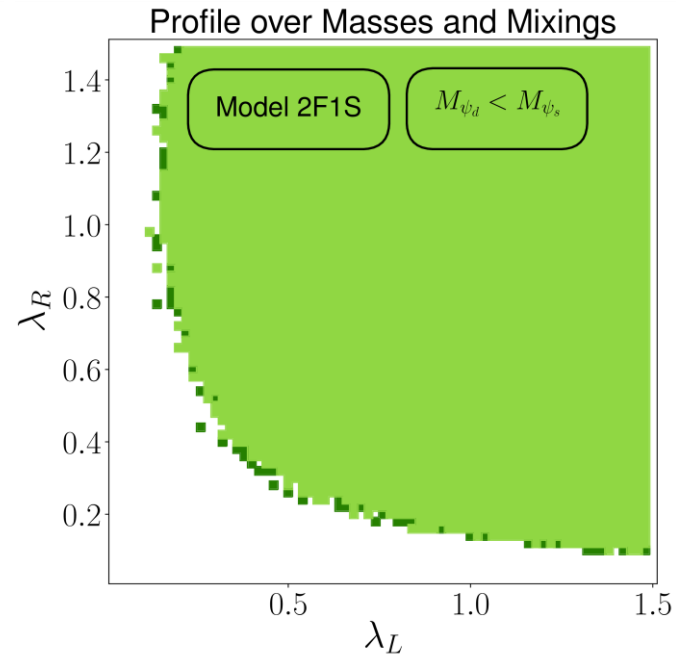
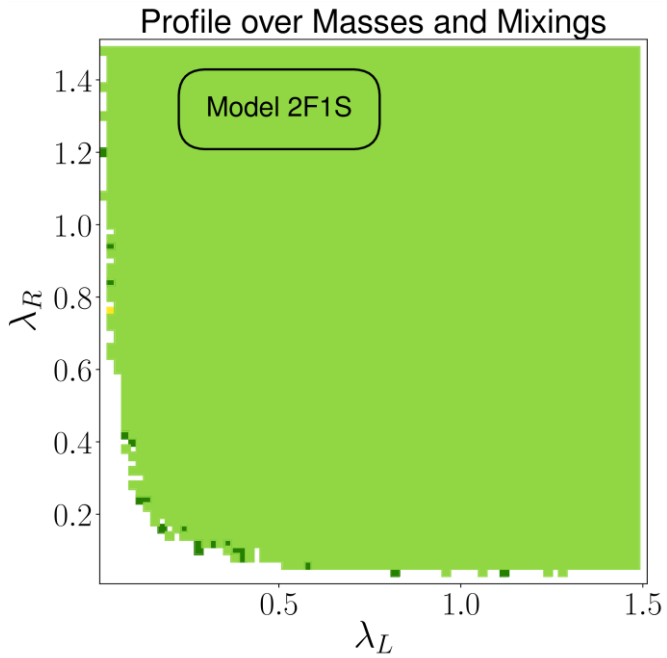
Exclusions:

~~Over Abundant~~

Direct Detection

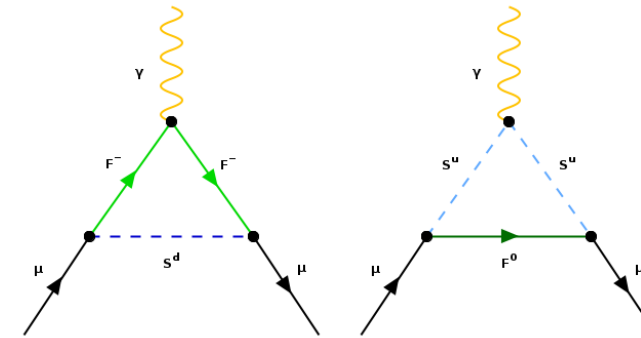
Three Fields with Dark Matter

Pair of New Fermions + Scalar



$$|\lambda_L \lambda_R| \gtrsim 0.036$$

Contributes to muon g-2



Now Ruled Out

Still Viable

Newly Viable

Exclusions:

