0-π Transition in Magnetic Triplet Superconductor Josephson Junctions

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We examine a Josephson junction involving two arbitrary equal-spin-pairing unitary triplet superconductors and a ferromagnetic tunneling barrier. Using perturbation theory, we show how the interaction of the barrier moment with the spin of the tunneling triplet Cooper pairs can reverse the sign of the Josephson charge current. This also results in a Josephson spin current, which contains a phase-independent contribution due to reflection processes at the barrier. We verify our analytic predictions using a nonperturbative Bogoliubov–de Gennes method.

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Introduction.—The interplay of superconductivity and magnetism is an enduring enigma of condensed matter physics. Many fascinating insights into this problem have been made in the study of singlet superconductor (SC) Josephson junctions with ferromagnetic (FM) tunneling barriers [1]. For example, as the barrier width is increased, the usual Josephson current I_J vs phase relationship $I_J = |I_0|\sin(\phi)$ becomes $I_J = |I_\pi|\sin(\phi+\pi)$. This so-called 0-$\pi$ transition is evidence of oscillations of the singlet SC correlations in the tunneling region [1,2]. A remarkable feature of such junctions is the presence of triplet SC (TSC) correlations induced by a proximity effect [1,3,4], with the realized triplet pairing states dictated by the details of the FM barrier and the bulk SCs. Because of the likely intimate connection between triplet superconductivity and magnetism [5], it is interesting to consider the case where the TSC pairing state can be chosen independently of the FM barrier. Despite the growing interest [4,6–9] in TSC Josephson junctions prompted by the discovery of Sr_2RuO_4 [10], the study of such TSC-FM-TSC (TFT) junctions is still in its infancy. Recently, a novel 0-π transition in a specific TFT junction was predicted [7,8], where the dependence of I_J upon the *orientation* of the FM moment indicates that it couples to the spin of the tunneling Cooper pairs.

In this Letter, we use perturbation theory [11] to obtain the Josephson charge current through a TFT junction for arbitrary choice of unitary equal-spin-pairing TSCs. We predict that the 0-π transition found in Refs. [7,8] is always present for sufficiently large magnetization, and is due to the spin flipping of tunneling triplet Cooper pairs. This also produces a Josephson spin current [6,9], which has a novel phase-independent contribution due to reflection processes. A nonperturbative Bogoliubov–de Gennes theory is used to demonstrate the universal character of our predictions, and that resonant tunneling through an Andreev bound state (ABS) does not qualitatively change the understanding of the 0-π transition [12,13].

Perturbation theory.—The Hamiltonian of the TFT Josephson junction is written $\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_{\rm tun} + \mathcal{H}_{\rm ref}$. $\mathcal{H}_L$ and $\mathcal{H}_R$, respectively, describe the bulk TSCs on the left and right side of the barrier:

$$\mathcal{H}_\nu = \frac{1}{2}\sum_{\mathbf{k}} \psi^\dagger_{\nu,\mathbf{k}} \begin{pmatrix} \epsilon_{\nu,\mathbf{k}}\hat{1} & i\mathbf{d}_{\nu,\mathbf{k}}\cdot\boldsymbol{\sigma}\hat{\sigma}_y \\ (i\mathbf{d}_{\nu,\mathbf{k}}\cdot\boldsymbol{\sigma}\hat{\sigma}_y)^\dagger & -\epsilon_{\nu,\mathbf{k}}\hat{1} \end{pmatrix} \psi_{\nu,\mathbf{k}}, \quad (1)$$

where $\psi_{\nu,\mathbf{k}} = (c_{\nu,\mathbf{k},\uparrow}, c_{\nu,\mathbf{k},\downarrow}, c^\dagger_{\nu,\mathbf{k},\uparrow}, c^\dagger_{\nu,\mathbf{k},\downarrow})^T$ and $c^\dagger_{\nu,\mathbf{k},\sigma}$ ($c_{\nu,\mathbf{k},\sigma}$) are fermion creation (annihilation) operators, $\epsilon_{\nu,\mathbf{k}}$ is the bare dispersion in the ν-hand TSC, and $\mathbf{d}_{\nu,\mathbf{k}} = \Delta_{\nu,\mathbf{k}}\hat{\mathbf{x}}$ are the triplet order parameters of the two TSCs. Both TSCs are in an equal spin-pairing state with respect to the z axis and are unitary (i.e., the triplet condensate has no net spin) [14]. The gap in each spin sector is $\Delta_{\nu,\mathbf{k},\sigma} = -\sigma|\Delta_{\nu,\mathbf{k}}|e^{i(\phi_\nu+\theta_{\nu,\mathbf{k}})}$ where ϕ_ν is the global phase of the ν-hand TSC and $\theta_{\nu,\mathbf{k}}$ is an internal phase specifying the pairing state, obeying $\theta_{\nu,-\mathbf{k}} = \theta_{\nu,\mathbf{k}} + \pi$. As our results depend *only* on the spin state of the Cooper pairs, any variation of the orbital part of the gaps near the barrier will not qualitatively alter our conclusions.

