Possible magnetic-field-induced voltage and thermopower in diluted magnetic semiconductors

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In diluted magnetic semiconductors, the carrier concentration and the magnetization of local moments are strongly coupled, since the magnetic interaction is mediated by the carriers. It is predicted that this coupling leads to an electric polarization due to an applied magnetic-field gradient and to the appearance of a magnetic-field-dependent voltage. An expression for this voltage is derived within Landau theory and its magnitude is estimated for (Ga,Mn)As. Furthermore, a large contribution to the thermopower based on the same mechanism is predicted. The role of fluctuations is also discussed. These predictions hold both if the magnetization is uniform and if it shows stripelike modulations, which are possible at lower temperatures.

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I. INTRODUCTION

Among magnetically ordered materials, diluted magnetic semiconductors[1–3] (DMS) are unique in the possibility of tuning the interaction between local moments *in situ*. Several groups[4–6] have demonstrated that the magnetization and Curie temperature T_c in a DMS can be changed by applying a gate voltage in a field-effect-transistor geometry. The gate potential affects the concentration of carriers, usually holes, in the DMS sample, which leads to a change of the magnetic interactions, since these are mainly mediated by the carriers. A strong dependence of the magnetization on carrier concentration is also seen in experiments on series of samples with different dopand concentrations.[7–9]

On a microscopic level, each local moment magnetically polarizes the carriers in its vicinity due to the exchange interaction between carriers and localized (usually *d*-shell) electrons of impurity ions. The other local moments are affected by these carrier spin polarizations, leading to an effective magnetic interaction between local moments. This is the essence of the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction.[10] It is here complicated by band-structure effects and spin-orbit coupling in *p*-type DMS,[11–14] screening due to disorder,[15] the small Fermi energy compared to the effective Zeeman splitting,[16] and competing short-range interactions. The RKKY-type interaction depends on carrier concentration essentially through the density of states at the Fermi energy. This dependence can be expanded to linear order in the concentration, in agreement with experimental observations.

The present author has recently suggested that this coupling may stabilize an equilibrium state in DMS with periodic but strongly anharmonic modulations of carrier concentration and magnetization below a certain temperature $T^* < T_c$.[17] The temperature T^* strongly depends on $\eta \equiv \partial T_c / \partial n$, the relative change of T_c with carrier concentration. In Landau theory we find $T_c - T^* \propto 1/\eta^2$. While some experimental results[8] are consistent with a small value of $T_c - T^* \sim 10$ K, which would make the effect observable, it is not clear whether this value is typical for DMS. In the present paper we put the main emphasis on the uniform phase. While these results apply to the case of weak spin polarization of the carriers, valid sufficiently close to T_c, Korenblit[18,19] had earlier studied the case of a half-metallic ferromagnet, i.e., complete spin polarization of the holes. Interestingly, this system also shows an instability toward a modulated state, but, in addition, a reentrant transition back to the homogeneous state at low temperatures.

In the present paper, we explore the reverse effect, compared to the gate control of magnetism: Can a change in magnetization lead to a change in carrier concentration? Or, if the sample is electrically isolated, can it lead to the appearance of a voltage? Microscopically the dependence of the effective spin interaction on the carrier concentration means that there is an additional contribution to the carrier energy depending on spin orientations. In particular, if the impurity spins are partially aligned, there is an *attractive* potential for carriers. This effect is discussed and its magnitude is estimated in Sec. II. In addition, the magnetization of course also changes with temperature, which should generate a contribution to the thermopower. This possibility is discussed in Sec. III. The paper is summarized in Sec. IV.

II. MAGNETIC-FIELD-INDUCED VOLTAGE

Let us consider the setup sketched in the inset in Fig. 1: A metallic DMS sample is placed in an inhomogeneous magnetic field and the voltage between opposite surfaces is measured. We assume the temperature to be close to T_c so that the magnetization is far from saturation. The magnetization is larger in the region with strong magnetic field. This region thus *attracts* carriers due to the mechanism discussed above. An electric current will flow for only a short time until the accumulated charge produces an electric field that prevents further charge accumulation. For positive (negative) carrier charge the electric potential in the strong-field region will be higher (lower) than in the weak-field region. The voltage measured between the two regions divided by the magnetic field difference will thus have the same sign as the carrier charge.

