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Spontaneous spin current due to triplet superconductor-ferromagnet interfaces

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We examine the appearance of a spontaneous bulk spin current in a triplet superconductor in contact with a metallic ferromagnet. The spin current results from the spin flip of Cooper pairs upon reflection from the interface with the ferromagnet, and is shown to display strong similarities to the spontaneous charge current in a Josephson junction. We express the spin current in terms of the Andreev reflection coefficients, which are derived by the construction of the quasiclassical scattering wave functions. The dependence of the spin current upon a number of parameters is investigated, in particular the orientation of the magnetic moment of the ferromagnet, the exchange splitting, the temperature, and the orbital pairing state of the triplet superconductor.

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I. INTRODUCTION

The interface between a singlet superconductor (SSC) and a ferromagnet (FM) is an ideal setting to explore the antagonistic relationship between these two phases.[1,2] It is now more than 25 years since the first theoretical investigations,[3] but the physics of SSC-FM interfaces continues to fascinate and surprise.[1,2,4–14] A key feature of such systems is the existence of an unconventional proximity effect:[2,8–11,13,14] in contrast to the spin-singlet pairing state of the bulk SSC, the spin splitting of the Fermi surface induces spin-triplet *correlations* in the FM. Although this effect occurs at all metallic FM interfaces, the character of the induced triplet pairing correlations is determined by the strength of the exchange-splitting in the FM,[2,8,10,13] whether the system is in the ballistic or the diffusive limits,[2,10,11,14] and also the particular geometry of the heterostructure.[2,11,14]

The unconventional proximity effect at SSC-FM interfaces clearly evidences an intimate connection between magnetism and spin-triplet pairing. Such devices can provide only a limited understanding of this interplay, however, as the FM determines the triplet pairing correlations. To be able to control the spin-triplet pairing independently, it would be necessary to replace the SSC with a triplet superconductor (TSC). Since the discovery of triplet superconductivity in Sr_2RuO_4,[15,16] there has been steadily growing interest in the properties of TSC heterostructures.[17–35] Despite the likely intimate connection between the two phases, the study of devices combining TSCs and FMs is still in its infancy.[19,20,29–35] Even so, several exotic effects have already been predicted, such as a 0-π transition in a TSC-FM-TSC Josephson junction caused by the misalignment of the vector order parameters (the so-called **d** vectors) of the TSCs with the moment **M** of the FM tunneling barrier.[25,32,35]

The origin of this unconventional behavior is the coupling of the FM moment to the spin of the triplet Cooper pair.[35] More generally, the extra degree of freedom provided by the Cooper pair spin is responsible for novel spin transport properties of TSC heterostructures. For example, a number of authors have demonstrated that a Josephson *spin current* flows between two TSCs when their **d** vectors are misaligned.[23,24,27] It has recently been established that a spin current may also be produced by the inclusion of a FM tunneling barrier in a TSC Josephson junction.[32–35] Two basic mechanisms have been identified: the barrier can act as a spin filter, preferentially allowing the tunneling of one spin species of Cooper pair over the other; alternatively, the barrier moment can flip the spin of a tunneling Cooper pair, which then acquires an extra spin-dependent phase. However produced, the tunneling spin currents are always dependent upon the phase difference between the TSC condensates on either side of the junction.

A bulk *phase-independent* contribution to the spin current in a TSC-FM-TSC Josephson junction was predicted in Ref. 35, and subsequently also identified in a SSC-FM heterostructure in Ref. 13. The origin of this spin current was shown to be the spin-dependent phase shift acquired by the flipping of a triplet Cooper pair's spin upon reflection at a δ-function-thin FM barrier. A number of properties were deduced: the spin current is polarized along the direction $\mathbf{d}\times\mathbf{M}$; the current in the tunneling limit is $\propto\sin(2\alpha)$, where α is the angle between **d** and **M**; and the sign of the current displays a pronounced dependence upon the orbital structure of the bulk TSC, due to the orbital-dependent phase shift experienced by the reflected Cooper pairs. Most remarkable is that although this spin current is carried by Cooper pairs, and hence is indistinguishable from the tunneling Josephson spin current, it is independent of the material on the other side of the FM barrier.

It is a natural question to ask if this effect also occurs at the interface between a bulk TSC and FM. In studying such interfaces, most authors have only addressed the case $\mathbf{d}\parallel\mathbf{M}$ when the spin current described above is not expected to occur.[20,30,31] Although Hirai and co-workers considered arbitrary orientation of **d** and **M** in their study of the TSC-FM interface, they did not examine the spin transport properties of the device.[19] It is therefore the purpose of this paper to examine the occurrence of a spontaneous spin current in a TSC-FM junction for arbitrary alignment of the TSC and FM vector order parameters. Using a quasiclassical technique, we demonstrate the existence of a spin current with the same dependence upon the bulk TSC orbital structure and the relative misalignment of **d** and **M** as predicted in Ref. 35. The spin current is also found to display a strong dependence upon the exchange splitting of the FM, which is explained due to the angular-dependence of the spin-flip reflection

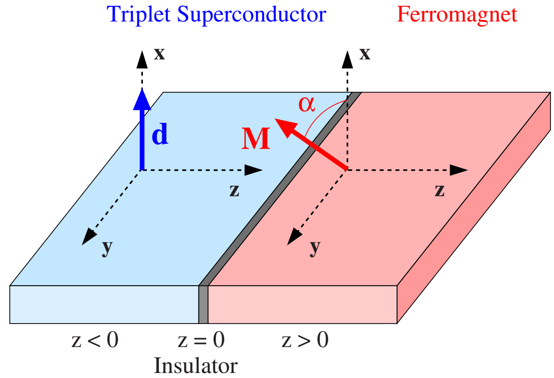

FIG. 1. (Color online) Schematic representation of the device studied in this work. We consider a junction between a bulk triplet superconductor and a bulk ferromagnet, separated by a thin insulating tunneling barrier. The **d** vector of the TSC defines the x axis, while the moment **M** of the FM lies in the x-y plane at an angle α to the x axis.

probability. We discuss similarities and differences to the usual Josephson effect, in particular arguing that low-temperature anomalies in the spin current imply a role for zero energy states at the interface in the transport.