The two TSCs on each side of the barrier are linked by the tunneling Hamiltonian

$$\mathcal{H}_{\rm tun} = \sum_{\nu=L,R}\sum_{\mathbf{k},\mathbf{k}'}\sum_{\sigma,\sigma'} T^{\sigma,\sigma'}_{\nu,\mathbf{k},\mathbf{k}'} c^\dagger_{-\nu,\mathbf{k},\sigma} c_{\nu,\mathbf{k}',\sigma'}, \quad (2)$$

where the subscript $-\nu = R(L)$ when $\nu = L(R)$. For a magnetically active barrier, we must also include reflection prcoesses [15]:

$$\mathcal{H}_{\rm ref} = \sum_{\nu=L,R}\sum_{\mathbf{k},\mathbf{k}'}\sum_{\sigma,\sigma'} R^{\sigma,\sigma'}_{\nu,\mathbf{k},\mathbf{k}'} c^\dagger_{\nu,\mathbf{k},\sigma} c_{\nu,\mathbf{k}',\sigma'} \quad (3)$$

Although reflection processes do not contribute to the charge current, *spin-flip* reflection may contribute to a Josephson spin current, as the spin flip of a reflected Cooper pair changes the total spin in the TSC by $\pm 2\hbar$.

In general, the matrix elements for spin-preserving tunneling $T^{\sigma,\sigma}_{\nu,\mathbf{k},\mathbf{k}'}$, spin-flip tunneling $T^{\sigma,-\sigma}_{\nu,\mathbf{k},\mathbf{k}'}$, and spin-flip reflection $R^{\sigma,-\sigma}_{\nu,\mathbf{k},\mathbf{k}'}$ are different. It is possible to derive expressions for the matrix elements from a more fundamental Hamiltonian [15], but here we will motivate a

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phenomenological form. By Fermi's golden rule we have $\mathcal{T}^{\sigma,\sigma'} \sim |T^{\sigma,\sigma'}_{\nu,\mathbf{k},\mathbf{k}'}|^2$ and $\mathcal{R}^{\sigma,-\sigma} \sim |R^{\sigma,-\sigma}_{\nu,\mathbf{k},\mathbf{k}'}|^2$ in the tunneling limit, where $\mathcal{T}^{\sigma,\sigma'} \ll 1$ and $\mathcal{R}^{\sigma,-\sigma} \ll 1$ are the transmissivity and spin-flip reflectivity of the barrier, respectively. We also require that a tunneling or reflected quasiparticle acquires the same phase as in the exact solution. Following Ref. [7], we consider the example of a purely FM barrier of δ-function width (appropriate for an atomically thin barrier) at $z = 0$. We assume that the FM barrier moment $\mathbf{M}$ lies in the x-y plane at an angle η to the x axis. We hence use the ansatz

$$T^{\sigma,\sigma}_{\nu,\mathbf{k},\mathbf{k}'} = (T_{\text{sp}}/M^2)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(k_z k'_z), \tag{4}$$

$$T^{\sigma,-\sigma}_{\nu,\mathbf{k},\mathbf{k}'} = -\nu i e^{-i\sigma\eta}(T_{\text{sf}}/M)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(k_z k'_z), \tag{5}$$

$$R^{\sigma,-\sigma}_{\nu,\mathbf{k},\mathbf{k}'} = \nu i e^{-i\sigma\eta}(R_{\text{sf}}/M)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(-k_z k'_z), \tag{6}$$

