Concepts in Strong-field QED

IKTP - Institutsseminar WS 2022/23

January 12th, 2023 // Uwe Hernandez Acosta

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Who am I?

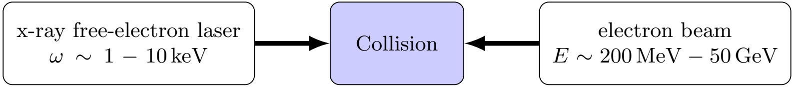
- Uwe Hernandez Acosta
- particle physicist by training
- Phd in physics 2021 from TU Dresden/HZDR
  - topic: strong-field QED
- affiliation: CASUS - Center for Advanced Systems Understanding @ HZDR
- interested in:
  - theoretical particle physics/quantum field theory
  - strong-field physics
  - event generation
  - multivariate mathematical modelling
  - Julia programming language

Contents

Part I:

Motivation

Physics use-case



Phenomena

- multi-photon scattering
- non-perturbative effects
- vacuum polarisation effects
- electromagnatic cascades

Applications

- Magnetars
- high-luminosity e^-e^+ collider
- Dirac/Weyl semi-metals
- relativistic plasma physics

Prelude:

Quantum harmonic oscillator

Prelude: the quantum harmonic oscillator (in 1D)

Hamiltonian and eigenstates

- Conventions: $\hbar = c = 1$ and $\omega = m = 1$
- Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2} + \frac{\hat{x}^2}{2}$$

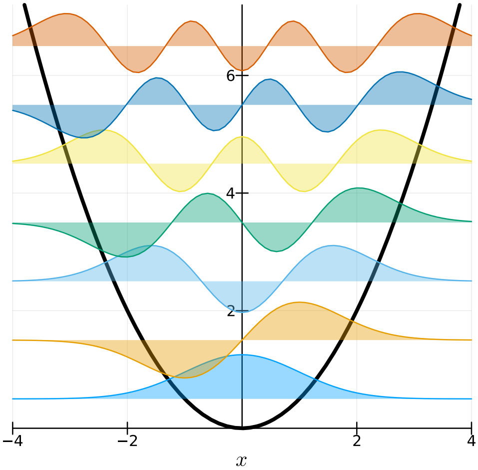
- Schrödinger's Equation

$$\hat{H}\,|\psi_n\rangle = E_n\,|\psi_n\rangle$$

- Eigenstates (position basis)

$$\psi_n(x) = \langle x\,|\,\psi_n\rangle = \frac{\pi^{-\frac{1}{4}}}{\sqrt{2^n n!}}\mathrm{e}^{-\frac{x^2}{2}}\mathcal{H}_n(x)$$

$$E_n = n + \frac{1}{2}$$



Prelude: the quantum harmonic oscillator (in 1D)

Expression with latter operators

- Annihilation and creation operators

$$\hat{a} := \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p}) \qquad \hat{a}^\dagger := \frac{1}{\sqrt{2}}(\hat{x} - i\hat{p})$$

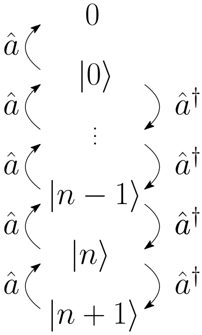
- canonical commutator

$$[\hat{x}, \hat{p}] = i\hat{\mathbb{1}} \qquad \Longleftrightarrow \qquad [\hat{a}, \hat{a}^\dagger] = \hat{\mathbb{1}}$$

- occupation number representation

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle\,, \qquad \hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$\hat{a}\,|0\rangle = 0, \qquad |n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n}}\,|0\rangle$$



Prelude: the quantum harmonic oscillator (in 1D)

- Heisenberg's uncertainty principle

$$\Delta x[\psi]\Delta p[\psi] \geq \frac{1}{2}$$

- operator variance

$$\Delta\mathcal{O}[\psi] = \sqrt{\langle\psi|\hat{\mathcal{O}}^2|\psi\rangle - \langle\psi|\hat{\mathcal{O}}|\psi\rangle^2}$$

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- Which state reaches minimal uncertainty?

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- Which state reaches minimal uncertainty?
  $\rightarrow$ The ground state $|0\rangle$!
- But ist this a unique property?
  $\rightarrow$ No!

Prelude: the quantum harmonic oscillator (in 1D)
Coherent states [Glauber. Phys. Rev. 131.6 (1963): 2766.]