II. THEORETICAL FORMULATION

A schematic of the device studied here is shown in Fig. 1. It consists of a bulk TSC and FM, separated by a thin insulating barrier at $z=0$, which we approximate by a delta function of height U. Both materials are assumed to be in the clean limit. The Bogoliubov–de Gennes (BdG) equation describing the quasiparticle states with energy E is written in Nambu-spin space as

$$\begin{pmatrix} \hat{H}_0(\mathbf{r}) & \hat{\Delta}(\mathbf{r}) \\ \hat{\Delta}^{\dagger}(\mathbf{r}) & -\hat{H}_0^T(\mathbf{r}) \end{pmatrix} \Psi(\mathbf{r}) = E\Psi(\mathbf{r}), \tag{1}$$

where the caret indicates a 2×2 matrix in spin-space. The noninteracting Hamiltonian is

$$\hat{H}_0(\mathbf{r}) = \left[-\frac{\hbar^2\boldsymbol{\nabla}^2}{2m} + U\delta(z)\right]\hat{\mathbf{1}} - g\mu_B\hat{\boldsymbol{\sigma}}\cdot\mathbf{M}\Theta(z) \tag{2}$$

In the interests of simplicity, we will assume that the effective mass m is the same in the TSC and the FM.[36] The moment of the FM is parameterized as $\mathbf{M}=M[\sin(\beta)\cos(\alpha)\mathbf{e}_x + \sin(\beta)\sin(\alpha)\mathbf{e}_y + \cos(\beta)\mathbf{e}_z]$. Since we do not include spin-orbit coupling, we can assume without loss of generality that $\beta=\pi/2$, i.e., the moment lies in the x-y plane. Other spin orientations can be obtained by appropriate rotation of the system about the x axis in spin space, which leaves the spin state of the TSC unchanged.

The gap matrix in Eq. (1) is $\hat{\Delta}(\mathbf{r})=i[\hat{\boldsymbol{\sigma}}\cdot\mathbf{d}(\mathbf{r})]\hat{\sigma}_y$ where $\mathbf{d}(\mathbf{r})$ is the vector order parameter of the TSC. We will restrict ourselves here to equal-spin-pairing unitary states, for which $\mathbf{d}(\mathbf{r})=\Delta(\mathbf{r})\Theta(-z)\mathbf{e}_x$ is a suitable choice. In such a state, the triplet Cooper pairs have z component of spin $S_z = \pm\hbar$ but the condensate has no net spin. The magnitude of the gap is assumed to be constant throughout the TSC. Due to the triplet spin state, the gap must reverse sign across the Fermi surface, implying an odd-parity orbital wave function. We will mostly be concerned with three different orbital pairing states: p_y-wave, $\Delta_{\mathbf{k}}=\Delta(T)k_y/k_F$; p_z-wave, $\Delta_{\mathbf{k}} =\Delta(T)k_z/k_F$; and a chiral p_z+ip_y-wave state $\Delta_{\mathbf{k}}=\Delta(T)[k_z +ik_y]/k_F$. These different gaps are illustrated in Fig. 2. The gap magnitude $\Delta(T)$ displays weak-coupling temperature dependence, with $T=0$ value Δ_0.

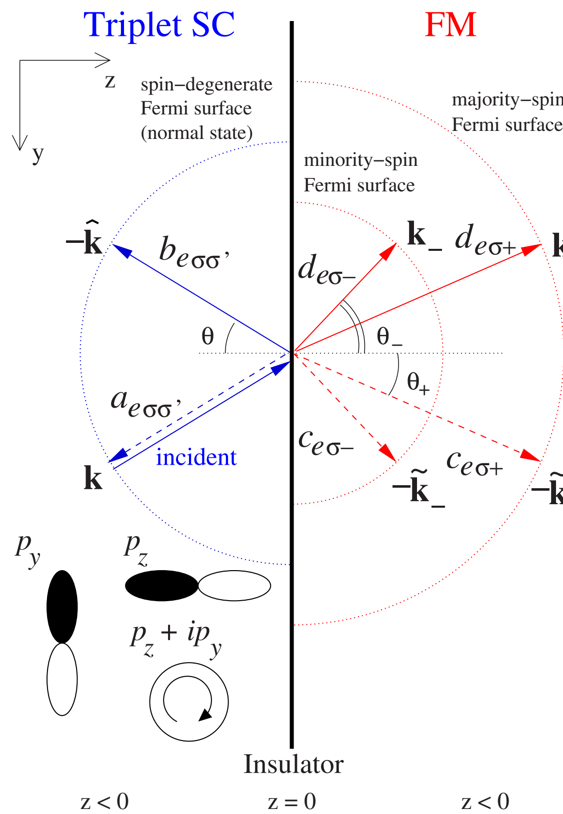

FIG. 2. (Color online) Schematic representation of the wave function $\Psi_{e\sigma}$ for a spin-σ electronlike quasiparticle with wave vector **k** incident upon the FM from the TSC. Note the different lengths of the wave vectors in the FM due to the exchange splitting of the Fermi surface. In the bottom left corner, we show the orbital arrangement of our three choices of pairing symmetry. In the p_y and p_z cases, the black and white lobes of the p-wave orbital indicate opposite signs; for the p_z+ip_y state, the gap magnitude is constant and the arrow indicates the direction of increasing phase.