Following Ref. 17 we start from a Landau theory for the coupled magnetization and carrier density, defined by the Hamiltonian $H = H_m + H_{\delta n}$ with

$$H_m = \int d^3r \left\{ \frac{\alpha}{2} m^2 + \frac{\beta}{4} m^4 + \frac{\gamma}{2} \partial_i \mathbf{m} \cdot \partial_i \mathbf{m} - B m_z \right\}, \qquad (1)$$

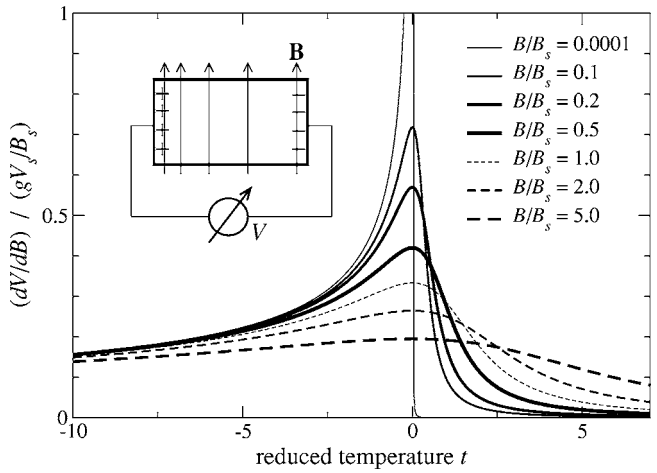

FIG. 1. Linear response of the voltage for a weakly nonuniform magnetic field, $\partial V/\partial B \propto m_z \partial m_z/\partial B$, as a function of reduced temperature for various magnetic fields, assuming positive carrier charge. Scaling factors and reduced temperature are defined in Eqs. (4)–(6). Inset: Sketch of DMS sample in a nonuniform field.

$$H_{\delta n} = \frac{1}{2}\int d^3r\, d^3r' \frac{e^2}{4\pi\epsilon_0\epsilon}\delta n(\mathbf{r})\delta n(\mathbf{r}')\frac{e^{-|\mathbf{r}-\mathbf{r}'|/r_0}}{|\mathbf{r}-\mathbf{r}'|} + \int d^3r\, q\,\delta n(\mathbf{r})V. \tag{2}$$

The first part is the usual Landau functional for a Heisenberg ferromagnet in a magnetic field $\mathbf{B}$ directed along the z axis. Summation over $i=x,y,z$ is implied. The second part contains the screened Coulomb interaction and an applied electric potential. The carriers are assumed to have charge $q=\pm e$. We impose the constraint $\int d^3r\ \delta n(\mathbf{r})=0$. The two parts are coupled through the coefficient α in H_m, which depends on temperature and vanishes at the Curie temperature. We write, to leading order, $\alpha=\alpha'(T-T_c-\eta\delta n)$, where T_c is now the Curie temperature for $\delta n=0$.

The equilibrium states can be found from the Euler equations $\delta H/\delta \mathbf{m}=0$ and $\delta H/\delta(\delta n)=0$.[17] We find a paramagnetic phase for $T\geq T_c$, a uniform ferromagnetic phase for $T^*\leq T<T_c$, and a phase with periodic modulation of $\mathbf{m}$ and δn for $T<T^*$, where $T^*\equiv T_c - e^2\beta\gamma/\epsilon_0\epsilon\alpha'^3\eta^2$. By rescaling of length, magnetization, and energy it is possible to reduce the number of parameters. For example, the magnetization can be written in the scaling form

$$\frac{\mathbf{m}(\mathbf{r})}{m_s} = \mathbf{M}\left(g,t,\frac{B}{B_s};\frac{\mathbf{r}}{r_0}\right), \tag{3}$$

where

$$m_s \equiv \frac{1}{r_0}\sqrt{\frac{\gamma}{\beta}}, \quad B_s \equiv \frac{1}{r_0^3}\sqrt{\frac{\gamma^3}{\beta}} \tag{4}$$

are the natural units of magnetization and magnetic field, respectively,

$$t \equiv \frac{\alpha'(T-T_c)r_0^2}{\gamma} \tag{5}$$

is a dimensionless reduced temperature, and

$$g \equiv \frac{\alpha'\eta\sqrt{\epsilon_0\epsilon}}{\sqrt{\beta}r_0 e} \tag{6}$$

is a dimensionless measure of the coupling η between carrier concentration and magnetization. $\mathbf{M}$ is a dimensionless vector scaling function.