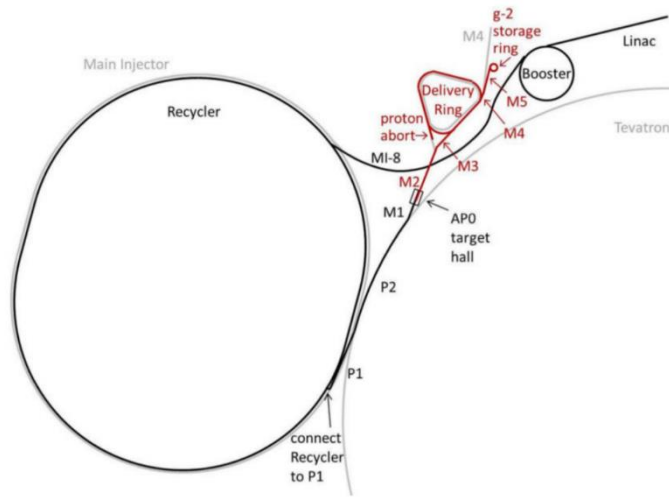
~~Over Abundant~~

Direct Detection

Future Developments

Upcoming Experiments Measurements

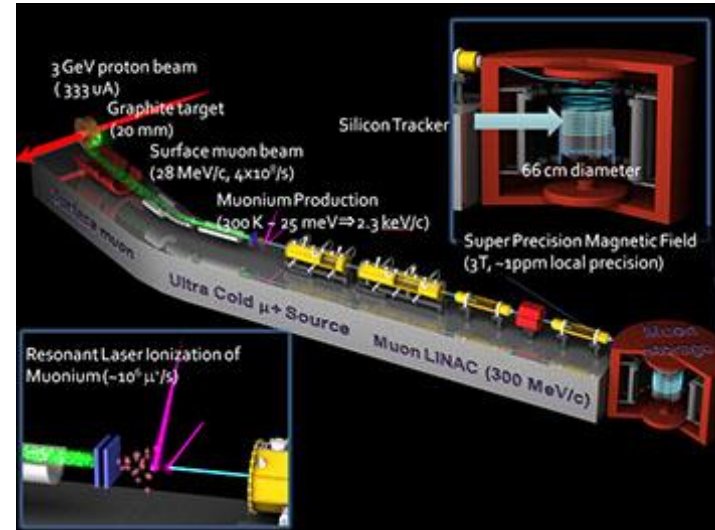
Fermilab



Run-4 now completed. Runs-5 & 6 planned.

Run-4 experimental precision: 0.14ppm

J-PARC



Upcoming.

Final experimental precision: 0.1ppm

Conclusions

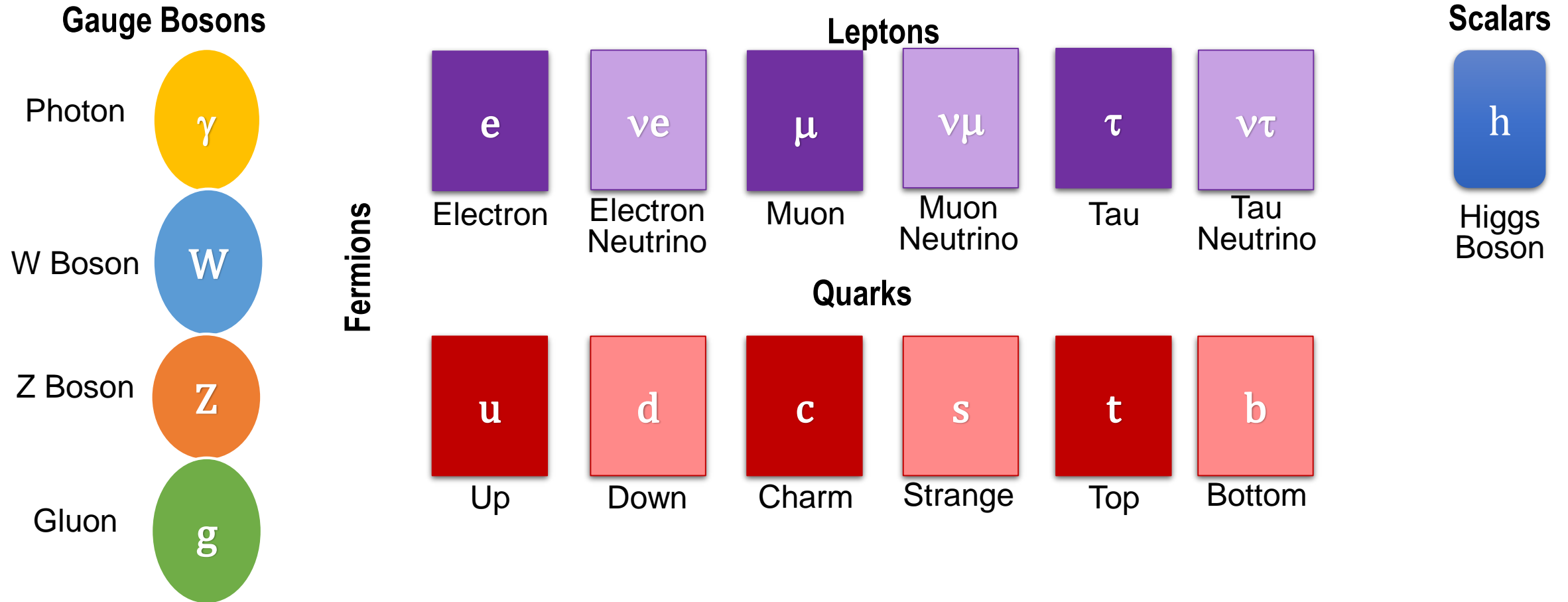
The anomalous muon magnetic moment, muon $g-2$

- **Current state of muon $g-2$**
- New muon $g-2$ value from Fermilab disagrees with SM prediction by 4.2σ .
- Many simple BSM theories cannot produce a contribution that is both positive and large.

- **Outlook**
- Upcoming muon $g-2$ experiments are set to further increase the precision, and if the measured value stays the same or increases, then disagreement between the SM and experiment will increase.

Thank you for Listening!

Standard Model of Particle Physics



Backup Slides

The sensitivity of g-2 to new particles

Coupling Strength of Muon to New Particles

The contribution to a mass m particle's g-2 from a diagram with a single loop of particles with mass of order M are proportional to

$$\Delta a \propto \frac{m^2}{M^2}$$

Since the muon is 207 times larger than the electron, contributions from new particles are of order $m_\mu^2/m_e^2 \approx 40000$ times larger to the muon g-2 than the electron g-2. So even though we can measure the electron g-2 more precisely, we expect to find evidence of new contribution in the muon g-2 first.