- Coherent state (or Glauber state):

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle\,, \qquad \text{where } \alpha \in \mathbb{C}$$

- unitary displacement operator:

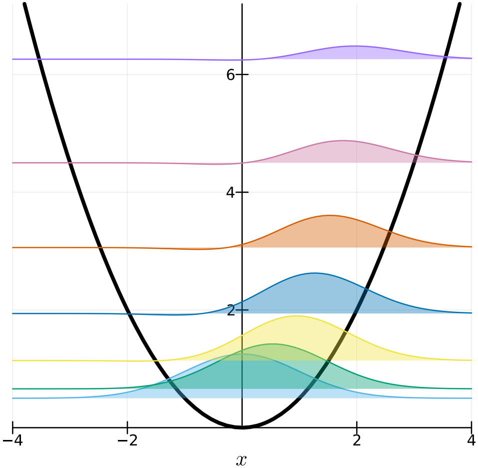
$$\hat{D}_\alpha = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}) \quad \Rightarrow \quad |\alpha\rangle = \hat{D}_\alpha\,|0\rangle$$

- minimal balanced uncertainty

$$\Delta x[\alpha] = \Delta p[\alpha] = \frac{1}{\sqrt{2}}$$

$\Longrightarrow$ "Most classical-like states."

[Schrödinger. Naturwissenschaften 14.28 (1926): 664-666]



Part II:

QED and coherent states

Quantum electrodynamics
The photon field [Itzykson, Zuber. Courier Corporation, 2012.]

- Lagrangian

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \overline{\psi}(ie\gamma^\mu \mathcal{A}_\mu)\psi - \frac{1}{2\xi}(\partial^\mu \mathcal{A}_\mu)^2$$

- quantised photon field: Gupta-Bleuler $\langle \partial^\mu \hat{\mathcal{A}}_\mu \rangle = 0$

$$\hat{\mathcal{A}}^\mu(x) = \sum_{\lambda=1}^{2} \int \frac{d^3k}{(2\pi)^3 2k^0} \left(\epsilon^\mu_\lambda(k)\hat{a}_\lambda(k)\mathrm{e}^{-ikx} + \epsilon^{*\mu}_\lambda(k)\hat{a}^\dagger_\lambda(k)\mathrm{e}^{ikx} \right) = \hat{\mathcal{A}}^\mu_{(+)}(x) + \hat{\mathcal{A}}^\mu_{(-)}(x)$$

- base polarisations

$$\epsilon^\mu_\lambda k_\mu = 0, \qquad \epsilon^\mu_\lambda \epsilon_{\tau\mu} = -\delta_{\lambda\tau}$$

- Ladder operators

$$\left[\hat{a}_\lambda(k), \hat{a}^\dagger_\tau(k')\right] = -(2\pi)^3 2k^0 g_{\lambda\tau}\delta^{(3)}(k-k')$$

Quantum electrodynamics

Semi-coherent states [Fradkin, Gitman, Shvartsman. Springer-Verlag, 1991.]

- multi-mode coherent state

$$\hat{A}^{\mu}_{(+)}(x)\,|C\rangle = C^{\mu}(x)\,|C\rangle$$

- Displacement operator

$$\hat{D}_C = \exp\left\{\sum_{\lambda=1}^{2}\int \frac{d^3k}{(2\pi)^3 2k^0}\left[C_\lambda(k)\hat{a}^{\dagger}_{\lambda}(k) - C^{*}_{\lambda}(k)\hat{a}_{\lambda}(k)\right]\right\} \qquad \Longrightarrow \qquad |C\rangle = \hat{D}_C\,|0\rangle$$

$\Rightarrow$ $\hat{D}_C$ "shifts" the photon field

$$\hat{D}^{\dagger}_{C}\hat{A}^{\mu}\hat{D}_C = \hat{A}^{\mu} + \underbrace{\left(C^{\mu}(x) + C^{*\mu}(x)\right)}_{:=A^{\mu}(x)}\hat{\mathbb{1}}$$

- semi-coherent state

$$|N;C\rangle = \hat{D}_C\,|N\rangle \qquad \text{with a Fock state } |N\rangle = |n_1,\dots,n_N\rangle$$

Quantum electrodynamics
Background field

$\Rightarrow$ "classical part of the photon field"

$$\frac{\langle N;C|\hat{A}^\mu(x)|N;C\rangle}{\langle N|N\rangle} = C^\mu(x) + C^{*\mu}(x) =: A^\mu(x)$$

with

$$\partial_\mu A^\mu(x) = 0 \qquad \text{(classical Lorenz gauge)}$$
$$\partial^\mu \partial_\mu A^\nu(x) = 0 \qquad \text{(Maxwell's equation)}$$

$\Rightarrow$ photon number on top of the classical field

$$\frac{\langle N;C|\hat{n}_i|N;C\rangle}{\langle N|N\rangle} = n_i + |C_{\lambda_i}(k_i)|^2$$