A. Uniform phase

In the uniform ferromagnetic and paramagnetic phases, we can drop the gradient term in H_m. We obtain the energy density $h=h_m+h_{\delta n}$ with

$$h_m = \frac{\alpha}{2}m^2 + \frac{\beta}{4}m^4 - Bm_z, \tag{7}$$

$$h_{\delta n} = \frac{e^2 r_0^2}{2\epsilon_0\epsilon}\delta n^2 + q\,\delta n V - q\,\delta n V_0. \tag{8}$$

The last term implements the constraint of charge conservation with the Lagrange multiplier $-qV_0$. From the Euler equation $\partial h/\partial(\delta n)=0$ we obtain

$$\delta n = -\frac{\epsilon_0\epsilon}{qr_0^2}(V-V_0) + \frac{\epsilon_0\epsilon}{e^2 r_0^2}\frac{\alpha'\eta}{2}m^2. \tag{9}$$

The constraint $\delta n=0$ leads to

$$V - V_0 = \frac{\alpha'\eta}{2q}m^2. \tag{10}$$

Here, V_0 is a reference potential, which is irrelevant for the measured voltages. From $\partial h/\partial \mathbf{m}=0$ together with $\delta n=0$ we obtain $m_x=m_y=0$ and the standard result $\alpha'(T-T_c)m_z+\beta m_z^3-B=0$. The elementary solution for both $T<T_c$ and $T\geq T_c$ is

$$\frac{m_z}{m_s} = \frac{-2\times 3^{1/3}t + 2^{1/3}(9b+\sqrt{81b^2+12t^3})^{2/3}}{6^{2/3}(9b+\sqrt{81b^2+12t^3})^{1/3}}, \tag{11}$$

where $b\equiv B/B_s$. For weakly inhomogeneous magnetic fields we obtain the *linear response* of the voltage to the magnetic field gradient by expanding Eq. (10),

$$\frac{\partial V}{\partial B} = \frac{\alpha'\eta}{q}m_z\frac{\partial m_z}{\partial B}. \tag{12}$$

An analytical expression can be obtained from Eq. (11). Using dimensionless quantities, we find

$$\frac{\partial V}{\partial B}\left(\frac{V_s}{B_s}\right)^{-1} = \pm g\frac{m_z}{m_s}\frac{\partial}{\partial b}\frac{m_z}{m_s} \tag{13}$$

with $V_s\equiv\gamma/(r_0\sqrt{\beta\epsilon_0\epsilon})$. The sign is given by the sign of the carrier charge. Results for different magnetic fields are shown in Fig. 1. In the limit of vanishing magnetic field we obtain, in the *ferromagnetic* phase, $\partial V/\partial B(V_s/B_s)^{-1}=\pm(g/2)1/\sqrt{-t}$. The linear-response coefficient thus diverges as T_c is approached from below.

We next estimate the order of magnitude of $\partial V/\partial B$. While the Landau theory does not apply down to $T=0$, extrapolating to $T=0$ results in a magnetization of the correct order of magnitude. For a rough estimate we thus write

$$\left.\frac{\partial V}{\partial B}\right|_{B=0} \approx \frac{\eta}{2q}\frac{m_0(T=0)}{\sqrt{T_c(T_c-T)}}. \tag{14}$$

Here, $\eta \approx 5.4\times 10^5$ K Å^3 has been estimated[17] from Ref. 8, for the magnetization we take the maximum value for 5% Mn in (Ga,Mn)As, $m_0(T=0)\approx 5.13\times 10^4$ A/m, and T_c is set to 110 K. This gives a typical scale of $\partial V/\partial B|_{B=0}\approx 7.9\times 10^{-4}$ V/T at low temperatures and the coefficient increases like $(T_c-T)^{-1/2}$ as the temperature is increased. This result suggests that the effect should be clearly measurable.