Both the TSC and FM are assumed to have circular Fermi surfaces. For the sake of clarity, we assume the TSC and FM to have identical Fermi energies E_F.[36] In the TSC, the Fermi surface in the normal state is spin degenerate with radius $k_F=\sqrt{2mE_F/\hbar^2}$. If the exchange splitting of the bands is less than the Fermi energy in the FM, we have a majority spin (aligned parallel to **M**, $s=+$) and a minority spin (aligned antiparallel to **M**, $s=-$) Fermi surface of radius

$$k_F^s = \sqrt{\frac{2m}{\hbar^2}(E_F + sg\mu_B M)} = k_F\sqrt{1+s\lambda}, \tag{3}$$

where

$$\lambda = g\mu_B M/E_F \tag{4}$$

is the ratio of the exchange splitting to the Fermi energy. The minority spin Fermi surface disappears when the exchange-splitting exceeds E_F, i.e., the FM is a half-metal.

Solving the BdG equations Eq. (1), we construct the wave function for a spin-σ electronlike quasiparticle with wave vector $\mathbf{k}$ incident upon the FM layer from the TSC using the Ansatz

$$\begin{aligned}\Psi_{e\sigma}(\mathbf{r}) = \Theta(-z)\{&\Phi^{TSC}_{\mathbf{k},e,\sigma}e^{i\mathbf{k}\cdot\mathbf{r}} \\ &+ \sum_{\sigma'=\uparrow,\downarrow}[a_{e\sigma\sigma'}\Phi^{TSC}_{\mathbf{k},h,\sigma'}e^{i\mathbf{k}\cdot\mathbf{r}} + b_{e\sigma\sigma'}\Phi^{TSC}_{-\hat{\mathbf{k}},e,\sigma'}e^{-i\hat{\mathbf{k}}\cdot\mathbf{r}}]\} \\ &+ \Theta(z)\sum_{s=\pm}\{c_{e\sigma s}\Phi^{FM}_{h,s}e^{-i\tilde{\mathbf{k}}_s\cdot\mathbf{r}} + d_{e\sigma s}\Phi^{FM}_{e,s}e^{i\mathbf{k}_s\cdot\mathbf{r}}\}.\end{aligned} \tag{5}$$

A schematic representation of the wave function $\Psi_{e\sigma}$ is shown in Fig. 2. For $z<0$, the Ansatz Eq. (5) describes an Andreev-reflected spin-σ' holelike quasiparticle with wave vector $\mathbf{k}$ and reflection probability amplitude $a_{e\sigma\sigma'}$, and a spin-σ' electronlike quasiparticle undergoing specular reflection with wave vector $-\hat{\mathbf{k}}=(k_x,k_y,-k_z)$ and probability amplitude $b_{e\sigma\sigma'}$. Note that we make the standard "quasiclassical" assumption that $E\ll E_F$, and so the magnitude of the wave vectors for the electronlike and holelike quasiparticles are approximated to be identical.[37,38] For $z>0$, the transmission probability amplitudes for hole and electron quasiparticles with spin projection $s=\pm$ along the direction of the ferromagnetic moment are $c_{e\sigma s}$ and $d_{e\sigma s}$ respectively. As shown in Fig. 2, due to the spin splitting of the FM Fermi surface, the trajectories of transmitted $s=+$ and $s=-$ quasiparticles are not coincident. Since translational invariance is satisfied along the x and y directions, the component of the wave vector parallel to the interface is preserved during the scattering. We hence have $k_F\sin(\theta)=k_F^s\sin(\theta_s)$. For $\sin(\theta)<k_F^s/k_F$ propagating solutions for both spin polarizations exist, and the z-component of $\mathbf{k}_s$ and $\tilde{\mathbf{k}}_s$ is then $k_{s,z}=k_F\sqrt{\cos^2(\theta)+s\lambda}$. Since $k_F^-<k_F<k_F^+$, there is however a critical angle $\theta_c=\arcsin(k_F^-/k_F)$ such that for $\theta_c<|\theta|<\pi/2$ the z component of the wave vector of the transmitted $s=-$ quasiparticles is purely imaginary. The resulting evanescent wave is exponentially suppressed on an inverse length scale $\kappa=k_F\sqrt{\lambda-\cos^2(\theta)}$.

The wave function ansatz Eq. (5) is expressed in terms of the spinors for the bulk TSC and FM phases. In the FM we have the spinors for electrons and holes with spin $s=\pm$,

$$\Phi^{FM}_{e,s} = (se^{-i\alpha}/\sqrt{2},1/\sqrt{2},0,0)^T, \tag{6}$$

$$\Phi^{FM}_{h,s} = (0,0,se^{i\alpha}/\sqrt{2},1/\sqrt{2})^T. \tag{7}$$

For the TSC, the spinors $\Phi^{TSC}_{\mathbf{k},e(h),\sigma}$ for an electronlike (holelike) quasiparticle with spin σ and wave vector $\mathbf{k}$ are

$$\Phi^{TSC}_{\mathbf{k},e,\uparrow} = (s_{\mathbf{k}}u_{\mathbf{k}},0,-v_{\mathbf{k}},0)^T, \tag{8}$$

$$\Phi^{TSC}_{\mathbf{k},h,\uparrow} = (s_{\mathbf{k}}v_{\mathbf{k}},0,-u_{\mathbf{k}},0)^T, \tag{9}$$