The colour confinement of quarks makes it difficult to measure their g-2. The short lifetimes of the tau particle and particles of similar mass make it difficult to measure their g-2 at a level of precision close to that of the muon and electron.

Single Field Extensions

Second Higgs Doublet Model (2HDM)

New Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
H_2	$(\mathbf{1}, \mathbf{2}, 1/2)$	+1,0

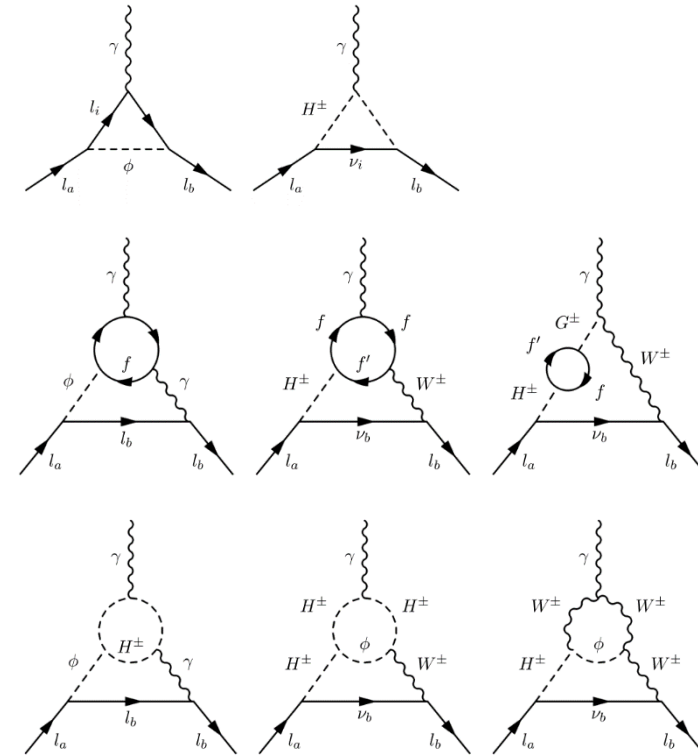
$$\Phi_1 = \begin{pmatrix} \varphi_1^\pm \\ \frac{v_1 + \varphi_1 + i\sigma_1}{\sqrt{2}} \end{pmatrix}, \Phi_2 = \begin{pmatrix} \varphi_2^\pm \\ \frac{v_2 + \varphi_2 + i\sigma_2}{\sqrt{2}} \end{pmatrix}$$

Interacts with the standard model through:

$$\mathcal{L}_{Yuk} = \frac{1}{v} \sum_{\varphi=h,H,A} \sum_f (Y_f^\varphi m_f \varphi \bar{f} P_R f + h.c.)$$

Contributes to muon g-2

H_2



Single Field Extensions

Second Higgs Doublet Model (2HDM)

New Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
H_2	$(\mathbf{1}, \mathbf{2}, 1/2)$	+1,0

$$\Phi_v = \begin{pmatrix} G^\pm \\ \frac{v + H_1 + iG^0}{\sqrt{2}} \end{pmatrix}, \Phi_\perp = \begin{pmatrix} H^\pm \\ \frac{H_2 + iA}{\sqrt{2}} \end{pmatrix}$$

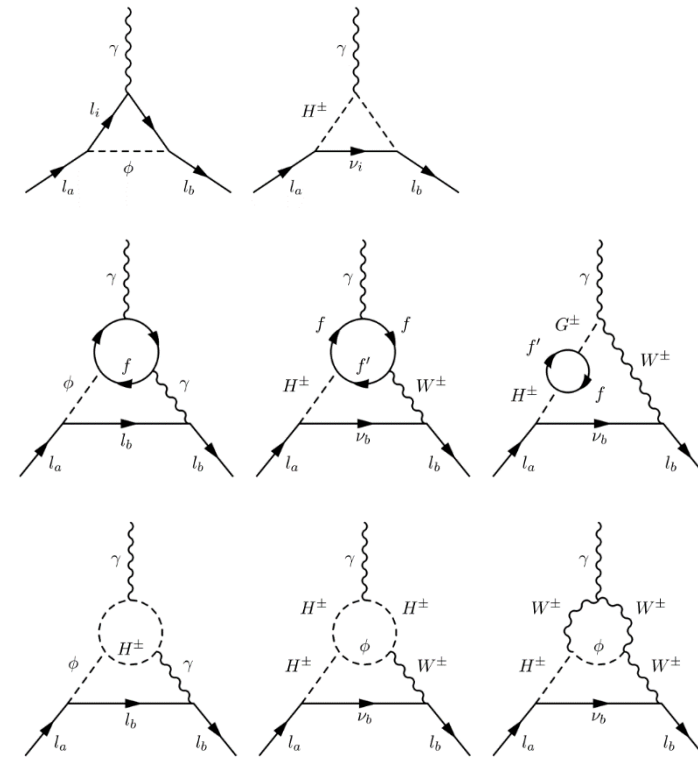
$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos(\beta - \alpha) & -\sin(\beta - \alpha) \\ \sin(\beta - \alpha) & \cos(\beta - \alpha) \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

Interacts with the standard model through:

$$\mathcal{L}_{Yuk} = \frac{1}{v} \sum_{\varphi=h,H,A} \sum_f (Y_f^\varphi m_f \varphi \bar{f} P_R f + h.c.)$$