The coupling strength η can in principle be determined by measuring T_c and the carrier concentration simultaneously for a series of samples with varying doping level[7,8] or, even better, for a single sample in the field-effect-transistor geometry.[5] For (Ga,Mn)As, one can infer $\eta\approx 5.4\times 10^5$ K Å^3 from Ref. 8 and $\eta\approx 1.5\times 10^5$ K Å^3 from Ref. 7. The growth procedure evidently strongly affects this parameter, probably because the underlying RKKY-type interaction is sensitive to disorder in both the hole and spin systems. From gate doping of a II-VI DMS sample,[5] one infers a significantly larger value of $\eta\approx 6.2\times 10^6$ K Å^3, but T_c is low. It should be noted that η only changes prefactors and does not affect the qualitative results regarding temperature and field dependance.

In the *paramagnetic* phase, the magnetization m is, to leading order, linear in magnetic field. Thus the voltage Eq. (10) is quadratic in B and there is no linear response at zero field. Right at the critical point the Landau-theory result for the magnetization is $m_z=(B/\beta)^{1/3}$, leading to $V=(\alpha'\eta/2q)\times(B/\beta)^{2/3}$.

Next, we briefly discuss the voltage due to a *large* change in magnetic field. It may be possible to measure this voltage by first grounding a metallic DMS sample in zero magnetic field, removing the ground connection, and then applying a strong, uniform field. Since higher magnetization *attracts* carriers, a negative (positive) potential difference between sample and ground is expected for positive (negative) carrier charge. Equation (10) gives for vanishing magnetic field $-V_0=(\alpha'\eta)/(2q)m_0^2$, where m_0 is the (uniform) magnetization in zero magnetic field, which is $m_0=\sqrt{-\alpha'(T-T_c)/\beta}$ in the ferromagnetic phase and $m_0=0$ in the paramagnetic phase. Consequently,

$$V=\frac{\alpha'\eta}{2q}(m^2-m_0^2). \tag{15}$$

V is the potential that would be necessary to maintain $\delta n=0$ if the sample were still connected to a charge reservoir. If the sample is isolated, a voltage $-V$ with the reverse sign is measured. In terms of dimensionless quantities we obtain

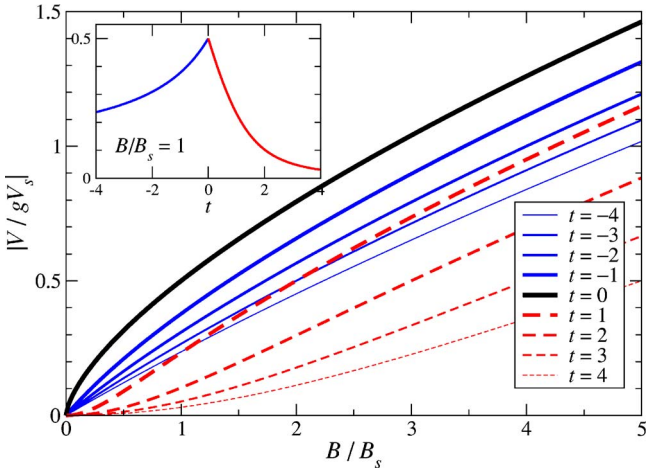

FIG. 2. (Color online) Magnitude of voltage $-V$ between sample and ground as a function of magnetic field for various temperatures for a DMS sample in the uniform ferromagnetic or paramagnetic phase. Here, $t=\alpha'(T-T_c)r_0^2/\gamma$ is the dimensionless reduced temperature. If the voltage is scaled with $1/g$, as it is plotted here, the curves do not depend on g. Inset: Voltage as a function of t for $B/B_s=1$.

$$-\frac{V}{V_s}=\mp\frac{g}{2}\left(\frac{m^2}{m_s^2}-\frac{m_0^2}{m_s^2}\right), \tag{16}$$

where the upper (lower) sign applies to positive (negative) carrier charge. For small magnetic fields we get the same linear-response result as above. The full nonlinear response is obtained by inserting Eq. (11) into Eq. (15). The resulting voltage is plotted as a function of magnetic field for various temperatures in Fig. 2. In the ferromagnetic phase, the voltage crosses over from a linear B dependence to $B^{2/3}$ at $B/B_s\sim t^{3/2}$ [see Eq. (11)]. In the paramagnetic phase there is a crossover from B^2 to $B^{2/3}$ at the same scale. The inset in Fig. 2 shows the dependence of $|V|$ on temperature at a finite magnetic field. The cusp at $T=T_c$ stems from the *zero-field* magnetization m_0 in Eq. (15).

