Supplemental Material for
Negative longitudinal magnetoconductance at weak fields in Weyl semimetals

Andy Knoll, Carsten Timm, and Tobias Meng

I. SEMICLASSICAL LIMIT

In this section, we derive the condition for the magnetic field range in which in the semiclassical limit is justified. For a magnetic field along the k_z-direction, the dispersion of the m-th Landau level of positive energy with $m > 0$ is given by

$$\epsilon_m(k_z) = v_F\sqrt{2eBm + (v_F k_z)^2}. \tag{S1}$$

The number of occupied Landau levels must be large in the semiclassical limit, $n \gg 1$. Equivalently, the energy splitting

$$\Delta\epsilon(B) \equiv v_F\sqrt{2eB(n+1)} - v_F\sqrt{2eBn} \tag{S2}$$

between the last occupied and the first unoccupied Landau level for $k_z = 0$ should be small compared to the chemical potential,

$$\Delta\epsilon(B) \ll \mu. \tag{S3}$$

The energy splitting can be estimated as

$$\Delta\epsilon(B) = v_F\sqrt{2eBn}\left(\sqrt{1+\frac{1}{n}} - 1\right) \cong v_F\sqrt{2eBn}\left(1 + \frac{1}{2n} - 1\right) = v_F\sqrt{\frac{eB}{2n}}. \tag{S4}$$

Since n is the index of the last occupied Landau level, we have

$$v_F\sqrt{2eBn} < \mu < v_F\sqrt{2eB(n+1)}. \tag{S5}$$

Using Eq. (S5), we obtain

$$n = \left\lfloor \frac{1}{2eB}\left(\frac{\mu}{v_F}\right)^2 \right\rfloor, \tag{S6}$$

where $\lfloor x \rfloor$ is the largest integer smaller or equal to x. By combining Eqs. (S3), (S4), and (S6), we obtain the condition

$$B \ll \frac{1}{e}\left(\frac{\mu}{v_F}\right)^2 \tag{S7}$$

for the magnetic field to be considered weak and the semiclassical approximation to be valid.

II. DETERMINATION OF THE COEFFICIENTS λ^χ AND δ^χ

The ansatz for the vector mean free path given in the main text,

$$\Lambda^\chi_\mu(\theta) = -\tau^\chi_\mu(\theta)\left(-h^\chi_\mu(\theta) + \lambda^\chi + \chi\delta^\chi\cos\theta\right), \tag{S8}$$

contains the four real coefficients λ^χ and δ^χ. Recall that $\chi = \pm$ denotes the chirality of the Weyl node. In this section, we present details on their determination, as there is a subtlety. We have to solve the equation

$$h^\chi_\mu(\theta) - \frac{\Lambda^\chi_\mu(\theta)}{\tau^\chi_\mu(\theta)} = -\sum_{\chi'}\frac{n}{4\pi}\int d\theta'\sin\theta'\,\frac{(k^{\chi'})^3}{|\boldsymbol{v}^{\chi'}_{\boldsymbol{k}'}\cdot\boldsymbol{k}'|}\,D^{\chi'}(\boldsymbol{k}')\left|V^{\chi\chi'}\right|^2(1+\chi\chi'\cos\theta\cos\theta')\,\Lambda^{\chi'}_\mu(\theta'). \tag{S9}$$

By inserting Eq. (S8), we obtain a system of equations for the four coefficients λ^+, λ^-, δ^+, and δ^-:

$$\begin{pmatrix} R_1^+ \\ R_1^- \\ R_2^+ \\ R_2^- \end{pmatrix} = \begin{pmatrix} C_1^{++}-1 & C_1^{+-} & C_2^{++} & C_2^{+-} \\ C_1^{-+} & C_1^{--}-1 & C_2^{-+} & C_2^{--} \\ C_2^{++} & C_2^{+-} & C_3^{++}-1 & C_3^{+-} \\ C_2^{-+} & C_2^{--} & C_3^{-+} & C_3^{--}-1 \end{pmatrix} \begin{pmatrix} \lambda^+ \\ \lambda^- \\ \delta^+ \\ \delta^- \end{pmatrix}, \tag{S10}$$

where

$$p^{\chi\chi'}(\theta) = \frac{n}{4\pi} \sin\theta \, \frac{(k^{\chi'})^3}{|\boldsymbol{v}_{\boldsymbol{k}}^{\chi'} \cdot \boldsymbol{k}'|} D^{\chi'}(\boldsymbol{k}) \left|V^{\chi\chi'}\right|^2, \tag{S11}$$

$$R_1^\chi = \sum_{\chi'} \int d\theta' \, p^{\chi\chi'}(\theta') \, h_\mu^{\chi'}(\theta'), \tag{S12}$$

$$R_2^\chi = \sum_{\chi'} \int d\theta' \, p^{\chi\chi'}(\theta') \, \chi' \cos\theta' \, h_\mu^{\chi'}(\theta'), \tag{S13}$$

$$C_1^{\chi\chi'} = \int d\theta' \, p^{\chi\chi'}(\theta'), \tag{S14}$$

$$C_2^{\chi\chi'} = \int d\theta' \, p^{\chi\chi'}(\theta') \, \chi' \cos\theta', \tag{S15}$$

$$C_3^{\chi\chi'} = \int d\theta' \, p^{\chi\chi'}(\theta') \, \cos^2\theta'. \tag{S16}$$