Contributes to muon g-2

H_2



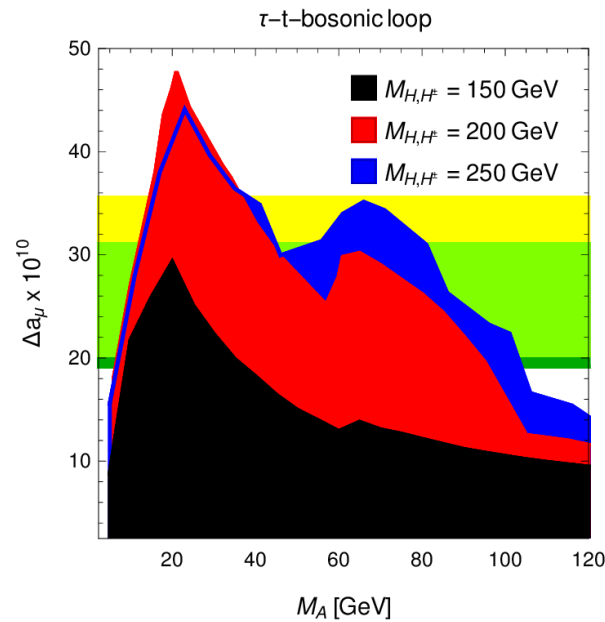
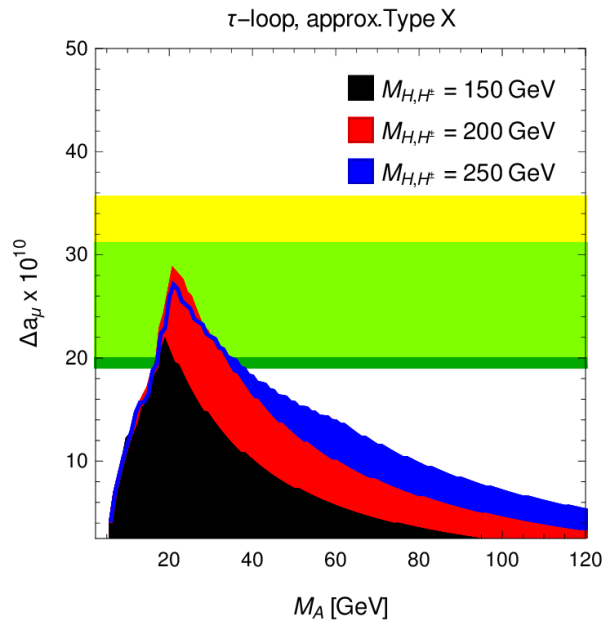
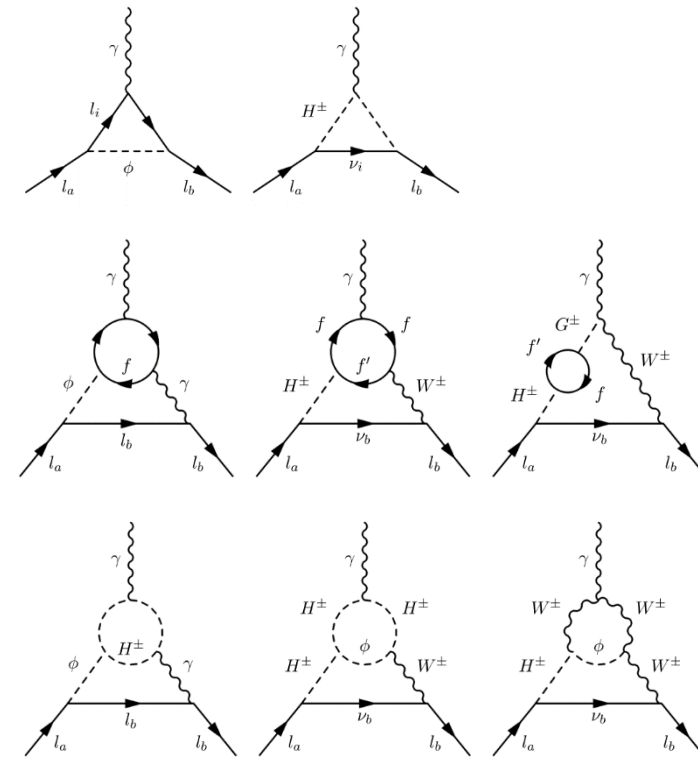
Single Field Extensions

Second Higgs Doublet Model (2HDM)

New Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Electric Charge
H_2	$(1, 2, 1/2)$	+1,0

H_2

Contributes to muon g-2



Now Ruled Out

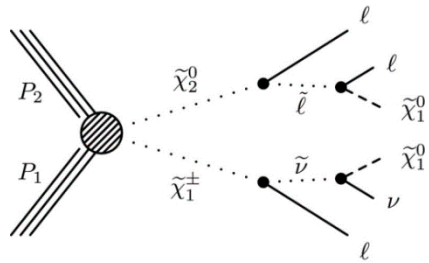
Still Viable

Newly Viable

Backup Slides

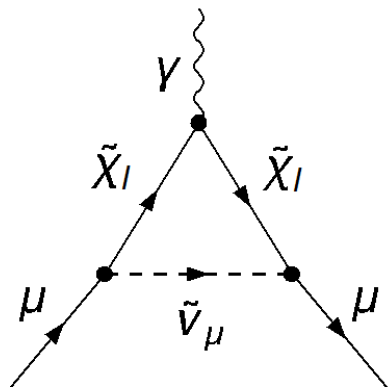
Detecting BSM Particles

Direct Observation



Direct detection of new physics through the change in momentum of some search particle, or observation through scattering.