All results up to this point have been obtained from Landau mean-field theory. The question arises whether the results survive if fluctuations are taken into account. After all, the average square magnetization $\overline{m^2}$ becomes an *analytical* function of temperature through T_c.

We restrict ourselves to temperatures $T\approx T_c$ and to small magnetic fields. Let us assume that charge fluctuations are fast compared to fluctuations in the magnitude of **m**. This seems reasonable since charge density fluctuations have a typical timescale of the inverse plasma frequency. In this case the carrier concentration instantaneously follows the magnetization fluctuations. Equation (12) then becomes

$$\frac{\partial V}{\partial B}=\frac{\alpha'\eta}{2q}\frac{\partial\overline{m^2}}{\partial B}. \tag{17}$$

Equation (1) shows that the averages of m_z and $\underline{m}^2$ are obtained from the *exact* free energy density f by $\overline{m_z}=-\partial f/\partial B$ and $\overline{m^2}=2\partial f/\partial\alpha$. This implies the Maxwell relation

$$\frac{\partial \overline{m^2}}{\partial B} = -2\frac{\partial \overline{m_z}}{\partial \alpha}. \tag{18}$$

For vanishing magnetic field and $T<T_c$, one finds $\overline{m_z} \propto (T_c - T)^{\underline{\beta}}$, where $\underline{\beta}$ is the usual critical exponent (not to be confused with the coefficient β in the Landau functional) and T_c is the *true* Curie temperature, taking fluctuations into account. Since α is a linear function of T we obtain

$$\left.\frac{\partial V}{\partial B}\right|_{B=0} \propto (T_c - T)^{\underline{\beta}-1}. \tag{19}$$

This quantity still diverges as T_c is approached from below. Note that for the three-dimensional Heisenberg model, $\underline{\beta} \approx 0.37$.[23] Fluctuations thus make the divergence *stronger*. In the paramagnetic phase, $\overline{m_z}$ of course vanishes for $B=0$ and we still do not find a linear response.

B. Stripe phase

Below T^* the Landau theory predicts an equilibrium state with periodic modulations of magnetization and carrier concentration.[17] We briefly review the pertinent properties of this state. The first two terms in the Landau functional H_m, Eq. (1), can be interpreted as a potential for a fictitious particle with coordinates $\mathbf{m}$. The uniform equilibrium states are determined by the minima of this potential. In the stripe phase the coefficients are renormalized,[17] leading to the energy density $h_0 = (\tilde{\alpha}/2)m^2 + (\tilde{\beta}/4)m^4$ with

$$\tilde{\alpha} \equiv \alpha'(T - T_c) + \frac{\alpha'^2 \eta^2 \epsilon_0 \epsilon}{2e^2 r_0^2}\overline{m^2}, \tag{20}$$

$$\tilde{\beta} \equiv \beta - \frac{\alpha'^2 \eta^2 \epsilon_0 \epsilon}{2e^2 r_0^2}, \tag{21}$$

where m^2 has to be calculated self-consistently. It turns out that a modulation of m about the minimum of h_0 is stable and energetically favorable for $m > m_{\mathrm{sing}}$, where $m_{\mathrm{sing}} \equiv \sqrt{\gamma e}/\alpha' \eta \sqrt{\epsilon_0 \epsilon} = m_s/g$. Modulations occur between two magnetizations with equal potential $h_0(m)$. For weak coupling, i.e., small η or g, m_{sing} is larger than the magnetization at the minimum of $h_0(m)$ and stable modulations are not possible. For larger coupling, m_{sing} is smaller than m at the minimum, allowing modulations to occur. The minimum-energy solution is obtained for the *maximum* possible modulation amplitude. In this case the lower turning point of the modulation approaches m_{sing}.[17]

In the present case we add the Zeeman term $-Bm_z$ to the energy density, which tilts the effective potential, as shown in the right inset in Fig. 3. Numerical integration shows that at any temperature the modulation with maximum amplitude still has the lowest energy. Figure 3 shows a typical phase diagram in the temperature-magnetization plane. A typical minimal-energy solution is shown in the left inset.