where $M = g\mu_B|\mathbf{M}|/\hbar\sqrt{v^L_{F,z}v^R_{F,z}}$ with $v^\nu_{F,z}$ the Fermi velocity along the (001) direction in the ν-hand TSC, T_{sp}, T_{sf}, and R_{sf} are real constants and the $\theta(\pm k_z k'_z)$ guarantees that the transmitted or reflected quasiparticle moves away from the barrier [16]. The $\mathbf{k}$ dependence of T_{sp}, T_{sf}, and R_{sf} is irrelevant for our argument and is neglected. As rotating the spin coordinates about the x axis leaves the TSCs unchanged, our results for the charge current hold for any $\mathbf{M}$ making an angle η with the $\mathbf{d}_{\nu,\mathbf{k}}$. The spin current results also hold, but with corresponding rotation of the polarization. A diagram of the junction is shown in Fig. 1(a).

We define particle currents in the two spin sectors of each TSC by $I_{\nu,\alpha} = -\nu\langle\partial_t N_{\nu,\alpha}(t)\rangle$, where $N_{\nu,\alpha}(t) = \sum_{\mathbf{k}} c^\dagger_{\nu,\mathbf{k},\alpha}(t)c_{\nu,\mathbf{k},\alpha}(t)$ and $\nu = L(R)$ as a subscript implies $\nu = -1(+1)$ elsewhere. We calculate $I_{\nu,\alpha}$ by expanding the S matrix to lowest order in $\mathcal{H}_{\text{tun}} + \mathcal{H}_{\text{ref}}$, hence treating the tunneling and reflection processes as a perturbation of $\mathcal{H}_0 = \mathcal{H}_L + \mathcal{H}_R$ [11], which is justified for small $T^{\sigma,\sigma'}_{\nu,\mathbf{k},\mathbf{k}'}$ and $R^{\sigma,\sigma'}_{\nu,\mathbf{k},\mathbf{k}'}$. The Kubo formula then gives $I_{\nu,\alpha} = -i\nu\int_{-\infty}^t dt'\langle[\partial_t N_{\nu,\alpha}(t), \mathcal{H}_{\text{tun}}(t') + \mathcal{H}_{\text{ref}}(t')]\rangle$. Working within the interaction picture, we write $\partial_t N_{\nu,\alpha}(t) = i\{B^{-\alpha,\alpha}_\nu(t) - B^{\alpha,-\alpha}_\nu(t)\} + i\sum_\sigma\{A^{\sigma,\alpha}_\nu(t) - A^{\alpha,\sigma}_{-\nu}(t)\}$ where $A^{s,s'}_\nu(t) = \sum_{\mathbf{k},\mathbf{k}'}T^{s,s'}_{\nu,\mathbf{k},\mathbf{k}'}c^\dagger_{-\nu,\mathbf{k},s}(t)c_{\nu,\mathbf{k}',s'}(t)$ and $B^{s,s'}_\nu(t) = \sum_{\mathbf{k},\mathbf{k}'}R^{s,s'}_{\nu,\mathbf{k},\mathbf{k}'}c^\dagger_{\nu,\mathbf{k},s}(t)c_{\nu,\mathbf{k}',s'}(t)$. In the zero-bias case the current through the spin-α sector of the ν-hand TSC is due only to the Josephson effect and may be conveniently expressed as $I^J_{\nu,\alpha} = 2\nu\text{Im}\{\Phi^r_{\nu,\alpha}(0) + \Psi^r_{\nu,\alpha}(0)\}$. The retarded correlation functions $\Phi^r_{\nu,\alpha}(\omega)$ and $\Psi^r_{\nu,\alpha}(\omega)$ give the tunneling and reflection contributions, respectively. After using Wick's theorem to expand the two-particle correlators in the corresponding Matsubara functions $\Phi_{\nu,\alpha}(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n\tau}\sum_{\sigma,s,s'}\langle T_\tau A^{\sigma,\alpha}_\nu(\tau)A^{s,s'}_\nu(0)\rangle$ and $\Psi_{\nu,\alpha}(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n\tau}\langle T_\tau B^{-\alpha,\alpha}_\nu(\tau)B^{-\alpha,\alpha}_\nu(0)\rangle$, we make the analytic continuation $i\omega_n \to \omega + i0^+$ to obtain the retarded functions. Substituting Eqs. (4)–(6) into the expressions for the retarded correlation functions at $\omega = 0$, we obtain the particle current