Adjustment of Physical Observables



Discovery of new physics through contributions causing a deviation in the measured value of some physical observable compared to the standard model prediction.

The Road to the Results

Muon G-2

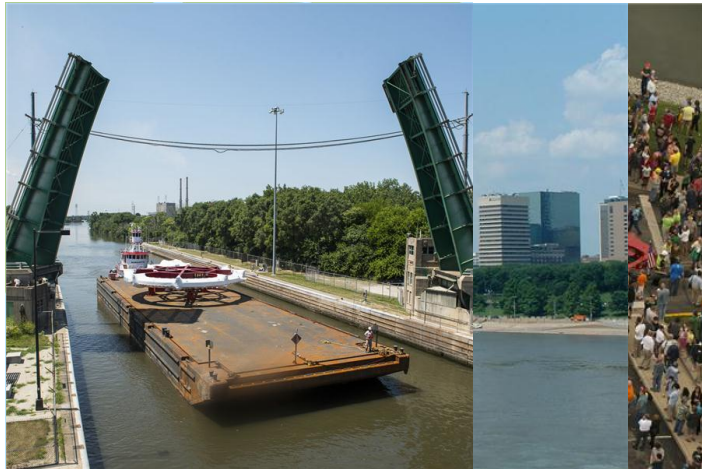
2006: Brookhaven Final Results Released

2013: Big Move

2018: Run-1 Begins

2019: Run-2 Begins

2020: Muon g-2 Theory Initiative Releases
White paper on SM prediction



Backup Slides

Goldstone Diagrams

Gauge-Fixing

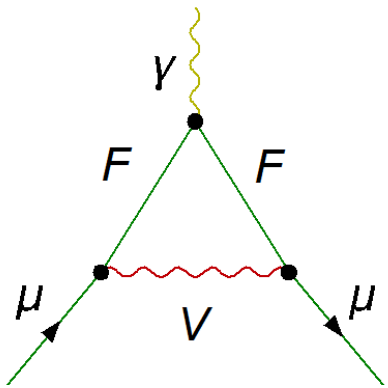
$$\begin{aligned}
 & \frac{1}{2} V_\nu (g^{\nu\lambda} \partial^2 - \partial^\nu \partial^\lambda) V_\lambda - e' v V^\nu \partial_\nu \phi_2 + \frac{1}{2\xi} (\partial_\nu V^\nu - \xi e' v \phi_2)^2 \\
 &= \frac{1}{2} V_\nu (g^{\nu\lambda} \partial^2 - (1 - 1/\xi) \partial^\nu \partial^\lambda) V_\lambda - e' v V^\nu \partial_\nu \phi_2 + e' v V^\nu \partial_\nu \phi_2 + \xi e'^2 v^2 \phi_2^2 \\
 &= \frac{1}{2} V_\nu (g^{\nu\lambda} \partial^2 - (1 - 1/\xi) \partial^\nu \partial^\lambda) V_\lambda + \xi e'^2 v^2 \phi_2^2
 \end{aligned}$$

Gauge	Vector Propagator	Goldstone Propagator
R_ξ gauge	$-i(g^{\nu\lambda} - k^\nu k^\lambda / m_V^2) / (k^2 - m_V^2) - i(k^\nu k^\lambda / m_V^2) / (k^2 - \xi m_V^2)$	$i / (k^2 - \xi m_V^2)$
Feynman gauge	$-i g^{\nu\lambda} / (k^2 - m_V^2)$	$i / (k^2 - m_V^2)$
Landau gauge	$-i(g^{\nu\lambda} - k^\nu k^\lambda / k^2) / (k^2 - m_V^2)$	i / k^2
Unitary Gauge	$-i(g^{\nu\lambda} - k^\nu k^\lambda / m_V^2) / (k^2 - m_V^2)$	0