The field-dependent transition temperature $T^*(B)$ to the stripe phase is determined by the condition $m = m_{\mathrm{sing}}$, where m is the uniform magnetization, Eq. (11). The solution is

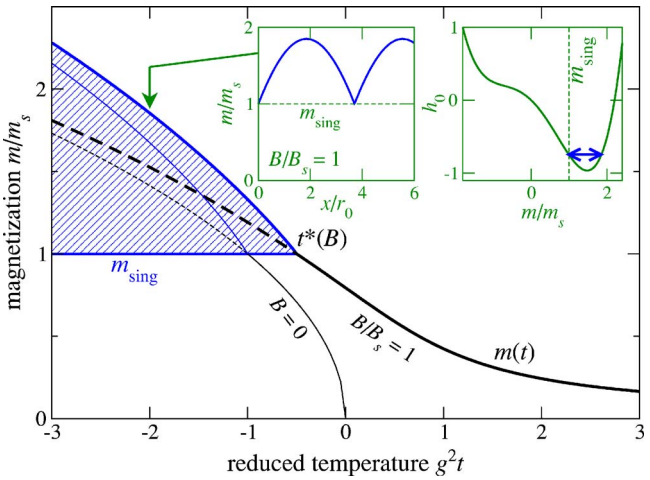

FIG. 3. (Color online) Diagram showing the equilibrium phases as a function of reduced temperature $g^2 t$ in a nonzero magnetic field $B/B_s = 1$. For $t > t^* = \alpha'(T^* - T_c) r_0^2/\gamma$ the equilibrium state is uniform with the magnetization given by the heavy solid line. For $t < t^*$ the magnetization shows periodic modulations spanning the cross-hatched magnetization interval. The lower limit of the modulations is $m = m_{\mathrm{sing}}$. The heavy dashed line shows the magnetization of the uniform solution for $t < t^*$, which exists but has higher energy. The thin lines denote the uniform magnetization and upper limit of modulations for $B=0$, for comparison. The left inset shows the magnetization as a function of position for $g^2 t = -2$ and $B/B_s = 1$. The right inset shows the effective potential h_0 as a function of magnetization for the same parameter values. The double-headed arrow denotes the magnetization modulation. The detailed numerical values depend on the choice of g ($g=1$ in this plot) and of B/B_s, but the topology of the diagram does not.

$$T^* = T_c - \frac{\beta \gamma e^2}{\alpha'^3 \eta^2 \epsilon_0 \epsilon} + \frac{\eta \sqrt{\epsilon_0 \epsilon}}{\sqrt{\gamma e}} B = T^*(B=0) + \frac{\eta \sqrt{\epsilon_0 \epsilon}}{\sqrt{\gamma e}} B. \tag{22}$$

Using Eqs. (4)–(6), this is equivalent to

$$t^* \equiv \frac{\alpha'(T^* - T_c) r_0^2}{\gamma} = -\frac{1}{g^2} + g\frac{B}{B_s}. \tag{23}$$

Thus the transition temperature depends linearly on B for arbitrary B. Of course, for large magnetic fields Landau theory becomes inapplicable. Note that while there is no sharp paramagnet-ferromagnet transition in nonzero field, the transition to the stripe phase remains sharp. The reason is that translational symmetry is preserved by the applied field and thus can be spontaneously broken at this transition. The inset in Fig. 4 shows the resulting phase diagram.

The analog of Eq. (10) in the stripe phase is

$$V - V_0 = \frac{\alpha' \eta}{2q}\overline{m^2}. \tag{24}$$

If fluctuations are neglected, the average is a spatial average over the static modulation. The linear response $\partial V/\partial B = (\alpha' \eta/2q)\,\partial \overline{m^2}/\partial B$ is calculated numerically and the result for $B=0$ is shown in Fig. 4. The linear-response coefficient shows a *discontinuity* at T^* within Landau theory. This phe-