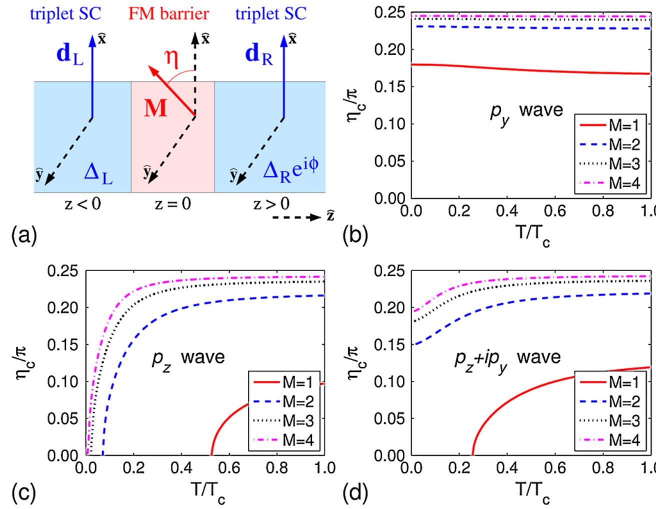

FIG. 1 (color online). (a) Schematic diagram of the TFT junction. (b)–(d) The critical line η_c between the 0 and π states for various values of M in the (b) p_y, (c) p_z, and (d) $p_z + ip_y$ junctions. The π state is realized for $0 \leq |\eta|$, $|\pi - \eta| < \eta_c$, while the 0 state occurs for $\eta_c < |\eta| < \pi - \eta_c$. The $M \gg 1$ behavior of η_c at high T reflects the dominance of spin-flip processes; at low T, details of the bulk TSCs are important.

$$\begin{aligned} I^J_{\nu,\alpha} = &-\sum_{\mathbf{k},\mathbf{k}'}\frac{T^2_{\text{sp}}}{M^4}\frac{|\Delta_{-\nu,\mathbf{k}}\Delta_{\nu,\mathbf{k}'}|}{E_{-\nu,\mathbf{k}}E_{\nu,\mathbf{k}'}}F_{-\nu,\nu,\mathbf{k},\mathbf{k}'}\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(k_z k'_z)\sin(\phi + \nu[\theta_{\nu,\mathbf{k}'} - \theta_{-\nu,\mathbf{k}}]) \\ &+ \sum_{\mathbf{k},\mathbf{k}'}\frac{T^2_{\text{sf}}}{M^2}\frac{|\Delta_{-\nu,\mathbf{k}}\Delta_{\nu,\mathbf{k}'}|}{E_{-\nu,\mathbf{k}}E_{\nu,\mathbf{k}'}}F_{-\nu,\nu,\mathbf{k},\mathbf{k}'}\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(k_z k'_z)\sin(\phi + 2\nu\alpha\eta + \nu[\theta_{\nu,\mathbf{k}'} - \theta_{-\nu,\mathbf{k}}]) \\ &+ \nu\sum_{\mathbf{k},\mathbf{k}'}\frac{R^2_{\text{sf}}}{M^2}\frac{|\Delta_{\nu,\mathbf{k}}\Delta_{\nu,\mathbf{k}'}|}{E_{\nu,\mathbf{k}}E_{\nu,\mathbf{k}'}}F_{\nu,\nu,\mathbf{k},\mathbf{k}'}\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(-k_z k'_z)\sin(2\alpha\eta - [\theta_{\nu,\mathbf{k}} - \theta_{\nu,\mathbf{k}'}]), \end{aligned} \tag{7}$$

where $E_{\nu,\mathbf{k}} = \sqrt{(\epsilon_{\nu,\mathbf{k}} - \mu)^2 + |\Delta_{\nu,\mathbf{k}}|^2}$ is the excitation spectrum in the ν-hand TSC, $\phi = \phi_R - \phi_L$ and $F_{\nu,\nu',\mathbf{k},\mathbf{k}'} = \sum_\pm[n_F(\pm E_{\nu,\mathbf{k}}) - n_F(E_{\nu',\mathbf{k}'})]/[E_{\nu,\mathbf{k}} \mp E_{\nu',\mathbf{k}'}]$ with $n_F(E)$ the Fermi distribution function.