$$\Phi^{TSC}_{\mathbf{k},e,\downarrow} = (0,s_{\mathbf{k}}u_{\mathbf{k}},0,v_{\mathbf{k}})^T, \tag{10}$$

$$\Phi^{TSC}_{\mathbf{k},h,\downarrow} = (0,s_{\mathbf{k}}v_{\mathbf{k}},0,u_{\mathbf{k}})^T, \tag{11}$$

where $u_{\mathbf{k}}=\sqrt{(E+\Omega_{\mathbf{k}})/2E}$, $v_{\mathbf{k}}=\sqrt{(E-\Omega_{\mathbf{k}})/2E}$, $\Omega_{\mathbf{k}}=\sqrt{E^2-|\Delta_{\mathbf{k}}|^2}$ and $s_{\mathbf{k}}=\Delta_{\mathbf{k}}/|\Delta_{\mathbf{k}}|$.

The probability amplitudes in Eq. (5) are determined by the boundary conditions obeyed by the wave function at the TSC-FM interface. In particular, we require that the wave function is continuous at the interface, i.e.,

$$\Psi_{e\sigma}(\mathbf{r})|_{z=0^-} = \Psi_{e\sigma}(\mathbf{r})|_{z=0^+}, \tag{12}$$

and also that the first derivative of the wave function obeys the condition

$$\left.\frac{\partial\Psi_{e\sigma}(\mathbf{r})}{\partial z}\right|_{z=0^+} - \left.\frac{\partial\Psi_{e\sigma}(\mathbf{r})}{\partial z}\right|_{z=0^-} = Zk_F\Psi_{e\sigma}(\mathbf{r})|_{z=0}, \tag{13}$$

where $Z=2mU/\hbar^2k_F$ is a dimensionless constant characterizing the height of the insulating barrier. These conditions yield eight coupled equations for the probability amplitudes. In general, the probability amplitudes are functions of the incident wave vector $\mathbf{k}$ and the quasiparticle energy E.

It is straightforward to modify the ansatz Eq. (5) for the wave functions $\Psi_{h\sigma}(\mathbf{r})$ describing the scattering of a holelike quasiparticle incident upon the FM from the TSC. The probability amplitudes $a_{h\sigma\sigma'}$, $b_{h\sigma\sigma'}$ etc. for this case are determined by applying the same boundary conditions as for $\Psi_{e\sigma}(\mathbf{r})$.

The currents in the TSC may be evaluated using the generalization of the Furusaki-Tsukuda formula to triplet pairing.[24,39] For the currents flowing perpendicular to the interface, we hence express the charge current I_C and the z and y components of the spin current, $I_{S,z}$ and $I_{S,y}$, respectively, in terms of the Andreev reflection probability amplitudes $a_{e(h)\sigma\sigma'}$ with energy argument analytically continued to $i\omega_n$,

$$\begin{aligned}I_C = \frac{e}{2\hbar}\int_{|\mathbf{k}|=k_F} dk_z dk_y \Theta(k_z)\frac{k_z}{k_F}\frac{1}{\beta\hbar}\sum_n\sum_\sigma \\ \times\left\{\frac{|\Delta_{\mathbf{k}}|}{\Omega_{n,\mathbf{k}}}a_{e\sigma\sigma}(\mathbf{k},i\omega_n) - \frac{|\Delta_{-\hat{\mathbf{k}}}|}{\Omega_{n,-\hat{\mathbf{k}}}}a_{h\sigma\sigma}(\mathbf{k},i\omega_n)\right\},\end{aligned} \tag{14}$$

$$\begin{aligned}I_{S,z} = \frac{1}{4}\int_{|\mathbf{k}|=k_F} dk_z dk_y \Theta(k_z)\frac{k_z}{k_F}\frac{1}{\beta\hbar}\sum_n\sum_\sigma \sigma \\ \times\left\{\frac{|\Delta_{\mathbf{k}}|}{\Omega_{n,\mathbf{k}}}a_{e\sigma\sigma}(\mathbf{k},i\omega_n) - \frac{|\Delta_{-\hat{\mathbf{k}}}|}{\Omega_{n,-\hat{\mathbf{k}}}}a_{h\sigma\sigma}(\mathbf{k},i\omega_n)\right\},\end{aligned} \tag{15}$$

$$\begin{aligned}I_{S,y} = \frac{i}{4}\int_{|\mathbf{k}|=k_F} dk_z dk_y \Theta(k_z)\frac{k_z}{k_F}\frac{1}{\beta\hbar}\sum_n\sum_\sigma \sigma \\ \times\left\{\frac{|\Delta_{\mathbf{k}}|}{\Omega_{n,\mathbf{k}}}a_{e\sigma\bar{\sigma}}(\mathbf{k},i\omega_n) - \frac{|\Delta_{-\hat{\mathbf{k}}}|}{\Omega_{n,-\hat{\mathbf{k}}}}a_{h\sigma\bar{\sigma}}(\mathbf{k},i\omega_n)\right\},\end{aligned} \tag{16}$$

where $\omega_n=(2n-1)\pi/\beta$, $\Omega_{n,\mathbf{k}}=\sqrt{\omega_n^2+|\Delta_{\mathbf{k}}|^2}$, and $\bar{\sigma}=-\sigma$. The expression for the x component of the spin current is vanishing, which reflects the fact that the Cooper pairs in the TSC do not have a spin component parallel to the $\mathbf{d}$ vector.[33] As required for the TSC-FM junction, the charge current is found to vanish in all considered circumstances. Further-