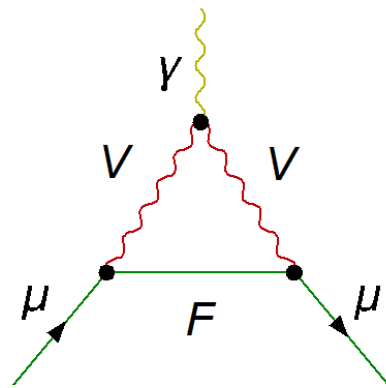
Contributions to g-2

General Contributions to g-2

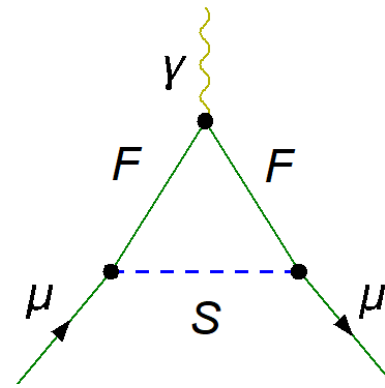
FFV Diagram



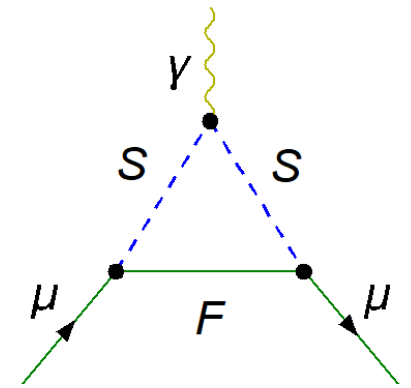
VVF Diagram



FFS Diagram



SSF Diagram



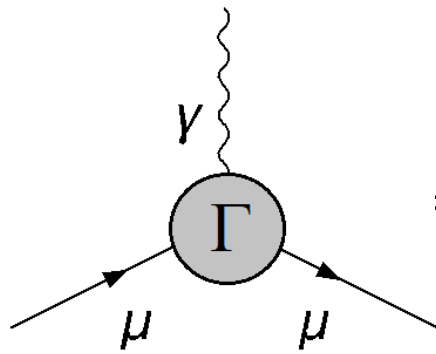
Coupling of muon to fermions/vectors:

$$\mathcal{L}_{\mu f V} = \bar{f} \gamma_\nu V^\nu (V F_L P_L + V F_R P_R) \mu + h.c$$

Coupling of muon to fermions/scalars:

$$\mathcal{L}_{\mu f \phi} = \bar{f} \phi (S F_L P_L + S F_R P_R) \mu + h.c$$

Backup Slides


$$= \bar{u}(p')ie\Gamma^\nu u(p)$$

Form Factors

Vertex Correction Function:

$$\Gamma^\nu = A \times \gamma^\nu + \delta\Gamma^\nu$$

$$\Gamma^\nu = A \times \gamma^\nu + B \times p^\nu + C \times p'^\nu$$

$$\Gamma^\nu = f_1(q^2) \times \gamma^\nu + f_2(q^2) \times (p^\nu + p'^\nu) + f_3(q^2) \times q^\nu$$

Ward Identity:
 $q_\nu \Gamma^\nu = 0$

$$\Gamma^\nu = (f_1(q^2) + 2mf_2(q^2)) \times \gamma^\nu - 2mf_2(q^2) \times i \frac{\sigma^{\nu\lambda} q_\lambda}{2m}$$

$$\Gamma^\nu = F_1 \times \gamma^\nu + F_2 \times i \frac{\sigma^{\nu\lambda} q_\lambda}{2m}$$

Gordon Identity:

$$\bar{u}(p') \frac{p^\nu + p'^\nu}{2m} u(p) = \bar{u}(p') (\gamma^\nu - i \frac{\sigma^{\nu\lambda} q_\lambda}{2m}) u(p)$$

Backup Slides

General Contributions

General 1-Loop Diagram Contributions to Muon g-2

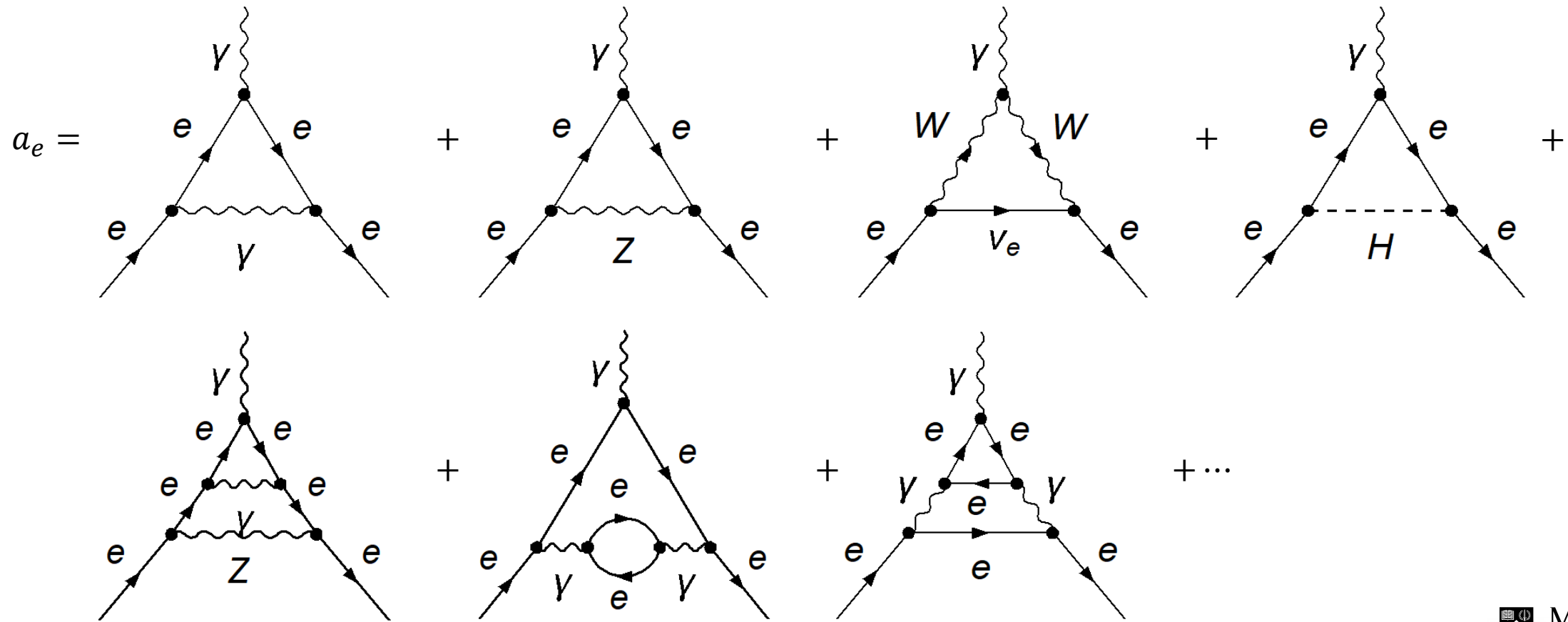
Working in the Feynman ($\xi \rightarrow 1$) Gauge:

$$\begin{aligned}
 a_{\mu}^{FFS} &= \frac{q_F m_F m_{\mu} (SF_L^2 + SF_R^2)(3 - 4x + x^2 + 2 \log x)}{16m_S^2 \pi^2 (1-x)^3} - \frac{q_F m_{\mu}^2 SF_L SF_R (2 + 3x - 6x^2 + x^3 + 6x \log x)}{24m_S^2 \pi^2 (1-x)^4} \\
 a_{\mu}^{SSF} &= \frac{q_S m_F m_{\mu} (SF_L^2 + SF_R^2)(1 - x^2 + 2x \log x)}{16m_S^2 \pi^2 (1-x)^3} - \frac{q_S m_{\mu}^2 SF_L SF_R (1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x)}{24m_S^2 \pi^2 (1-x)^4} \\
 a_{\mu}^{FFV} &= \frac{q_F m_{\mu}^2 (VF_L^2 + VF_R^2)(4 - 9x + 5x^3 + (6 - 12x) \log x)}{48m_V^2 \pi^2 (1-x)^4} - \frac{q_F m_F m_{\mu} VF_L VF_R (1 - x^2 + 2x \log x)}{4m_V^2 \pi^2 (1-x)^3} \\
 a_{\mu}^{VVF} &= \frac{q_V m_{\mu}^2 (VF_L^2 + VF_R^2)(7 - 33x + 57x^2 - 31x^3 + 6x^2(3x - 1) \log x)}{96m_V^2 \pi^2 (1-x)^4} \\
 &+ \frac{3q_V m_F m_{\mu} VF_L VF_R (1 - 4x + 3x^2 - 2x^2 \log x)}{16m_V^2 \pi^2 (1-x)^3}
 \end{aligned}$$

Backup Slides

Contributions of Feynman diagrams to $g-2$

Contributions to electron $g-2$



Backup Slides

Standard Model of Particle Physics

Standard Model Multiplets

$L_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$	e_{iR}	$Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$	u_{iR}	d_{iR}	W_ν^a	Z_ν	A_ν	$H = \begin{pmatrix} \phi^+ \\ \nu + h + \phi^0 \end{pmatrix}$
Left-handed SU(2)L lepton doublet	Right-handed SU(2)L lepton singlet	Left-handed SU(2)L quark doublet	Right-handed SU(2)L up quark singlet	Right-handed SU(2)L down quark singlet	W vector boson	Z vector boson	Photon	Higgs SU(2)L doublet

a denotes the W boson index.

i denotes family index (e.g. e, μ, τ).

L, R denote left, right-handed fermion.

Standard Model Lagrangian (Colourless)

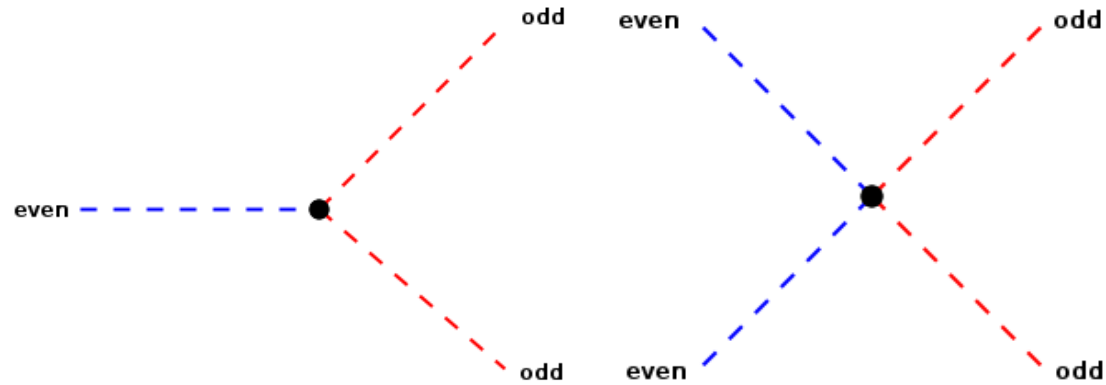
$$\begin{aligned} \mathcal{L} &= W^{a\mu\nu}W_{\mu\nu}^a + Z^{\mu\nu}Z_{\mu\nu} + A^{\mu\nu}A_{\mu\nu} + \bar{L}_{iL}(i\gamma^\nu D_\nu + m_{Li})L_{iL} + \bar{e}_{iR}(i\gamma^\nu D_\nu + m_{ei})e_{iR} \\ &+ \bar{Q}_{iL}(i\gamma^\nu D_\nu + m_{Qi})Q_{iL} + \bar{u}_{iR}(i\gamma^\nu D_\nu + m_{ui})u_{iR} + \bar{d}_{iR}(i\gamma^\nu D_\nu + m_{di})d_{iR} + |D_\nu H|^2 + V(H) \end{aligned}$$