Equation (7) is our first important result, as it contains all contributions to the current. The first term describes spin-preserving tunneling, where the Cooper pairs preserve their spin during the tunneling event, giving the usual Josephson result. The second term describes spin-flip tunneling, where the spin of the Cooper pair is reversed by the coupling to the FM moment. Relative to spin-preserving tunneling, these Cooper pairs acquire a phase shift of $2\alpha\nu\eta$ due to the spin flip itself, and a further π shift arising from the intrinsic phase difference between the spin-$\uparrow$ and spin-$\downarrow$ condensates in the TSC. Last, we have the current due to Cooper pairs undergoing a spin flip when they are reflected at the tunneling barrier. As such, this term is independent of the TSC on the other side of the barrier, depending only upon the phase due to the spin flip itself and the gap experienced by the reflected Cooper pairs.

Charge current.—From Eq. (7) we obtain the Josephson charge current $I_J = -e(I^J_{\nu,\uparrow} + I^J_{\nu,\downarrow})$:

$$I_J = 2e\left(\frac{T^2_{\rm sp}}{M^4} - \cos(2\eta)\frac{T^2_{\rm sf}}{M^2}\right)\sum_{\mathbf{k},\mathbf{k}'}\frac{|\Delta_{R,\mathbf{k}}\Delta_{L,\mathbf{k}'}|}{E_{R,\mathbf{k}}E_{L,\mathbf{k}'}} \times F_{R,L,\mathbf{k},\mathbf{k}'}\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(k_z k'_z)\sin(\phi + \theta_{R,\mathbf{k}} - \theta_{L,\mathbf{k}'}) \qquad (8)$$

The first term in brackets corresponds to the spin-preserving contribution, while the second term is due to the spin-flip tunneling. The η dependence of the latter is due to the extra phase shifts for spin-flip tunneling: ignoring orbital effects, a spin-α Cooper pair incident from the left-hand side (lhs) undergoing a spin flip during tunneling experiences an effective phase difference $\phi_\alpha = \phi + \pi - 2\alpha\eta$ between the two TSCs. The spin-flip current vs phase relationships in each spin channel are hence shifted by $\pm 4\eta$ with respect to one another. The interference between the two spin channels results in the modulation of the total spin-flip current by $\cos(2\eta)$; this is analogous to the effect of the spin-dependent phase shifts for tunneling between TSCs with misaligned $\mathbf{d}$ vectors [6–9].

Spin-flip tunneling dominates I_J for large M: in this case, relevant for a half-metallic barrier, we find $I_J \propto \cos(2\eta)$, and so the current changes sign at $\eta = \pi/4$ as the moment is rotated about the z axis; i.e., there is a 0-π transition controlled by the orientation of the moment. As this originates solely from the spin structure of the triplet Cooper pairs it is a *universal* feature of unitary equal-spin-pairing TFT junctions, our second important result. To test this prediction, we consider TFT junctions where both TSCs are made of the same material, for the three choices of p_y, p_z, and $p_z + ip_y$ orbital pairing symmetry (the latter of most relevance to Sr_2RuO_4 [10]). In a model with spatially constant TSC gaps, we can include tunneling and reflection processes to all orders [12,15,17] by solving the Bogoliubov–de Gennes equations to obtain the ABS energies $E_{\pm,\mathbf{k}}(\phi, \eta, M, T)$ at temperature T. The free energy of the junction is then given by $F = -\beta\sum_{\mathbf{k}}\sum_{p=\pm}(|k_z|/k_F)\log(2\cosh(\beta E_{p,\mathbf{k}}/2))$; we assume a 2D circular Fermi surface in the y-z plane. The ground state of the junction is found by numerically minimizing F with respect to ϕ. For each junction, the minimum of F lies at $\phi = 0$ or π; we find a 0-π transition when the global minimum shifts from one of these values to the other. In Figs. 1(b)–1(d) we plot the critical angle η_c at which this occurs in each junction for fixed M; according to Eq. (8), in the tunneling limit $\eta_c = \pi/4$.