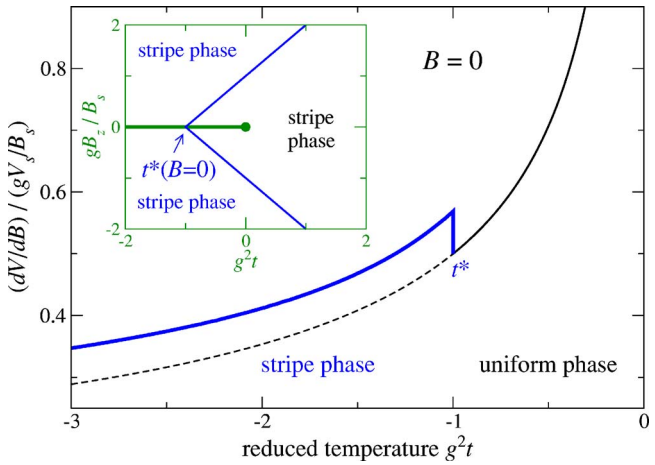

FIG. 4. (Color online) Linear response $\partial V/\partial B$ for small magnetic field, as a function of reduced temperature. The jump in $\partial V/\partial B$ at $t=t^*$ is discussed in the text. For $t<t^*$, the result for the unstable uniform state would continue along the dashed line. The numerical calculation has been performed for the coupling $g=1$. Inset: Phase diagram in the temperature–magnetic-field plane. The heavy solid line denotes the first-order transition of the ferromagnet, ending in a critical point at T_c (i.e., $t=0$). The thin solid lines emanating from $t=t^*(B=0)$ denote the second-order transition between uniform and stripe phases.

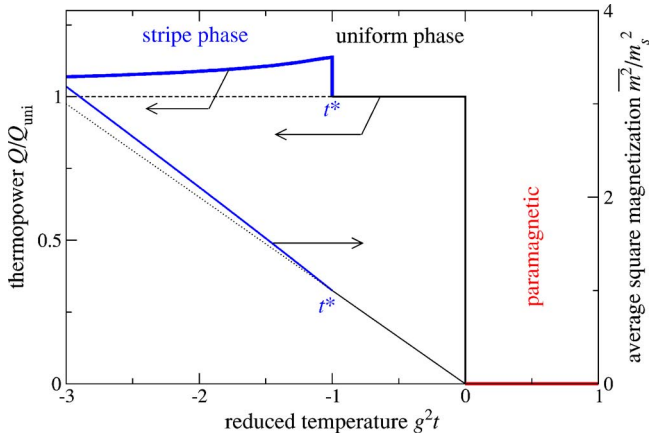

FIG. 5. (Color online) Left axis: Thermopower Q in units of the constant thermopower $Q_{\rm uni}$ in the uniform phase for vanishing magnetic field, as a function of reduced temperature. Also shown is the average square magnetization $\overline{m^2}$ (right axis). The numerical calculation for $t<t^*$ has been performed for $g=1$.

nomenon is accompanied by a similar jump in the susceptibility at T^* (not shown). Physically, the system is more easily polarizable in the stripe phase, since it has an additional parameter, the wavelength λ, that can be adapted to the applied field. In agreement with this interpretation, the inset in Fig. 4 shows that the stripe phase is *stabilized*by a magnetic field.

III. THERMOPOWER

Since the magnetization changes with temperature and is coupled to the carrier concentration, there should also be a change in carrier concentration or potential with temperature. In the case of an electrically isolated sample one expects a nonzero thermopower

$$Q \equiv -\left.\frac{\partial V}{\partial T}\right|_{I=0}. \tag{25}$$

Consider the following setup. One end of a DMS sample is kept at a temperature T_1, the other at temperature $T_2>T_1$. A discussion of the standard origin of the thermopower can be found in textbooks.[24] The result is that Q has the same sign as the carrier charge q. For this reason the thermopower is often measured to obtain the sign of this charge in semiconductors, including DMS.[20,21]

Here, we predict an additional contribution in DMS. In the same setup, with $T_1<T_2<T_c$, the magnetization m is larger in the cool region. Carriers are attracted to high-magnetization regions, leading to a current which results in the accumulation of positive (negative) charge in the cool region for positively (negatively) charged carriers. This charge generates an electric field that prevents further charge flow. The electric potential ϕ is higher (lower) in the cool region for positive (negative) carrier charge q. With the explicit minus sign in Eq. (25) this leads to a thermopower of the *same sign* as the normal contribution. Proceeding as in Sec. II we obtain

$$V = \phi(T_2) - \phi(T_1) = \frac{\alpha'\eta}{2q}\left[\overline{m^2}(T_2) - \overline{m^2}(T_1)\right]. \tag{26}$$

We restrict ourselves to vanishing magnetic field. In the uniform phase we can write, neglecting fluctuations, $\overline{m^2}=m^2$. In Landau theory, Eq. (26) gives

$$Q \equiv Q_{\rm uni} = \frac{\alpha'^2\eta}{2q\beta}. \tag{27}$$

Thus the thermopower is *independent of temperature* for $T^*<T<T_c$. On the other hand, for $T\geq T_c$ there is no magnetization and this contribution to the thermopower vanishes. We thus find a jump at T_c.