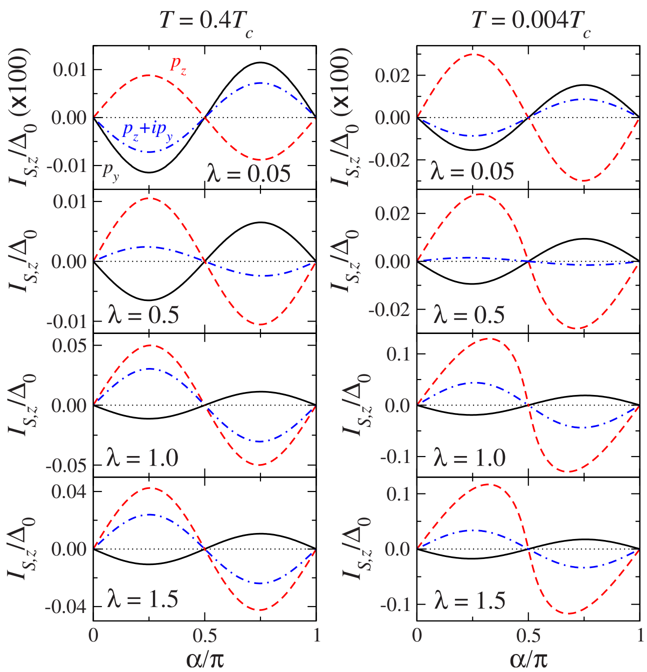

FIG. 3. (Color online) Spin current as a function of the angle α for the p_y (black solid curve), p_z (red dashed curve), and p_z+ip_y-wave (blue dot-dash curve) TSCs for various values of the exchange-splitting λ. The left column shows the spin currents at $T=0.4T_c$ while the right column shows the spin currents for $T=0.004T_c$.

more, the y component of the spin current is also vanishing for the choice of $\mathbf{M}$ adopted here. As mentioned above, any orientation of $\mathbf{M}$ can be rotated into the x-y plane without changing the pairing state of the TSC. Since the polarization of the spin current must also be correspondingly rotated under such a transformation, we hence conclude that the spin current is always polarized along the direction $\mathbf{d}\times\mathbf{M}$.[35]

Due to the time-reversal-symmetry-breaking pairing state in the p_z+ip_y-wave TSC, here we expect to find surface charge and spin currents flowing parallel to the interface.[17,20,34] We will not be concerned with such currents in what follows, presenting only results for the spin current flowing perpendicular to the interface. For convenience, we shall also adopt units where $\hbar=1$. In all plots we take the interface parameter $Z=1$.

III. RESULTS

We find that the spin current is an odd periodic function of α with period π. As shown in Fig. 3, for most situations the spin current obeys $I_{S,z}\propto\sin(2\alpha)$. This dependence upon α is reminiscent of the usual Josephson charge current vs phase relationship for Josephson junctions, suggesting that the angle of misalignment between $\mathbf{d}$ and $\mathbf{M}$ plays a role similar to the phase difference. As was argued in Ref. 35, this analogy is valid: a spin σ Cooper pair incident upon the barrier acquires a phase shift $-2\sigma\alpha$ when undergoing a spin flip at reflection. Interpreting spin-flip reflection as "tunneling" between the spin-$\uparrow$ and spin-$\downarrow$ condensates of the TSC, the phase shift $-2\sigma\alpha$ is therefore the effective phase difference for a Cooper pair "tunneling" between the spin σ and spin $-\sigma$ condensates. Naturally, this drives a Josephson charge current of equal magnitude but opposite sign in each spin sector of the TSC, thus producing a finite spin current but vanishing total charge current. Note, however, that the phase difference in a Josephson junction is a property of the wave functions of the bulk condensates on either side of the tunneling barrier; in the TSC-FM junction, in contrast, the effective "phase difference" results entirely from a property of the interface.

Higher harmonics in 2α clearly appear in the spin current for the p_z-wave case at low temperatures and moderate to strong exchange splitting $\lambda\gtrsim0.5$. Continuing the Josephson junction analogy, we interpret this as evidence of coherent spin-flip reflection of multiple Cooper pairs. In a Josephson junction, such processes are associated with the formation of a zero energy bound state at the tunneling barrier, which allows the resonant tunneling of multiple Cooper pairs. It is well known that zero energy states form at unconventional superconductor interfaces when the orbitals are aligned such that a quasiparticle specularly reflected at the interface experiences a sign reversal of the superconducting order parameter.[38] Such a state does not therefore occur in the p_y junction, while it is present for all $\mathbf{k}$ in the p_z junction, and present only for $\mathbf{k}=k\mathbf{e}_z$ in the p_z+ip_y junction. This is consistent with the absence of higher harmonics in the spin current for the p_y and p_z+ip_y junctions. Although suggesting an important role for the zero energy states in the spin current, it does not explain the absence of the higher harmonics at low λ. This indicates that the coherent reflection of multiple Cooper pairs is also controlled by the bulk properties of the FM.

It is clear from Fig. 3 that the spin current is strongly influenced by the orbital structure of the TSC; in particular, the spin currents in the p_z and p_y junctions always have opposite sign. In the former case, the reflected Cooper pairs acquire an additional π phase shift due to the reversed sign of the superconducting gap, thus reversing the spin current relative to the p_y case. This can be strikingly demonstrated by examining the variation of the spin current as the p-wave orbital in a time-reversal-symmetric TSC is rotated from the p_y-wave to the p_z-wave configurations. We parameterize the orientation of the orbital by the angle γ between the p-wave orbital maximum and the y axis, see Fig. 4(a). Only Cooper pairs with incident angle $0<\theta<\gamma$ experience a reversal of the gap sign upon reflection, and hence contribute a spin current of opposite sign relative to the Cooper pairs with incident angle $\gamma<\theta<\pi/2$. As shown in Fig. 4(b), the former contribution to the spin current comes to dominate the latter as γ is increased from 0 to $\pi/2$, with the spin current consequently changing sign at some critical value of γ.