For T sufficiently close to the transition temperature T_c, η_c always approaches the tunneling limit results as M is increased. For the p_y junction [Fig. 1(b)], η_c shows only weak T dependence at fixed M, consistent with Eq. (8). In the p_z and $p_z + ip_y$ junctions [Figs. 1(c) and 1(d), respectively], however, η_c varies significantly with T. The key difference between the p_y junction and the p_z and $p_z + ip_y$ junctions is the absence of a zero energy ABS in the former. In the latter, there is a zero energy ABS at $\phi = \pi$ for *any* choice of tunneling barrier, which raises the free energy of the π state, thereby suppressing the 0-π transition. The strongest deviations from Eq. (8) therefore occur for the p_z junction, as here a zero energy ABS forms for all $\mathbf{k}$; for the $p_z + ip_y$ junction, in contrast, a zero energy ABS forms only when $\mathbf{k} \parallel \hat{\mathbf{z}}$. Our perturbation theory results are recovered at higher T in these junctions due to the suppression of multiple-Cooper pair tunneling processes, which are a key feature of tunneling through a zero energy ABS [12,18].

Spin current.—The spin of a triplet Cooper pair also allows a zero-bias Josephson spin current to flow across the junction [6,8,9]. The spin current is polarized along the z axis [9], and so from Eq. (7) we obtain the spin current in the ν-hand TSC $I^{S,z}_{\nu,J} = \frac{\hbar}{2}(I^J_{\nu,\uparrow} - I^J_{\nu,\downarrow})$:

$$I^{S,z}_{\nu,J} = \hbar\nu\sin(2\eta)\sum_{\mathbf{k},\mathbf{k}'}\frac{T^2_{\rm sf}}{M^2}\frac{|\Delta_{R,\mathbf{k}}\Delta_{L,\mathbf{k}'}|}{E_{R,\mathbf{k}}E_{L,\mathbf{k}'}}\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(k_z k'_z) \times \cos(\phi + \theta_{R,\mathbf{k}} - \theta_{L,\mathbf{k}'})F_{R,L,\mathbf{k},\mathbf{k}'} + \hbar\nu\sin(2\eta)\sum_{\mathbf{k},\mathbf{k}'}\frac{R^2_{\rm sf}}{M^2}\frac{|\Delta_{\nu,\mathbf{k}}\Delta_{\nu,\mathbf{k}'}|}{E_{\nu,\mathbf{k}}E_{\nu,\mathbf{k}'}}\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}\theta(-k_z k'_z) \times \cos(\theta_{\nu,\mathbf{k}} - \theta_{\nu,\mathbf{k}'})F_{\nu,\nu,\mathbf{k},\mathbf{k}'}. \qquad (9)$$

The first term is from spin-flip tunneling, while the second ϕ-independent term is due to spin-flip reflection [19]. The spin-dependent phase shifts of the spin-flipping Cooper pairs are responsible for driving the spin current, again in analogy to the spin current between TSCs with misaligned $\mathbf{d}$ vectors [6,8,9]. This implies $I^{S,z}_{\nu,J} = 0$ for $\eta = n\pi/2$, $n \in \mathbb{Z}$, as the relative phase between the spin-flip currents in each spin channel is then $2n\pi$. As shown in Fig. 2(a), $I^{S,z}_{\nu,J}$ reverses sign across the barrier: the spin current due to the tunneling Cooper pairs reverses on the spin flip, and the spin flip reflected Cooper pairs in each TSC carry opposite spin current as they move in opposite directions.

Equation (9) may be simplified for the three junctions introduced above. By energy conservation we have $\mathbf{k}' = \mathbf{k}$ in the tunneling term and $\mathbf{k}' = \tilde{\mathbf{k}} = (k_x, k_y, -k_z)$ in the reflection term. Furthermore, we set $T_{\rm sf} = R_{\rm sf}$ as here $\mathcal{T}^{\sigma,-\sigma} = \mathcal{R}^{\sigma,-\sigma}$; the amplitude of the cosine term in each contribution to Eq. (9) is then identical. We hence find $I^{S,z}_{\nu,J} \propto \gamma + \cos(\phi)$ where γ is an orbital-dependent