In the stripe phase, for $T<T^*$, we find numerically that the thermopower increases in magnitude with temperature. At T^* it shows a downward jump, since the average square magnetization $\overline{m^2}$ changes slope as a function of temperature at the transition. Thermopower and $\overline{m^2}$ are shown in Fig. 5.

To estimate the order of magnitude of the thermopower, we again replace α'/β by $m^2(T=0)/T_c$, leading to $Q=-\beta\eta m^4(T=0)/2qT_c^2$. The only additional parameter needed beyond those given in Sec. II is β. By comparing the mean-field result for the gain in energy density due to magnetic ordering of the Heisenberg model on a simple cubic lattice to the corresponding gain for the Landau theory, we obtain

$$\beta \approx \frac{1}{(g\mu_B)^4}\frac{6k_BT_c}{S^3(S+1)n_{\rm Mn}^3} \approx 2.1\times 10^{-12}\ \text{J m/A}^4, \tag{28}$$

where $g\approx 2$ is the g factor for the impurity spins with $S=5/2$ and $n_{\rm Mn}$ is the concentration of impurities, again assumed to be 5% of cation sites. This gives $Q\approx 2.0$

$\times 10^{-3}$ V/K. In nonmagnetic semiconductors showing thermally activated conduction, the magnitude of the thermopower is of the order of $10^{-5}-10^{-4}$ V/K. In metals it is a few times 10^{-6} V/K. Thus the thermopower generated by the carrier-magnetization coupling is expected to be larger than the normal contribution.

The thermopower has been measured for various DMS,[20,21] but usually only to infer the sign, which is not changed by the physics discussed here. A study of the thermopower in (Ga,Mn)As is under way.[22] It would be worthwhile to look for an anomalously large thermopower in DMS. However, the main signature of the magnetization-induced effect would be a downward jump at T_c.

Finally, we briefly turn to the qualitative effect of fluctuations. The average square magnetization becomes analytic in temperature through T_c so that we expect the jump in Q to be replaced by a continuous crossover. Its width should be given by the Ginzburg criterion for the fluctuation-dominated temperature interval. In real DMS, disorder plays an important role,[25] which may lead to additional broadening. However, for high-quality metallic samples of (Ga,Mn)As, the magnetization curves close to T_c show a sharp decrease[7,26–29] similar to the Heisenberg model on a regular lattice, suggesting that disorder is not dominant.

IV. SUMMARY AND CONCLUSIONS

The strong coupling between carrier concentration and magnetization in ferromagnetic DMS has been found to lead to a dependence of the electric potential on the magnetization. If the magnetization is controlled by an applied magnetic field, a magnetic-field-induced voltage is expected. Two possible setups are discussed in this paper, which allow one to study the linear response for weakly nonuniform magnetic fields and the nonlinear voltage induced by a strong field. The linear-response coefficient in the limit of small field is estimated to be of the order of 10^{-3} V/T for (Ga,Mn)As at low temperatures. It diverges as T_c is approached from below. This singularity persists if fluctuations are taken into account.

On the other hand, if the magnetization is changed by varying the temperature, the same physics leads to a variation of electric potential with temperature, i.e., a thermopower. Within Landau theory, the thermopower is temperature independent in the uniform ferromagnetic phase and shows a discontinuity at the Curie temperature T_c, which is smeared out by fluctuations. For (Ga,Mn)As the estimate for this contribution to the thermopower is of the order of 10^{-3} V/K, which is what one would have guessed from the magnitude of the magnetic-field-induced voltage. All these results are obtained for a uniform equilibrium magnetization.

A nonuniform, stripelike equilibrium magnetization is possible in DMS at lower temperatures $T<T^*$.[17] The phase diagram of uniform and stripe phases is obtained within Landau theory. If the stripe phase were realized, the magnetic-field-induced voltage would be qualitatively similar to that in the uniform case, but the linear response $\partial V/\partial B$ would show a discontinuity at T^*. The effect is larger in the stripe phase because the magnetization can adapt more easily to the applied magnetic field by varying the wavelength of the stripe pattern. The thermopower in the stripe phase is temperature dependent, unlike in the uniform phase, and shows a downward jump at T^*.

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