A similar effect occurs in the p_z+ip_y case, but here the additional orbital phase shift experienced by the reflected Cooper pairs is $\pi-2\arctan(k_y/k_z)$, i.e., it depends upon the incident trajectory. Normally incident Cooper pairs thus undergo an additional π phase shift as in the p_z junction, whereas a trajectory grazing the interface has no additional phase shift as in the p_y junction. The angle of incidence therefore determines the sign of the spin current contributed by the reflected Cooper pair. Making the change of variables

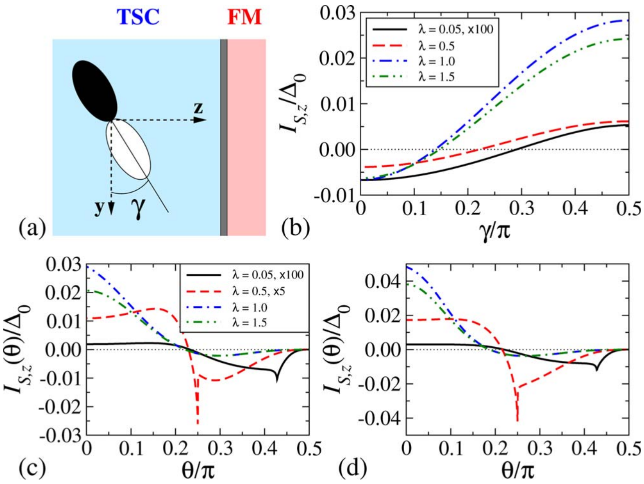

FIG. 4. (Color online) (a) Definition of the angle γ parameterizing the alignment of the p-wave orbital in the TSC with the interface. $\gamma=0$ gives the p_y-wave orbital state, while $\gamma=\pi/2$ gives the p_z-wave orbital state. (b) Spin current as a function of γ for various values of λ. We take $\alpha=0.1\pi$ and $T=0.4T_c$. (c) The spin current as a function of the incident angle θ in the p_z+ip_y junction for $\alpha=0.1\pi$ and $T=0.4T_c$; (d) shows the same curves at $T=0.004T_c$.

$k_z \to k_F\cos(\theta)$, $k_y \to k_F\sin(\theta)$ in Eq. (15), we define the angle-resolved spin current $I_{S,z}(\theta)$ by

$$I_{S,z}=\int_{-\pi/2}^{\pi/2} d\theta I_{S,z}(\theta). \tag{17}$$

We plot $I_{S,z}(\theta)$ in Figs. 4(c) and 4(d), where we see that it changes sign at $\theta\approx 0.2\pi$. Note that for $\lambda<1$ the angle-resolved spin current is sharply peaked at the critical incident angle θ_c, where the transmitted $s=-$ quasiparticle has vanishing z component of its wave vector.

The variation of the spin current with λ is shown in Fig. 5(a). There are some common features for all choices of the TSC orbital: at low λ the spin current goes as $\sim\lambda^2$; a maximum value of the spin current is reached at (p_z and p_z+ip_y) or close to (p_y)$\lambda=1$, where the minority spin Fermi surface disappears; and for $\lambda>1$ the magnitude of the spin current displays slow monotonic decrease. Despite these similarities, the three curves nevertheless have some key distinguishing features at $\lambda<1$. In the p_y case, the spin current increases much more slowly with λ than for the other junctions, while the maximum is smooth and occurs before the disappearance of the minority Fermi surface. Although the spin currents in the p_z and p_z+ip_y junctions show similar λ dependence for $\lambda>0.5$, the spin current in the p_z+ip_y junction changes sign at $\lambda\approx 0.39$ [see Fig. 5(b)]. We further note that the spin current in the p_y junction has greater magnitude than that in the p_z junction for $\lambda\lesssim 0.2$, but the smallest magnitude of all the currents for $\lambda\gtrsim 0.65$.

The transport properties of a junction can be formulated in terms of the normal state scattering matrix.[7] Insight into the complicated dependence of the spin current upon λ can therefore be gained by examining the $T>T_c$ spin-flip reflection probability $|R_{sf}|^2$, given by

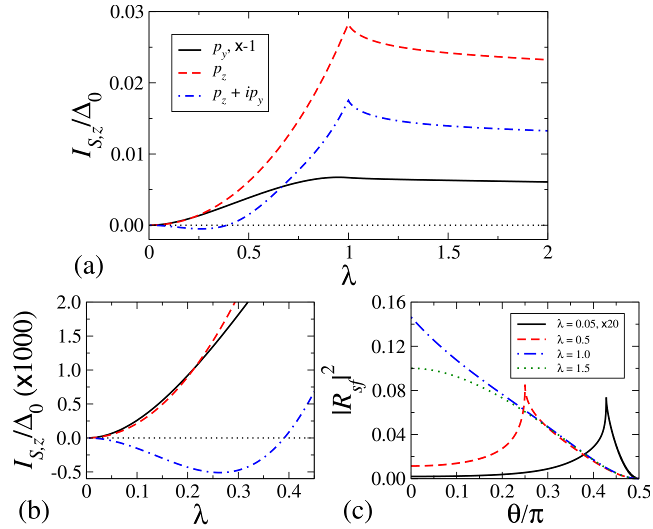

FIG. 5. (Color online) (a) Spin current as a function of the ratio $\lambda=g\mu_B M/E_F$. Note that the current for the p_z+ip_y-wave TSC changes sign at $\lambda\approx 0.4$. (b) Spin current at low λ. Curves have identical meaning as in panel (a). In both (a) and (b) we set $\alpha=0.1\pi$ and $T=0.4T_c$. (c) $T>T_c$ normal state spin-slip reflection probability as a function of the incident quasiparticle angle θ.