$\Rightarrow$ $A^\mu(x)$ is called background field

Quantum electrodynamics
Interaction and Furry picture

- Scattering matrix with semi-coherent states [Kibble. Phys. Rev. 138, B740 (1965)] [Frantz. Phys. Rev. 139, B1326 (1965).]

$$\left\langle f;C\middle|\hat{S}\left[\hat{\psi},\hat{\bar{\psi}},\hat{\mathcal{A}}^\mu\right]\middle|i;C\right\rangle = \left\langle f\middle|\hat{D}_C^\dagger\hat{S}\left[\hat{\psi},\hat{\bar{\psi}},\hat{\mathcal{A}}^\mu\right]\hat{D}_C\middle|i\right\rangle = \left\langle f\middle|\hat{S}\left[\hat{\psi},\hat{\bar{\psi}},\hat{\mathcal{A}}^\mu + A^\mu\hat{\mathbb{1}}\right]\middle|i\right\rangle$$

- Interaction picture with classical background: the Furry picture [Furry. Phys. Rev. 81, 115 (1951).]

$$\begin{aligned} & \mathcal{A}^\mu && \rightarrow && \mathcal{A}^\mu + A^\mu \\ \Rightarrow \quad & (i\gamma^\mu\partial_\mu - m)\psi = 0 && \rightarrow && (i\gamma^\mu\partial_\mu + ie\gamma^\mu A_\mu - m)\psi_A = 0 \end{aligned}$$

- Analytical solutions for ψ_A only exist for special cases of $A^\mu(x)$

  → e.g. $A^\mu(x) = A^\mu(\phi := kx)$ with $k^2 = 0$

  [Volkov. Z. Phys. 94, 250 (1935)] [Berestetzki, Lifschitz, Pitajewski. Akademie Verlag (1980).]

time for a breathing spell

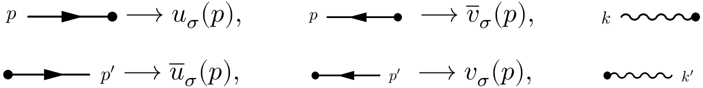
Part III:

Feynman rules and applications

Feynman rules in a background field

[UHA. PhD thesis (2021)] [Meuren,Keitel,Di Piazza. Phys. Rev. D 88, 013007 (2013)] [Mitter. Acta Phys. Austriaca Suppl. 14, 397–498 (1975)]

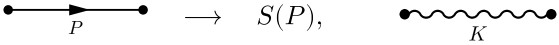
- external legs



$$p \longrightarrow u_\sigma(p), \quad p \longrightarrow \overline{v}_\sigma(p), \quad k \longrightarrow \epsilon_\lambda(k),$$

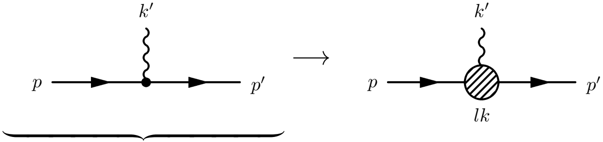
$$p' \longrightarrow \overline{u}_\sigma(p), \quad p' \longrightarrow v_\sigma(p), \quad k' \longrightarrow \epsilon^{*\mu}(k)$$

- Propagators



$$P \longrightarrow S(P), \quad K \longrightarrow D_{\mu\nu}(K)$$

- (dressed) vertex



$$= -ie\Gamma^\mu(l)(2\pi)^3\delta^{(4)}(p + lk - p' - k')$$

$$-ie\gamma^\mu(2\pi)^4\delta^{(4)}(p+p'-k')$$

The dressed vertex [UHA. PhD thesis (2021)]

- vertex function

$$\Gamma^\mu(l,p,p',k) = \pi\mathcal{G}\delta(l)\gamma^\mu + \left[\Gamma_1^{\mu\nu} - \mathcal{P}\frac{\gamma^\mu\alpha^\nu}{l}\right] B_{1\nu}(l) + \left[\Gamma_2^{\mu} - \mathcal{P}\frac{\gamma^\mu\alpha_2}{l}\right] B_2(l)$$

- elementary vertices

$$\Gamma_1^{\mu\nu}(p,p',k) := e\left(\frac{\gamma^\mu \not{k}\gamma^\nu}{2(kp)} + \frac{\gamma^\nu \not{k}\gamma^\mu}{2(kp')}\right), \qquad \Gamma_2^{\mu}(p,p',k) := -e^2\frac{\not{k}}{2(kp)(kp')}k^\mu$$

- Phase integrals

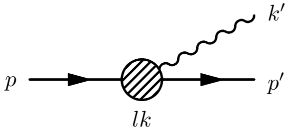
$$\left.\begin{matrix} B_1^\mu(l) \\ B_2(l) \end{matrix}\right\} = \int_{-\infty}^{\infty} \mathrm{d}\phi\ \exp(il\phi + iG(\phi)) \left\{\begin{matrix} A^\mu(\phi) \\ A^\mu(\phi)A_\mu(\phi) \end{matrix}\right.$$