Backup Slides

Z2 Symmetry

Z2-odd fields interact only in pairs:

$$\begin{aligned}\psi_{\text{even}} &\rightarrow \psi_{\text{even}} \\ \psi_{\text{odd}} &\rightarrow \psi_{\text{odd}} e^{i\pi}\end{aligned}$$

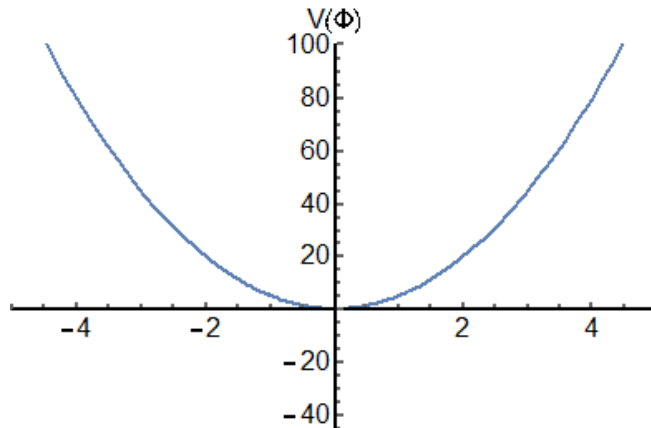


Constraints

- Muon g-2 Contributions: FlexibleSUSY
- Dark Matter Limits: MicrOmegas
- Direct Detection Limits: DDCalc
- Particle Collider Constraints: SModelS

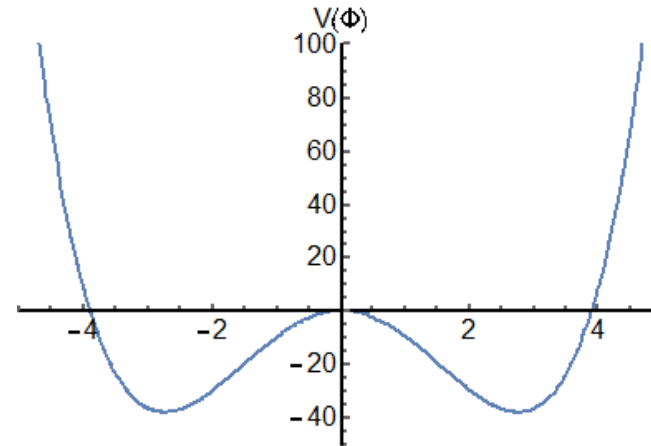
Backup Slides

Symmetry Breaking



$$\Phi \rightarrow (\phi_1 + i\phi_2)$$

$$\text{VEV: } \langle \Phi \rangle = 0$$



$$\Phi \rightarrow (v + \phi_1 + i\phi_2)$$

$$\text{VEV: } \langle \Phi \rangle = v$$

$$e'v \rightarrow m_V$$

The kinetic terms of a complex scalar singlet Φ :

$$\frac{1}{2} (|\partial_\nu \phi_2|^2 + |\partial_\nu \phi_1|^2 - e'V^\nu (\phi_1 \partial_\nu \phi_2 - \phi_2 \partial_\nu \phi_1) + e'^2 V^\nu V_\nu (\phi_1^2 + \phi_2^2))$$

$$\frac{1}{2} (|\partial_\nu \phi_2|^2 + |\partial_\nu \phi_1|^2 - e'V^\nu (\phi_1 \partial_\nu \phi_2 - \phi_2 \partial_\nu \phi_1) + e'^2 V^\nu V_\nu (\phi_1^2 + 2v\phi_1 + \phi_2^2) + e'^2 v^2 V^\nu V_\nu - e'v V^\nu \partial_\nu \phi_2)$$

Gauge-fixing term removes the mixing between vectors and Goldstones: $\frac{1}{2\xi} (\partial_\nu V^\nu - \xi e'v\phi_2)^2$

Backup Slides

Contributions to g-2

Symmetry Breaking

The kinetic terms of a complex scalar Φ :

$$\mathcal{L}_{D\Phi} = |D_\nu \Phi|^2 = \left| (\partial_\nu - ie'V_\nu) \frac{1}{\sqrt{2}} (\phi_1 + i\phi_2) \right|^2$$

$$= \frac{1}{2} \left(|\partial_\nu \phi_2|^2 + |\partial_\nu \phi_1|^2 + e'^2 V^\nu V_\nu (\phi_1^2 + \phi_2^2) - e' V^\nu (\phi_1 \partial_\nu \phi_2 - \phi_2 \partial_\nu \phi_1) \right)$$

$$\mathcal{L}_{D\Phi} = \frac{1}{2} \left(|\partial_\nu \phi_2|^2 + |\partial_\nu \phi_1|^2 + e'^2 V^\nu V_\nu (\phi_1^2 + 2v\phi_1 + \phi_2^2) + e'^2 v^2 V^\nu V_\nu - e' V^\nu (\phi_1 \partial_\nu \phi_2 - \phi_2 \partial_\nu \phi_1) - e' v V^\nu \partial_\nu \phi_2 \right)$$

$$\begin{aligned} \Phi &\rightarrow (v + \phi_1 + i\phi_2) \\ e'v &\rightarrow m_V \end{aligned}$$

We add a gauge-fixing term to our Lagrangian to remove the mixing between vectors and Goldstones:

$$\frac{1}{2\xi} (\partial_\nu V^\nu - \xi e' v \phi_2)^2$$