$$|R_{sf}|^2 = \begin{cases} \dfrac{k_z^2(k_{+,z}-k_{-,z})^2}{[Z^2k_F^2+(k_z+k_{-,z})^2][Z^2k_F^2+(k_z+k_{+,z})^2]} & |\theta|<\theta_c \\ \dfrac{k_z^2(k_{+,z}^2+\kappa^2)}{[k_z^2+(Zk_F+\kappa)^2][Z^2k_F^2+(k_z+k_{+,z})^2]} & |\theta|\geq\theta_c \end{cases} \tag{18}$$

Note that $|R_{sf}|^2$ is finite when $Z=0$, and so the spin current survives when the insulating barrier is removed. We plot $|R_{sf}|^2$ for $Z=1$ in Fig. 5(c). When $\lambda\leq 1$ the spin-flip reflection probability is sharply peaked at the critical angle θ_c, and grows in magnitude as λ is increased. For $\lambda>1$ the reflection probability remains peaked at $\theta=0$, although it decreases from the $\lambda=1$ maximum. For larger values of Z, $|R_{sf}|^2$ may very slightly increase with increasing λ at $\theta\neq 0$ (not shown). Comparison of the angle-resolved spin current $I_{S,z}(\theta)$ in Figs. 4(c) and 4(d) with Fig. 5(c) clearly shows the connection of $|R_{sf}|^2$ to the transport: the spin-flip reflection of Cooper pairs with incident angle θ close to θ_c is strongly favored, and tends to dominate the spin current. The sign change of the spin current in the p_z+ip_y junction with increasing λ can thus be understood as arising from the shift in the peak of $|R_{sf}|^2$: at $\lambda\ll 1$ this strongly favors grazing trajectories experiencing only small orbital phase shift, while at $\lambda\sim 1$ almost-normal trajectories with orbital phase shift close to π dominate.

The gap anisotropy in the p_z and p_y junctions implies a greater contribution to the spin current for trajectories along which the gap magnitude is maximal. The peak in $|R_{sf}|^2$ at grazing trajectories for $\lambda\ll 1$ therefore results in a greater spin current in the p_y junction than the p_z junction. As λ is

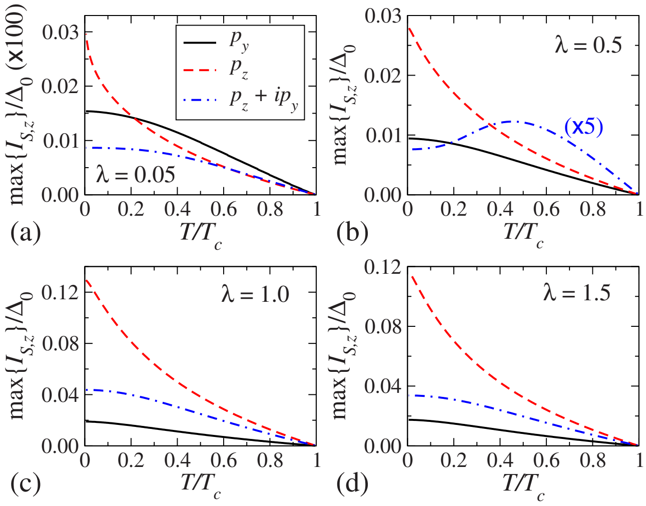

FIG. 6. (Color online) The maximum value of the spin current obtained by varying α for (a) $\lambda=0.05$, (b) $\lambda=0.5$, (c) $\lambda=1.0$ and (d) $\lambda=1.5$. The curves in panels (b)–(d) have the same meaning as those in panel (a); note that the p_z+ip_y curve is multiplied by 5 in (b) for clarity.

increased, the reduction of θ_c leads to the observed strong enhancement of the spin current in the p_z junction. Because the reflection probability at high θ also grows with λ, the spin current continues to increase in the p_y junction, albeit at a much slower rate than in the p_z junction. The reduction of the spin current at $\lambda>1$ is due to the reduction of the peak in $|R_{sf}|^2$ at $\theta=0$.

The last quantity of interest is the maximum spin current max$\{I_{S,z}\}$ obtained by varying α. This is plotted as a function of T for fixed λ in Fig. 6. As can be seen, for all λ the magnitude of the spin current in the p_y junction shows a moderate enhancement with decreasing temperature, with an apparent plateauing at $T<0.1T_c$. In the p_z junction, in contrast, there is a strong enhancement of the spin current magnitude with decreasing temperature. The temperature dependence of max$\{I_{S,z}\}$ in the p_z and p_y cases is reminiscent of the critical current in Josephson junctions with and without zero energy states, respectively,[21,38] again indicating a role for such states in the device studied here. The temperature dependence of max$\{I_{S,z}\}$ in the p_z+ip_y junction is determined by λ: at low and high λ we find the moderate increase with decreasing T characteristic of the p_y junction, and also seen in the critical current of nonmagnetic Josephson junctions between p_z+ip_y TSCs.[18] At intermediate λ, however, we find a nonmonotonic dependence of max$\{I_{S,z}\}$ upon T. This can be understood by examining the angle-resolved spin current in Figs. 4(c) and 4(d): for $\lambda=0.5$, we observe a particularly strong enhancement of the current for trajectories near θ_c as the temperature is lowered. Since the current for such trajectories has opposite sign to that of the total current, there is consequently a reduction of the total spin current. For a small range of $\lambda\sim0.4$, the spin current in the p_z+ip_y junction reverses sign with decreasing temperature.