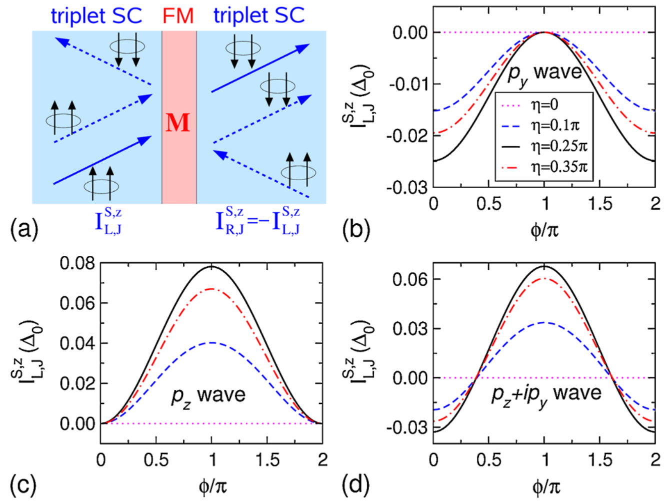

FIG. 2 (color online). (a) Cartoon of spin-flip tunneling (solid line) and reflection (broken line) processes contributing to the spin current. (b)–(d) The z component of spin current on the lhs for various values of η at $M = 2$ and $T = 0.4T_c$ in the (b) p_y, (c) p_z, and (d) $p_z + ip_y$ junctions. The legend in (b) is for all plots. Δ_0 is the maximum gap magnitude at $T = 0$.

constant due to the phase shift $\Delta\theta_{\nu,\mathbf{k}} = \theta_{\nu,\mathbf{k}} - \theta_{\nu,\bar{\mathbf{k}}}$ experienced by specularly reflected Cooper pairs. For the p_y junction there is no extra phase shift upon reflection, giving $\gamma = 1$. For the p_z junction, in contrast, all reflected Cooper pairs experience a π phase shift and therefore $\gamma = -1$. In the $p_z + ip_y$ junction, $\Delta\theta_{\nu,\mathbf{k}} = \pi - 2\arctan(k_y/k_z)$ depends upon $\mathbf{k}$; integrating across the Fermi surface we find $-1 < \gamma < 0$. We again verify these predictions within the Bogoliubov–de Gennes theory for spatially constant TSC gaps. Solving for the scattering wave functions [8,12,20], we obtain the Andreev reflection amplitudes $a^{\mathrm{eh(he)}}_{\nu,\sigma,\sigma'}$ for a spin-σ electronlike (holelike) quasiparticle incident from the ν-hand-side Andreev-reflected as a spin-σ' holelike (electronlike) quasiparticle. Following Ref. [20], we write the spin current in terms of the $a^{\mathrm{eh(he)}}_{\nu,\sigma,\sigma'}$

$$I^{S,z}_{\nu,J} = -\frac{\nu}{8}\int_{|\mathbf{k}|=k_F} d\mathbf{k}\,\frac{|k_z|}{k_F}\frac{1}{\beta\hbar}\sum_n \frac{|\Delta_{\nu,\mathbf{k}}|}{\sqrt{\omega_n^2 + |\Delta_{\nu,\mathbf{k}}|^2}} \times \sum_\sigma \sigma\{a^{\mathrm{eh}}_{\nu,\sigma,\sigma}(\mathbf{k}, i\omega_n) - a^{\mathrm{he}}_{\nu,\sigma,\sigma}(\mathbf{k}, i\omega_n)\}. \quad (10)$$

We verify the relation $I^{S,z}_{L,J} = -I^{S,z}_{R,J}$ (not shown), and also find excellent agreement with the tunneling Hamiltonian predictions for γ in all three junctions; see Figs. 2(b)–2(d). The role played by reflection processes in the spin transport is our third important result.

We have not accounted for the transfer of spin to the barrier moment when the spin current is nonzero. This can be physically justified if the barrier is in contact with a spin reservoir, allowing the diffusion of the transferred spin. In the absence of such a reservoir, we speculate that the barrier moment will precess about the x axis, as the spin current only has a spin polarization $\parallel \mathbf{d}_{\nu,\mathbf{k}} \times \mathbf{M}$. This very interesting matter requires a nonequilibrium treatment, which is beyond the scope of the present work. As $I^{S,z}_{\nu,J} = 0$ for $\eta = 0$ and $\eta = \pi/2$, the different sign of I_J at these angles is, however, a robust equilibrium feature.

Conclusions.—We have analyzed the Josephson currents through a TFT junction for any choice of unitary equal-spin-pairing TSCs. We predict that the sign of the charge current is controlled by the relative importance of spin flip to spin-preserving tunneling. Spin-flip processes also produce a Josephson spin current, with a phase-independent term due to reflection. Our results reveal the importance of the Cooper pair spin as a novel degree of freedom in TSC Josephson junctions.

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