Explicit evaluation shows that the coefficient matrix in Eq. (S10) has rank 3. Consequently, it has a one-parameter family of solutions. The origin of this apparent arbitrariness is that the solution of the *linearized* Boltzmann equation and hence of Eq. (S9) is only determined up to a constant: if $\Lambda_\mu^\chi(\theta)$ solves Eq. (S9) then $\Lambda_\mu^\chi(\theta) + c$ with c an arbitrary constant does so as well. The physical solution is found by imposing electron-number conservation,

$$\sum_{\chi,\boldsymbol{k}} g_{\boldsymbol{k}}^\chi = 0. \tag{S17}$$

By solving Eqs. (S9) and (S17) simultaneously we obtain the results given in the main text. For completeness, the coefficients λ^χ and δ^χ are plotted in Fig. S1 as functions of $\alpha = eBv_F^2/2\mu^2$ for the dotted curves in Fig. 2 ($V_{\text{inter}} = V_{\text{intra}}/2$) of the main text. For the solid curves in Fig. 2 ($V_{\text{inter}} = V_{\text{intra}}$) the four coefficients λ^χ and δ^χ vanish.

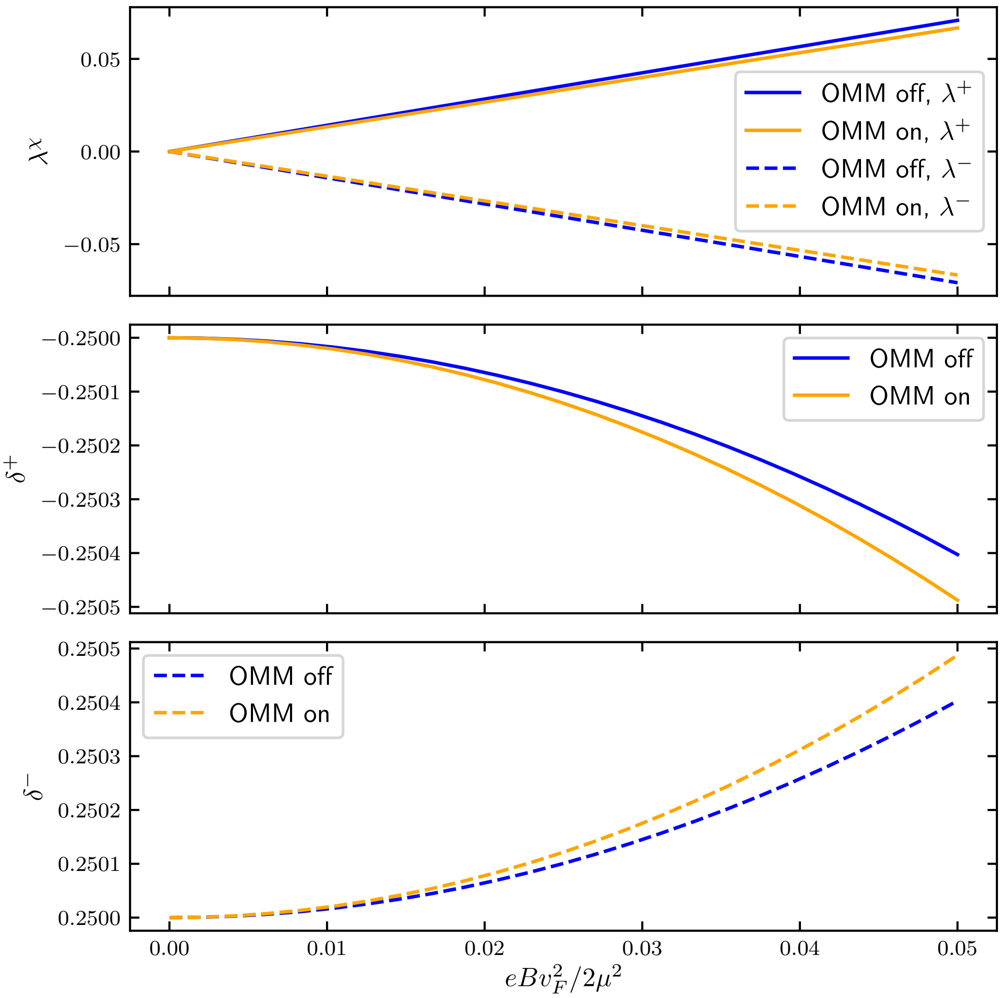

Figure S1. Coefficients λ^χ and δ^χ in the presence and the absence of the OMM for $V_{\rm inter} = V_{\rm intra}/2$.