IV. CONCLUSIONS

This work presents an analysis of the spontaneous spin current generated in a TSC by contact with a bulk metallic FM. The essential requirement for the appearance of this spin current is that the vector order parameters of the TSC and FM are misaligned but not mutually perpendicular. Following Ref. 35, the spin current has been interpreted as a Josephson-like effect, where the phase shift picked up by Cooper pairs undergoing spin-flip reflection at the interface plays the role of the phase difference between the superconductors in a Josephson junction. This analogy is supported by the temperature evolution of the maximum spin current, and the low-temperature dependence upon α, which suggests that resonant reflection of multiple Cooper pairs through zero energy states plays a significant role in the current.

The spin current nevertheless possesses several properties which are not anticipated by the Josephson junction analogy. As it arises from reflection processes, the choice of orbital pairing state in the TSC can determine the sign of the spin current, due to the orbital phase shift experienced by the reflected Cooper pairs. In the case of the p_z+ip_y junction, this produces the interesting result that spin-flip reflected Cooper pairs with different incident trajectories can carry spin currents of opposite sign. The exchange-splitting of the FM also influences the spin current, most significantly when the FM has both a minority and majority spin Fermi surface. This is closely connected to the orbital structure of the TSC, as the exchange splitting determines the angular-dependence of the spin-flip reflection probability.

A few critical remarks upon our method are necessary. Considering that interface effects play the essential role in the generation of the spin current, it is reasonable to question our approximation that the TSC and FM order parameters are both constant up to the insulating barrier. It is well known that there is a strong suppression of the p_z-wave gap at partially transparent interfaces,[17,26] but as this does not change the spin structure of the TSC it is only likely to renormalize the results presented here. More interesting is a possible change in the orientation of $\mathbf{d}(\mathbf{r})$ and $\mathbf{M}(\mathbf{r})$ close to the interface due to an unconventional proximity effect. Although the analysis of Ref. 13 indicates that the spin current will survive a self-consistent treatment of the superconductor, it is desirable that the TSC and FM be treated on an equal level. This is possible, for example, within a real-space Hartree-Fock analysis.[6,20,22,31] The TSC-FM interface has already been studied using this method,[20,31] but only for the case $\mathbf{d}\parallel\mathbf{M}$ which we predict to display vanishing spin current. Much scope therefore remains for further investigations.

Another important assumption is that the magnetic moment of the FM is constant. As pointed out in Ref. 35, however, the spin-flip reflection of the Cooper pair imparts a spin $\delta\mathbf{S}\parallel\mathbf{d}\times\mathbf{M}$ to the FM. If $|\delta\mathbf{S}|\ll|\mathbf{M}|$, the effect of the imparted spin should be to slightly rotate the magnetic moment about the axis defined by $\mathbf{d}$, without significantly altering its magnitude, see the cartoon representation in Fig. 7. This is a generalization of the well-known spin-transfer torque effect of spintronics to a superconducting device.[40] For a FM without anisotropy, we speculate that the cumulative effect of many spin-flip reflection events is to cause the precession of

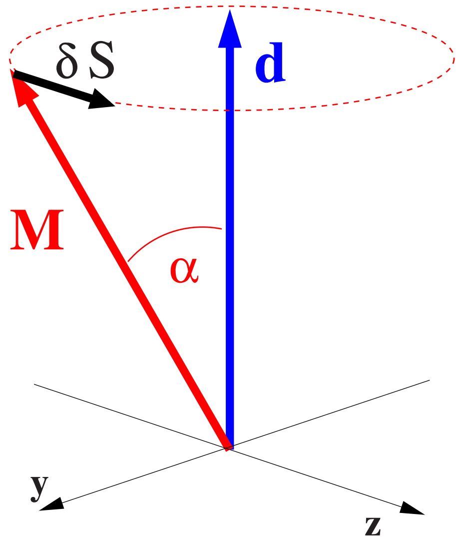

FIG. 7. (Color online) The spin-flip reflection of the Cooper pairs imparts the spin change $\delta\mathbf{S}\parallel\mathbf{d}\times\mathbf{M}$ to the FM. We speculate that this results in a precession of the magnetic moment around the axis defined by the **d** vector.

the magnetic moment. If this precession is sufficiently slow, the results presented here should still remain valid, and we anticipate a periodic modulation of the polarization of the spin current due to the rotating moment. Rigorous verification of these claims requires a more sophisticated analysis than our argument above, and is left for future work.

Finally, we consider the prospects for the experimental verification of our predictions. Direct measurement of spin currents in superconductors is unlikely to be easy, although several proposals exist, e.g., spin-resolved neutron scattering[41] or ARPES with circularly polarized light.[42] Alternatively, one could search for evidence of a spin accumulation at the edge of the TSC opposite to the interface with the FM. If our speculation above proves to be well founded, it might also prove possible to deduce the existence of a spin current by measuring the precession of **M**. Initial characterization of a TSC-FM device would, however, almost certainly involve measurements of the LDOS at the interface[30] and the tunneling conductance.[19] The unique behavior of these quantities at TSC-FM interfaces is a signature of the unconventional interplay of ferromagnetism and triplet superconductivity, of which the spontaneous spin current is only one manifestation.

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