- non-linear Volkov phase

$$G(\phi) = \alpha_1^\mu \int_{-\infty}^{\phi} A_\mu(\phi')\mathrm{d}\phi' + \alpha_2 \int_{-\infty}^{\phi} A_\mu(\phi')A^\mu(\phi')\mathrm{d}\phi'$$

Radiating electron in background field

- Feynman diagram (tree level)



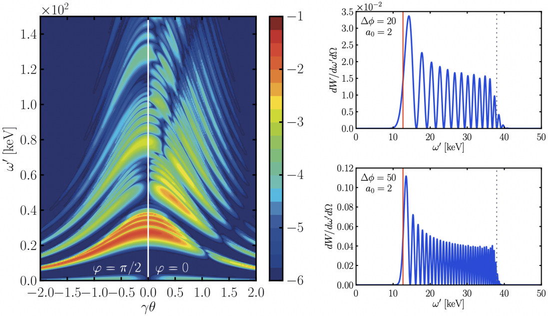
- Matrix element

$$S_{\mathsf{sfC}} = -ie \int \frac{\mathrm{d}l}{2\pi} \overline{u}(p')\Gamma^{\mu}(l)\epsilon'^{*}_{\mu}(k')u(p) \times \delta^{(4)}(p + lk - p' - k'),$$

- differential emission probability

$$\frac{\mathrm{d}W}{\mathrm{d}\omega'\mathrm{d}\Omega} = \frac{e^2\omega'}{64\pi^3(kp)(kp')}\frac{1}{2}\sum|M|^2$$

Pulsed optical laser (1.44 eV) vs e^- (40 MeV)



[Seipt. arXiv:1701.03692v1 (2017)]

"Decay of a photon" in a laser field

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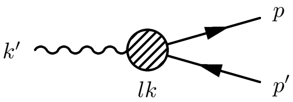
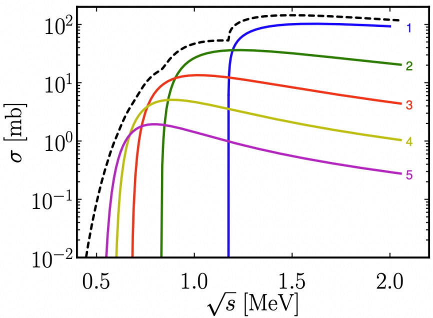
- Feynman diagram (tree level)



- infinitely extended laser field
  $a_0 = 0.8$, $s = (k + k')^2$



- Matrix element

$$S_{\mathsf{sfBW}} = -ie \int \frac{\mathrm{d}l}{2\pi} \overline{u}(p')\Gamma^{\mu}(l)\epsilon_{\mu}(k')v(p) \times \delta^{(4)}(k' + lk - p' - p),$$

- differential emission probability

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}E'\mathrm{d}\Omega} = \frac{1}{I_L}\frac{e^2E'}{64\pi^3(kp)(k'p)}\frac{1}{2}\sum|M|^2$$

[Nousch, Seipt, Kämpfer, Titov PLB 715 (2012) 246–250]

Highlights and open questions

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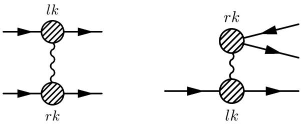
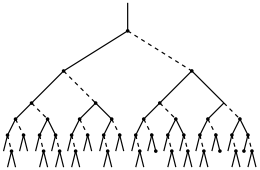
Higher multiplicity



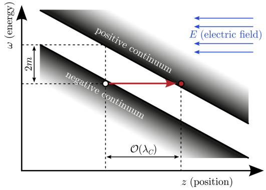
- Higher order phase integration
- New sampling methods
- Factorisation ?

Laser driven cascades



- Self amplification ?
- Bohr's conjecture ???

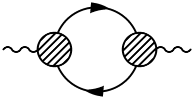
Schwinger pair production



- Critical field $E_{\text{cr}} = \frac{m^2}{e}$
- Observability ???

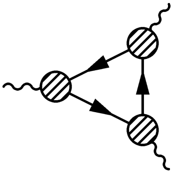
Highlights and open questions

Loop corrections



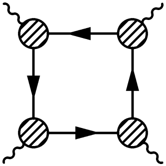
- General renormalisation theory ?
- Radiation reaction ?
- Resummation ?

Photon splitting



- Frequency shift
- Double-slit vacuum polarisation ??

Photon-photon scattering



- Rate enhancement ?
- Vacuum birefringence ???